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Vol. IV JANUARY, 1920 Number 1

THE IMPORTANCE OF THE NODAL POINTS IN LENS TESTING.*

BY
G. W. MOFFITT.

THE use of a collimator for obtaining a distant effective source to be used in the optical bench examination of lenses is accompanied by so many disadvantages and difficulties that it is usually dispensed with and an illuminated pinhole at some convenient distance employed in its stead. This procedure at once eliminates all the aberrations which would arise from the use of a collimator lens however well corrected it might be. But it also introduces complications in the geometry of the test which, if disregarded, may sometimes lead to quite erroneous interpretations of some of the results. The nature of the problem can best be seen by first developing the general formulæ and afterward studying the effects of convenient approximations, thereby determining under what conditions the errors introduced by the proposed approximations will have a negligible effect on the results.

This paper deals with the interpretation of lens measurements made with finite object distance on an optical bench equipped with a nodal slide turntable provided with means for holding the lens thereon and for adjusting its position relative to the axis of rotation of the nodal slide. It is further assumed that for the purposes of viewing and locating the image with respect to the axis of the nodal slide a micrometer microscope of good power is mounted on a second carriage, movable on the main track of the bench. Both nodal slide support and microscope carriage should

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* Communication No. 89, from the Research Laboratory of the Eastman Kodak Company.

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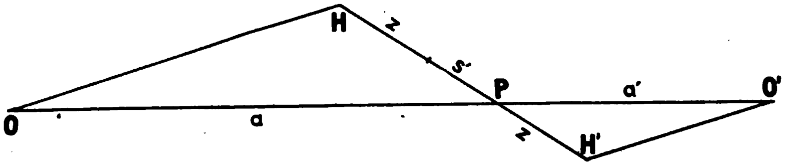
be provided with transverse micrometer motions as it is necessary to measure lateral displacements as well as axial ones when studying the characteristics of a lens system.

CARDINAL POINT DETERMINATIONS.

While the nodal slide has long been known, its properties for magnifications other than zero and infinity have not until recently been clearly stated. Within the past few years several papers have appeared on this subject.[1] From these the general law of the nodal slide as developed by Professor Anderson may be briefly stated as follows: An axial image will suffer no lateral displacement for a small rotation of the lens system provided the rotation takes place about a point, called the nul point, so located that it divides the distance between the nodal points into parts whose ratio is equal to the lateral magnification of the system. This will be at once apparent from the similar triangles of Fig. 1. From this it is clear that the axis of rotation of the nodal slide will pass through a nodal point of the system only for the cases of parallel incidence or emergence—on the second nodal point in the first case, on the first nodal point in the second. Thus it is seen that the separation of the nodal points must be taken into account in cases where the magnification is not zero or infinity, and that a knowledge of the location of these points with respect to each other must be had before a true reduction of a set of bench readings can be made, and before even the equivalent focal length can be determined. This knowledge of the relative location of the nodal points may be available from previous experience with other lenses of the same construction. Otherwise a determination of

Fig. 1.



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[1] Prof. A. Anderson, *Phil. Mag.*, 33, Jan., 1917, pp. 157–159; Prof. A. Anderson, *Phil. Mag.*, 34, July, 1917, pp. 76–80; Prof. A. Anderson, *Phil. Mag.*, 34, Sept., 1917, pp. 174–177; R. E. Baynes, *Phil. Mag.*, 33, April, 1917, pp. 357–358; J. A. Tompkins, *Phil. Mag.*, 35, Jan., 1918, pp. 21–27.

the apparent separation of these points must be made. This must be done before it is possible to determine whether the error in reducing the readings on the assumption that the lens was pivoting on the second nodal point is negligible or not.

The general formulæ may be easily obtained from a consideration of Fig. 1. The angles are, of course, exaggerated for the purpose of making the drawing clear. It is assumed that the actual rotations are so small that distortion effects are inappreciable, this being the assumption that is always made in this connection. The axis of rotation is at P, the object being at O and the image at O'. When the lateral shift of the image has been eliminated for a small rotation the triangle OPH and $O'PH'$ are similar. On rotating the slide and lens as a whole through an angle of 180° and again eliminating lateral motion for a small rotation of the lens system all that happens is an interchange of the positions of the two nodal points. That is, the geometry of the figure remains unchanged. But in order to go from one setting to the other, it is necessary to move the lens on the slide through a distance s'—the apparent separation of the nodal points—such that

$$s' = s - 2z$$

in which s is the actual separation of the nodal points and z is the difference between the actual and the apparent position of a nodal point. The formulæ at once follow:

$$s = \frac{a + a'}{a - a'}\, s'$$

$$z = \frac{a's'}{a - a'} = \frac{a's}{a + a'}$$

$$f = \frac{aa'\,(a + a' - s)}{(a + a')^2} = \frac{aa'(a - a' - s')}{a^2 - a'^2}$$

$$= \frac{aa'}{a + a'} - \frac{aa's'}{a^2 - a'^2} = \frac{aa'}{a + a'}\left(1 - \frac{s'}{a - a'}\right).$$

The distance, z, from the pivot, P, to the second nodal point is the error made in locating the point on the assumption that it was coincident with the pivot. And $2z$ is the error made in determining the separation of the nodal points on the same assumption. It is to be noticed that for converging lenses, which are the only ones considered in this paper, the error is always such as to make the nodal points appear to be closer together than they really are.

The last form of the focal length formula is especially convenient as it enables one to quickly estimate the percentage error that would be made in a determination of the focal length by assuming that the nodal points were together on the pivot and using the simple approximate formula with a as the object distance and a' as the image distance. For as soon as s', a, and a' are known the second term in the parenthesis gives the fractional error resulting from the approximation. For instance, if it is desired to determine a focal length of about 100.00 mm. to an accuracy of 0.1 mm., then

$$\frac{s'}{a - a'} \text{ must be considerably less than } 0.001$$

If such proves to be the case it is permissible to use the simpler approximate formula,

$$f = \frac{aa'}{a + a'}$$

thereby simplifying the calculations.

With such object distances as are usually used the errors resulting from this approximation—although negligible in the case of short focus lenses whose nodal points happen to be close together—are not to be neglected in the case of medium or long focus lenses having considerable separation of the nodal points. The formula shows that the percentage error increases with increasing focal length and also with increasing separation of the nodal points. But if these points be coincident, then no error is introduced for any focal length as in such a case the assumptions of the approximation are realized for all magnifications. The error in reducing the focal length readings on a long focus lens with nodal points close together may easily be less than that for one of medium focal length having considerable separation of the nodal points. Therefore, we cannot always say that since the focal length is not large the error in reducing the readings will be negligible if carried out on the assumption that the lens was pivoting on the second nodal point.

Since the second term in the focal length formula is negative, it follows that the approximate computation gives a value of the focal length that is too large for converging lenses with nodal points normal and too small when the nodal points are reversed, or crossed. It is also of interest to note that when $a = a'$ neither the separation of the nodal points nor the focal length can be determined by this method.

An example will show what the magnitude of the error may be in a case which would hardly be considered exceptional. A Ross Telecentric lens of 13-inch nominal focal length was measured with $a = 8600.0$ mm. The value of a' was 343.1 mm. and for s' it was 61.36 mm.

$$f(\text{approx.}) = \frac{aa'}{a + a'} = 329.94 \text{ mm.}$$

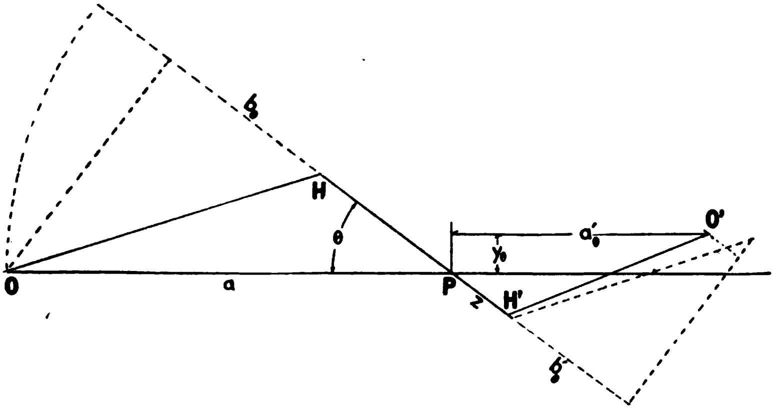
$$f(\text{true}) = 327.48 \text{ mm.}$$

$z = 2.55$ mm., the error in locating H or H' by the approximate method.

FIELD DETERMINATIONS.

To derive the formulæ for the field characteristics assume the lens to be in position for the determination of the cardinal points

FIG. 2.



as just described. The lens may then be rotated about the axis of the nodal slide through any desired half-field angle, θ, as in Fig. 2. Let the image of O be at O'. The Cartesian coördinates, (a'_θ, y_θ), of this point are measured directly with the bench microscope. The normal equation of the theoretical image plane is

$$x \cos\theta + y \sin\theta = z + b'_\theta.$$

It is convenient to express the field aberrations in terms of the length of the perpendicular from O' to the theoretical image plane. This distance is

$$\Delta f = a'_\theta \cos\theta + y_\theta \sin\theta - (z + b'_\theta),$$

the algebraic sign being negative when the image is between the theoretical image plane and the lens. The quantity, z, has already been expressed in terms of the measured quantities in the discussion of cardinal point determinations. To similarly express b'_θ, we have

$$b_\theta = a \cos\theta - (s - z)$$

Therefore,

$$\frac{1}{a \cos\theta - (s - z)} + \frac{1}{b'_\theta} = \frac{1}{f}$$

from which

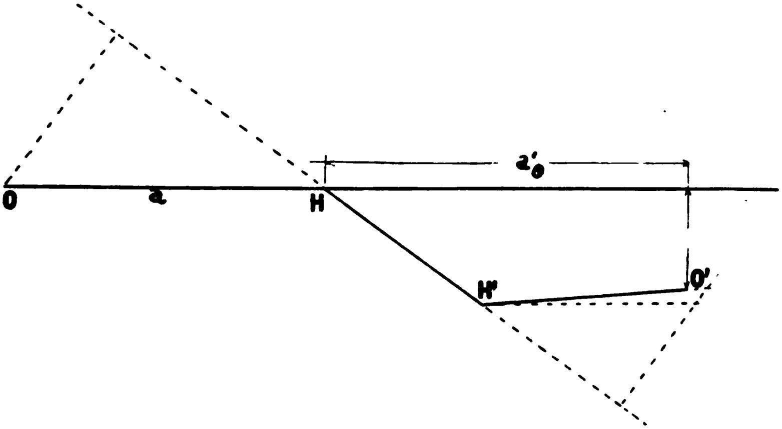
$$b'_\theta = \frac{fa \cos\theta - f(s - z)}{a \cos\theta - (s - z) - f}$$

It will be noticed that the actual half-field angle is somewhat larger than the measured angle, θ, when the nodal points are normal, as in Fig. 2, so that a correction must be applied to the angle readings as made on the nodal slide circle if the true half-field angles for which the readings were taken are to be had, although this correction does not enter into the calculation of the field curvature and the astigmatic difference according to the formulæ derived above.

Other methods of procedure in field determinations lead to slightly different formulæ. When the first nodal point is placed on the pivot certain advantages in determining the distortion are realized and the apparent half-field angles are then the same as the actual ones (Fig. 3). When the second nodal point is on the pivot, as in Fig. 4, slight simplification of the field formula results. But it is seldom that the gain to be realized by either of these procedures warrants their use, as it is necessary to determine the actual location of the nodal points before making the field readings in order to properly adjust the lens on the slide.

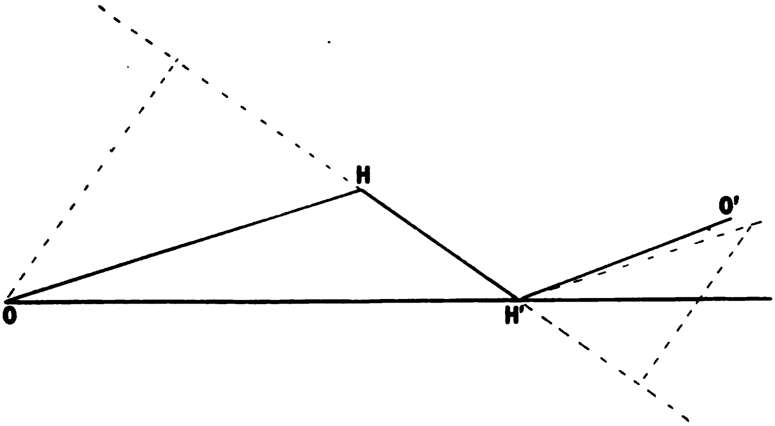
With these formulæ the reduction of a set of readings leads to reliable values for the field aberrations of the lens under the conditions of the test. What is actually mapped by the lens is not the image of a plane object perpendicular to the axis of the lens but that of a spherical object surface whose centre lies on the axis of rotation of the nodal slide. Strictly speaking, we cannot deduce from such a set of readings what the field would have been for a plane object at any distance unless we know how the curvature of field and the astigmatic difference vary with object distance for the lens under consideration. Usually this is not known.

FIG. 3.



With respect to photographic distortion there is no reduction that rigidly satisfies the problem in general, but an approximation sufficiently good for practical purposes is obtainable in nearly all cases. Erroneous conclusions regarding distortion are most

FIG. 4.



likely to be met when dealing with systems having considerable separation and displacement of the nodal points with respect to the lens, as in the case of telephoto combinations. But whenever the emergent pencil is symmetrical with respect to the emergent principal ray the distortion may be exactly evaluated.

While the refinements here discussed are not required in the great majority of cases, nevertheless a knowledge of their possible importance is necessary for the satisfactory operation of the lens bench method of testing.

ROCHESTER, N. Y.,
January 14, 1920.

THE MEASUREMENT OF DIFFUSE REFLECTION FACTORS AND A NEW ABSOLUTE REFLECTOMETER.

BY

A. H. TAYLOR.

I. INTRODUCTION—NATURE OF REFLECTION.

THE reflection factor of a surface is defined as the ratio of the total luminous flux reflected by the surface to the total luminous flux incident upon it.

The surface may be illuminated by a narrow beam of light, by light from several directions, by totally diffused light, or by some combination of these. Reflection may take place in many ways, *e.g.,* specular reflection, in which case an incident cone of light is reflected as a cone, the angles of incidence and reflection being equal; perfectly diffused reflection, in which case the light is reflected in all directions in accordance with Lambert's cosine law; mixed specular and diffused reflection, in all possible combinations between the extremes.

No surface obeys Lambert's cosine law perfectly, and most surfaces are very far from being perfect diffusers.

The numerical value of the reflection factor of a surface may depend on the color of the incident light and the manner of its incidence.

In the practical application of light and illumination it is often desirable to be able to determine the reflection factors of various surfaces. In the design of a lighting installation a knowledge of the reflection factors of the walls and ceilings enables the illuminating engineer, with the aid of prepared tables, to estimate the size and number of lamps which will be required to produce the desired illumination. The determination of reflection factors with precision is one of the most difficult feats in photometry, and previous to this time, so far as the author knows, no method has been proposed which will give accurate results and which may be applied to the measurements of surfaces in place. All previous methods, with the exception of one, involve laboratory measurements, and the one exception does not give correct results.

II. EARLIER REFLECTOMETERS.

If one considers the way in which the light flux is reflected, it is evident that in all cases, except that of pure specular reflection, any method of measuring reflection factors must inevitably involve an integration of the reflected flux. This practically limits the method to some application of the integrating sphere.

A search of the scientific literature reveals only a few articles on this subject. The information obtained shows that several methods have been proposed, but apparently nothing has been done as yet to standardize the method. In all except one of the methods of which descriptions were found the test surface is compared in some way with a standard surface, but little is said about the initial standardization of the reference standard surface. Even though the method of comparison may be faultless, the values obtained are merely relative to that of the standard surface, and if it is incorrectly evaluated they will evidently be subject to the same error. A number of the methods which have been used in the past have been seriously in error because of the assignment of an incorrect value to the standard surface used. A few of the methods used will be briefly described.

Apparently the first definite proposal of an instrument for the measurement of diffuse reflection factors was made by Doctor Nutting[1] in 1912. His instrument consists of a highly polished nickeled ring 150 mm. in diameter and 32 mm. high, through which projects the nose of a Martens-Koenig polarization photometer. An illuminated diffusing glass plate is placed on one side of the ring and the test surface on the other, and the photometer is arranged to view the two surfaces at an angle of about 75° from the normal. He describes it thus: "The principle of the method is that of two parallel infinite planes, one of which is a diffuse illuminator and the other the surface whose reflecting power is to be determined. The relative brightness of the two planes is then the reflecting power of the non-luminous plane." Judging by the low value assigned to magnesium carbonate by this instrument, it apparently gives results which are very seriously in error. Some of the factors which may contribute to this error are as follows:

(*a*) The instrument is based on the principle of parallel infinite planes, whereas it employs planes of very limited area, bounded by a nickeled ring which is far from a perfect reflector.

(*b*) The distribution of illumination over the two planes is seriously disturbed by the presence of the nosepiece of the polarization photometer, and does not have the ideal distribution which is assumed.

(*c*) No surface obeys Lambert's cosine law perfectly, and the brightness of the surface at a very oblique angle is usually very appreciably lower than that of a perfect diffuser emitting the same total flux.

A method devised and used by Mr. W. F. Little [2] at the Electrical Testing Laboratories consists in the projection of a beam of light through a small hole in the wall of an integrating sphere onto the test surface, placed near the centre of the sphere. In this method the brightness of the observation window when the test surface is in place, to that when the standard surface is used, is substantially the same as the ratio of the reflection factors of the two surfaces. Evidently this method is limited by the accuracy of the value assigned to the standard surface, but a slight modification would make it an absolute method. If the light beam is first projected onto the sphere wall at a point unscreened from the observation window and is next projected onto the test surface, screened from the window, the ratio of the brightness of the window in the second case to that in the first case is the numerical value of the reflection factor of the test surface. In this method the area of the test surface should be small with respect to the sphere surface. Another method of using the sphere would be to determine the reflection factor of its surface by a method which will presently be described, then to determine the relative values of test and sphere surface by projecting the beam first on one then on the other, the illuminated spot being screened from the observation window in each case. The greatest practical difficulty in the application of any one of these three methods of using the sphere is the realization of a narrow beam of light which is of a sufficiently high intensity and at the same time is so concentrated that none of it is incident anywhere except on the test surface. The two modifications of Mr. Little's methods, as pointed out by the author, require the sphere surface to be uniform in reflecting power.

In 1916 Doctor Rosa [3] and the present author described and applied a method of measuring the reflection factor of the surface of an integrating sphere. It consists in the determination of the ratio of the average illumination received by the sphere surface from the test lamp, to the total illumination of the sphere surface by both direct and reflected light, the numerical value of this ratio being the absorption factor of the sphere surface. The absorption factor of an 88″ sphere at the Bureau of Standards, surfaced with a white cement, when the surface was fresh, was found to be 7.5 per cent. Since magnesium oxide and carbonate have long been considered, and probably are, the whitest substances in existence, this test set the lowest possible limit for their reflection factors, and definitely established the fact that the value of 88 per cent. given by Doctor Nutting's reflectometer was considerably in error. This sphere method of determining the reflection factor of the sphere surface is by far the most precise method which it is possible to devise, but it is evidently very limited in its application.

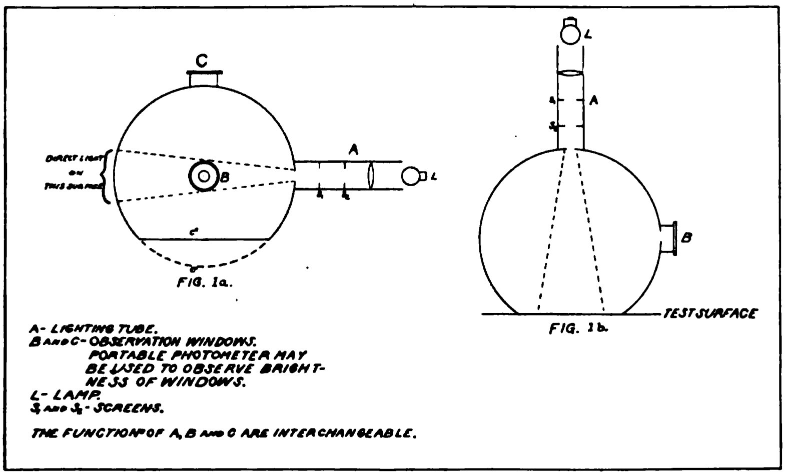
In 1917 Mr. M. Luckiesh[4] described a new relative method for measuring reflection factors. An opal glass globe, such as is used in lighting fixtures, is mounted in a white box. The globe is surrounded by four or more lamps, symmetrically placed. The globe has an opening at the bottom, against which is placed the object to be tested. A brightness photometer views the test object, its brightness being compared with that of a standard surface of known reflection factor. The results may be in error because of the fact that the photometer views the test object at a fixed angle, and that the brightness at that angle may depend very largely on the amount of specular reflection of the object, but it is probable that this error is not very large. This, like other relative methods, will be in error if the standard surface is incorrectly evaluated.

III. THE NEW ABSOLUTE REFLECTOMETER.

In 1916 the author worked out the complete theory of a reflectometer which was to be an absolute instrument, and shortly thereafter the experimental instrument used in these tests was constructed. The few tests which were made gave good results, but for lack of time very little more work was done with the instrument until within the past few months. No publication of the theory was made because insufficient work had been done to completely verify it, but it has now been verified by extensive experiments, as will be shown later.

At the convention of the Illuminating Engineering Society in Chicago in October, 1919, in the discussion of two of the papers presented, the author called attention to the fact that the value of 88 per cent. for magnesium carbonate was much too low, and also briefly described the instrument which it is the object of this paper to describe more fully. Because of delay in publication, that discussion has not appeared in print at the time of writing. In a revision of his discussion, prepared shortly after the convention, the author stated that the reflection factor of magnesium carbonate was approximately 96 per cent., that being slightly lower than a value which he obtained by one measurement by the point-by-point method.*

FIG. 1.


C
A
B
L
DIRECT LIGHT ON THIS SURFACE
S_1
S_2
c'
c
FIG. 1a.
L
A
B
TEST SURFACE
FIG. 1b.
A- LIGHTING TUBE.
B AND C- OBSERVATION WINDOWS.
PORTABLE PHOTOMETER MAY BE USED TO OBSERVE BRIGHTNESS OF WINDOWS.
L- LAMP.
S_1 AND S_2 - SCREENS.
THE FUNCTION OF A, B AND C ARE INTERCHANGEABLE.

The reflectometer consists of a small sphere arranged as shown in Fig. 1. At *B* is a small hole through which the opposite wall of the sphere can be viewed by a brightness photometer.

---

* The absolute method for the determination of the reflection factor of magnesium carbonate which was described by another author in a recent publication is merely a modification of the method which he heard the present author describe in Chicago, and is strictly limited to the determination of the reflection factor of any surface which can be embodied in a hollow sphere. His method uses an incomplete sphere divisible into two fractional parts, whereas the author described the use of the complete sphere and one fractional part.

The segment of sphere surface c' is cut off by a plane, leaving the opening c''. The test surface is placed over this opening, and the direct light is projected onto the sphere wall or the test surface, depending on whether the lighting tube is placed at A or C. The attachments are so constructed that their positions are interchangeable. In the experimental instrument constructed and used in the tests the area c'—the portion cut off— was 10 per cent. of the total sphere area.

IV. THEORY OF REFLECTOMETER.

If a plane surface is illuminated by an infinite plane of uniform brightness, b (candles per unit area), the flux received by unit area of the illuminated plane is πb lumens.[5]

A perfect diffuser of uniform brightness b emits πb lumens per unit area.

The interior illumination of a hollow sphere with diffusely reflecting walls may be considered as composed of two parts: (a) the light received directly from the light source, and (b) the illumination made up of light diffusely reflected from the sphere walls. The part (b) is the same in value at all points in an empty sphere, in accordance with the theory of the integrating sphere.

Assume the hollow sphere arranged as shown in Fig. 1 *a*. Let the area of the complete sphere be unity.

Let $c' =$ ratio of sphere area cut off to total sphere area.
$c'' =$ ratio of area of hole to total sphere area.
$a = 1 - c' =$ fraction of total sphere area which remains.
$m =$ dffuse reflection factor of sphere surface.
$m_x =$ diffuse reflection facor of test surface.
$F =$ total light flux (lumens) received from lamp.
$b_0 =$ average brightness (candles per unit area) of sphere wall, due to *reflected light only* when hole is uncovered.
$b_x =$ brightness when hole is covered with a flat test surface having a diffuse reflection factor m_x.
$b =$ brightness when test surface has reflection factor m.

When the hole is uncovered the escaping light flux may be considered as composed of two parts. The portion of first reflected flux which escapes through the hole is $c'mF$, since it is the once reflected flux which would ordinarily be received by the portion of the surface which has been removed. The second part is the flux due to the average brightness, b_0, of the sphere walls, which in turn is due to reflected light *only*. Its effect in the plane

c'' is equivalent to that of an infinite luminous plane of brightness b_o. Hence this second part is $\pi b_o c''$, and the total flux escaping through the hole is

$$(1) \quad F' = c'\,mF + \pi b_o c''$$

The total flux received by the remaining portion of the sphere surface is

$$\frac{\pi b_o a}{m} + F$$

the amount being absorbed being

$$(2) \quad (1 - m)\left[\frac{\pi b_o a}{m} + F\right]$$

Neglecting the small amount of light flux lost through the lighting and observing windows, the sum of (1) and (2) must be equal to the total flux received, since it all escapes through the opening or is absorbed. Therefore, we have

$$(3) \quad \pi b_o c'' + c'mF + (1-m)\left[\frac{\pi b_o a}{m} + F)\right] = F$$

Solving (3) for b_o,

$$(4) \quad b_o = \frac{m^2\, F(1-c')}{\pi[mc'' + a(1-m)]} = \frac{m^2 Fa}{\pi\,[mc'' + a(1-m)]}$$

If the hole is covered with a flat surface having a reflection factor m_x, then $(1-m_x)$ times the flux incident on the test surface will be absorbed. In that case we find that

$$(5) \quad (1-m_x)(\pi b_x c'' + c'mF) + (1 - m)\left[\frac{\pi b_x a}{m} + F\right] = F$$

Solving for b_x, we have

$$(6) \quad b_x = \frac{m^2 F(a + c'm_x)}{\pi[c''m(1-m_x) + a(1-m)]}$$

If $m_x = m$, *i.e.*, if the test surface has the same reflection factor as the sphere surface, then (6) becomes

$$(7) \quad b = \frac{m^2 F(a + c'm)}{\pi[(1-m)(c''m + a)]}$$

It is possible to measure the relative brightness of the sphere wall under the various conditions, and from these measurements m and m_x can be computed.

$$\text{Let } \frac{b}{b_o} = K \quad \text{and} \quad \frac{b_x}{b_o} = K_x$$

Then

$$(8)\quad K = \frac{\dfrac{m^2F\,(a + c'm)}{\pi(1-m)\;(c''m+a)}}{\dfrac{m^2Fa}{\pi[mc'' + a(1-m)]}} = \frac{(mc' + a)[mc'' + a(1-m)]}{a(mc'' + a)\;(1-\;m)}.$$

Clearing of fractions and collecting,

$$(9)\quad m^2[c'(c'' - a) + Kac''] + ma[c' + (c'' - a)\;(1 - K)] + a^2(1 - K) = 0.$$

Since c', c'' and a are dimensional constants of the sphere, and K can be measured, m, the reflection factor of the sphere surface, can be evaluated from this equation.

$$(10)\quad K_x = \frac{\dfrac{m^2F(a + c'm_x)}{\pi[c''m\,(1 - m_x) + a(1 - m)]}}{\dfrac{m^2Fa}{\pi[mc'' + a(1 - m)]}} = \frac{(a + c'm_x)[mc'' + a(1 - m)]}{a[mc''\,(1 - m_x) + a(1 - m)]}$$

$$(11)\quad m_x = \frac{a(K_x - 1)\;[c''m + a(1 - m)]}{ac'(1 - m) + mc''(c' + K_xa)}$$

The above equations have been derived for the case where the direct light from the lamp is incident on the sphere wall. While it is possible to use the reflectometer with that arrangement, it is much more sensitive to changes of reflection factor of the test surface, and therefore more accurate, if it is arranged so that the direct light is incident on the test surface. The most convenient method is to use the former arrangement and determine the value of m from equation (9), then arrange the instrument so that the light is incident directly on the test surface, as shown in Fig. 1 *b*. By using for a reference standard a surface painted with the sphere paint, it is then possible to evaluate in absolute measure the reflection factor of any test surface. The theory of the instrument when so used is as follows:

Let c', c'', a, m, m_x and F have the same meaning as before.

b' = brightness of sphere surface when the hole is covered with a test surface of factor m_x.

$$R = \frac{\text{value of } b' \text{ when hole is covered with surface } m''}{\text{value of } b' \text{ when hole is covered with surface } m'}.$$

Since unit area of the sphere surface of brightness b' emits $\pi b'$ lumens, it must be receiving $\frac{\pi b'}{m}$ lumens. Hence the sphere area, a, absorbs $(1-m)\;\frac{\pi b'a}{m}$ lumens. The test surface receives $F + c''\pi b'$ and absorbs $(1-m_x)\;(F + c''\pi b')$ lumens. Hence, since all the light is absorbed except that negligible amount which escapes through the lighting and observing holes, we have

$$(13)\quad (1-m)\,\frac{\pi b'a}{m} + (1-m_x)\,(F + c''\pi b') = F$$

Solving for b',

$$(14)\quad b' = \frac{m m_x F}{\pi\,[a(1-m) + c''m\,(1-m_x)]}$$

$$(15)\quad R = \frac{\dfrac{mm''F}{\pi[a(1-m) + c''m\,(1-m'')\,)}}{\dfrac{mm'F}{\pi[a(1-m) + c''m\,(1-m')]}} = \frac{m''[a(1-m) + c''m(1-m')]}{m'[a(1-m) + c''m(1-m'')]}$$

In the special case where $m' = m$, we have

$$(16)\quad m'' = m_x = \frac{Rm[a(1-m) + mc'']}{(1-m)\,(a + c''m) + Rm^2c''}$$

V. EXPERIMENTAL RESULTS.

In order to verify the theory of this instrument a graded series of test objects was made up. Neutral gray objects were obtained by mixing black drawing ink and lampblack with a white cement (Keene's Fine). They were surfaced with coarse sandpaper, resulting in fairly good diffusers. Those having reflection factors below 50 per cent. were better diffusers than those above that value. This appears to be characteristic of this material, possibly of others, and will be treated more fully in a future paper.

The test objects were next tested for reflection factors by means of the apparatus shown in Fig. 2. The objects were illuminated normally by a 60-watt vacuum tungsten lamp, and the surface brightness was measured by point-by-point observations. The amount of flux reflected was then calculated by applying the proper factors, and since the incident flux was known, the ratio of the reflected to the incident flux gave the reflection factors of the test surfaces. The greatest difficulty in that method of measurement is the standardization of the apparatus to read surface brightness, as it involves very considerable practical difficulties, chief among which is the great intensity step which must be taken. Much work was done on this part of the problem, and it is believed that the uncertainty in that standardization was not very large.

A number of the cement standards and a block of magnesium carbonate were measured by both methods of using the instrument as described above. By the use of equation (9) the reflection factor of the sphere surface was found to be 88.3 per cent. This value was then substituted in the equations and the reflection factors of the various test objects were computed. The results of the measurements in the three different ways are shown in Table I.

TABLE I.

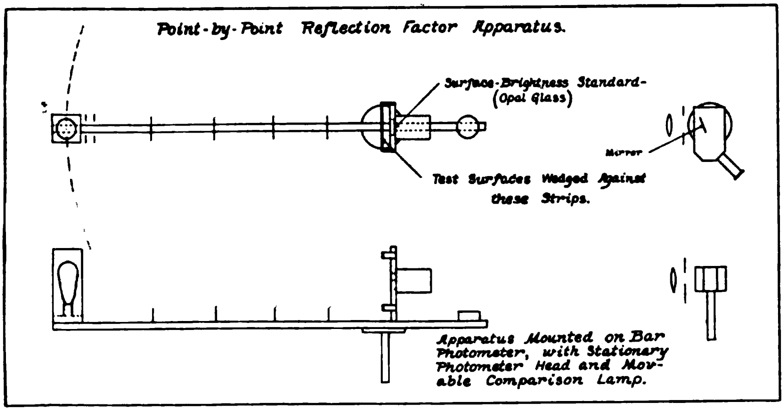
Reflection Factors by Three Methods of Measurement.

Test object	Point-by-point measurement	Sphere reflectometer	
		Direct light on test objects	Direct light on sphere wall
	per cent.	*per cent.*	*per cent.*
Cement standard No. 1	17.7	18.4	21.9
Cement standard, No. 2	23.7	23.9	28.2
Cement standard, No. 3	31.5	31.6	35.2
Cement standard, No. 4	36.3	37.1	39.6
Cement standard, No. 5	42.9	42.9	46.4
Cement standard, No. 6	60.0	60.7	62.8
Cement standard, No. 7	67.4	67.3	70.2
Cement standard, No. 8	81.9	81.4	82.7
Cement standard, No. 9	86.5	85.5	86.8
Cement standard, No. 10	90.8	90.4	89.6
Magnesium carbonate	99.3	99.1	98.7

It will be noted that the agreement betwen columns 2 and 3 is almost perfect, and that for reflection factors above 80 per cent. the fourth column is in satisfactory agreement. For factors below 80 per cent. the method represented by the fourth column is rather insensitive, the error in the result being larger than the error of the photometer reading with the actual conditions existing in the experimental instrument. This is clearly shown by the curves in Fig. 3. When the direct light is incident on the test surface, the error in the factor is always less than the error of the photometer reading. Hence, it may safely be stated that at least a part of the discrepancy between columns 2 and 4 may be attributed to experimental error. The fact that the differences are all in the same direction for the objects having low reflection factors might lead to the belief that not all the discrepancy could be explained this way, and that it might be necessary to look farther for an additional reason. It is possible that the reflection factor may be higher for diffused light than for light having a large normal component. If this is true, the differences are in the direction to be expected. The sensitivity of the instrument when used with diffused illumination of the test surfaces would be greatly increased if the reflectometer were painted with a paint having a higher reflecting power and it might then be possible to determine whether this is a real departure from the theory as developed. The author has made up paints having a factor of about 94 per cent., which would be much better for this application.

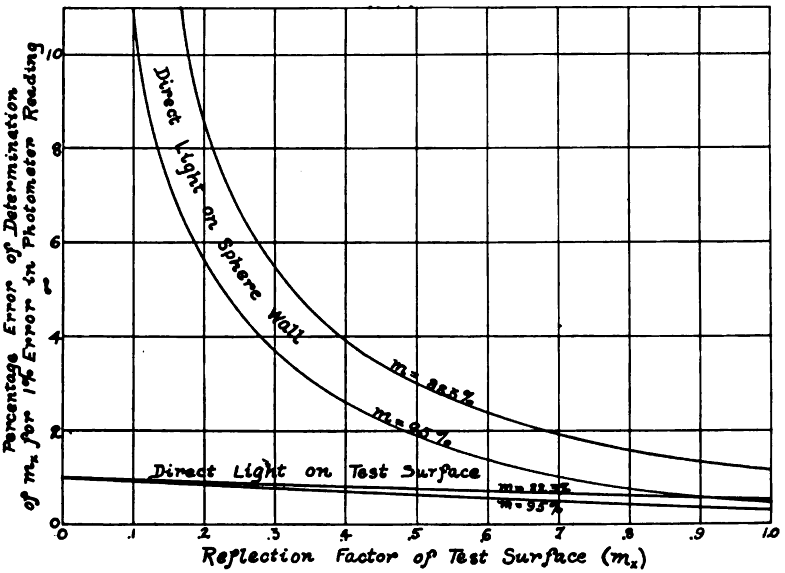
It will be noted that the measurements by the three methods give about 99 per cent. for magnesium carbonate, whereas the previously accepted value was only 88 per cent. It did not seem possible that its value could be so high, and the author was reluctant to accept that value as reliable, in spite of the agreement of the three methods, unless verified by still another method. Hence, steps were taken to verify it by a fourth absolute method, which was described above as a modification of the sphere method used by Mr. Little. For this purpose a small disk of magnesium carbonate was made up from the block previously tested. It was placed in a sphere and a very narrow beam of light was projected through a small hole in the sphere wall onto the sphere surface at a point unscreened from the observation window, then onto the magnesium carbonate disk so placed that none of the first reflected light from it could reach the observation window. The ratio of the brightness of the window in the second case to that in the

FIG. 2.


Point-by-Point Reflection Factor Apparatus.
Surface-Brightness Standard-(Opal Glass)
Mirror
Test Surfaces Wedged Against these Strips.
Apparatus Mounted on Bar Photometer, with Stationary Photometer Head and Movable Comparison Lamp.

first case is the reflection factor of the test surface. This measurement gave a factor of 98.9 per cent., which, on the basis of the consistency of the photometric readings, appeared to be thoroughly reliable. Hence, it appears that the reflection factor of this particular block of magnesium carbonate is 99 per cent. Another block, obtained about three weeks later from the same source, has a factor of approximately 98 per cent., while another block which has been in the laboratory for about two or three years is appreciably darker than either of these. The author has not investigated the reproducibility of such surfaces for precision standards.

FIG. 3.



It should be pointed out that magnesium carbonate is not a perfect diffuser, hence this must be taken into account if it is desired to use it as a standard of surface brightness. The block which was tested by point-by-point measurements was surfaced by scraping with a sharp glass edge. When illuminated normally its brightness at about 50° from the normal to the surface was the same as that of a theoretically perfect diffuser emitting the same total flux.

VI. EFFECT OF SPECULAR REFLECTION FROM TEST SURFACES.

As mentioned above, the surfaces with which these tests were carried out were fairly good diffusers. Some other surfaces which have been examined by point-by-point measurements have been ordinary semi-matt surfaces. An examination of these data shows that when the light is incident normally the angle at which the surface has the same brightness as a perfect diffuser emitting the same total flux is usually about 45° to 50° from the normal, which is also approximately the angle between the normal and the line joining the observation window and the centre of the test surface in the instrument used in these tests. The difference between the surface brightness of the test surface at 50° and a perfect diffuser emitting the same total flux does not exceed 3 to 5 per cent. in the case of all the surfaces tested, and is usually below 3 per cent. The effect of a deviation of 5 per cent. from perfect diffusion at 50° for a surface of reflection factor 80 per cent. has been calculated for a reflectometer having 10 per cent. of its area cut off, and painted with a paint having a reflection factor of 90 per cent. The calculation shows that the error of the determination would be less than .5 per cent. Hence, it appears safe to state that the error of determination due to specular reflection will not exceed 1 or 2 per cent in a reflectometer having these dimensions, unless an excessive amount of the specular reflection of direct light is incident on the observation window. This will not hold true unless the specular reflection takes place only at the first surface.

If it is desired to determine the reflection factor of a mirror, silvered on the back, this may be done by first directing a narrow beam of light into the opening of the reflectometer, then letting the same beam be reflected from the mirror and be directed into the reflectometer when the mirror is placed at an appreciable distance from the reflectometer. The ratio of the brightness of the observation window in the second case to that in the first will be the reflection factor of the mirror.

VII. PRECAUTIONS IN USE OF REFLECTOMETER.

In the use of the reflectometer as described above, certain precautions are necessary. Some of them are as follows:

(*a*) The dimensions should be precisely determined in order to fix the values of the constants c', c'' and a.

(*b*) In painting the sphere and the flat surface by means of which its reflection factor is determined, care must be taken to make the flat surface as nearly as possible the same in reflecting power as the sphere surface.

(*c*) When calculating the reflection factor of the sphere surface by the use of equation (9), the figures should be carried out as far as possible, as the equation involves differences of numbers which are nearly equal, and a small error in calculation may make an appreciable error in the result. The use of logarithms for this step is recommended, though slide rule calculations are sufficiently accurate in working up test data for observations taken with direct light on the test surfaces.

(*d*) When the direct light is incident on the sphere wall, care must be taken to prevent any direct light from escaping through the large opening, c''.

The most symmetrical arrangement of the reflectometer is obtained if the observation hole and the side lighting hole are about 90° from the centre of the portion cut off, and 90° from each other. This is the arrangement shown in Fig. 1 *a*.

VIII. CONCLUSION.

In conclusion the most important points brought out by this paper may be summarized as follows:

1. Five "absolute" methods of measuring reflection factors are described, at least three of which are apparently new. Measurements on magnesium carbonate by four of these methods give values which are in excellent agreement.

2. The numerical value of the reflection factor for magnesium carbonate which has been in use for many years, *viz.*, 88 per cent., is in error. This fact was first publicly pointed out by the author at Chicago in October, 1919. The actual value of its diffuse reflection factor is approximately 99 per cent., but the degree of reproducibility of this value with materials from different sources is unknown.

3. The method described above for the use of an incomplete sphere as a reflectometer furnishes two new absolute methods for the determination of diffuse reflection factors. The determination of the reflection factor of the sphere surface is only an incidental step in the use of the instrument, and is not its principal object.

4. The instrument just described should find a large field of usefulness in photometry and illuminating engineering, and furnishes a method of measuring the reflection factors of surfaces *in situ*. Apparently no other instrument which is accurate and portable has been proposed for this purpose. It can be adapted for use with any good type of portable photometer.

5. It should be strongly emphasized that the reflection factor of any *colored* surface is dependent on the color of the incident light and that measurements by this or any other method will give its reflection factor only under the particular conditions of the test. Hence, under such conditions the principal value of the measurement is to indicate the approximate value of the factor, but this is all that is usually required.

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[3] "The Integrating Photometric Sphere, Its Construction and Use," E. B. Rosa and A. H. Taylor, *Trans. Illuminating Engineering Society*, Vol. 11, p. 453, 1916.

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BUREAU OF STANDARDS.

A NEW MICROPHOTOMETER FOR PHOTOGRAPHIC DENSITIES.

BY

W. F. MEGGERS and PAUL D. FOOTE.

I. INTRODUCTION.

PROBLEMS involving the necessity of measuring the photographic density of a minute portion of a photographic plate have called into existence several types of microphotometers, such as the well-known Hartman[1] instrument, the Koch[2] registering microphotometer and the new microphotometer recently described by Fabry and Buisson.[3]

In the course of some recent experiments on low voltage arc spectra of cæsium, the writers required the use of a microphotometer to measure photographic densities of spectral lines. None of the above-mentioned instruments being at hand, the instrument which was employed by the writers over three years ago in some photographic density work by the addition of a few slight changes, proved so satisfactory, accurate, and efficient a microphotometer as to merit serious consideration as a piece of laboratory apparatus. The instrument (Fig. 1) is essentially the micropyrometer described by Burgess,[4] except that a microscope of higher power was employed. The photographic plate was mounted just below the objective of the microscope on a horizontal bed, movable with a graduated screw, and was illuminated beneath by an intense beam of light from a tungsten ribbon gas-filled lamp. Light transmitted by a small portion of the photographic plate, the image of which was adjacent to that of the tip of the photometer lamp, was matched with equal filament brightness by adjusting the current through the lamp. The ammeter readings are readily translated into measurements of photographic density.

II. CALIBRATION.

To define the state of blackness of a developed photographic plate, or in general of any absorbing layer, a beam of light is passed through it and the intensity I of the incident beam is compared with the intensity T of the transmitted beam. The ratio $O = I/T$ is called the opacity, and the value of this ratio is 1 for a perfectly transparent plate. It is more convenient, however, to use the logarithm (taken to the base 10) of this quantity and call it density, D. Then:

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[1] *Zeitschr. f. Inst.* 19, p. 97, 1899.

[2] *Ann. de Phys.* (4) 39, p. 705, 1912.

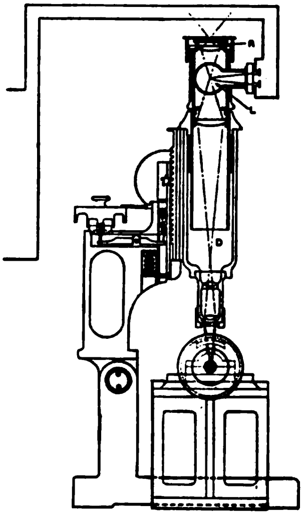
[3] *J. de Phys.* (5) 9, p. 37, 1919.

[4] *Bull. Bureau of Standards,* 9, p. 475, 1913.

$$D = \log O = \log I/T$$

so that a perfectly transparent plate has zero density and a density of 1 indicates that 1/10 of the incident light is transmitted by a plate. In general, slightly different values of density are found from measurements with different-colored lights. It is, therefore, advantageous to make density measurements with homogeneous light rather than with white light, since the phenomena then obey the above simple law and the difficulties inherent in heterochromatic photometry are evaded.

FIG. 1.



Microphotometer.

These advantages are possessed by the microphotometer in which the usual "monochromatic" red-glass screen used in optical pyrometers is employed. Of course, a suitable monochromatic screen of any other color could be used in the same way. The densities measured with red light will be the same as for any other color if the photographic plate is non-selective.

There are several methods of calibration and use of this type of photometer, of which the following appeared the most satisfactory: The instrument with a red glass ocular was calibrated as a pyrometer by sighting upon a small black body, the temperature of which was measured by an optical pyrometer, thus obtaining the relation between the temperature of the source and the current through the photometer lamp sufficient for a match. If the red glass screen is sufficiently monochromatic, we may neglect the variation of the "effective wave-length" of transmission with the temperature of the source.[5] The particular glass employed had an effective wave-length of approximately $\lambda = 0.65\mu$.

In use the illuminating source is maintained at any constant brightness sufficiently high to permit observation on lines of the highest density. The measurements of the current required to match against any spectral line and against the clear film in the neighborhood of this line are made and are converted into "temperatures" by use of the calibration plot of the instrument. If θ_1 is the absolute temperature observed for the line and θ_2 the corresponding temperature for the clear film (fog) the true density D of the line, fog correction being considered, follows at once from Wien's law:

$$D = \frac{c_2 \log e}{\lambda}\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right) = 9590\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)$$

For convenience of use, instead of plotting current through the lamp *versus* temperature, we plot current *versus* $9590/\theta$, obtaining for the particular instrument employed, the curve shown in Fig. 2. Thus, suppose the current i_a is observed for any fog reading on clear film and i_b when sighted upon the line. The true density of the line is given by the difference in ordinates of the points A and B.

Another method of calibration, in some respects more satisfactory, is to sight upon the illuminating source and vary the intensity by means of sectored disks. Still another method is to use screens which have been measured by other photometers. Screens calibrated for white light may be employed when plates similarly treated are measured, thus eliminating the slight difference in density arising from measurements in the usually employed white light and red light on account of any slight selectivity of the gelatine. However, the effect of scattering of light by the photographic film is many times greater than that of the selective absorption and requires special attention to the type of illumination employed, as discussed later.

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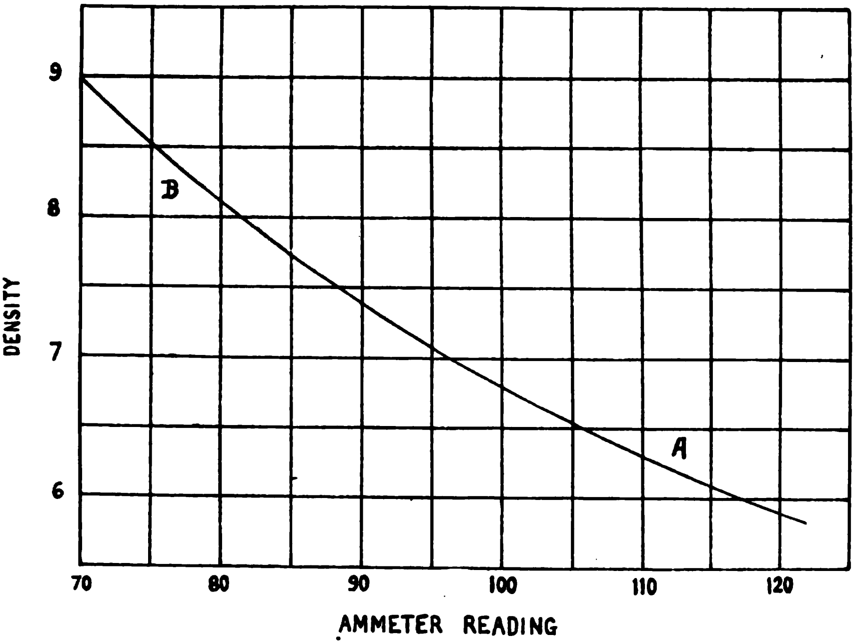
[5] *Bull. Bur. of Stand.* 12, p. 483, 1915.

FIG. 2.



Calibration curve of microphotometer.

III. APPLICATION.

The primary object for which this microphotometer was assembled and calibrated was to measure the photographic densities of narrow spectral lines. To accomplish this, the photographic plate was mounted in the object plane of the microscope and moved horizontally to bring the images of different spectral lines or parts of the plate in coincidence with the image of the tip of the pyrometer lamp filament. This shift was effected by turning a screw which was provided with a revolution counter, and a head divided so that 1 division corresponds to 5μ in linear motion of the bed on which the photographic plate is mounted. An important feature of this arrangement is that the tip of the pyrometer lamp filament may be regarded as a filar in the eye-piece and the relative positions or wave-lengths of spectral lines on a plate are measured at the same time that their photographic densities are measured. The measurement of densities has been explained above, and it only remains to give an example of the simultaneous double use of this instrument as a photo-measuring micrometer and a microphotometer. Table I gives the results obtained from a one-minute exposure of a dicyanin stained plate to a six-volt cæsium arc spectrum formed by a small prism spectograph.

TABLE I.

Wave Lengths, Photographic Densities, and Estimated Intensities in Cæsium Arc Spectrum.

Micrometer readings	Wave length		Photographic densities	Estimated intensities	I/D
	Computed	Accepted			
178.81	4555.30*	4555.30	0.78	5	6.4
179.47	4592	4593	.73	5	6.9
181.64	5656	5664	.58	4	6.9
2.92	5837	5845	1.01	7	7.0
4.01	6006	6010	1.08	8	7.4
4.15	6029	6034	.40	3	7.5
5.19	6211	6213	1.16	9	7.8
5.93	6353	6355	.93	7	7.5
7.04	6586	6587	1.04	8	7.7
7.64	6723.18*	6723.18	1.31	9	6.9
8.05	6825	6827	.59	4	6.8
8.26	6878	6873	.93	7	7.5
8.67	6975	6973	1.38	10	7.2
9.53	7229	7229	1.03	8	7.8
199.72	7286	7280	.93	7	7.5
200.72	7615	7610	.76	5	6.6
1.61	7945	7944	.45	3	6.7
1.80	8024	8017	.26	2	7.7
1.95	8086	8080	.17	1	6.0
2.92	8521.12*	8521.12	1.16	9	7.7
203.73	8946	8943	.28	2	7.2

The first three columns of Table I illustrate the use of the microphotometer for wave-length measurements at the same time that photographic densities (column 4) of the spectral lines are measured. Three wave-lengths marked with asterisks were used in wave-length computations by the Hartmann Formula.

In column 5 the visually estimated intensities of the spectral lines are given. These estimates were made according to the usual practice in spectroscopy of assigning values from 1 to 10 to the intensities of spectral lines recorded on a photographic plate. According to Fechner's law, that "the intensity of sensation increases as the logarithm of the stimulus," the natural tendency in formulating a scale of opacities is to choose a geometric series so that the ratio of the transmissions of any two adjacent steps is constant throughout the series. In other words, the apparent contrasts between successive steps are equal only if the logarithms of the transmissions differ by equal amounts. Visually estimated intensities of spectral lines appear to obey this law very approximately, as is shown in the last column of Table I, in which the ratios of visually estimated intensities to measured photographic densities is given. If the estimated intensities are divided by 7, in this particular example, the quotients will be in close agreement with the photographic densities or logarithms of opacities.[6]

Fig. 3 is an illustration of the possible use of the microphotometer in measuring the distribution of energy in a very wide spectrum line. The photograph represents the appearance of the infra-red line 8521.12 *A* in the 120-volt arc spectrum of cæsium as photographed on a dicyanin stained plate with a concave grating giving a scale of 10 *A* per millimetre. The curve below was obtained from photographic density measurements with the microphotometer.

This instrument is also well adapted for the measurement of spectral sensitivity of various types of photographic plates after being exposed in a spectograph to a continuous spectrum. A typical example of this is shown in Fig. 4, which represents microphotometer measurements of density on a Seed 23 plate stained with dicyanin and developed after an exposure of one minute in a prism spectograph which was directed at a black body furnace at 1450° C. The sensitivity of this stained plate to red light appears to be enormously greater than its blue sensitiveness, but the curve should really be corrected to equal energy distribution in the spectrum. To accomplish this, "characteristic curves" for different wave-lengths were drawn from microphotometer measurements on exposures of different durations and then the plate sensitivity was reduced to the relative densities which would be expected if the spectrum possessed equal energy for all wave-lengths. This is shown in Fig. 5.

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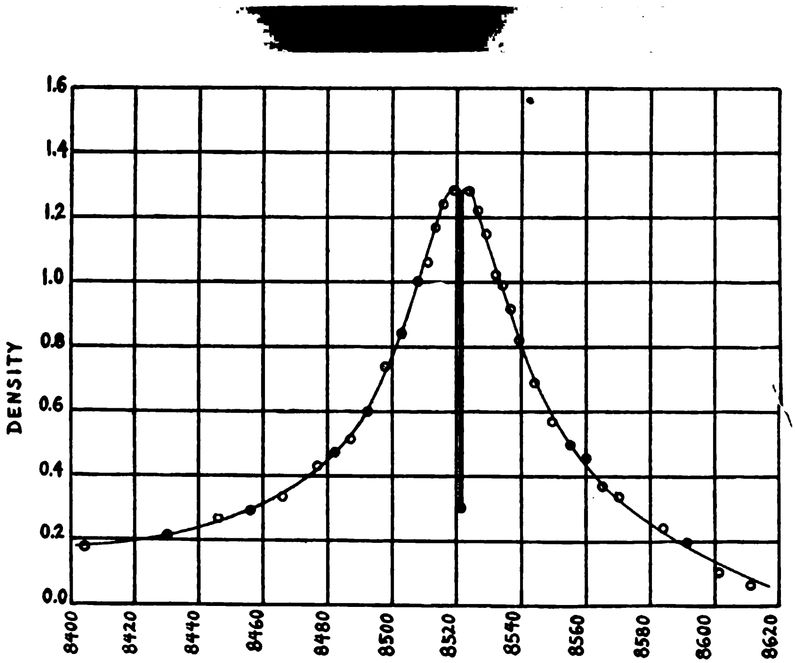
[6] It is readily possible, often advisable and sometimes the practise of spectroscopists, to use more than 10 degrees of opacity in estimating intensities of spectral lines. The average spectrogram perhaps does not give a density of more than 2 for the strongest line on the plate, and this suggests that the upper limit of the scale of estimated intensities might conveniently be extended to 20, or even to 200. Then a larger number of steps would be available in a finer discrimination of opacities, and if the estimates follow Fechner's law, the factor 10 or 100 will readily permit a rough, but often sufficiently precise, transformation from visually estimated intensities to photographic densities.

FIG. 3.



Photographic density of the cæsium line 8521 *A*.

As a test of the accuracy of density measurements obtained with the microphotometer, some strips of photographic plates prepared in a sensitometer for the purpose of obtaining the *H* and *D* characteristic curves[7] were very carefully measured with a

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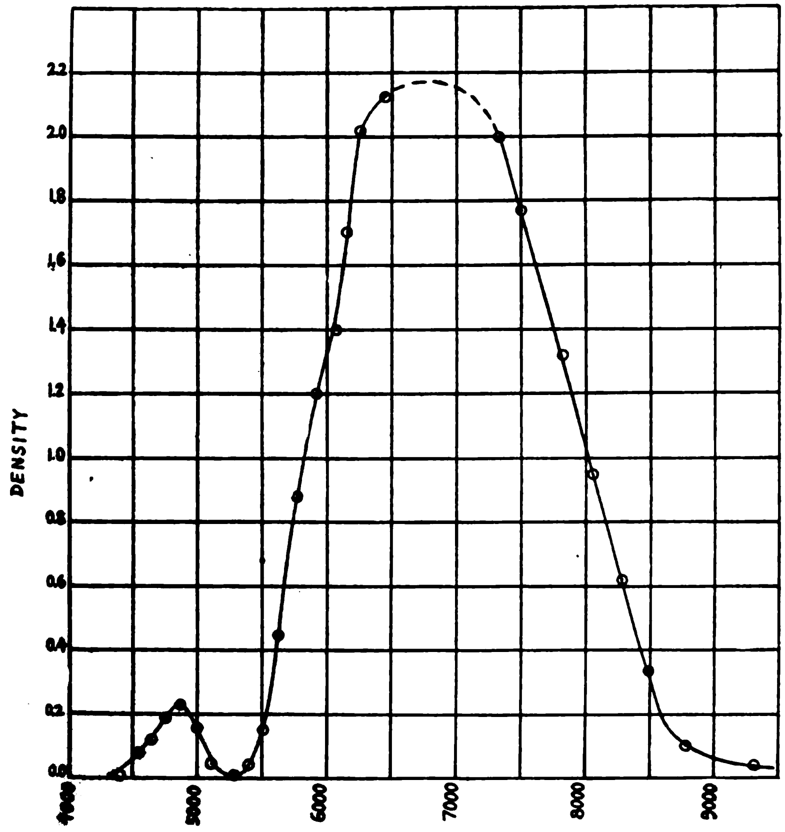
[7] Hurter and Driffield: *Jour. Soc. Chem. Ind.*, May, 1890, p. 455.

Koenig-Martens polarization photometer and then with the new microphotometer. The densities measured with the microphotometer were found to be considerably larger than those given by the polarization instrument and the chief reason for this was

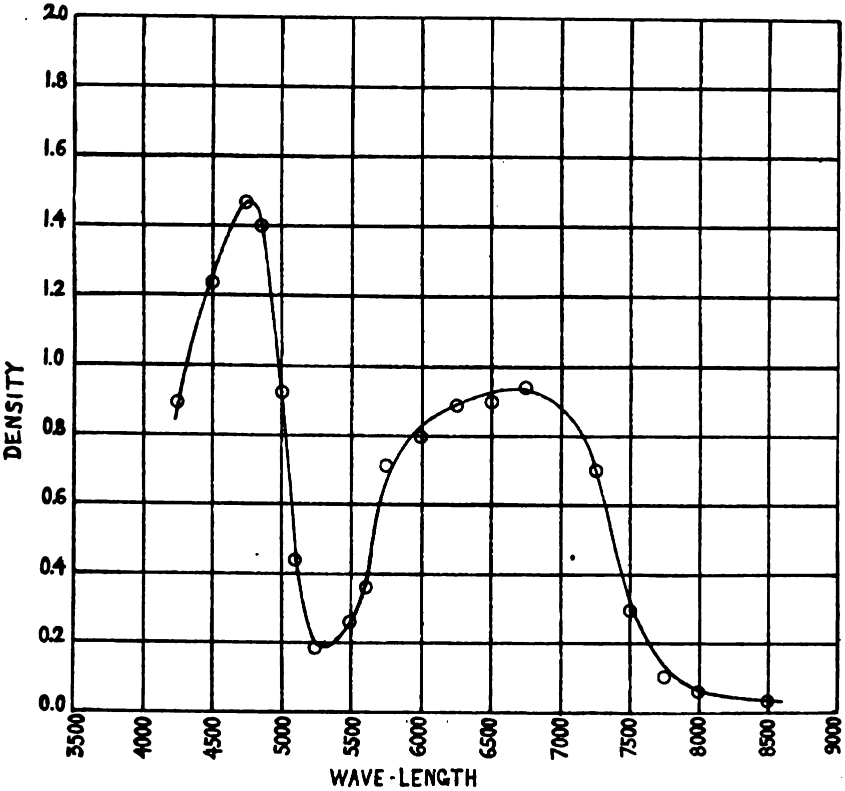
Fig. 4.



Sensitivity of a Seed 23 plate stained with dicyanin.

traced to the manner in which the photographic plate was illuminated in the two instruments. In the polarization photometer the photographic strip is lighted by lamps in a box of opal glass so that very diffuse light illuminates the plate. This diffuse light is scattered and partly reradiated by developed silver particles in the photographic plate, and gives the plate apparently a higher transmission and lower density than it really possesses. Practically the same low densities were obtained with the microphotometer when two layers of diffusing ground glass were placed just behind the photographic plate. This is illustrated in Fig. 6, in which the densities of a sensitometer plate as measured with the polarization photometer are plotted as ordinates, and those measured with the pyrometer microphotometer as abscissæ. If both instruments gave the same densities the values would lie on a dashed line at 45°. Curve *A* shows densities determined with the microphotometer when the photographic plate was illuminated

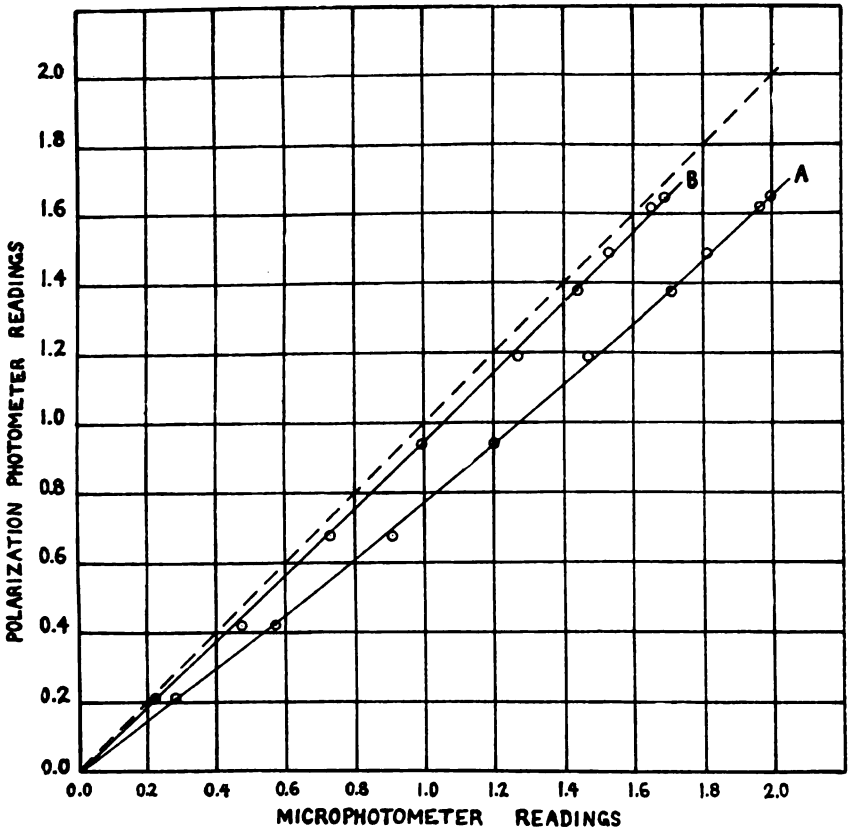
FIG. 5.



Sensitivity of dicyanin stained plate to equal energy spectrum.

with parallel light and curve *B* when diffuse light from two layers of ground glass illuminated the plate. This experience calls attention to the importance of specifying the character of the illumination when measurements are made on the light transmission of diffusing media.[8]

FIG. 6.



Microphotometer *vs.* polarization photometer readings.

SUMMARY.

The new microphotometer for measuring photographic densities is essentially the micropyrometer described in the *Bulletin* of the Bureau of Standards (Vol. ix, p. 475, 1913), except that a microscope of higher power is used. The photographic plate is mounted just below the objective of the microscope on a horizontal bed, movable with a graduated screw, and is illuminated beneath by an intense beam of light from a tungsten ribbon lamp. Light transmitted by a small portion of the photographic plate, the image of which is adjacent to that of the tip of the pyrometer lamp, is matched with equal filament brightness by adjusting the current through the lamp. The ammeter readings are readily translated into measurements of photographic density. If the tip of the pyrometer lamp filament be regarded as a filar in the eyepiece, the relative positions or wave-lengths of spectral lines on a plate are measured at the same time that their photographic densities are measured.

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[8] Absorption and diffusion of light in the developed photographic plate have been studied by Callier, who also presents some of the photographic consequences of the scattering of light. *Zeitschr. f. Wiss. Phot.* 7, p. 257, 1909.

An example of the use of this microphotometer for measuring wave-lengths and densities of spectral lines is given, and it appears that the visually estimated intensities of such lines form a geometric series in accordance with Fechner's law.

Illustrations of this microphotometer's application to the measurement of energy distribution in a broad spectral line and to the measurement of spectral sensitivity of a photographic plate are given.

Measurements of photographic density when the plate is illuminated by parallel light are found to be much larger than when the illumination is diffuse. This experience calls attention to the importance of specifying the character of the illumination when measurements are made on light transmission of diffusing media.

BUREAU OF STANDARDS,
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THE RESPONSE OF THE AVERAGE PUPIL TO VARIOUS INTENSITIES OF LIGHT.

BY

PRENTICE REEVES.

THE eye is probably the most remarkable product of Nature's process of evolution. It is a sensory and a motor organ as we receive impressions from the world about us through the stimulation of the retina and also from the functioning of the eye muscles. As a sensory organ the eye ranks first, and even with the average imperfections it remains very efficient. As a motor organ the eye is capable of extremely fine discriminations. In addition to these facts the eye may be considered as a physical instrument. The eye and the photographic plate are the most widely used physical instruments in dealing with problems in light and in the majority of such problems the final decisions are rendered by the eye. It has been shown that the eye functions efficiently over a much wider range than any other known instrument, and, furthermore, it possesses the property of continually renewing its sensitiveness, except in the extreme upper limits of stimulation. Although we know quite a little about the dioptrics of the eye, we are as yet uninformed as to the exact nature of the retinal image and the physiological or psychological factors in visual perception. While we can not measure visual perceptions directly, we can accurately control the physical stimuli and determine the reaction of the retina to various intensities of illumination such as have been expressed as threshold sensibility, contrast sensibility, and glare sensibility. These sensibilities vary with several factors, the principal one of which is the brightness under which the eye is working.

For each brightness level used the pupil has a different diameter, as the pupil is automatically adjusted so as to tend to maintain constant light energy on the retina. That is, when the intensity of illumination is increased the pupil contracts and *vice versa,* or if the distance between the eye and stimulus is changed, the pupil is also changed to keep the retinal illumination constant. The literature on this subject of pupillary reactions deals largely with pathological cases and the results from normal eyes are few and show a lack of agreement.[1] In this study all subjects were normal and the data are offered as such.[2]

The procedure used by Blanchard[3] on the consensual reflex, *i.e.,* the effect of exposing one or both eyes to the light, was repeated, and although the actual pupillary diameters are different, the shape of the various curves is the same as Blanchard's. In this part of the experiment the subject's eye was focused on the ground glass of a 5 x 7 view camera and his head fixed in a head rest. A small millimetre scale was placed in the plane of the pupil so it would be magnified to the same extent as the pupil and simplify the reading of the records. After fifteen minutes' adaptation to total darkness a flashlight photograph was taken. The procedure was repeated for several brightness levels ranging from total darkness to a just tolerable reflection of direct sunlight from white drawing paper. The effect of exposing one or both eyes was studied throughout the entire range of brightness. Fig. 1 and Table I show the results obtained from one of the subjects, and it will be seen that in the range of ordinary intensities there is a marked effect of closing one eye.

From these results it is seen that the amount of contraction of the pupil for brightnesses in excess of 100 millilamberts is slight, so the photographs of six other subjects were taken for six brightness levels chosen from the curve just shown. In this way the minimum diameter obtained is near the actual minimum and the subjects were not exposed to the very unpleasant glare.

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[1] Blakesley, *Phil. Mag.,* 1910, **29,** 960; Nutting, *Trans. Ill. Eng. Soc.,* 1916, **11,** 1; Reeves, *Astrophys. J.,* 1917, **46,** 167; Langfeld, *Zsch. f. Sinnesphysiol.,* 1908, **42,** 349; Stevenson, *J. of Brit. Astron. Assn.,* 1916, **26,** 303.

[2] Similar treatment of data in *Psychol. Rev.,* 1918, **25,** 330.

[3] *Phys. Rev.,* 1918, **11,** 81.

In this part of the experiment a motion picture camera was used with an extension for "close ups" so that a full-sized image of the pupil could be recorded. A bank of lamps was used instead of the flash powder. Several pictures were taken of each subject on different days and the results averaged. Fig. 2 and Table II show the results from the six subjects and we see individual differences in the separate curves with marked variations in the maximum diameters. The average eye is represented by the heavy line and from this curve we are able to determine the

FIG. 1.

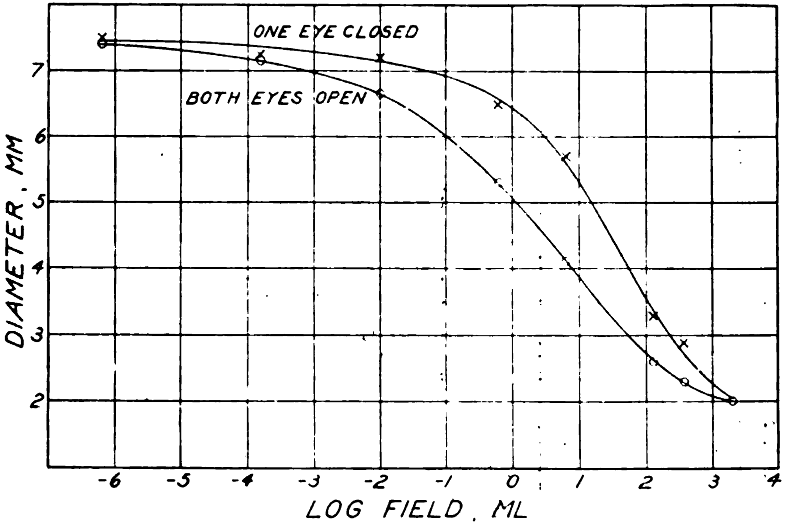
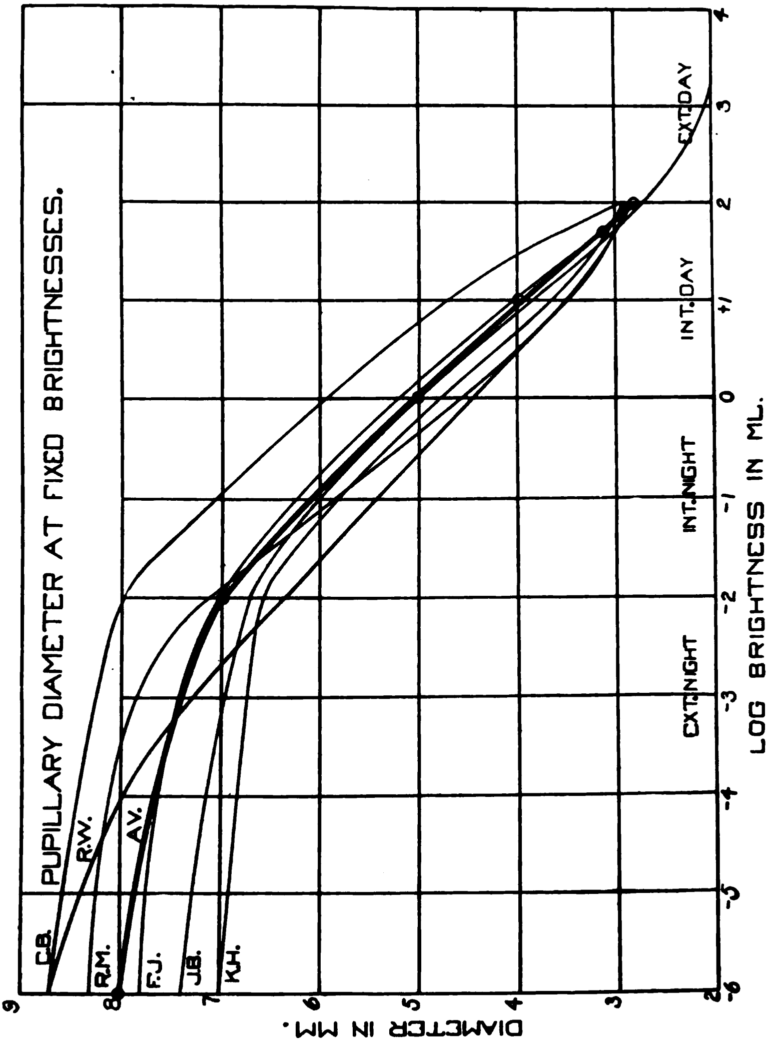


TABLE I.

Effect of Closing One Eye on Pupillary Diameters.

Brightness in millilamberts	Log B	Both eyes exposed		One eye closed		Difference	
		J. B.	*P. R.*	*J. B.*	*P. R.*	*J. B.*	*P. R.*
0	Taken as −6.0	7.4	8.6	7.5	8.6	0.1	0
0.00015	−3.82	7.1	8.5	7.2	8.5	0.1	0
0.01	−2.0	6.7	8.3	7.2	8.5	0.5	0.2
0.6	−0.22	5.3	7.0	6.5	7.8	1.2	0.8
6.3	0.8	4.1	4.3	5.7	5.9	1.6	1.6
126.0	2.1	2.6	2.9	3.3	3.3	0.7	0.4
355.	2.55	2.3	2.4	2.9	2.9	0.6	0.5
2000.	3.3	2.0	1.9	2.0	2.0	0	0.1

Fig. 2.


PUPILLARY DIAMETER AT FIXED BRIGHTNESSES.
C.B.
R.M.
F.J.
J.B.
K.H.
R.W.
A.V.
EXT. NIGHT
INT. NIGHT
INT. DAY
EXT. DAY
9
8
7
6
5
4
3
2
DIAMETER IN MM.
-6
-5
-4
-3
-2
-1
0
+1
2
3
4
LOG BRIGHTNESS IN ML.

dimensions of an average pupil for any given brightness. In ordinary practice we seldom use either the maximum or minimum diameters which may be taken as 8 mm. and 2 mm., but ordinarily take the range from 2.5 to 7 mm. It might be well to add at this point that the diameters we measure in these photographs are really the apparent diameters when viewed through the cornea and aqueous humor and that the actual dimensions are about seven per cent. less.

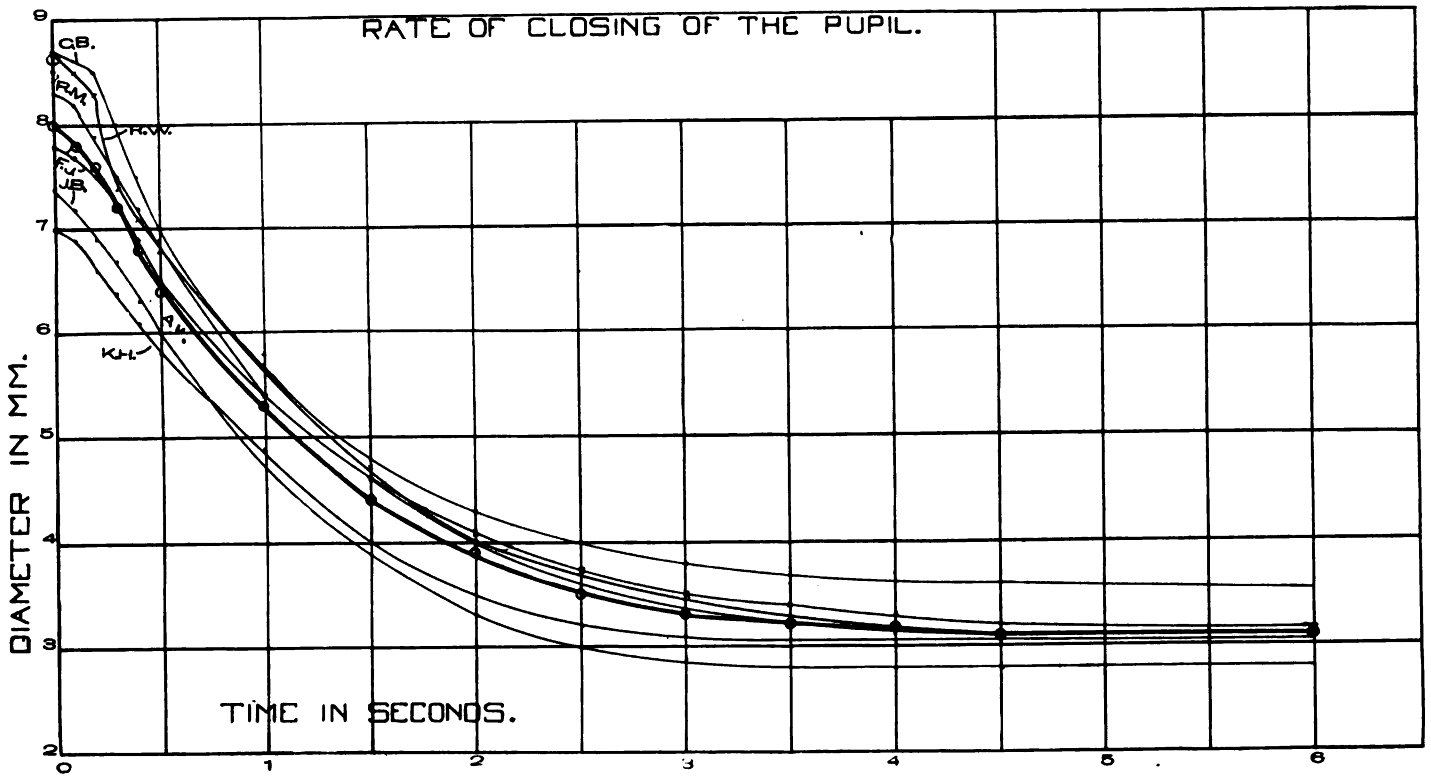
When changing from one brightness to another the retinal sensibility changes and has been shown to occupy an appreciable period of time for even slight changes in illumination. This change consists of two factors—a retinal process (whatever it may be) and the change in pupillary diameter. The former is explained by various theories but the latter is capable of being measured. To determine the rate of closing of the pupil the motion picture camera was driven by a controlled motor so as to take a picture every tenth of a second. The subject was fixed in the head rest as before, and after being fully dark adapted so as to obtain a maximum pupil the camera was started, the taking lights turned on, and pictures were taken as the pupil closed from its maximum to the minimum diameter as chosen. The same six subjects were used in this experiment, as well as two others, and Fig. 3 and Table III show the results from the six subjects with the average in the heavy line. The average pupil reached its minimum in less than five seconds and the greater part of the contraction occurred within the first two seconds. Here again we see individual differences in actual values but similarity in shapes of the curves.

TABLE II.

Pupillary Diameters at Fixed Brightnesses.

Brightness in millilamberts	Log B	J.B.	C. B.	K. H.	F. J.	R. M.	R. W.	Average of 6 subjects
0	Taken as −6	7.4	8.7	7.0	7.8	8.3	8.7	8.0
0.00015	−3.8	7.1	8.4	6.8	7.6	8.1	7.9	7.6
.01	−2	6.7	8.0	6.7	7.0	7.1	6.3	7.0
1.0	0	5.0	5.9	4.8	5.2	4.6	4.5	5.0
10.	1.0	3.9	4.8	3.6	4.6	3.5	3.5	4.0
55.	1.7	3.0	3.5	3.1	3.1	3.0	2.9	3.1
100.	2	2.7	2.9	2.7	2.9	2.8	2.9	2.8
2000.	3.3	2.0	....	....	....	....	....	2.0

Fig. 3.


RATE OF CLOSING OF THE PUPIL.
G.B.
R.M.
R.W.
F.J.
J.B.
A.L.
K.H.
9
8
7
6
5
4
3
2
DIAMETER IN MM.
0
1
2
3
4
5
6
TIME IN SECONDS.

In order to get data on the rate of opening of the pupil it was necessary to make a rather elaborate electrical device which would turn off the lights for any given time, to allow the pupil to dilate, then start the motion picture camera and turn on the taking lights in time to photograph the pupil after the required time in darkness. A preliminary series was run on one subject in order to obtain the proper time intervals for opening and eleven intervals were chosen, ranging from one-half second to five minutes. The same six subjects were photographed and Fig. 4 and Table IV show the results obtained. The opening of the pupil to a maximum diameter requires as many minutes as it required seconds for it to close to the minimum. (The explanation of this difference would probably involve quite a philosophical discourse.)

TABLE III.

Rate of Closing of the Pupil.

Number of picture	Time in seconds	J. B.	C. B.	K. H.	F. J.	R. M.	R. W.	Average of 6 subjects
1	0	7.4	8.7	7.0	7.8	8.3	8.7	8.0
2	.1	7.2	8.6	6.9	7.7	8.2	8.5	7.8
3	.2	6.9	8.5	6.6	7.5	7.9	8.3	7.6
4	.3	6.7	8.0	6.4	7.2	7.5	7.4	7.2
5	.4	6.3	7.5	6.1	6.9	7.2	7.1	6.8
6	.5	6.0	7.0	5.8	6.0	6.9	6.8	6.4
11	1.0	4.7	5.7	4.9	5.4	5.4	5.8	5.3
16	1.5	3.9	4.8	4.0	4.6	4.6	4.7	4.4
21	2.0	3.3	4.3	3.6	4.1	4.0	4.0	3.9
26	2.5	3.0	4.0	3.2	3.7	3.7	3.7	3.5
31	3.0	2.9	3.8	3.1	3.5	3.3	3.5	3.3
36	3.5	2.8	3.6	3.0	3.4	3.3	3.2	3.2
41	4.0	2.8	3.7	3.1	3.3	3.1	3.2	3.2
46	4.5	2.8	3.7	3.0	3.2	3.1	3.1	3.1
51	5.0	2.8	....	....	....	....	3.1	....
	1 minute	2.8	3.4	3.1	3.1	3.1	3.1	3.1

As the pupil varies with changes in convergence, this factor was kept constant by having a constant fixation point 35 cm. directly in front of the subject. When working in the fainter intensities and darkness an illuminated pinhole was used, and it was kept at a just perceptible brightness for a few seconds before the photograph was to be taken, by the subject himself. The data plotted in all curves represent the average of at least four series on different days from each subject and as high as twenty series from some. The daily variation, however, was

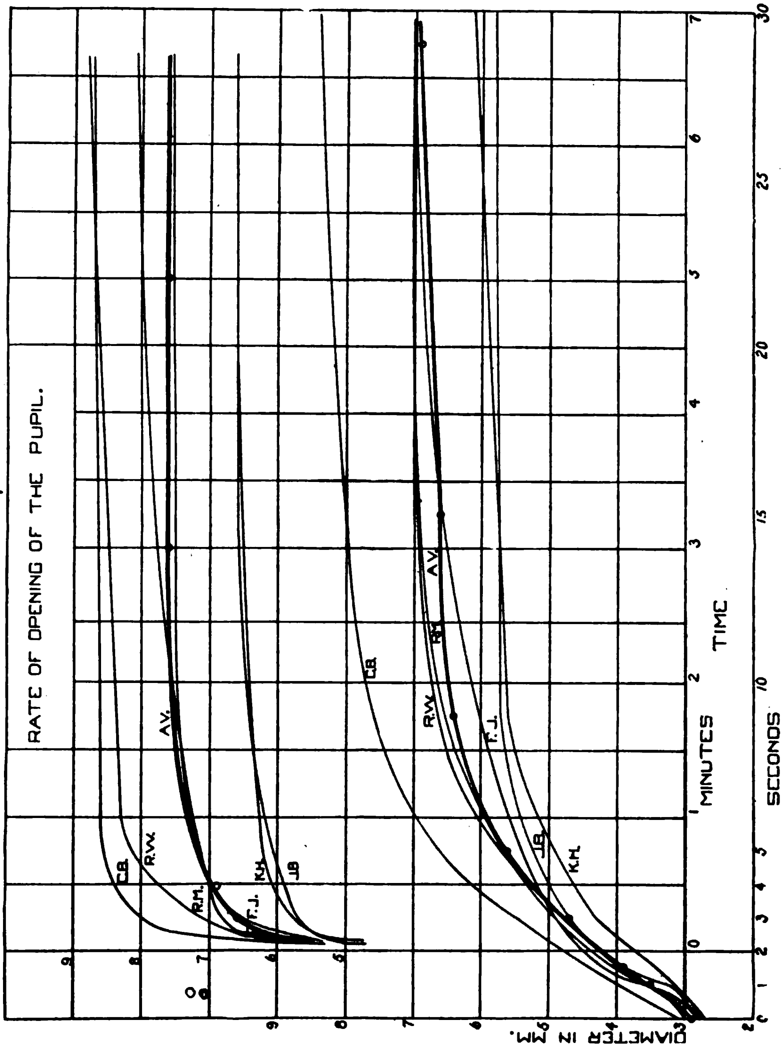

RATE OF OPENING OF THE PUPIL.
C.B.
R.W.
A.V.
R.M.
T.J.
K.H.
J.B.
DIAMETER IN MM.
TIME
MINUTES
SECONDS
0
1
2
3
4
5
6
7
10
15
20
25
30
9
8
7
6
5
4
3
2

Fig. 4.

slight. Age has been found to be a factor in pupillary reflex,[4] but none of the subjects were old enough to expect any age effects.

When considering the results on the opening and closing of the pupil, it must be remembered that these data were obtained from changing from one definite brightness to another, darkness and 100 millilamberts. We could expect different results if we used any other brightness range. The light sources used in the entire experiment were sunlight and tungsten filament lamps, so that we might expect other values if colored (especially monochromatic) light were used.

TABLE IV.

Rate of Opening of the Pupil.

Time in darkness	J. B.	C. B.	K. H.	F. J.	R. M.	R. W.	Average of 6 subjects
0	2.7	3.1	2.7	2.9	2.8	3.0	2.9
.5 seconds	3.0	3.5	2.9	3.1	3.0	3.2	3.1
1.0 seconds	3.2	4.0	3.2	3.6	3.6	3.7	3.5
1.5 seconds	3.9	4.4	3.4	4.2	3.9	3.8	3.9
3.0 seconds	4.7	5.3	4.3	4.7	5.0	4.4	4.7
5.0 seconds	5.3	6.6	4.9	5.4	5.8	5.7	5.6
9. seconds	5.7	7.6	5.6	6.0	6.6	6.7	6.4
15. seconds	5.7	8.0	5.5	6.6	6.9	6.9	6.6
29. seconds	5.8	8.4	6.1	7.0	7.0	7.0	6.9
1 minute	6.2	8.6	6.3	7.3	7.2	8.3	7.3
3 minutes	6.6	8.7	6.5	7.5	7.8	8.5	7.6
5 minutes	6.6	8.6	6.6	7.3	8.0	8.7	7.6
10 minutes	....	8.9	....	....	....	....	....

In experiments where it is essential to know the light energy striking the retina, the diameter of the pupil is an important item. In choosing artificial pupils we should know at least the minimal working pupil. The designer of optical instruments is very directly interested in pupillary dimensions. The ophthalmologist often determines the diameter of the patient's pupils and examines the pupillary reflexes. In many pathological cases one of the earliest symptoms to appear is a change in the pupillary reflex.

This study has merely touched a very interesting field and valuable information will be found in the treatment of other combinations of brightnesses and also in the use of colored light.

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[4] Kanngieser, F., *Arch. f. Augenhk.*, 1909, 63, 78; Langfeld, *op. cit.*

A NEW SOFT FOCUS SCREEN FOR PORTRAIT PHOTOGRAPHY.

BY

C. W. FREDERICK.

In portrait work it is not always desirable that a lens should give sharp definition because marks and blemishes are brought out too clearly. Lines and similar defects must be smoothed out to resemble more nearly what the casual eye sees under ordinary white light. This is usually done by retouching the negative, or by employing a lens with soft definition, or both. Soft definition may be obtained in many ways, but its character makes a great difference in the appearance of a picture. It may be produced by merely throwing the lens out of focus, but the effect is not pleasing because the definition is obliterated too completely. We want something that will leave the main features of the picture clearly sketched and at the same time smooth out or obliterate unimportant or disfiguring details.

The usual method of producing soft definition is to modify the design of the lens so that it will fall off in spherical aberration. Thus the image formed by light passing through an outer zone of the lens will not be at the same distance as that formed by light passing through the central portion of the lens, and intermediate images will be formed at intermediate points, so within a certain range a part of the light will form a sharp image and the remainder a series of superposed fuzzy images. The effect aimed at is definition with a mellow outline.

It may be readily understood that the character of the effect will depend upon the distribution of light in the sharp and the fuzzy images, which distribution may be influenced by lens type and different adjustments as to spherical aberration. But the nature of spherical aberration places certain limitations upon the problem. We can not, for example, adjust the aberration of a lens so that three-fourths of the light will form a sharp image and the remaining one-fourth a diffuse image; thus we are cut off from many effects that may be desirable. It usually happens that there is too much light in the diffuse images, so that double lines appear in certain portions of the picture where there is a great change from white to black, as a white collar against a black coat, or a well-lighted cheek against a dark background.

The natural distribution of light due to spherical aberration may be modified by interposing an opaque diaphragm with a small central opening to allow the sharp image to be formed and radial openings to control the amount of light in the diffuse image. But such a device has the disadvantage of slowing down the lens to an inconvenient degree. However, it was a study of the action of such a diaphragm that suggested the new diffusing screen which is the subject of this paper.

The necessity for a means of diffusion independent of the lens arose in connection with a new enlarging camera which was being developed. This camera was fitted with a device which kept the lens automatically in focus as the enlargement was varied. It was to be used in enlarging portraits, and it was necessary that both sharp definition and soft definition could be obtained at will. It would have been an easy matter to design a mount for the lens which by unscrewing one component would alter the relative positions of the lens elements and introduce spherical aberration. But this would cause a change in the focal length of the lens, and any change in this would prevent its proper functioning. Thus it was necessary to find something independent of the lens.

What at first appeared to be a hardship led eventually to emancipation. Many things were thought of and tried. Diffraction was invoked and rejected. Semitransparent material such as cloth and ground glass were tried but found to slow down the lens too much. Glass slides, coated with transparent varnish giving an undulatory surface, failed to prove satisfactory because the diffusion was not controlled, some of the light being scattered so far that the entire picture was flattened.

We were finally led to try glass discs with fine grooves polished in them. If the grooves were accurately cut the diffusion would be controlled so that no ray of light could deviate more than a limited angle, a few minutes, from its undisturbed path, and thus the picture would not be flattened by stray light. Of course the smoothing out of fine detail will give a certain appearance of flattening which can not be avoided, but this is not noticeable in portrait photography. The discs were placed directly in front of the lens, and being entirely transparent caused no slowing down of the combination.

A great many experiments were tried with different devices for cutting the grooves, different depths and widths of grooves, and different patterns cut upon the disc. The possibilities were endless, and each change had an individuality of its own in its effect upon a photograph.

First we tried concentric circular grooves cut upon the outer portion of the glass disc, leaving the central portion clear for about half the aperture of the lens. The light passing through the central portion of the screen would give a sharp definition, and that passing through the outer portion of the screen diffuse definition. Then by merely stopping the lens the amount of light in the diffuse image could be reduced, and the relative proportion of light in the sharp and diffuse images altered at will. But this screen proved unsatisfactory because it was found to give double lines along boundaries of strong contrast, there being too much light in the rays of maximum deviation. Also the circular grooves were hard to cut.

To get around these difficulties we changed to cylindrical grooves ground in intersecting linear patterns. It was expected the intersecting grooves would diminish the area of maximum slopes, and thus reduce the amount of light of maximum diffusion. But this, too, was a failure, as it was found the successive systems of grooves simply superposed themselves upon the glass surface whether flat or undulating. At the intersection of two grooves the glass was abraded to twice the depth for one groove, and no advantage was obtained.

The grooves could be readily examined by means of interference fringes. When the ground surface of one disc was placed in contact with the unground surface of another, interference bands would appear, giving a very beautiful contour map of the whole system of cuts. When seen under a mercury lamp these contour lines became very sharp indeed, making it easily possible to estimate the depth of the cuts to one or to two millionths of an inch.

Finally, after many experiments, a screen was settled upon in which there were three systems of linear grooves ground at angles of one hundred and twenty degrees from each other, and the grooves of each system were spaced at equal intervals apart but were progressively of increasing depth from the centre outward. The finished pattern showed hexagonal areas of clear glass constituting about two-thirds of the area of the entire screen. Thus two-thirds of the light was allowed to form a sharp image, and the remaining one-third a succession of soft images varying in diffuseness according to the depth of the successive grooves of the screen. The screen gives soft effects without a trace of double lines, and at the same time it gives definition that does not fail under a magnifier. Its effect may be slightly modified by stopping down the lens.

The results obtained with this screen were very good indeed and distinctly better than anything that could be done with a soft focus lens. In fact, they were so promising that we hastened to apply for a patent upon the idea.*

The screens are capable of many unusual effects. One screen was ground with a single system of parallel grooves, which when used with the diffusion vertical in a portrait had a tendency to make the face appear longer, or if the diffusion were horizontal the face would appear broader. In landscapes it gave similar effects. If the diffusion were vertical the corresponding detail, such as grass or tree trunks, would remain sharp in the picture, while horizontal detail as limbs and twigs of trees would be fuzzed out. Thus it is possible that with proper care effects could be obtained similar to those seen in the paintings of Corot.

A rectangular system of grooves gives a very satisfactory screen for landscapes, and also for portraits. If placed with the diffusion at about forty-five degrees the detail of a landscape will be streaked out in a way suggestive of etching. It is possible that a rectangular system with one set of grooves deeper than the other would be found useful.

One screen of the hexagonal type was ground with one system of grooves strong and the other two weak, but we have not had time to give it a serious test. It might be used to alter the length of a face, or might be turned so as to bring out better detail in hair, or to alter the appearance of cloth in a dress or coat. A satin dress should preserve its peculiar sheen much better if the greater diffusion were directed parallel to the grain of the cloth.

A fundamental peculiarity of all the screens consisting of systems of straight lines is that the diffusion occurs at right angles to the various systems, and does not occur equally in all directions. If a small hole be pricked in a negative its image in the enlargement will be seen to consist of a point with short radial streamers extending out from it in a star-like pattern. This is probably beneficial in helping to preserve the contrast between the blacks and whites in fine detail, as one set of streamers is likely to coincide closely with the direction of the detail, thus adding to the already preponderant light in the sharp image, while subtracting nothing from the softening effect upon the picture.

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* [EDITOR'S NOTE: Unfortunately, no known printing process can reproduce the striking and beautiful effects shown by the pictures submitted by Mr. Frederick in illustration of the work done by means of this new diffusing screen. These include a landscape and a dozen or more large portraits.]

While most of our efforts have been directed toward developing a diffusing screen for enlarging cameras, it is possible that they may be useful also in direct portrait photography. A few large screens for this purpose were ground at haphazard, and tried in a tentative way. The results seemed to indicate that the problem of direct taking was somewhat different from that of enlarging, and would require a different type of screen. In direct taking the screen causes the white areas to encroach upon the black, producing an effect similar to halation, while in the case of enlarging, the black areas encroach upon the white and do not produce the effect of halation. Thus in direct photography a white collar will irradiate upon a black coat and spoil the appearance of the picture, while in an enlargement the black coat will encroach upon the white collar with no unpleasant effect. However, by limiting the diffusion and modifying its distribution it is quite likely that a satisfactory screen may be developed for direct photography. At least the action of the screen in this case is similar to that of the soft focus lenses now in use, and the greater adaptability of the screen should be in its favor.

HAWK EYE WORKS, EASTMAN KODAK COMPANY,
ROCHESTER, N. Y., Feb., 1920.

THE IONIZATION AND RESONANCE POTENTIALS OF NITROGEN, OXYGEN AND HYDROGEN.*

BY

F. L. MOHLER and PAUL D. FOOTE.

THE methods[1] used by the authors in the measurement of critical potentials for electron currents in metallic and non-metallic vapors have been applied, with some modifications, to a study of several gases. A four-electrode tube was employed containing a hot-wire cathode surrounded by two cylindrical grids and a plate. One grid was mounted close to the cathode and the other close to the plate, with a relatively large distance between grids. The electrodes were of platinum sheet and gauze. The tube was connected to pumps and to a gas reservoir so that the gas could be either streamed through the tube or maintained stagnant.

Of the several possible methods for applying potential and for measuring current, the following was found the most satisfactory. A variable accelerating field was applied between the cathode and inner grid; the two grids were maintained at the same potential; a retarding field was fixed between the outer grid and plate; and the current reaching the plate was observed. Thus, if the cathode were at zero potential, the grids were at $+V_1$ and the plate at $V_1 - V_2$.

When V_2 is small, measurement of the electron current reaching the plate furnishes a sensitive indication of potentials of inelastic impact. If V_2 is greater than V_1, so that electrons from the cathode are unable to reach the plate, positive current flows to the plate, above the ionization potential. This is the Lenard method for detecting ionization. Davis and Goucher[2] have shown that the radiation emitted at the resonance potential produces a photo-electric emission from the plate, giving rise to a current easily mistaken for ionization. As these investigators have demonstrated, it is possible to so connect a four-electrode tube as to differentiate these two effects. We have found that while the ionization effect was of the same magnitude as the partial electron current when V_2 was small, the radiation effect was very much less.

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* Abstract.

[1] Tate and Foote, *Phil. Mag.*, 36, p 64, 1918; Bur. Stds. S 317; Foote and Mohler, *Phil. Mag.*, 37, p 33, 1919; *J. Wash. Acad. Sci.*, 8, p 513, 1918; Foote, Rognley and Mohler, *Phys. R.*, 13, p 59, 1919; Mohler, Foote and Stimson, Bur. Std. S 368; Mohler and Foote, Abstract of Non-metallic Vapors, *Phys. R.* (in press).

[2] Davis and Goucher, *Phys. R.*, 10, p 101, 1917.

The usual procedure, as in the earlier work, was to determine the resonance potential from the applied potentials at successive inflections in the curve, plate current *versus* accelerating field, and to add to the observed potential, at ionization, the value of the initial potential determined from the partial current curve.

NITROGEN.

The work of Franck and Hertz gave 7.5 volts for the ionization potential but Davis and Goucher[3] showed that radiation without ionization was emitted at 7.5 volts and that the true ionization potential was about 18 volts. Smyth,[4] using a three-electrode tube and the Lenard method, observed the resonance potential as 8.29 volts and confirmed Davis and Goucher's value for ionization.

In our work, commercial nitrogen purified by hot copper, calcium chloride, and liquid air traps was employed. Plate currents with V_2 small showed as many as three sharp inflections due to successive resonance collisions. By the Lenard method the ionization point as well as the radiation effect at the resonance potential were readily detected. Ionization was measured by three methods:

1. Measuring the potential at which the total electron current increased sharply and correcting for initial potential determined from a partial current curve showing resonance points.
2. Measuring by the Lenard method, using the same galvanometer sensitivity as with partial current curves and correcting as before.
3. Measuring by the Lenard method, with high sensitivity, the potential difference between the occurrence of the radiation and ionization effects.

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[3] Davis and Goucher, *Phys. R.*, 13, p 1, 1919.

[4] Smyth, *Phys. R.*, 14, p 409, 1919.

The three methods were in close agreement, giving the final mean as follows:

Resonance potential = 8.18 ± 0.1 volts.
Ionization potential = 16.9 ± 0.5 volts.

Smyth's conclusion that the principal resonance potential corresponds, by the quantum relation, to the nitrogen doublet, observed by Lyman $\lambda = 1492.8$ A and $\lambda = 1494.8$ A is confirmed by our work. The theoretical value is $V = 8.26$ volts in excellent agreement with our observed value of 8.18 volts. Accordingly it appears likely that this line is the first term of a series converging at $\lambda = 730$ A (equivalent to 16.9 volts).

OXYGEN.

Other observers using the Lenard method have obtained results for the ionization potential of oxygen as follows:

Franck and Hertz	9 volts
Mayer	9.5 volts
Hughes and Dixon	9.2 volts
Bishop	9 volts

However, it is to be suspected that the above values are for the resonance potential and that the ionization potential is considerably higher.

Oxygen was made electrolytically and dried as in the work with nitrogen. Plate currents showed the inflections due to successive resonance collisions very distinctly, as many as four reversals of nearly equal sharpness being readily obtained. Ionization was measured by the Lenard method with the correction for initial potential determined from the partial current curves. The radiation method did not give results of much value, suggesting that platinum in oxygen may be less photo-sensitive than in nitrogen.

The following final results were obtained for oxygen:

Resonance potential = 7.91 ± 0.1 volts
Ionization potential = 15.5 ± 0.5 volts

These values indicate that $\lambda = 1570$A is the single line spectrum of nitrogen and the first member of a series converging at $\lambda = 800$ A.

HYDROGEN.

Since the development of the Bohr theory the study of hydrogen is of especial interest. From this theory it would follow that the ionization potential for a hydrogen atom should be determined by the limit of the Lyman series, *i.e.*, the Rydberg number, giving $V = 13.5$ volts and that the resonance potential should be three-fourths of this value, corresponding to an electron displacement from the first to the second orbit. The theory for the normal hydrogen molecule is not so definite. It seems quite likely, however, that the ionization potential for the molecule should be greater than that for the atom by an amount equivalent to the work required to dissociate a molecule. Thus the simplest ionization of a molecule would result in a neutral atom and an ionized atom.

The work of numerous investigators has led to the value 11 volts for ionization, thereby raising a very serious difficulty in the acceptance of Bohr's theory. In addition to the ionization at 11 volts, Davis and Goucher observed ionization at 15.8 volts and inelastic collision *without* ionization at 13.6 volts.

Since our work described below was completed, a paper by Horton and Davies [5] has appeared in which they have arrived at conclusions very similar to ours, and it now seems certain that Bohr's theory of atomic hydrogen satisfactorily interprets the results. Less conclusive is the evidence for molecular hydrogen, since the heat of dissociation amounts to but 2.7 volts on Bohr's theory and to 3.9 volts as experimentally observed by Langmuir,[6] the latter value being frequently cited as an objection to Bohr's hypothesis.

On Bohr's theory there may exist at least three critical potentials in the range 10 to 17 volts, which if present would be quite difficult to accurately isolate and measure.

The hydrogen was generated electrolytically and streamed through hot copper, drying tubes and liquid air traps. Measurements by the Davis and Goucher method showed radiation without ionization near 10 volts, some ionization near 13 volts and strong ionization near 16 volts. Having shown that the 10-volt inelastic collision resulted in radiation without ionization, and that there were two ionization potentials, more accurate measurements were made by the methods employed with the other gases.

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[5] *Proc. Roy. Soc. Lond.*, March, 1920.

[6] *J. Am. Chem. Soc.*, 37, p 457, 1915, 90,000 cal. per g. mol. at constant pressure.

Partial current curves were obtained with two reversals, indicating successive resonance collisions. Often a pronounced break in the curves was occasioned by the second type of ionization. From these curves both the resonance potential and the second ionization potential may be obtained. The Lenard method showed both ionization points sharply. The following is a summary of our results:

Resonance potential = 10.4 volts.
First ionization potential = 13.3 volts.
Second ionization potential = 16.5 volts.

Without doubt the first ionization potential is for the atom and the second for the molecule. Hence the difference should give the energy required to dissociate hydrogen. This difference of 3.2 volts corresponds to a heat of dissociation per gram molecule at constant pressure of 74,000 calories, whereas Langmuir's observed value is 90,000 calories and the value derived from Bohr's theory of the molecule is 65,000 calories. The second ionization point may be observed with far greater precision than the first. Hence, if we assume the first point as 13.5 volts we obtain $16.5 - 13.5 = 3.0$ volts, or 69,000 calories, in excellent agreement with the value computed from Bohr's theory of molecular hydrogen.

We have observed a slight evidence of a resonance collision between 2.5 and 3.5 volts, but have not been able as yet to secure curves with sufficiently pronounced inflections to permit the measurement of this voltage carefully. The work, however, indicates that hydrogen may be dissociated without ionization. This fact would not follow from the presence in the tube of monatomic hydrogen, as indicated by the resonance collisions, since the hydrogen is rapidly dissociated at the temperature at which the hot wire was operated. No doubt, even if hydrogen is dissociated without ionization by low voltage electronic impact, the greater part of the monatomic gas in the ionization tube is produced by the catalytic action of the heated filament.

The following table summarizes our data on the three gases:

Gas	Resonance potential		Ionization potential	
	Observed	Theoretical	Observed	Theoretical
Nitrogen	8.18	8.26	16.9	....
Oxygen	7.91	....	15.5	....
H_2	....	....	16.5	16.2
H	10.4	10.1	13.3	13.5

Dissociation of H_2

$16.5 - 13.3 = 3.2$ volts

$16.5 - 13.5 = 3.0$

Langmuir (experimental) $= 3.9$

Bohr (theoretical) $= 2.7$

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
March 19, 1920.

1919 REPORT OF STANDARDS COMMITTEE ON VISUAL SENSITOMETRY.

BY

P. G. NUTTING, Chairman.

PRELIMINARY REPORT ON DATA.

VISION is accomplished through the influence of certain kinds of radiation upon the retina. The various responses of the retina to radiation correspond with those of a number of different kinds of physical instruments, each giving its particular kind of scale reading and each scale reading representing a certain sensibility or ratio of increment of scale reading to increment of stimulus producing it. The practical usefulness of our eyes is a matter of the sensibility and range of response of our retina to the stimulus of light in its various forms.

The quantitative study of the response of the retina to light practically began with the work of A. König twenty-five years ago. Subsequent advances in our knowledge of radiation made much more precise work possible and during the past fifteen years the investigation of the various responses of the retina to radiation has been vigorously prosecuted. At the present time, preliminary surveys of all the more important fields have been made, but much remains to be done in determining the properties of the average eye by observations on large numbers of subjects and in working out the numerous applications of this information to problems in photometry, illumination, colorimetry, etc., and to the design of optical instruments. This summary of our present knowledge of the subject covers: I. The Visibility of Radiation. II. Intensity and Contrast Sensibility. III. Chromatic Sensibility. IV. Rates of Adaptation. V. Absolute Sensibility.

I. THE VISIBILITY OF RADIATION.

The amount of radiant energy (measured in watts) necessary to produce a fixed luminous sensation of brightness (in lumens or lamberts), varies enormously with the wave length of the radiation. It is least in the mid-green, about .0015-watt lumen, at a wave length of about 556 millimicrons. The ratio is ten times as large in the blue-green on the one hand and in the red on the other hand, and it is ten thousand times as large in the extreme violet (wave length 400) and extreme red (750). An eye well dark adapted perceives a light sensation from radiation at least as far out as 330 in the ultra-violet and 900 in the extreme red, if sufficiently intense.

Various methods have been employed by numerous observers in determining the relative visibility of radiation. A number of the earlier observers [1] measured the relative energies necessary to produce the threshold of light sensation. Another group [2] of observers used as a fixed standard of luminosity, that just sufficient to permit the correct reading of tables of numbers. König [3] used a white comparison field of fixed known brightness, that brightness varying in nine different sets of observations from the threshold of vision up to about 600-metre candles. Later observers [4] avoided the direct comparison of white with colored fields by using the flicker photometer for this purpose. Allen [5] obtains a visibility curve with no comparison field at all by simply noting the critical frequencies at which flicker just disappears at the various wave lengths. Hyde [6] has recently obtained excellent visibility curves by the step-by-step method, the brightness of each part of the spectrum being compared, not with a white standard but with a neighboring portion of the spectrum. The relative merits of these various methods have yet to be carefully investigated. There has been no lack of normal subjects; Ives had 19, Nutting 21, Hyde 29 and Coblentz and Emerson 125. The spectral energy of the acetylene flame, upon which most of the best data are based, is fairly well known, though several times revised. Improvements are to be looked for in the use of carefully studied sources, larger spectral apparatus of higher dispersion and a carefully chosen method of observation and of reduction of observations. All the modern data refer to visibility at moderate and high intensities (cone vision?). König's investigation of visibility at low intensities (rod vision?) should be repeated with modern apparatus and methods.

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[1] König and Dieterici, *Zs Psy. Sinnesorgans,* **4,** 241–347, 1893; A. Pflüger; *Ann. Physik.,* **9,** 185, 1902.

[2] S. P. Langley, *Am. Jour. Sci.,* **36,** 359, 1888.

[3] A. König, *Ges. Abh. Leipzig,* 1903.

[4] H. E. Ives, *Phil. Mag.,* **24,** 149, 1912; *Phys. Rev.,* **6,** 329, 1915; P. G. Nutting, *Trans. Ill. Eng. Soc.,* **9,** 633, 1914; Coblentz and Emerson, *Bull. Bu. Standards,* **14,** 167, 1918.

[5] F. Allen, *Phys. Rev.,* **9,** 257, 1900; *Phil. Mag.,* **38,** 85, 1919.

[6] E. P. Hyde, *Astroph. Jour.,* **48,** 87, 1918.

König's early data on relative visibility at various intensities, as reduced by the writer,[7] are given in the following table. There exists considerable uncertainty in these data due to uncertainties in (1) the correction for the dispersion in the prism employed and in (2) the spectral energy of the source (a gas flame). This latter determination was made for König by Langley in Washington on a somewhat similar gas flame.

König's Data on Visibility.

Intensities (m.c./mm^2 pupil) Ratio to preceding step	0.00024 (threshold)	.00225 9.38	.0360 16	.575 16	2.30 4	9.22 4	36.9 4	147.6 4	590.4 4
Wave length	Visibility								
430	0.081	0.093	0.127	0.128	0.114	0.114	....	....	....
450	.33	.30	.29	.31	.23	.175	.16	....	....
470	.63	.59	.54	.58	.51	.29	.26	.23	....
490	.96	(.89)	(.76)	(.89)	(.83)	.50	.45	.38	.35
505	1.00	1.00	1.00	1.00	.99	(.76)	.66	.61	.54
520	0.88	0.86	0.86	0.94	0.99	(.85)	0.85	0.85	0.82
535	.61	.62	.63	.72	.91	(.98)	.98	.99	.98
555	.26	.30	.34	.41	.62	.84	.93	.97	.98
575	.074	.102	.122	.168	(.39)	(.63)	(.76)	(.82)	(.84)
590	.025	.034	.054	.091	.27	.49	.61	.68	.69
605	0.008	0.012	0.024	0.056	0.173	0.35	(.45)	0.54	0.55
625	.004	.004	.011	.627	.098	.20	.27	.35	.35
650	.000	.000	.003	.007	.025	.060	.085	.122	.133
670	.000	.000	.001	.002	.007	.017	.025	.030	.030
Maximum....	.503	.504	.504	.508	.513	.530	.541	.543	.544

The shift of the visibility curve toward longer wave lengths with increasing intensities (the basis of the Purkinje effect) is clearly shown. The threshold visibility curve is reproduced in the accompanying figure.

In the following table are summarized the best data on the visibility of radiation of moderate and high intensities (cone vision). Absolute values of visibility or ratio of light to energy, in lumens per watt, are given in the final column. These are calculated from the visibilities on the basis of a mechanical equivalent of 0.0015 watt per lumen or a luminous equivalent of 667 lumens per watt for maximum visibility (at wave length 556).

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[7] P. G. Nutting, *Bull. Bu. Standards*, 7, 235, 1910.

Data on Visibility.

(Ordinary Cone Vision.)

Wave lengths	Ives and Kingsbury	Nutting	Coblentz Emerson	Hyde, Cady, Forsythe	Adopted mean I. E. S.	Absolute visibility
400	0.0003	0.0021	0.006	0.00009	0.0004	0.27
10	.001	.0036	.011	.00062	.0012	0.80
20	.005	.0065	.015	.0041	.0040	2.7
30	.012	.0115	.020	.0115	.0116	7.72
40	.024	.022	.029	.022	.023	15.3
450	0.040	0.038	0.041	0.036	0.038	25.3
60	.060	.064	.058	.055	.060	40.0
70	.090	.101	.086	.087	.091	60.7
80	.137	.149	.128	.138	.139	92.6
90	.211	.215	.194	.216	.208	139
500	0.318	0.314	0.320	0.328	0.323	216
10	.473	.456	.503	.515	.484	322
20	.637	.646	.710	.698	.670	446
30	.801	.815	.862	.847	.836	557
40	.915	.925	.954	.968	.942	629
550	0.988	0.986	0.997	0.996	0.993	662
60	.996	.995	.998	.995	.996	663
70	.947	.949	.968	.944	.952	641
80	.859	.871	.898	.855	.870	580
90	.758	.762	.800	.735	.757	502
600	0.653	0.634	0.685	0.600	0.631	421
10	.534	.498	.555	.464	.503	336
20	.396	.368	.425	.341	.380	253
30	.283	.268	.305	.238	.262	175
40	.183	.166	.195	.154	.170	113
650	0.110	0.105	0.114	0.094	0.103	68.8
60	.068	.058	.062	.051	.059	39.3
70	.039	.032	.032	.026	.030	20.0
80	.026	.016	.0162	.0125	.016	10.7
90	.014	.0081	.0080	.0062	.0081	5.4
700	0.007	0.0036	0.0038	0.0031	0.0041	2.7
10	......	......	.00190	.0015	.0021	1.4
20	......	......	.00092	.00075	.0010	0.67
30	......	......	.00045	.00036	.00052	0.35
40	......	......	.00024	.00018	.00025	0.17
750	......	......	0.00015	0.00009	0.00012	0.08
60	......	......	......	.00005	.00006	0.04
Wave length of maximum .	556	556	557	556	556	

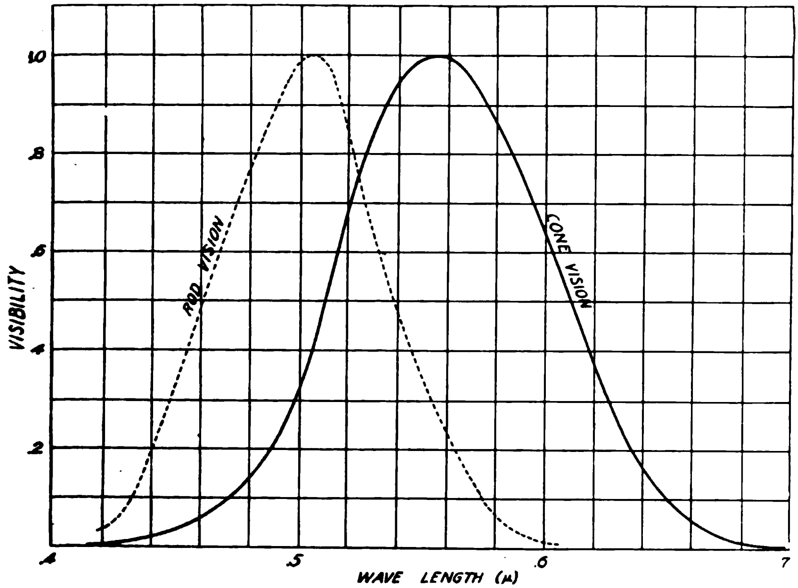
The visibility curve is roughly in the form of the well-known probability curve but slightly steeper on the violet side. It may be represented by the probability curve, but for mathematical purposes (converting radiant energy into light), it is more convenient to use the form

$$V = V_m R^a e^a (1-R) \text{ or}$$
$$\log(V/V_m) = a(\log R + 1-R)$$

in which $R = 556/\text{wave length}$, and $V_m = 667$ lumens/watt.

The absolute value of the visibility constant V_m in lumens per watt has been determined by a number of investigators. The method used is to measure some convenient illumination both as light and as energy. The monochromatic green mercury line 546.1 has been used, also the nearly white light from an acetylene flame or incandescent lamp (integrated) and filtered white light. Even the best determinations are not in very good agreement, ranging from 625 to 667 lumens per watt. The corresponding value, the ratio of the lumen to the watt for rod vision, is yet to be determined.

FIG. 1.



Visibility of radiation.

A knowledge of the visibility of radiation, as determined by the reaction of the average normal fovea to radiation, is of the utmost importance in many branches of optics and illuminating engineering. The data given above are essential in reducing radiant energy to light, for example, and in calculating the brightness of a heated body at a known temperature. It would be quite feasible to establish a standard of light in terms of radiant energy through similar data more accurately determined. In optical pyrometry and other problems it is necessary to calculate by means of the visibility curve the amount of light passed by a given light filter. It is to be hoped that the problems connected with the precise determination of visibilty may be attacked even more vigorously in the future.

II. COLOR SENSIBILITY.

The human retina is sensitive not only to differences in brightness, but to differences in the quality of that brightness, due essentially to the composition of the radiant energy reaching it. The differences in quality of the radiant energy are those of wave frequency or wave length. In the preceding paragraphs have been discussed the range of wave lengths to which the eye is sensitive; color discrimination depends upon the ability to distinguish between lights of different dominant wave lengths. It is, of course, impossible to set down the precise sensation corresponding to each wave length, but we can readily determine sensibility to differences in wave length and integrate this sensibility into a continuous color scale. Finally, the pure hue may be mixed with either white or black, and here again sensibility to differences may be observed and integrated into scales of tint or shade. Any one color may be specified by three and but three coördinates, namely (1) wave length of *dominant hue*, (2) *saturation* as regards admixture of white, and (3) *intensity* represented in the case of self-luminous bodies by emissivity and in other bodies by either reflecting or transmitting power for light.

Sensibility to differences in wave length may be readily determined by means of a modified simple spectometer having two collimators separately movable. While one is held fixed, the other is moved toward either longer or shorter wave lengths (usually each alternately), until the color difference is just noticeable. Then the movable one is held fixed while the other is loosened and moved to give a just noticeable difference in color. A good bright spectrum stepped off in this way gives from about 130 to 180 distinctly different pure hues between the red and violet, while from violet back to red through the purples and magentas there are about 20 additional steps as determined by their complementaries in the green. The variation of color sensation with wave length is by no means uniform, as may be seen by a glance at a spectrum. In two regions, namely in the blue-green and in the yellow, hue changes very rapidly with wave length, while in the middle of the green and red, hue changes very slowly with wave length. Beside these two pronounced maxima of sensibility at wave lengths of about 490 (blue-green) and 585 (yellow), there are lesser maxima at about 637 in the red and 440 in the blue-violet.

The first reliable data of this nature were published by Steindler[8] in 1906 and reduced and published by the writer.[9] Steindler had 12 subjects, all with normal color vision. Curves were averaged not by taking the average sensibility at each wave length, but by finding the average wave lengths of each maximum and minimum of sensibility, since averaging the sensibilities would have given a mean curve flatter than any individual curve.

More recently this work has been repeated by Mr. L. A. Jones and the writer, using several forms of special spectrometer for the purpose. Our data are in very close agreement with those of Steindler taken 10 years previously in Vienna. Mr. Jones further made determinations[10] with his left eye as well as right eye, showing that the two eyes were practically the same in color sensibility. These data are summarized in the following table and figure:

Hue Sensibility.

	A		*B*		*C*		*D*		*E*		*F*	
	λ	$d\lambda$	λ	$d\lambda$	λ	$d\lambda$	λ	$d\lambda$	λ	$d\lambda$	λ	$d\lambda$
At $\lambda=660$........	660	6.0	660	5.2	660	5.0	660	...	660	5.4	660	6.0
First minimum...	635	2.8	638	2.4	637	2.4	635	3.1	637	2.5	637	3.0
First maximum...	622	3.4	628	3.5	623	3.2	620	3.7	624	3.4	621	3.7
Second minimum..	584	1.4	585	1.2	586	1.1	584	1.4	585	1.2	580	1.4
Second maximum.	538	3.5	540	3.3	537	3.2	538	3.0	538	3.3	535	3.3
Third minimum...	496	1.2	490	1.1	495	1.1	496	1.0	494	1.1	492	1.3
Third maximum..	462	2.3	465	2.4	464	2.5	465	1.9	464	2.4	458	3.1
Fourth minimum..	442	1.8	443	1.8	445	1.8	454	1.5	443	1.8	440	2.4
At $\lambda=410$........	410	4.0	410	4.6	410	4.0	410	5.0	410	4.2	440	5.5

Column *A*.....First determination with right eye (discontinuous)
Column *B*.....Second determination with right eye (discontinuous)
Column *C*.....Third determination with right eye (continuous)
Column *D*.....First determination with left eye (discontinuous)
Column *E*.....Mean values of *A*, *B* and *C*.
Column *F*.....Mean values from Steindler data (12 subjects)

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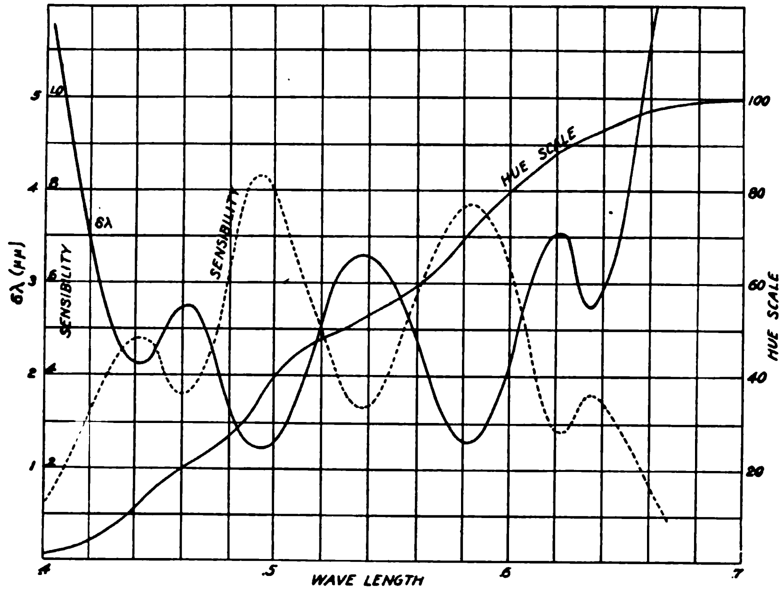
[8] O. Steindler, *Sitz. Ak. Wiss. Wien,* 115, 2A, Jan., 1906.

[9] P. G. Nutting, *Bull. Bu. Standards,* 6, 89, 1909.

[10] L. A. Jones, *Jour. Opt. Soc. Am.,* 1, 63, 1917.

The hue sensibility is taken as the reciprocal of the just noticeable difference in wave length. The hue scale is the integral of the sensibility with a factor to reduce the total range to a convenient number of units, 100 in this case. The scale obtained by Jones[11] by stepping off equal hue differences through the spectrum differs scarcely perceptibly from the scale obtained by integration. His data are given in the table on page 63.

FIG. 2.



Hue limen, sensibility and scale.

The amount of white that may be mixed with a pure hue before it becomes noticeable has been determined by the writer and L. A. Jones. Data are given in the following table:

Per cent. White	0,	10,	20,	30,	40,	50,	60,	70,	80,	90,	100
Per cent. Hue	100,	90,	80,	70,	60,	50,	40,	30,	20,	10,	0
Impurity noticeable	4.7,	4.6,	4.5,	4.4,	4.2,	4.0,	3.7,	3.4,	3.0,	2.5,	2.1

These data apply to red, green and blue; other hues have not been investigated. They apply to all moderate and high intensities

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[11] Jones, *loc. cit.*

Just Noticeable Hue Differences.

No.	λ	δλ	No.	λ	δλ	No.	λ	δλ
1	700.0	....	44	576.5	1.4	87	490.4	1.1
2	678.0	22.0	45	75.2	1.3	88	89.4	1.0
3	65.0	13.0	46	73.7	1.5	89	88.2	1.2
4	59.0	6.0	47	71.7	2.0	90	87.0	1.2
5	54.0	5.0	48	70.1	1.8	91	85.8	1.2
6	49.5	4.5	49	68.4	1.7	92	84.5	1.3
7	46.0	3.5	50	66.6	1.8	93	83.2	1.3
8	42.8	3.2	51	64.8	1.8	94	81.7	1.5
9	40.2	2.6	52	63.0	1.8	95	80.0	1.7
10	37.8	2.4	53	61.1	1.9	96	78.2	1.8
11	35.5	2.3	54	58.6	2.5	97	76.5	1.7
12	33.1	2.4	55	57.0	2.6	98	75.0	1.5
13	30.0	3.1	56	54.4	2.6	99	72.9	2.1
14	26.5	3.5	57	51.8	1.6	100	70.5	2.4
15	23.0	3.5	58	49.1	2.7	101	68.2	2.3
16	20.0	3.0	59	46.1	3.0	102	65.8	2.4
17	17.3	2.7	60	43.0	3.1	103	63.6	2.2
18	14.9	2.4	61	39.8	3.2	104	61.2	2.4
19	12.5	2.4	62	36.5	3.4	105	58.7	2.5
20	10.2	2.3	63	33.2	3.3	106	56.5	2.2
21	08.0	2.2	64	30.1	3.1	107	54.4	2.3
22	06.0	2.0	65	27.1	3.0	108	52.1	2.3
23	04.1	1.9	66	24.2	2.9	109	50.0	2.1
24	02.3	1.8	67	21.4	2.8	110	48.0	2.0
25	600.6	1.6	68	19.1	2.3	111	46.0	2.0
26	599.0	1.6	69	16.8	2.3	112	44.2	1.8
27	97.4	1.6	70	14.6	2.2	113	42.5	1.7
28	95.9	1.5	71	12.6	2.0	114	40.8	1.7
29	94.5	1.4	72	10.6	2.0	115	39.0	1.8
30	93.1	1.4	73	08.0	1.6	116	37.2	1.8
31	91.8	1.3	74	07.0	1.0	117	35.3	1.9
32	90.5	1.3	75	05.4	1.6	118	33.3	2.0
33	89.5	1.0	76	04.0	1.4	119	31.3	2.0
34	88.5	1.0	77	02.6	1.4	120	29.3	2.1
35	87.5	1.0	78	01.3	1.3	121	27.0	2.2
36	86.4	1.1	79	500.0	1.3	122	24.8	3.2
37	85.3	1.1	80	498.7	1.3	123	22.3	2.3
38	84.0	1.3	81	97.4	1.3	124	19.7	2.8
39	82.7	1.3	82	96.1	1.3	125	16.7	3.0
40	81.5	1.2	83	94.8	1.3	126	13.8	2.9
41	80.3	1.2	84	93.7	1.1	127	10.4	3.4
42	79.1	1.2	85	92.6	1.1	128	405.8	4.6
43	77.9	1.2	86	91.5	1.1	...	.....	...

(cone vision). Hue sensibility, previously discussed, is also independent of intensity over a wide range of moderate and high intensities.

Various color analyzers, properly called colorimeters, make analyses in terms of pure hue and saturation. The so-called monochromatic analyzers determine color in terms of wave lengths of dominant hue and per cent. impurity. Trichromatic analyzers determine colors as ratios of three primary hues, which, when mixed, will just match the known sample. On account of the difficulty of making absolute analyses, color comparators, sometimes improperly called colorimeters, are much used. These instruments merely serve to determine color matches between the unknown and some prepared standard of color. That standard may be a colored solution, a certain thickness of quartz between crossed nicols, or various mixtures of prepared pigments. One of the most extensive and most carefully prepared sets of comparison standards was prepared by Robert Ridgway of the Smithsonian Institution, gotten out by Hoen of Baltimore. Ridgway takes 36 spectral hues and purples and dilutes each with white in three steps and with black in three steps. Each primary hue is further diluted with gray and these in turn diluted with white and black. The whole set consists of 1113 colors differentiated from each other by about twice the least perceptible difference.

III. SENSIBILITY TO INTENSITY.

The eye operates normally under intensities ranging from about 10 lamberts (the brightness of white paper in full sunlight) down to about one millionth of a millilambert. An average brightness out of doors on a clear day is about 1 lambert, in interiors during the day about 10 ml, in interiors at night 0.1 ml, out of doors at night 0.001 ml. The adapted threshold of vision is about 0.0000007 ml. In operating over this enormous range of 20 billion to one, the eye far exceeds in range any other physical instrument. It is enabled to do this by automatically adjusting its sensibility to the brightness of the field viewed. The sensibility of the retina varies by a factor of about 10 million, while the brightness of the field viewed varies by a factor of approximately 10 billion. It is sometimes assumed that variation in size of pupil takes care of the widely varying range of intensity of field, but this is not the case, since the total range of pupillary diameters is but from about 2 mm to 7.5 mm, or in area from 3 mm^2 up to 30 $mm,^2$ a range of but 10 : 1.

It is of course impossible by direct measurement to determine the relation between brightness sensation and intensity of the stimulus producing that sensation. There are, however, various indirect means of determining that relation. With the retina adapted to any given field brightness, there is a corresponding (1) least perceptible brightness (threshold), (2) a least perceptible contrast in brightness, either in juxtaposed fields or in alternating fields, and (3) an intensity just uncomfortably glaring. Determinations of each of these quantities have been made for normal vision throughout the whole range of operating brightnesses.

Threshold sensibility is determined by means of the visual sensitometer.[12] A large cardboard illuminated to various known brightnesses is used to sensitize the eye. In the centre of this field is a spot independently illuminated. After the eye is fully adapted, the light on the large sensitized field is suddenly switched off and the brightness of the spot adjusted so that it is barely visible at the first instant of switching off the light of the large field.

The data taken by means of this method are given in the following table:

Adaptation in Brightness Sensibility.

(All Values are in Millilamberts.)

Field brightness	Instantaneous Threshold	T/F	T_0/T	Log F	Log T
0.00000071	0.00000071	1.00	1.00	−6.15	−6.15
0.00000100	0.00000093	0.93	0.77	−6.00	−6.03
0.0000100	0.0000042	0.42	0.17	−5.00	−5.38
0.000100	0.000019	0.19	0.038	−4.00	−4.72
0.00100	0.000093	0.093	0.0077	−3.00	−4.06
0.0100	0.00039	0.039	0.0018	−2.00	−3.41
0.10	0.00175	0.017	0.00041	−1.00	−2.76
1.00	0.0082	0.0082	0.000087	0.00	−2.09
10.0	0.036	0.0036	0.000020	+1.00	−1.44
100.	0.191	0.0019	0.0000037	+2.00	−0.72
1,000.	2.140	0.0021	0.00000033	+3.00	+0.33
2,000.	3.980	0.0020	0.00000018	+3.30	+0.60

The data for white light is the mean of that taken by Blanchard, Reeves and myself; the three subjects agree with each other in absolute sensibility to within the uncertainty in a single set of readings. The curve is a straight line except at the ends and with a very slight dip in the region of ordinary brightnesses, indicating that there the retina is a little more sensitive than normal. The bending off near the threshold is due to the test field being considerably smaller than the sensitizing field. The falling off from the linear relation at high intensities is due to failure of retinal adaptation at high intensities. If continued, the curve would cut the 45-deg. line at about 7.1 lamberts, indicating that that brightness is blinding. (See Fig. 4.)

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[12] P. G. Nutting, *Trans. Ill. Eng. Soc.*, 11, 7, 1916.

Data of a similar nature have been obtained for the four colors blue, green, yellow and red by Blanchard.[13] His determinations, after correction for the Purkinje effect in the specification of intensities, coincide with the curve of threshold sensibility obtained for white light to within the error of determination.

Contrast Sensibility, König and Brodhun.

Wave length	0.670	0.605	0.575	0.505	0.470	0.430	White
Field Br.	Least perceptible contrast						
1,000,000	....	....	....	....	....	....	0.0358
500,000	....	....	....	....	....	....	0.0273
200,000	....	0.0425	....	....	....	....	0.0267
100,000	....	0.0241	0.0325	....	....	....	0.0195
50,000	0.0210	0.0255	0.0260	....	....	....	0.0173
20,000	0.0160	0.0183	0.0205	0.0195	....	....	0.0175
10,000	0.0156	0.0163	0.0179	0.0181	....	....	0.0176
5,000	0.0176	0.0158	0.0166	0.0160	....	....	0.0179
2,000	0.0165	0.0180	0.0180	0.0175	0.0180	....	0.0181
1,000	0.0169	0.0198	0.0185	0.0184	0.0167	0.0178	0.0178
500	0.0202	0.0214	0.0180	0.0194	0.0184	0.0214	0.0192
200	0.0220	0.0225	0.0225	0.0220	0.0215	0.0245	0.0222
100	0.0292	0.0278	0.0269	0.0244	0.0225	0.0246	0.0298
50	0.0376	0.0378	0.0320	0.0252	0.0250	0.0272	0.0324
20	0.0445	0.0460	0.0383	0.0295	0.0320	0.0345	0.0395
10	0.0655	0.0610	0.0582	0.0362	0.0372	0.0396	0.0477
5	0.0918	0.103	0.0888	0.0488	0.0464	0.0494	0.0593
2	0.1710	0.167	0.136	0.0655	0.0715	0.0600	0.0939
1	0.258	0.212	0.170	0.0804	0.0881	0.0740	0.123
0.5	0.376	0.276	0.208	0.0910	0.096	0.0966	....
0.2	....	0.332	0.268	0.110	0.127	0.116	....
0.10	....	....	0.396	0.133	0.138	0.137	....
0.05	....	....	....	0.183	0.185	0.154	....
0.02	....	....	....	0.251	0.209	0.223	....
0.01	....	....	....	0.271	0.289	0.249	....
0.005	....	....	....	0.325	0.300	0.312	....
0.002	....	....	....	....	....	0.369	....
Threshold	0.060	0.0056	0.0029	0.00017	0.00012	0.00012	0.00072

Contrast Sensibility.—If two fields of view side by side differ in brightness, there exists a minimum difference in brightness which is just perceptible. The ratio of this brightness difference to the mean of the two fields is the well-known Fechner fraction which is a constant over a wide range of intensities.

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[13] Julian Blanchard, *Phys. Rev.*, 11, 81, 1918.

We are indebted to König and Brodhun [14] for the first investigation of contrast sensibility. Their data on white light and six pure hues are given in the table on page 66. Their unit of intensity was roughly 0.0040 millilamberts. Intensities for the colored lights are given as fractions of high intensities all equal. If corrected for the Purkinje effect, the thresholds and all contrast ratios for all colors would be sensibly equal at each intensity.

These data may be fairly accurately represented by the function $P = P_m + (1 - P_m)(F/F_o)^{-n}$. The values of the exponent n in this equation are given in the following table for the six different wave lengths:

	F_o	n	$P_m^{1/n}$	$P_m^{1/n} : F_o$
670	0.060	0.584	0.000832	0014
605	0.0056	0.388	0.0000234	0042
575	0.0029	0.364	0.0000116	0040
505	0.00017	0.335	0.00000434	0025
470	0.00012	0.323	0.00000276	0023
430	0.00012	0.323	0.00000276	0023

These values of course are only measures of the Purkinje effect. If the contrast sensibilities had been reduced to equal brightness sensations, the values for the different hues would have been sensibly equal and equal to the value for white.

Similar data have been obtained by Blanchard [15] and the writer using the visual sensitometer as devised by the writer. Contrast strips of varying densities were fastened across the test spot and adaptation curves run for contrasts of 0, .39, .67, .87 and .97. Our data check those of König and Brodhun, provided their unit of intensity was 0.0040 ml.

In the following table are summarized data on contrast sensibility over a wide range of field brightnesses. In the third column headed Discrimination Factor are plotted the reciprocals of the Fechner fraction, this being a measure of the relative ability of the eye to detect slight contrasts at the various intensities. The threshold limit is that previously given. In the fifth column are given data obtained by multiplying the difference fractions by the field brightnesses. This quantity is the difference in millilamberts between the two fields which will appear just perceptibly different in brightness at each intensity. It is to be noted that this difference over a wide range of moderate intensities is just

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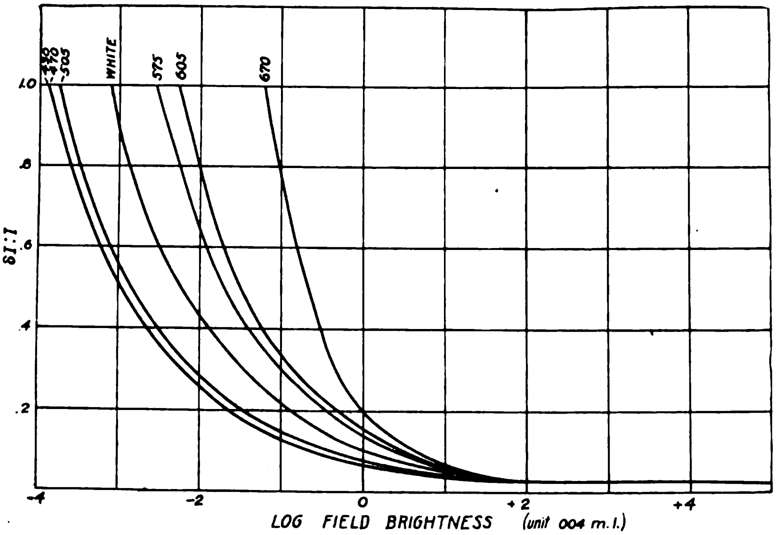
[14] König and Brodhun, *Sitz. Ak. Berlin,* July 26, 1888; P. G. Nutting, *Bull. Bu. Standards,* 5, 285, 1908.

[15] P. G. Nutting, *loc. cit.*

FIG. 3.



Contrast sensibility.

Threshold, Contrast Difference and Glare at Various Field Intensities.

Field brightness	Difference fraction	Discrimination factor	Threshold limit	Difference	Glare limit
0.000001 ml	(1.00)	1.0	0.00000093 ml	0.0000010	20.1 ml
0.00001	(0.66)	1.5	0.0000042	.0000066	40.7
0.0001	0.395	2.5	0.000019	.0000395	89.
0.001 (E.N.)	0.204	4.5	0.000087	.000204	86.
0.01	0.078	12.8	0.00039	.00078	400.
0.1 (I.N.)	0.037	27.0	0.00174	.0037	810. ml.
1.0	0.0208	48.2	0.0081	.0208	1.66 l.
10. (I.D.)	0.0174	57.5	0.036	.174	3.47
100.	0.0172	58.1	0.28	1.72	7.25
1000. (E.D.)	0.0240	41.7	2.15	24.0	14.45
10000.	(0.048)	(20.9)	(232.)	480.	30.90

0.001 ml (E. N.) Exteriors at night, street lighting.
0.01 ml (I. N.) Interiors at night, interior artificial lighting.
1.0 ml (I. D.) Interior daylight lighting.
1.0 l. (E. D.) Exterior full daylight lighting.

twice the threshold limit to within the error in measurement. In other words, the difference in millilamberts between fields just noticeably different in contrast is twice the minimum perceptible brightness for an eye sensitized to the same mean field brightness. For example, with a mean field brightness of 0.1 ml, a rough average for interiors at night, the least perceptible difference is about 0.0037 ml, while the threshold limit is 0.00174 ml. The intensities indicated by initials in parenthesis are rough averages of those of most particular interest.

In Fig. 4 are plotted the logarithms of the just perceptible thresholds and contrast differences each in millilamberts for various brightnesses of field viewed. Both are linear functions of the field brightness and they are sensibly parallel. The mean slope of each line is two-thirds as closely as it can be read.

Taking sensibility as inversely proportional to these just perceptible intensities, we have therefore sensibility, $B : : F^{-2/3}$. The constant of proportionality is quite arbitrary. If it be taken such that sensibility is unity at the threshold, then we should have $B = (F/F_0)^{-2/3}$.

We are now in a position to determine the brightness sensation as a function of the stimulus by integrating the sensibility. This integral of BdF gives for the sensation, $S = 1/3\, F_0^{2/3}(F^{1/3} - F_0^{1/3})$. Hence we have that the sensation is proportional to the cube root of the brightness of the field viewed, except at the lowest intensities, the constant of integration being so chosen that the sensation becomes zero at the threshold. This cube root sensation law rests primarily upon the balance between the catabolic and metabolic processes in the visual purple. The energy flux per unit area at the retina is proportional to the brightness as defined and used in this paper.

Glare Sensibility.—In response to the need of automobile headlight engineers for data on brightnesses just intense enough to appear uncomfortable, determinations were made by the author, Mr. Blanchard, and Mr. Reeves.[16] The retina was sensitized to a definite field brightness, then a much brighter field suddenly flashed on. The three observers agreed surprisingly closely in their glare determinations, although these were made at different times and each observer formed his own criterion for judging glare. The mean results are given in the last column of the preceding table and are plotted in Fig. 4. The logarithm of the glare is a linear function of the logarithm of the field brightness, the slope of the line being 0.32. From the slope and position of the glare line, brightness, $G = 1700F^{0.32}$. If the glare line be continued slightly, glare becomes equal to the field, or adaptation ceases at a field brightness of about 52 lamberts, five times as bright as white paper in direct sunlight. At night out of doors an average brightness is .001 ml and with the eye adapted to this average brightness, a field of 186 ml will just appear uncomfortably bright if suddenly flashed in the centre of vision.

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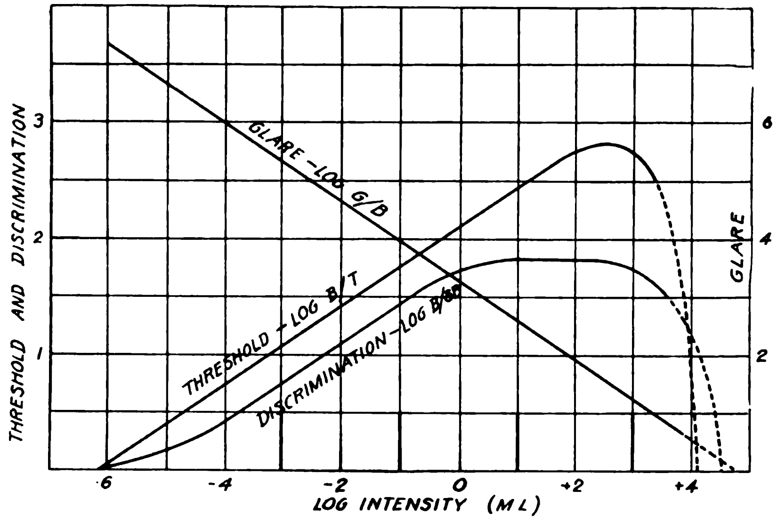
[16] P. G. Nutting, *loc. cit.*

FIG. 4.



Threshold, contrast difference and glare.

It is remarkable that both sensation and glare are proportional to the cube root of the brightness of the field viewed. This is less remarkable when we consider that the glare limit is a direct measure of the brightness sensation. From the last two equations it follows that if the unit of sensation is such that sensibility is unity and sensation zero at the threshold, then the glare limit is precisely 13 million times the sensation at each field brightness.

IV. RATE OF ADAPTATION.

If the eye is adapted to one mean brightness of field of view, that is to say, a given flux density of energy into the retina, then the mean brightness of that field abruptly increased or diminished to another general level, the retina adapts itself to the new level of brightness at a given rate. The bulk of the adaptation takes place in the first few seconds, but in extreme cases continues at a gradually decreasing rate for many hours.

The visual sensitometer previously mentioned was devised by the writer primarily to study the rate of adaptation from one level to another. With this instrument a wide variety of adaptations may be studied very rapidly, since intensities are varied by sliding along a black wedge whose density is quite accurately proportional to the distance from the thin end. Our observations thus far have been chiefly on rate of adaptation from light to darkness, using various initial brightnesses, contrasts and colors, but the instrument is equally well adapted for determining adaptation from darkness to light. An earlier set of observations indicates quite clearly the general character and range of the phenomena involved. In this the eye was sensitized to a mean brightness of 25 ml, then this light suddenly switched off and the rate of increase in sensibility determined. Then the eye was fully adapted to darkness and suddenly exposed to a field brightness of 25 ml and the rate of sensibility decrease determined. These rough results are given in the following table:

Rates of Increase and Decrease of Sensibility.

Time	$B_0=0$, $B=25$ *ml.* Sensibility decrease	$B_0=25$*ml.*, $B=0$ Increase
1 second	2.1 times	1.6 times
2 seconds	4.2 times	2.6 times
5 seconds	16.2 times	7.6 times
10 seconds	58. times	14.4 times
10 minutes	120. times	20.9 times

For example, if an eye adapted to darkness be suddenly exposed to 25 ml, its sensibility drops to 1/16 in 5 seconds, and a subsequent rest of 10 seconds will by no means restore its sensibility. An alternation of equal exposures to a light field will therefore produce a decided depression in sensibility. The application of such data in practise is obviously of importance as, for instance, in passing along an ordinary lighted street at night.

FIG. 5.

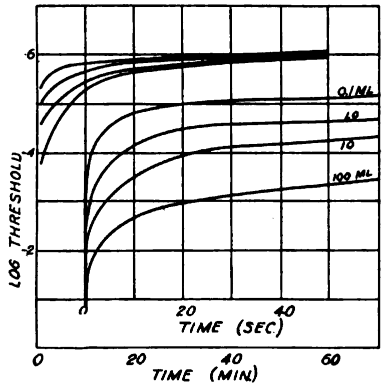
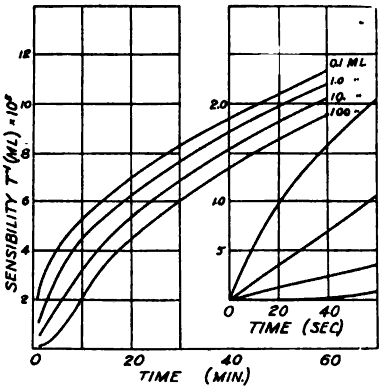
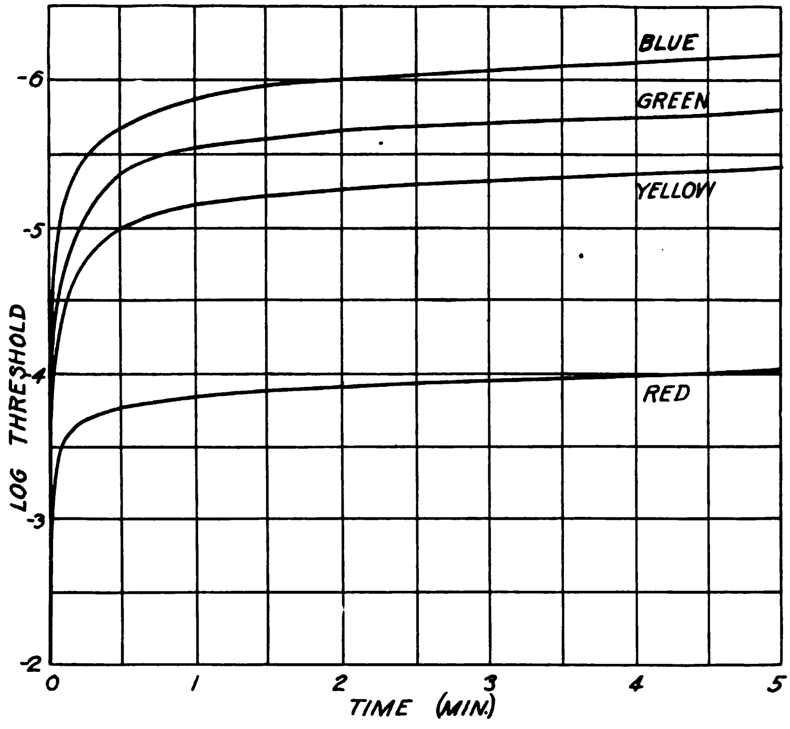


FIG. 6.



Figs. 5 and 6.—Dark adaptation, white light.

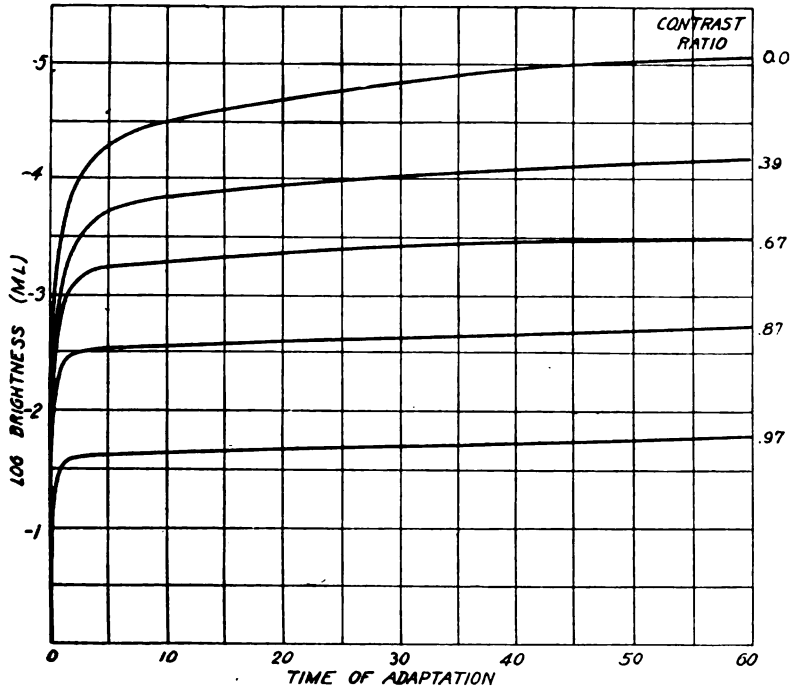
FIG. 7.



Dark adaptation, colored light.

The general phenomena of dark adaptation[17] are indicated in Figs. 5 and 6. The initial steady states from which adaptation proceeded were 0.1, 1.0, 10 and 100 ml. Curves are shown for each initial brightness, one set magnified in time to show the initial stages. These curves are not purely logarithmic as might have been anticipated. The sensibility taken as reciprocal of the just perceptible brightness of test spot is roughly proportional to the time of adaptation. This is shown in Fig. 6. When the pre-adaptation field brightness is small, the initial rise of sensibility is considerably greater than when the initial field is bright.

FIG. 8.



Adaptation to contrast.

The rate of adaptation varies considerably with the color of the light used, as shown in Fig. 7 for the colors red, yellow, green and blue. The adaptation to red light is more rapid but does not proceed as far as that for blue light. Green and yellow give intermediate rates of adaptation. The rate for white of the same initial intensity lies between those for yellow and for green.

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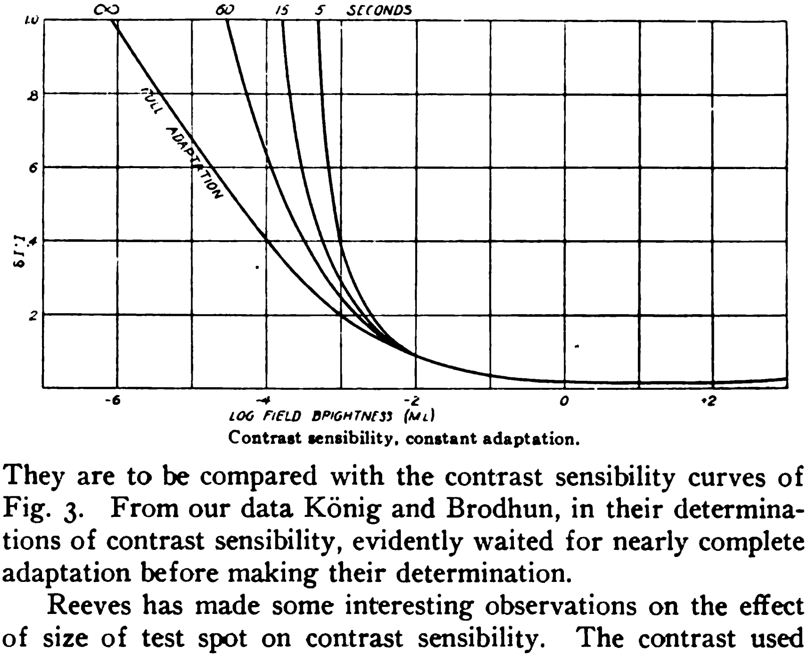
[17] P. G. Nutting, *loc. cit.*

Other determinations were made of the time necessary to just detect certain fixed contrasts: 0, .39, .67, .86 and .97. The results obtained are shown in Fig. 8.

These data were obtained by fixing across the test spot, strips of film whose transmission coefficients had the above values. Adaptation to slight (0.97 : 1) contrast is rapid but reaches nearly its final value in less than two seconds. Greater contrasts require longer and proceed farther in their adaptation. The same data plotted in a different way are shown in Fig. 9. These curves show the contrast just perceptible after a given time of adaptation.

FIG. 9.



Contrast sensibility, constant adaptation.

They are to be compared with the contrast sensibility curves of Fig. 3. From our data König and Brodhun, in their determinations of contrast sensibility, evidently waited for nearly complete adaptation before making their determination.

Reeves has made some interesting observations on the effect of size of test spot on contrast sensibility. The contrast used was 0.67. The test spots were square of the size indicated in the table below. These were viewed from a distance of 25 cm. The data are given in logarithms of brightness in ml.

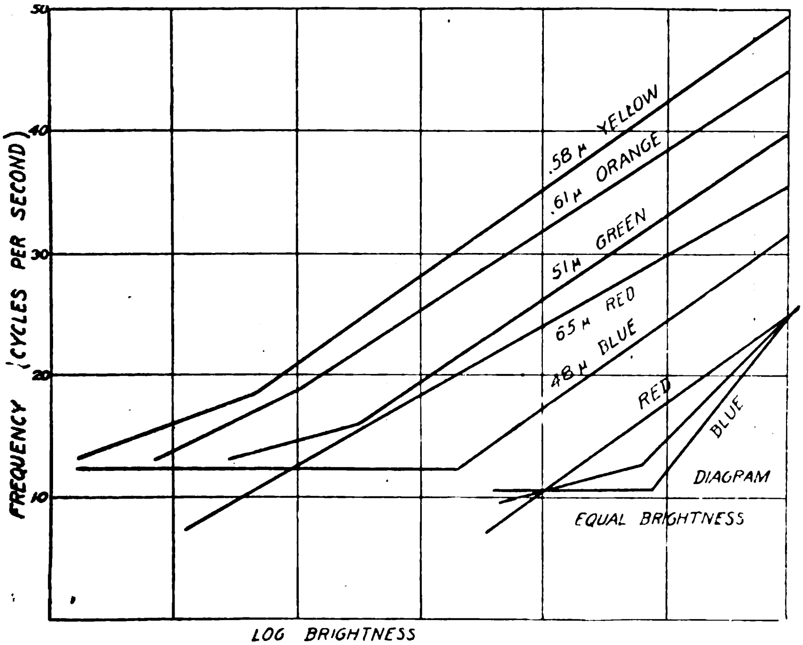
Effect of Size of Test Spot on Contrast Sensibility, Contrast 0.67.

Adaptation	3 cm. sq.	2.5 cm. sq.	1.5 cm. sq.	0.7 cm. sq.	0.3 cm. sq.	0.2 cm. sq.
0 second	−2.42	−2.10	−1.90	−1.69	−1.49	−1.31
2 seconds	−3.03	−2.64	−2.30	−1.91	−1.76	−1.46
10 seconds	−3.25	−2.93	−2.49	−2.11	−1.87	−1.58
60 seconds	−3.52	−3.25	−2.69	−2.27	−1.97	−1.69

It may be noted that the eye is nearly 100 times as sensitive for the 3 cm sq. test spot as for the 2 mm sq. test spot. This corresponds to the well-known fact, namely that signboards which are to be read quickly at night must be printed in large letters with heavy contrasts to be easily legible.

Critical Frequency.—The frequency at which flicker just disappears depends upon alternate light and dark adaptation of short period. Critical frequency is quite simply related to the brightness of the field viewed and has been used quite extensively as a method for research in visual sensitometry. Porter[18] found that for white light the critical frequency at moderate and high field brightnesses is a linear function of the logarithm of the field

FIG. 10.



Critical frequency.

$$N_1 = 12.4 \log F + 24.9$$
$$N_2 = 1.56 \log F + 19.6$$

The second relation holds at low intensities down to the threshold. The field brightnesses are here given in meter candles. The logarithms are common logarithms and the frequencies are alterations per second of equal light and dark exposures. As a means of measuring illumination, however, the critical frequency is rather insensitive.

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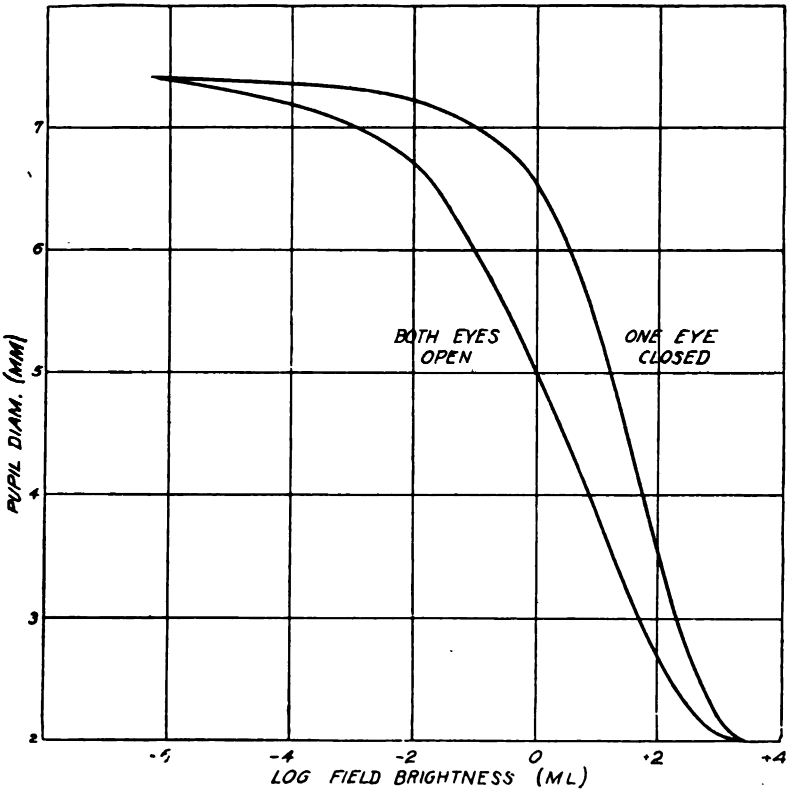
[18] T. C. Porter, *Proc. Roy. Soc.*, 70, 313, 1902.

Similar work has been done by Ives[19] for various monochromatic pure hues. The relation between critical frequency and field brightness is linear throughout for red light. For blue light, critical frequency is independent of brightness for all intermediate and low brightnesses. For other colors the relations are intermediate between these two extremes. Allen[20] has used a critical frequency method with great success in the investigation of after-images in relation to the color vision theory.

FIG. 11.



Diameter of pupil.

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[19] H. E. Ives, *Phil. Mag.*, Sept., 1912.

[20] Frank Allen, *Phil. Mag.*, July, 1919.

V. ABSOLUTE SENSIBILITY.

Diameter of Pupil.—All the data in the preceding sections of this paper were obtained with the normal pupil, with the exception of those on critical frequency for which a light in lumens or watts per sq. mm at the retina is directly proportional to the area of the pupil as well as to the brightness of the object viewed. All visual sensitometry involves the relation between size of pupil and mean brightness and angular size of object viewed.

Various pupilometers have been suggested and used for determining the size of pupil, but it remained for Blanchard and Reeves[21] to actually make precise determinations. This was accomplished by flashlight photography. The results of their work are shown graphically in Fig. 11.

Data were obtained for both eyes opened and for one eye closed during the sensitizing period. At a brightness of 1 ml, having the other eye closed makes a difference of fully 30 per cent. in the diameter of the pupil observed.

Other data are given in the following table:

Retinal Flux Densities.

Field, *Ml.* *F*	Observed	Effective	Area effective *S*	Lumens per mm^2 *E*	Lumens per mm^2/ml.
	Diameter of pupil				
0.00001	7.30	8.17	52.2	7.0×10^{-12}	7.0×10^{-7}
0.001	6.97	7.80	47.8	6.4×10^{-10}	6.4
0.01	6.65	7.44	43.4	5.8×10^{-9}	5.8
0.1	6.00	6.72	35.4	4.7×10^{-8}	4.7
1.0	5.06	5.66	25.1	3.3×10^{-7}	3.3
10.	3.86	4.32	14.6	1.9×10^{-6}	1.9
100.	2.72	3.04	7.25	9.7×10^{-6}	.97
1,000.	2.08	2.32	4.23	5.6×10^{-5}	.56
	2.00	2.24	3.94	1.1×10^{-4}	.11

(In the final column of this table are given the lumens per sq. mm. at the retina per ml. of field brightness. The calculations in the 5th column are based on the equation $L=(1/\pi) \times 10^{-5} FSn^2/v^2$, in which L is in lumens per sq. mm., F is field brightness in ml, S is the area of the effective pupil, n the mean refractive index of the eye media (taken as 1.34), and v is the equivalent back focal length of the eye (taken as 20.7 mm). The reduction factor of 1.14/1.02 was used to reduce from the observed apparent diameter of pupil to the actual entrance pupil of the system.)

Minimum Perceptible Radiation.—Reeves[22] has recently published some interesting data on the minimum energy necessary to excite the sensation of light. This is not a matter of light intensity alone, but involves both angular area of object

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[21] Frank Allen, *loc. cit.*

[22] Prentice Reeves, *Astroph. Jour.*, April, 1919.

(area of retina exposed) and the time of exposure. His data on the relation between threshold and angular area are given in the following table:

Variation of Absolute Threshold of the Retina with Size and Shape of Stimulus.

Stimulus	Log. threshold	Threshold	Energy entering Eye
		Ml	*Ergs. per sec.*
1—mm. star at 3 m.........	−2.14267	0.00720	17.1×10^{-10}
1—mm. star at 1.5 m........	−2.58503	.00260	24.8×10^{-10}
1—mm. star at 35 cm........	−3.61979	.00024	42.1×10^{-10}
2—mm. square at 35 cm.....	−4.54837	0.000028	25.3×10^{-10}
5—mm. square at 35 cm.....	−5.17881	66	37.
1—cm. square at 35 cm......	−5.61744	24	54.
2—cm. square at 35 cm......	−5.99012	102	91.
3—cm. square at 35 cm......	−6.34659	450	90.7
6—cm. square at 35 cm......	−6.58872	258	208.
12—cm. square at 35 cm......	−6.75647	175	564.

The energy entering the eye was calculated from the formula SFM^2/R^2 ergs per second in which S is the area of the test spot in cm^2, F its brightness in lamberts, M the mechanical equivalent of light; here taken as 1.59 ergs per second per cm^2, r the radius of the pupil, R the distance of the eye from the stimulus, in cm. As the mechanical equivalent of light has never been determined with any precision at these low intensities, the value used (1.59) may be subject to a very considerable correction factor.

Variation of Absolute Threshold of the Retina with Time of Exposure to Stimulus.

Time	Log. threshold	Threshold	$1/T$	T x Time
0.002	−3.44092	0.00036230	2.76	0.0672
.006	−4.01092	9752	10.3	58
.011	−4.35	4458	22.4	50
.020	−4.62	2394	41.7	48
.034	−4.91	1228	81.5	42
.160	−5.15	706	142.	.05113
.250	−5.29	512	195.	13
.500	−5.45	345	282.	18
1.000	−5.58	00262	382.	26
2.000	−6.11	77	1300.	15
4.000	−6.20	63	1588	25

Time is an essential factor in determining sensibility at these low intensities, as though the light were integrated in some way to produce the threshold effect, which is of course dependent upon the ultimate reactivity of the retina, just as the glare limit marks the minimum reactivity. The data in the above table were obtained by using a carefully calibrated focal plane shutter for determining the time of exposure necessary, the test spot being 3 cm^2 at a 35 cm distance. The approach to constancy in the energy density of the retina given in the final column is very interesting indeed.

If the time of exposure is very much decreased, similar effects have been noted even at moderate and high intensities. Blondel and Rey[23] found that the sensation was proportional to the energy entering the eye in the first fraction of a second of time of exposure. Much important work remains to be done in this entire field of research.

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[23] *Trans. Ill. Eng. Soc.* 7, 625–662, 1912.

FOURTH ANNUAL MEETING OF THE OPTICAL SOCIETY OF AMERICA.

The Fourth Annual Meeting of the Optical Society of America was held at Columbia University, New York City, February 26 and 27, 1920. This meeting was to have been held in St. Louis with the A.A.A.S. in December but was postponed on account of traffic conditions. About 50 members of the society were in attendance. The program was as follows:

THURSDAY MORNING AT 10:00.

G. W. Moffitt, Research Dept., Eastman Kodak Co.,
  1. The Importance of Nodal Points in Lens Testing.
  2. A Method for Determining the Photographic Absorption of Lenses.

L. T. Troland, Harvard University,
  The "All or None" Law in Visual Response.

Prentice Reeves, Research Dept., Eastman Kodak Co.,
  The Response of the Average Pupil to Various Intensities of Light.

A. H. Pfund, Johns Hopkins University,
  The Optical Properties of Paints. (Not read.)

A. H. Taylor, Bureau of Standards,
  A New Reflectometer for the Measurement of Diffuse Reflection Factors of Objects Diffusely Illuminated.

E. D. Tillyer, Research Dept., American Optical Co.,
  Observations on the Reflections from Foliage.

F. K. Richtmyer, Cornell University,
  The Relation of X-ray Phenomena to the Problems of Optics.

L. A. Jones, Research Dept., Eastman Kodak Co.,
  1. A Graphic Method for Solving Problems in Tone Reproduction.
  2. A Colorimeter Operating on the Subjective Principle.

Charles Sheard, Research Dept., American Optical Co.,
  Convergence Associated with the Act of Accommodation at the Reading Point. (Read by Mr. Tillyer.)

C. W. Frederick, Dept. of Lens Design, Eastman Kodak Co.,
  A New Soft Focus Screen for Portrait Photography.

THURSDAY AFTERNOON AT 2:00.

Address of the Retiring President, DR. F. E. WRIGHT, Geophysical Laboratory.

Progress in Crystal Optics During the Past Century.

Presentation and Discussion of the Preliminary Reports of the Committee on Nomenclature and Standards,

1. Colorimetry, I. G. Priest.
2. Pyrometry, W. E. Forsythe.
3. Photographic Materials, L. A. Jones.
4. Lenses and Optical Instruments, C. W. Frederick.
5. Photometry and Illumination, P. G. Nutting.
6. Polarimetry (no report).
7. Spectacle Lenses, E. D. Tillyer.
8. Projection Apparatus, H. G. Gage (no report).
9. Optical Glass, G. W. Morey.
10. Wave Lengths, W. F. Meggers.

THURSDAY EVENING AT 6:30.

Annual Dinner of the Society followed by the Business Meeting of the Executive Council.

FRIDAY MORNING AT 10:00.

P. D. FOOTE and F. L. MOHLER, Bureau of Standards,
Determination of Planck's Constant h from Metallic Vapors.

W. F. MEGGERS and P. D. FOOTE, Bureau of Standards,
A New Microphotometer for Photographic Densities.

FRED E. WRIGHT, Geophysical Laboratory,
The Resolving Power of a Microscope Using Polarized Light.

P. G. NUTTING, Research Dept., Westinghouse Co.,
Dispersion Formulas and the Testing and Listing of Optical Glass.

I. G. PRIEST, K. S. GIBSON, and H. J. MCNICHOLAS, Bureau of Standards, The Munsell Color System.

I. G. PRIEST, Bureau of Standards,
The Applications of Rotary Dispersion to Colorimetry, Photometry and Pyrometry.

I. G. PRIEST and P. D. FOOTE, Bureau of Standards,
A New Study of the Leucoscope and Its Application to Pyrometry.

F. E. Ross, Research Dept., Eastman Kodak Co.,
A New Formula for Photographic Resolving Power.
H. KELLNER, Research Dept., Bausch & Lomb,
A Simple Spectrophotometer.

FRIDAY AFTERNOON.

The Society visited the Keuffel and Esser Works at Hoboken.

The reports of the sub-committees of the Committee on Nomenclature and Standards were mostly of a preliminary nature, since considerable work and discussion will be required before such reports can be put in final form for adoption by the society. It was felt, however, that on account of the importance of such reports and their general usefulness to all workers in applied optics their preparation should be no longer delayed. After perhaps two more annual reports from each sub-committee, it has been planned to publish them in full in the JOURNAL. The discussion of the preliminary reports as presented was very animated and profitable.

P. G. NUTTING,
Acting Secretary.

65529

Journal
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Devoted to all Branches of Optics

Vol. IV MAY, 1920 Number 3

A METHOD FOR DETERMINING THE PHOTOGRAPHIC ABSORPTION OF LENSES.* †

BY

G. W. MOFFITT.

THE relations between image and object brightness and their dependence on the characteristics of the image forming system have been studied by a number of investigators whose work was carried out by visual methods and whose theoretical considerations were, in almost every case, based on the assumption of an instrument used visually. But the corresponding photographic problem has not been so fully investigated. While visual determinations of the percentage transmission would be expected to give values of the same order as those obtained by means of a photographic method, nevertheless since the theoretical conditions are not identical in the two cases and since the visual method gives no information regarding the relation of the photographic object brightness to the total photographic flux density in the image, it seemed worth while to develop a photographic method for determining the loss of light in a lens system.

Let it be assumed that in Fig. 1, *O* represents a small luminous object plane perpendicular to the axis of the lens system that is forming a real image of *O* at *O'*. Assume further that the object surface is radiating according to the Lambert cosine law and that the normal photographic flux density at its surface is I_0. Let *h* be the radius of the image of the stop as seen from the object position, and let U be the angle at the object subtended by this radius. Consider a zone of radius r and width dr in this effective stop. The total flux from ds_0 passing through such a zone is

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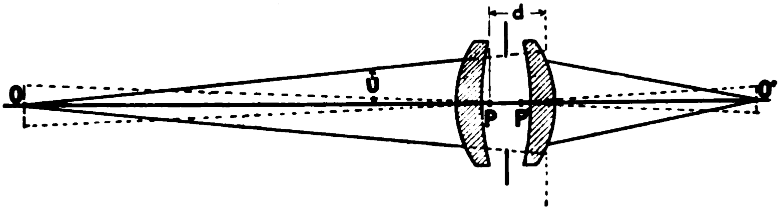
* Read at the St. Louis meeting of the American Physical Society, December, 1919.

† Communication No. 87 from the Research Laboratory of the Eastman Kodak Company.

$$dL = \frac{I_0 dS_0 \cos^2 U' \times 2\pi r dr}{r^2 + (u+d)^2}$$

in which u is the distance from the object to the first principal point of the lens system and d is the distance from this first principal point to the effective stop, considered positive when the effective stop is farther from the object than is the first principal point.

FIG. 1.



Photographic illumination by lens.

By substituting for $\cos^2 U'$ its value in terms of r, u, and d an expression for dL is obtained that is easily integrated for the total flux through the effective stop. The result may be written in the following convenient form:

$$L = \pi I_0 dS_0 \sin^2 U.$$

This total flux will ultimately pass through the element of image dS'_0. But $dS'_0 = Y^2 dS_0$ where Y is the lateral magnification. Therefore the flux density in the image will be

$$I_1 = \frac{\pi I_0 \sin^2 U}{Y^2}$$

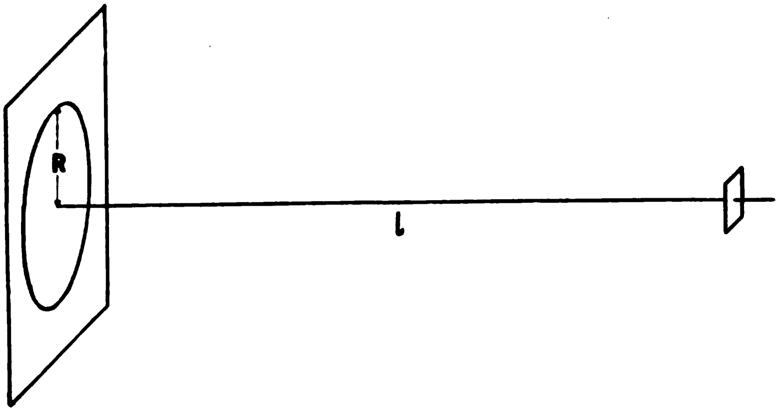
This is the law relating the theoretical photographic flux density in the image to the normal photographic flux density at the object plane. For magnifications approaching zero (infinite object distance) the equation becomes much simpler and may be written

$$I_1 = \frac{\pi I_0}{4 S^2}$$

where S is the ordinary " F-number " of the lens system.

The above equations assume no losses in passage through the system. They involve only the geometrical limitations of relative aperture and magnification, but have nothing to do with losses by absorption and scattering. In fact, they show what the flux density in the image would be for what might be called a perfect lens working under the same conditions. It is also to be noted that the angle of incidence on the plate for the part of the flux coming to the image from the zones of the lens has been neglected. For all ordinary lenses these angles are small and are considered negligible as far as the blackening of the photographic plate is concerned.

FIG. 2.



Photographic illumination by sensitometer.

Now assume the lens to be removed from the arrangement shown in Fig. 1, leaving the conditions as sketched in Fig. 2. Let the radius of the radiating plane be R and the perpendicular distance from the source to the photographic plate be l. The element of flux density at the plate due to a ring element, $2\pi r dr$, of the source is

$$dL = \frac{2\pi r I_0 dr \cos^2\theta}{r^2 + l^2}$$

$$= 2\pi I_0 l^2 r \frac{dr}{(r^2 + l^2)^2}$$

Integrating this expression over the area of the source the total flux density at the plate is found to be

$$I_s = \frac{\pi I_0 R^2}{R^2 + l^2}$$

Thus one has a sensitometric means of impressing a plate with a known flux density, and by comparing the resulting blackening of the plate with the one due to an exposure to the lens image of the same source the unknown photographic flux density in the lens image may be determined.

The axial photographic transmission coefficient of a lens system may now be defined for a given set of conditions as the ratio of the light flux of photographic quality in the image of a small object on the axis of the system to the light flux of photographic quality that would reach the image were there no losses of any kind in transmission through the system, other conditions remaining the same. If a photographic plate be exposed to the action of the lens image, the remainder of the plate being shielded from radiation, the exposure will be $E_l = I_l t_l$ where t_l represents the time of exposure. If an adjacent portion of the plate be exposed to the direct action of the source (lens and lens board removed) for a time interval, t_s, so adjusted that equal densities are obtained in lens image and sensitometer strip the exposure will be $E_s = I_s t_s$. The apparatus may be so adjusted that t_s and t_l are of about the same magnitude, so that the question of the failure of the reciprocity law does not enter here, and it may be assumed without further discussion that equal densities are produced by equal exposures for the range of time intervals used in this work. Equating the exposures for equal densities,

$$I_l t_l = I_s t_s,$$

$$I_l = \frac{\pi I_0 R^2}{R^2 + l^2} \times \frac{t_s}{t_l}$$

According to the definition already given the transmission coefficient of the lens is

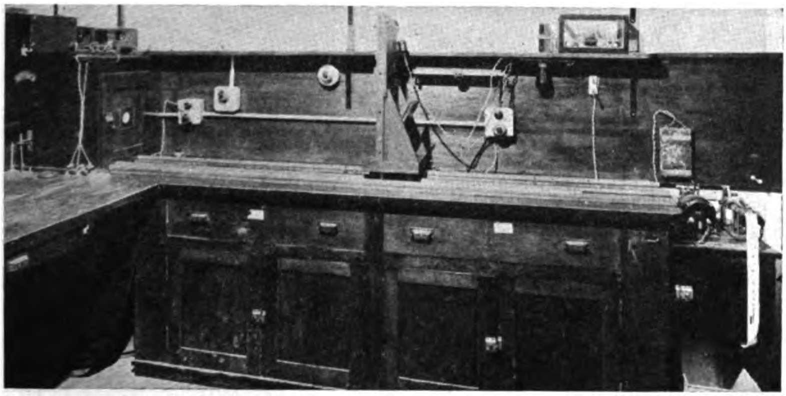
$$T = \frac{I_l}{I} = \frac{\dfrac{\pi I_0 R^2}{R^2 + l^2} \times \dfrac{t_s}{t_l}}{\dfrac{\pi I_0 \sin^2 U}{Y^2}}$$

$$= \frac{t_s}{t_l} \times \frac{R^2 Y^2}{(R^2 + l^2)\sin^2 U}$$

$$= \frac{t_s}{t_l} \times \frac{R^2}{R^2 + l^2}\left(Y^2 + \frac{[Y(f+d)+f]^2}{h^2}\right).$$

For the experimental work one of our standard sensitometers (Fig. 3) proved to be suitable without modification other than the addition of a lens carrying support and some screening for the best possible elimination of stray light. The instrument was equipped with a cubical lamp house (Fig. 4), having white diffusing anterior walls. A voltage-controlled tungsten lamp in the centre illuminated a window of pot opal glass in one side of the box. This window could be stopped down to any desired diameter by means of suitable metal plates having centred circular openings and, after being screened with the proper filter to render the emitted light of approximate daylight quality, served as the source. or object plane, in the experiments. The plate holding and exposing mechanism (Fig. 5) was placed at any convenient and suitable distance from the source. This device is designed to expose a plate in strips composed of a series of steps, the time of exposure in each step being automatically controlled as desired and continuous from beginning to end. By placing a lens carried on a suitable screening board between the lamp house and the sensitometer a focused image of the circular source was formed on the plate in the sensitometer and exposed in a series of stops, although the size of the image was such that usually only one stop, and never more than three, was exposed. Then the lens and lens board were removed and step strips adjacent to and on both sides of the lens image were impressed upon the plate by means of the direct illumination, the same control of the source being maintained throughout both lens and sensitometer exposures. From the developed plate the ratio of the times of exposure for

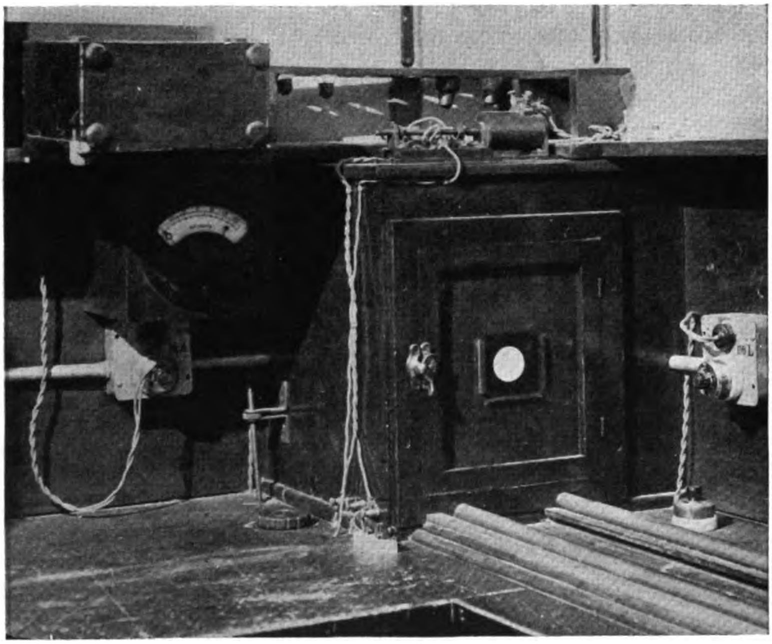
FIG. 3.



Constant intensity continuous exposure sensitometer.

equal densities in the lens image and the sensitometer strips could be determined. This ratio, together with the dimensions of the apparatus and the constants of the lens, gave sufficient data for the determination of the percentage transmission for light of photographic quality, according to the formula already derived. The distance, stop, size and intensity of source, and exposure times were so adjusted as to bring the density in the lens image approximately equal to that in the middle of the sensitometer strip. That is the important densities were all grouped together at the centre of the plate. With this procedure good results on the axial transmission were obtained.

FIG. 4.



Lamp house end of sensitometer

If in a discussion of the problem of oblique transmission an attempt be made to follow a line of reasoning similar to that just given for axial transmission difficulties will at once be encountered. The stop ceases to be of a simple nature. Not only the diaphragm but various lens rings as well become effective in limiting unsymmetrically the pencil passing through the lens. Since these lens rings have a great variety of sizes and positions in different types of lenses, it is evident that a general formula for oblique transmission would necessarily be very complicated. For this reason it seems advisable to express oblique transmission values in terms of the axial transmission coefficient. The experimental determination of oblique transmission would then consist of a comparison of the density obtained in the image of a small field object source with that obtained in an axial image of the same source, other conditions remaining unchanged. This amounts to a determination of the relative photographic action at a field image point as compared with that at the axial image

FIG. 5.

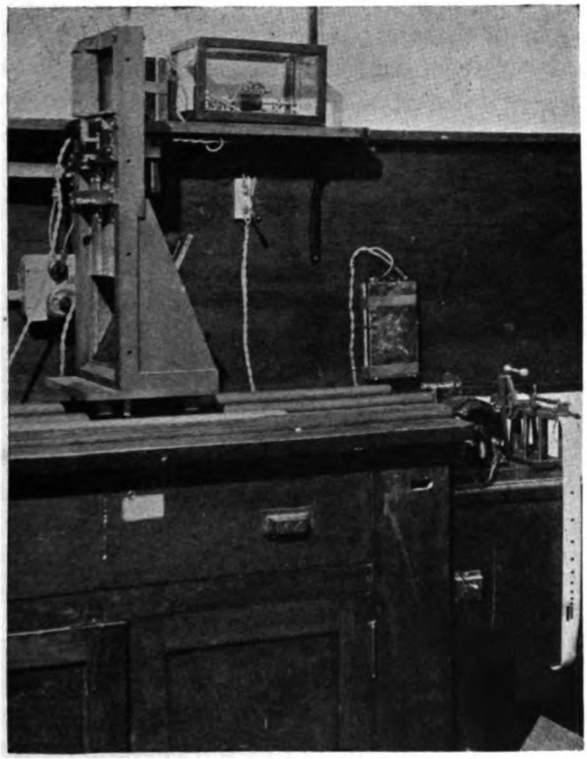


Plate exposing end of sensitometer.

point. It affords a satisfactory means of comparison of the variation of field illumination with variation of angle in different lenses.

Because of inherent irregularities in the photographic plate quite a number of exposures must be made in order to arrive at a good average value of the transmission coefficient. Considerable time is required to perform the experiments. Preliminary results obtained by this method have been found to be in good general agreement with those given by visual methods,[1] as well as with values deduced from the theory of reflection and absorption.

ROCHESTER, N. Y.,
March 10, 1920.

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[1] P. G. Nutting, *Astrophys. Jour.* 40, p. 33–42, 1914; R. W. Cheshire, *Proc. Opt. Con.*, p. 34–40, 1912.

DYES FOR PHOTOGRAPHIC SENSITIZING.

BY

W. F. MEGGERS and F. J. STIMSON.

In the past forty years photography has found such universal and varied applications, both popular and scientific, that a layman might be justified in believing that it is a perfected science. Only the chemist fully realizes how little is really known of the fundamental facts of photo-chemical action and only scientists who attempt to use photography for investigations in a large range of spectrum are thoroughly conscious of the shortcomings of commercial photographic materials.

It is well known that photographic emulsions which owe their light sensitiveness to the silver halides alone are strongly affected by the short waves of blue, violet, and ultra-violet light, but are practically insensitive to the longer waves. This fact led, in the early days of photography, to the conception of the so-called actinic or chemical rays. It is now known, however, that the chemically active rays are not necessarily the shortest waves in the spectrum, and it has actually become possible to make photographic emulsions more or less sensitive to all the visible colors and even to the waves of infra-red light or so-called heat rays.

Both the absolute and the spectral sensitivity of silver bromide are powerfully influenced by the action of so-called sensitizers, which have generally been grouped into two classes: (1) the chemical sensitizers and (2) the optical sensitizers.

There is no doubt that the principal factor in the action of all sensitizers is their greater absorption for light. The vibrations of light are considered to be of electro-magnetic nature and the absorption of light is regarded as being conditioned by resonance phenomena depending upon electro-magnetic and other physical properties of the resonators. The ultra-microscope has shown that gelatine consists of a homogeneous mass containing aggregates which, in a gelatine silver-halide emulsion, would be naturally associated with the halide in forming the above-mentioned resonators; thus offering an explanation of the sensitizing action of gelatine.[1] Silver bromide, either in collodion or in gelatine, always shows the maximum of light sensitiveness to blue light, but the gelatine emulsion is more than a hundred times as sensitive as the collodion emulsion. On the old chemical theory, the sensitizing action of easily oxidizable organic substances, *e.g.*, albumin, gelatine, etc., was explained as assisting in the reduction of silver halide to sub-halide when acted on by light and such substances were called chemical sensitizers by Doctor Vogel (*Photographic News,* p. 209, 1865). All experiments, however, with silver halides imbedded in various media have apparently been unable to increase the spectral range of sensitiveness. Exception must be made of some experiments by Captain W. de W. Abney,[2] who prepared a modification of silver bromide which was sensitive to red and infra-red light as well as to blue and violet light. The production of such red sensitive emulsions appears to have been too difficult or uncertain to continue successfully, but it by no means follows that all the possibilities of combining silver halides with chemical sensitizers have been exhausted. This field seems to have been more or less neglected since the use of photo-sensitizing dyes (optical sensitizers) has produced photographic emulsions of considerable color sensitiveness.

In 1873, H. W. Vogel was studying the effect of the solar spectrum on silver halide emulsions. In one of his experiments he used a silver bromide dry plate which had been stained yellow with aniline red to prevent halation. In addition to the blue and violet sensitiveness, this plate showed a second maximum of sensitiveness in the green which corresponded to an absorption band in the spectrum of a solution of aniline red. He then made experiments with other dyes[3] and stated: "From these experiments I feel justified in concluding with a good deal of certainty that we are able to make silver bromide sensitive to any desired color, or to increase its existing sensitiveness for certain colors. It is only necessary to add a substance which will promote the decomposition of silver bromide and which absorbs the rays in question, letting the others pass. The aggravating difficulty in regard to the photographic ineffectiveness of certain colors appears to be a thing of the past." This conclusion at first met with much opposition. In the *British Journal of Photography* for 1874, Vogel's discovery was derided and rejected. The Berlin Academy of Sciences, however, recognized the importance of these results and made a considerable grant to enable Vogel to carry on his investigations. The rejection of this discovery was partly due to the fact that very few people at that time were clear as to the connection between chemical action and absorption of radiant energy. Most of those who tried to duplicate Vogel's results failed because they dyed the films too much and exposed them too little. Furthermore, the dyes available at that time were so impure that they caused the collodion emulsions to spoil.

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[1] Sheppard and Mees, "Theory of the Photographic Process," p. 268.

[2] Abney, *Phil. Trans.*, 171, p. 653, 1880, and 177, p. 457, 1886.

[3] Vogel, "Handbuch der Photographie," 4th Ed., 1, p. 204, 1890.

So far as the introduction of this principle into commercial practice was concerned, the prospect seemed at first absolutely hopeless, Vogel himself obtaining only one good plate to five bad ones. About this time, a revolution was brought about in photography by the introduction of the silver bromide-gelatine process. The optical sensitizers with which gelatine emulsions were then dyed proved, in the beginning, of such doubtful value that Vogel regarded the apparent indifference of silver bromide-gelatine emulsion to such sensitizers as a characteristic of this "modification" of silver bromide. Somewhat later, the firm of Attout Tailfer and Clayton came forward with a French patent (Pat. No. 152645 of December 13, 1882, and March 29, 1883), according to which silver bromide-gelatine was made strongly sensitive to yellow by means of eosine. Plates made by this process were called "isochromatic." Following this notable event many color sensitizers were examined by Vogel, Schumann, and Eder, the latter alone investigating the sensitizing action of more than 140 coloring matters.[4]

Altogether a very large number of dyes have been examined for photo-sensitizing action, but relatively few have been found which exert any marked effect in making silver bromide-gelatine emulsions sensitive to the longer waves of the spectrum. H. W. Vogel discovered chilolin-red and cyanin as excellent sensitizers and with these introduced color-sensitive "azalin" plates into commerce. The very effective green sensitizing action of erythrosin. discovered by Eder[5] in 1884, has been used extensively in the production of so-called "orthochromatic" plates. Other dyes which have found wide application as photographic optical sensitizers are pinaverdol, pinacyanol, homocol, orthochrome T, pinachrome, pinachrome blue, pinachrome violet, pinacyanol blue, cyanin, dicyanin, dicyanin A, methyl violet, methyl green, etc.

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[4] Eder, "Ausführliches Handbuch der Photographie," 2, p. 443, 1898.

[5] Eder and Valenta, "Beiträge zur Photochemie und Spectralanalyse," Part iii, p. 78, Wien, 1904.

Our conceptions of the action of such optical sensitizers as well as that of the chemical sensitizers have undergone a change in recent years, the electron theory having displaced the older chemical theories. A large number of both solid and liquid substances have been found to lose negative electricity when exposed to ultra-violet light; *i.e.*, if negatively charged, they rapidly lose the charge, while if initially uncharged, they acquire a positive charge. The sensitiveness of this photo-electric effect depends upon the absorption, the effect being greatest for the spectral region which is most strongly absorbed. The silver halides and especially the dyes used as sensitizers have been found to be highly photo-electric. Under the action of light, electrons are liberated from the photographic emulsion, and the increased ionization perhaps brings about a chemical change which is the so-called latent image.[6]

Sensitizing dyes may be applied to photographic emulsions in two different ways—either by incorporating the dye in the emulsion before it is flowed on plates, or by staining a plate, coated with blue-sensitive emulsion, in a dilute solution of the dyestuff and then drying rapidly. Both methods have yielded plates which have found wide use in general photography. In either method only a relatively small quantity of dye is added to the silver salt to make it sensitive. As little as 1 part of cyanin in 5,000,000 parts of staining bath produces a noticeable effect, and too large a proportion of the dye materially reduces the total sensitiveness of an emulsion.[7] When incorporated during the emulsion making process, about 2 to 4 mg. of dye to 100 c.c. emulsion appear to give the best results. In the staining process the best proportion of dye in the bath ranges with different dyes from 1 part in 25,000 to 1 part in about 75,000. Very extensive and careful experiments on the technic of plate bathing were carried out in 1907 by R. James Wallace,[8] who found that the best results were obtained by rapidly drying the plate after staining with a dilute solution of the dye in water and alcohol, a small amount of ammonia being added. At the Bureau of Standards, where stained plates have been used extensively and successfully for spectroscopic investigations and for experiments in aerial photography, the practice is to prepare a stock solution of the dye by dissolving one part in 1000 or 2000 parts of ethyl alcohol and then mix a staining bath by adding 6 c.c. of this stock solution to a mixture of 60 c.c. alcohol and 85 c.c. of water, to which 6 c.c. of strong ammonia water is finally added. Various types of blue-sensitive photographic plates become sensitized by soaking from 3 to 5 minutes in the staining bath after which they are given a brief rinsing in alcohol and dried rapidly.[9]

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[6] Sheppard and Mees, "Theory of the Photographic Process," p. 270.

[7] Eder, "Beiträge zur Photochemie," Part iii, p. 95, Wien, 1904.

[8] Wallace, *Astroph. Jl.*, 26, p. 299, 1907.

The use of water and alcohol is a compromise between conflicting requirements for a staining bath. While water, by softening the gelatine, aids in the penetration of the dye, it retards drying, and rapid drying is essential. Again, in a water solution containing ammonia, the dyes flocculate easily, producing a worthless staining bath. The presence of alcohol tends to prevent this flocculation and also hastens drying; but, at the same time, by hardening the gelatine film it tends to prevent the dye from penetrating it. Because of these conflicting conditions the use of a water-alcohol-dye bath first, followed by an alcohol rinsing has been found to give the best results.

Ammonia plays an important part in the dyeing process to obtain the greatest sensitivity. Investigations at the Bureau of Standards[10] have shown that the sensitiveness of commercial panchromatic plates can be increased very considerably by bathing such plates in a dilute solution of ammonia in water and alcohol. It has also been found that staining baths are more permanently useful if they are free from ammonia, so that an excellent procedure is to bathe the plate first in a water-alcohol solution of dye and then in a separate solution of ammonia.

Plates stained by the operator just before use have an advantage over commercial color sensitive products in that they can be prepared so as to have the maximum sensitivity in any desired spectral regions without being limited to processes which give good keeping plates. Furthermore, many dyes of interest in scientific photography have never been incorporated in the emulsions which are obtainable on the market. These advantages are such that plate bathing will be resorted to in all photographic work for which commercial emulsions are insensitive or too slow.

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[9] See B. S. Cir., "Color-Sensitive Photographic Plates," Nov. 6, 1919.

[10] Burka, *Journal Franklin Institute,* p. 25, Jan., 1920.

Previous to 1914, practically all of the dyes which were most successfully used in photographic sensitizing were produced in the large dye factories in Germany. When this source of supply was cut off not only was the commercial color-sensitive plate manufacturer put to some trouble to find substitutes for the dyes he formerly imported, but the scientist also, who prepared bathed plates for special problems, seemed likely to have to discontinue his work because of the shortage of dyestuffs. Various laboratories in England and in the United States soon took up the problem of making the important dyes for photographic sensitizing which were formerly procured from Germany. For the manufacture of panchromatic photographic plates, pinacyanol in considerable quantities was required and the Ilford Company of London and the Eastman Company of Rochester were both successful in the production of this dye.

The synthesis of photo-sensitizing dyes was taken up in this country as a war problem by the Chemical Section of the Science and Research Division of the Bureau of Aircraft Production in 1917. Since the signing of the armistice this work was taken over and continued by the Color Laboratory of the Bureau of Chemistry (Department of Agriculture). The Spectroscopy Section of the Bureau of Standards (Department of Commerce) has carefully tested the photographic sensitizing action of practically all the dyes made for this purpose and up to the present time has tested over 150 different samples, most of which were prepared by the Bureau of Chemistry.

These dyes fall into three general classes: (1) green sensitizers (orthochrome, erythrosin, pinaverdol); (2) yellow-red sensitizers (pinacyanols) and (3) infra-red sensitizers (dicyanins, kryptocyanin). Pinaverdol[11] sensitizes strongly for green and yellow light with wave-lengths between 5000 A and 6000 A (1 A = 0.0000001 mm.), showing maxima at 5400 A and 5800 A. Pinacyanol is excellent for yellow and red from 5500 A to 7000 A and gives maxima at 5800 A and 6400 A. The most successful dye thus far discovered for infra-red sensitizing is dicyanin, which is now made by the Bureau of Chemistry. The sensitivity imparted by it to a silver bromide plate shows three maxima, one in the yellow at 5800 A, a small one in the red at 6400 A, and a strong red band at 7000 A, extending out into the infra-red. A new sensitizer for red light has recently been found by the Bureau of Chemistry. It is called Kryptocyanin and shows, in addition to a weak band at 6000 A, a strong band whose centre is at wave-length 7400 A, sensitizing in a narrow spectral region from 7200 A to 7700 A.

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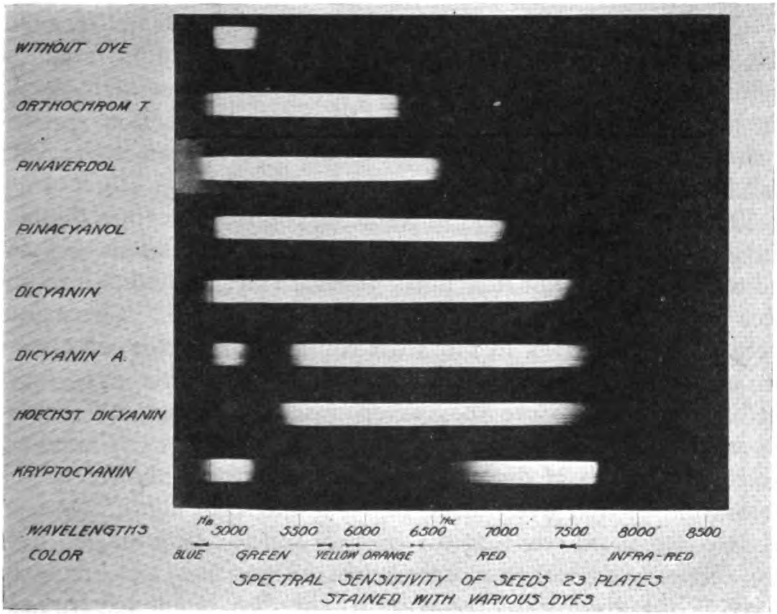
[11] The crystallography and optical properties of this dye have been described by E. T. Wherry and E. Q. Adams, *Journal Washington Academy of Sciences*, 9, p. 396, 1919.

FIG. 1.



Spectral sensitivity of Seed's 23 plates stained with various dyes.

It is obvious that the most direct and conclusive test of the photo-sensitizing action of dyes is to bathe a silver emulsion in a solution of the dye and expose it to a continuous spectrum. For a comparison of the spectral sensitivity of dyed plates, the spectrum of sunlight or skylight may be used, although it is often more convenient and desirable in the laboratory to use an artificial source like an incandescent lamp. Fig. 1 illustrates the photographic impression made by a continuous spectrum upon Seed's 23 plates before and after treatment with typical photo-sensitizing dyes. The plates were stained in dye baths as described above and then exposed in a small concave grating spectrograph[12] illuminated by light from an incandescent lamp. The violet and ultra-violet parts of the spectrum are not shown, since only waves longer than those of blue light are of great interest in connection with the sensitizing dyes.

Other methods of detecting optical sensitizers would be available if the chemical and physical properties of dyes could be definitely and regularly associated with sensitizing action. At the present time, the relationship between such action and the chemical constitution of the dyes does not appear to be very well known.

It is often true that the unstable or fugitive dyes are the best sensitizers. This is especially true with the best sensitizers for red and infra-red, but it is not a general rule, inasmuch as some dyes which sensitize strongly are also quite stable. The manufacture of these dyes in our own country is a great advantage to our scientific work, since it makes the unstable dyes more readily available before they deteriorate. The fact that before the war many scientists in this country discredited the excellent sensitizing action of dicyanin is perhaps partly explained by deterioration of that dye during importation from abroad.

It is a general law that the spectral sensitivity imparted by a dye corresponds rather closely with the position and shape of bands in the absorption spectrum of the dye, but it has already been pointed out that a comparatively small number of dyes act as photo-sensitizers, although they all have absorption bands. It was first announced by Eder[13] in 1885 that the wave-length for which the dye acted most strongly in sensitizing silver emulsion was 200 to 400 A greater than the wave-length corresponding to the centre of the absorption band of the dye in solution. It appears that it is necessary for a dye actually to stain the silver salt to become a sensitizer and the absorption band of the dyed silver salt seems to correspond exactly to the spectral band of sensitiveness imparted to the emulsion. This would be expected if resonance-absorption of energy and photo-electric effect are responsible for the sensitizing action. The absorption spectra of some typical photo-sensitizing dyes in alcoholic solution have been mapped by the Colorimetry Section of the Bureau of Standards

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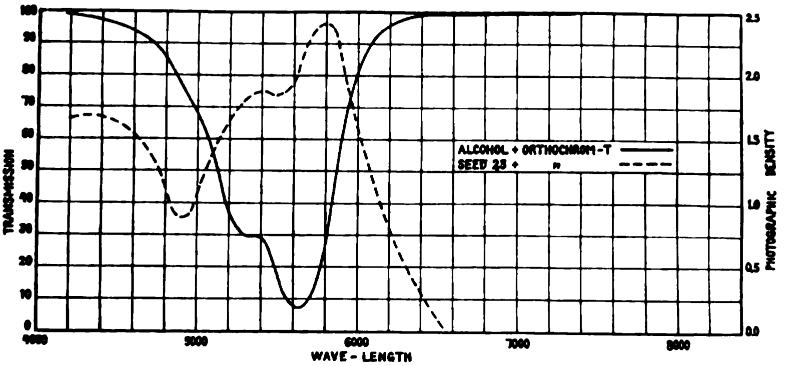
[12] *Journal Franklin Institute,* p. 26, Jan., 1920.

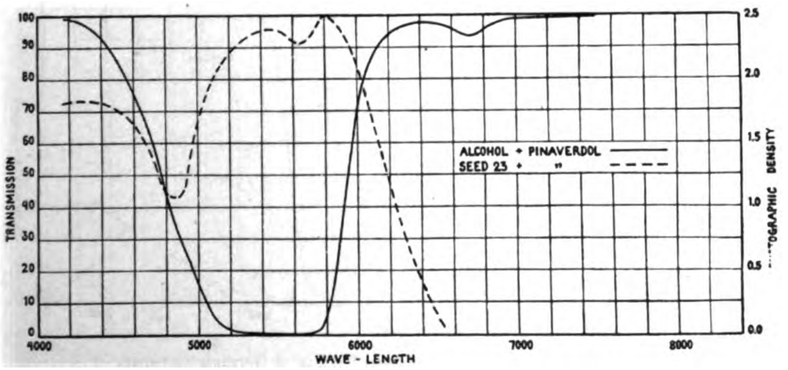
[13] Eder, " Beiträge zur Photochemie," Part iii, p. 35, Wien, 1904.

FIG. 2.



Spectral transmission and photo-sensitizing of orthochrom T.

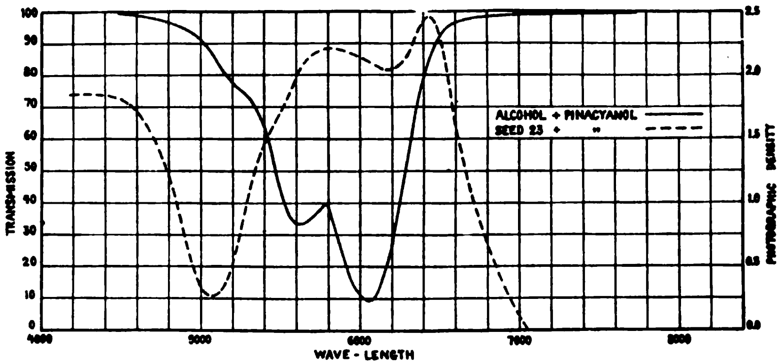
FIG. 3.



Spectral transmission and photo-sensitizing of pinaverdol.

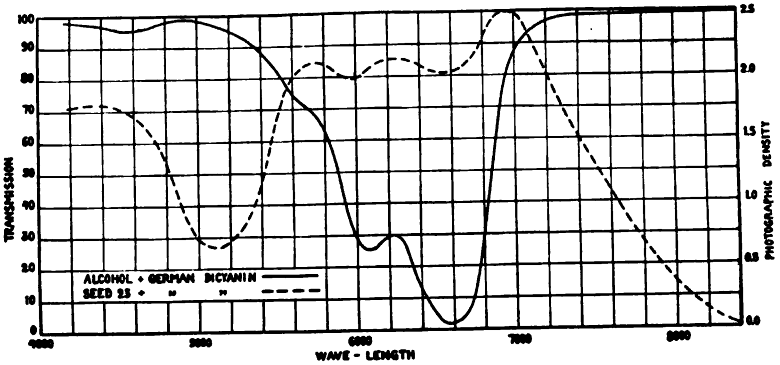
and Figs. 2, 3, 4, 5, 6, and 7 show the characteristic absorption and photo-sensitizing bands of these dyes. The continuous curves represent the percentage of light of various wave-lengths transmitted by dyes in alcoholic solution(left-hand ordinates), while the dashed curves give the relative photographic blackening of a silver bromide emulsion after being stained with the dyes, exposed to a continuous spectrum and developed (right-hand ordinates). The close relationship between the two curves is evident in each figure. If the transmission curve be inverted and displaced slightly toward the longer waves, it will roughly coincide with the photographic density curve (except in the blue end of the spectrum where the natural sensitivity of silver bromide is predominant). As suggested above, the bands shown in the two curves differ in wave-length slightly because in one case we observe the absorption of dye in alcoholic solution, and in the other the absorption of dyed silver bromide having a much larger index of refraction.

FIG. 4.



Spectral transmission and photo-sensitizing of pinacyanol.

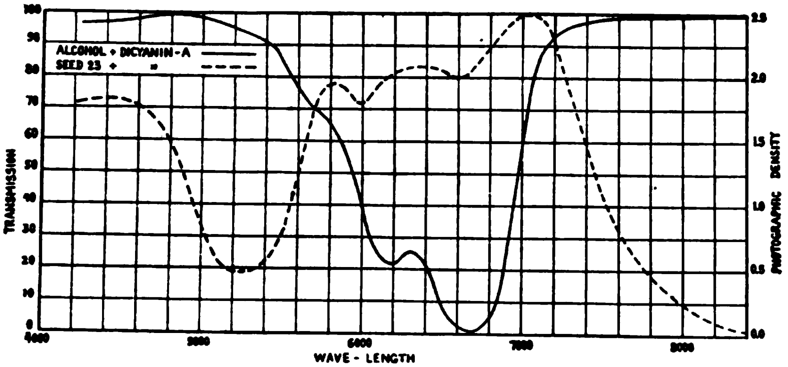
FIG. 5.



Spectral transmission and photo-sensitizing of German dicyanin.

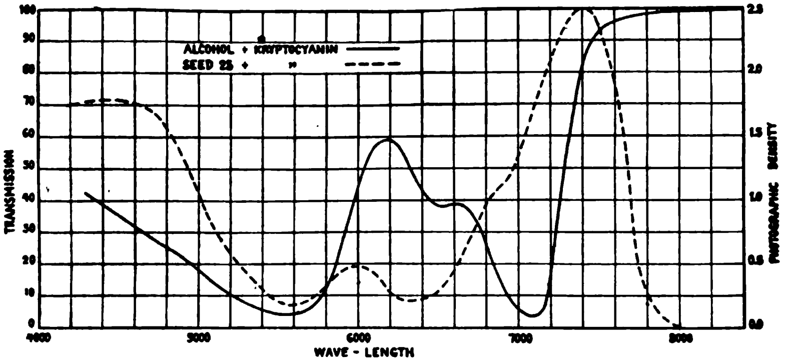
The important part which photography played in the recent war brought about some developments which will be of permanent interest to photographers, and especially to those who make use of photography as an aid in scientific research. Aerial photography from great altitudes was difficult because of atmospheric haze which is very rich in blue light and obscures the landscape.

FIG. 6.



Spectral transmission and photo-sensitizing of dicyanin A.

FIG. 7.



Spectral transmission and photo-sensitizing of kryptocyanin.

It was necessary to use color-sensitive photographic plates (orthochromatic or panchromatic) with ray filters to screen out this scattered blue light and extensive research was therefore carried on to improve the existing color sensitive emulsions. It appears that the Germans were successful in producing a remarkable orthochromatic plate which showed much more contrast and from four to fifteen times the speed of the best orthochromatic plates used by the Allies. The Allies, however, were probably more successful in the production of panchromatic plates for military purposes. The most notable success was by the English firm of Ilford, Ltd., which prepared a panchromatic plate with extraordinary red-sensitiveness and this is now on the market as Ilford Special Rapid Panchromatic. The special technic employed in obtaining these results appears to be quite new and a real step in advance of previous processes. It consists of a method of stimulating the sensitizing powers of certain dyes of the pinacyanol class. This was done by using auramine, a yellow dye of the coal-tar group, as an auxiliary agent; it gives increased speed and at the same time yields clean-working and reasonably stable plates.[14]

These researches on color-sensitive plates involving the use of optical sensitizers have made both England and America independent of Germany in the matter of photographic dyes. It has also stimulated research as to their constitution and properties, which is now bearing fruit in improved sensitizers for the longer light waves and which, if continued, will undoubtedly lead to further progress in photography.

Improvements in photographic emulsions are of vital importance to physicists, astronomers and professional photographers, not to mention the amateurs. In the first place, it seems probable that a simple and thoroughly practical method of photography in natural colors will be available as soon as the speed and the spectral sensitiveness of emulsions are materially increased. This is a problem which should find its solution in the near future.

From the scientific viewpoint, increased light sensitiveness throughout a very wide range of spectrum is desirable. Perhaps most people are unaware of the fact that the most sensitive photographic materials now available are many millions of times less sensitive to the visible spectrum than the eye. For this reason a great deal of scientific work, where the objects to be photographed are faint and where the work is of spectroscopic nature, is now rather narrowly limited by the characteristics of available photographic emulsions. An extension of the photographic possibilities lies in the hands of the chemist and there is no doubt that research work on both chemical sensitizers and optical sensitizers will open up new fields.

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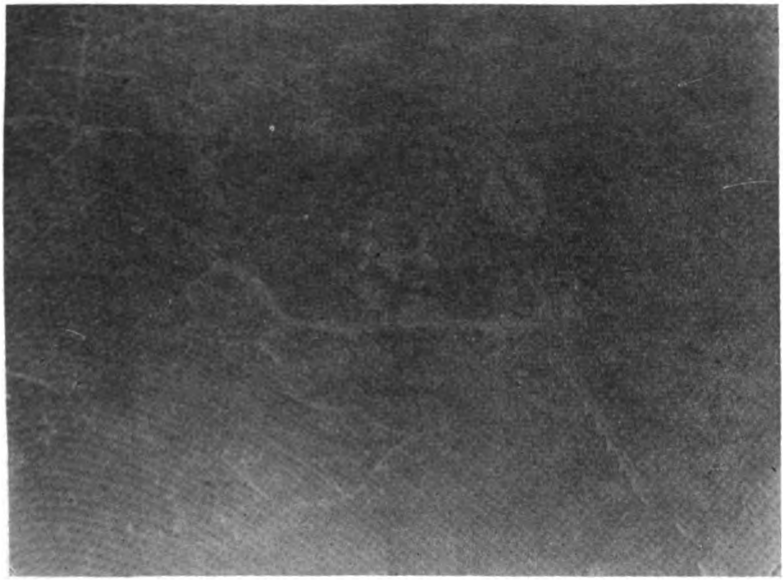
[14] Read also "Progress in Photography Resulting from the War," by Paul W. Merrill, in *Publications of the Astronomical Society of the Pacific*, No. 185, pp. 1–11, Feb., 1920.

Great advances in aerial photography and long-range landscape photography rest upon improvements in color sensitive emulsions which can be used in cameras with ray filters.[15] Figs. 8 and 9 illustrate the advantage of red-sensitive over blue-sensitive plates in haze penetration which is very important for aerial photography.

FIG. 8.



Seed's 30 plate exposed 1/100 second behind F/11 lens without color filter. Taken simultaneously with Fig. 9 in a multiple lens airplane camera at 10,000 feet altitude.

In physics, the extension of exact spectroscopic knowledge and the development of photographic photometry may be cited as cases in which improvements in photographic materials will be of great service.

Astronomy would benefit most of all. Finer grained and faster emulsions would effectively increase the power of the greatest telescopes. Better color sensitive emulsions would enlarge the field of stellar spectroscopy and permit more exact determination of the colors and temperature of stars. It might even be conceived that emulsions of extraordinary red or infra-red sensitiveness could be used with screens removing most of our skylight, so that many stars could be photographed in bright daylight, thus lengthening the astronomical observer's day to twenty-four hours and enabling him to test the Einstein prediction of solar deflection of light from the stars without waiting for an eclipse of the sun.[16] In fact, one's imagination can perhaps suggest only a very few of the future possibilities with photography after the chemist has sufficiently improved the photographic materials.

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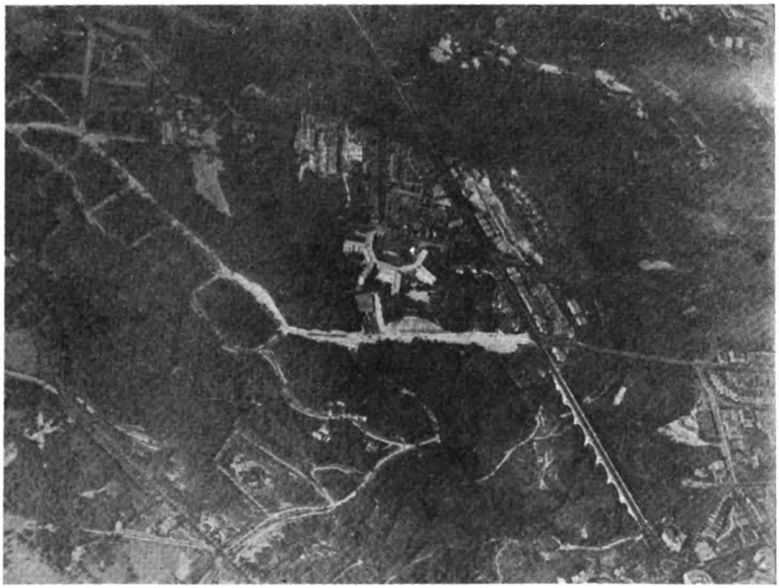
[15] *Pub., Astron. Soc., Pac.,* Vol. xxxii No. 185, 1920, Frontispiece and page 69.

FIG. 9.



Seed's 23 plate stained with pinacyanol and exposed 1/100 second behind F/6.8 lens with orange color filter. Taken simultaneously with Fig. 8 in a multiple lens airplane camera at 10,000 feet altitude.

WASHINGTON, D. C.,
March 30, 1920.

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[16] Lindemann, *Monthly Notices, R. A. S*, 77, pp. 140–151, 1916.

MEASUREMENTS ON THE THERMAL DILATATION OF GLASS AT HIGH TEMPERATURES.

BY

C. G. PETERS and C. H. CRAGOE.*

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VII. SUMMARY.

I. INTRODUCTION.

THE dilatation or dimensional change of glass with variation in temperature affects its thermal resistance, optical properties, and dimensional permanency, and is often the direct cause of difficulties encountered in working glass. The cracking or breaking of articles made of glass, when subjected to sudden temperature changes, is familiar to everyone. When glass is cooled rather quickly from the molten state, strains are set up due to the unequal expansions or contractions of the inside and outer surface.

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If these strains are present in the optical elements of an instrument, they may cause, for example, variation in the refractive index throughout a prism, changes in the curvature of a lens, distortion of the plane surface of a plate, or, in some cases, shattering of the glass. Proper annealing in order to remove the strains is therefore of prime importance.

But no annealing procedure will prevent sealed junctions of glasses having different expansitivities from cracking, or seals between metals and glass, as in the case of incandescent lamps, from presenting serious difficulties unless the expansitivities are nearly the same. The thermal expansitivity of materials known as " glass " depends on their chemical composition which may be extremely complicated and widely different. This necessitates a determination of the thermal expansitivity of each type before the glass can be used effectively and intelligently.

Previous investigations of this property have been confined to temperatures below 100° C., except in a few isolated cases when glasses of unknown composition were taken to 500° C. Since most of the strains in glass are introduced while cooling through the temperature region between 575° and 475° C., as will be shown later, these measurements throw little light on the subject of annealing. They do, however, furnish the rate of expansion below the annealing region. Hence, a brief review of the experimental methods and results of these earlier investigations is appropriate.

1. Micrometric Method.

The greater part of the measurements of thermal expansion have been made by direct observation of the elongation with micrometer microscopes focussed upon lines ruled near the ends of a rod of the material. The elongation ΔL is read directly from the micrometer head, the mean coefficient of expansion for a given temperature interval Δt being given by

$$C = \frac{\Delta L}{L \cdot \Delta t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where L is the original length of the specimen. This method was used by Callender[1] for determining the thermal expansion of hard glass (from a gas thermometer) in the temperature interval 17° to 494° C. A tube 63 cm. × 5 mm. which had been drawn down to a fine capillary at each end was heated in an iron tube gas furnace and then allowed to cool. The measurements were made while the specimen was cooling, because a permanent contraction due to change in structure of the glass took place at the higher temperature. This permanent contraction amounted to .052 per cent. of the length of the tube on the first heating and .026 per cent. on the second. Later in their work on the boiling point of sulphur, Callender and Griffiths[2] determined the thermal expansion of a specimen of glass tubing. A permanent contraction of .056 per cent. of the samples' length was found after the first of five heatings to 400° C. The amount of the contraction decreased upon successive heatings.

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[1] Callender, *Roy. Soc., Trans.*, Vol. 178, p. 161, 1887.

This same effect was observed by Holborn and Grüneisen[3] in their work on the thermal expansion of Jena 59^{III} glass. A rod (5.7×435.8 mm.) when held (in a horizontal electric furnace) at 550° C. for five hours, shortened 0.79 mm. With the temperature held at 500° C., readings taken in one-half hour intervals revealed successive contractions of 0.084, 0.038, 0.036, and 0.018 mm.

Thiesen, Sheel, Sell,[4] and Henning[5] used the micrometric method for the determination of the thermal expansion of thermometer glasses between −191° to +100° C.

M. So[6] investigated the thermal expansion of glass rods, 1 × 300 mm., which were suspended within a vertical, electrically heated, iron tube furnace. A 25-gram weight was fastened to the lower end of the rod and the expansion was measured with micrometer microscope fastened to the same iron pillar which supported the specimen. The expansion of this pillar which might have introduced an error in the determination was not considered. With this apparatus sixteen different kinds of glass were examined, the data and curves for four of which are presented in his paper. These curves show a nearly linear expansion from room temperature to 400° C. With the unannealed glass, a considerable contraction, not shown by the annealed glass, occurred during the next 50° rise in temperature. Beyond 450° a rapid elongation took place, due perhaps to a stretching of the plastic glass by the 25-gram weight.

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[2] Callender and Griffiths, *Roy. Soc. Trans.*, Vol. 182, p. 123, 1891.

[3] Holborn and Grüneisen, *Ann. der Phys.*, Vol. 311, p. 136, 1901.

[4] Thiesen, Sheel, Sell, *Zeits. f. Instk.*, Vol. 16, p. 49, 1896.

[5] Henning, *Ann. der Phy.*, Vol. 22, p. 631, 1907.

[6] M. So, *Tokyo Math. Phy., Soc. Proc.*, Vol. 9, p. 425, 1917–18.

2. Interference Method.

With the interference method originated by Fizeau [7] and later developed by Pulfrich,[8] the change in length of a ring or tripod of the material which acts as a separator for the two interferometer plates is determined from the shift of the straight interference fringes past a reference mark. This method has been employed by many observers for determining the expansion of small samples of various substances over a wide temperature range, but the measurements on glass have been confined to the lower temperature region. Reimerdes [9] reached a temperature of 220° in his work on three samples of Jena glass. Pulfrich, Winklemann, and Weidmann [10] determined the thermal expansion of a number of glasses of known chemical composition over the temperature range 5° to 95° C. This work was part of an extensive and thorough investigation of optical glass carried out at the factory of Schott and Genossen, Jena.

Dorsey [11] measured the expansion coefficient of glass tubing of unknown chemical composition between +20° and −170° C.

3. Summary of Previous Work.

The results of this previous work on the thermal expansion of glass are summarized in Table I. All determinations for temperatures above 100° C. have been recorded, although those for glasses of unknown composition are of little value. The measurements made in the temperature interval of 0° to 100° C., on different types of glass of known composition, were collected from Hovestadt. In a few cases measurements made during the present investigation are, for comparison purposes, also included in this table.

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[7] Fizeau, *Ann. der Phy.*, Vol. 128, p. 564, 1866.

[8] Pulfrich, *Zeits. f. Instk.*, Vol. 13, p. 365, 1893.

[9] Reimerdes, Dissertation Jena, 1896.

[10] Hovestadt, "Jena Glass," p. 234.

[11] Dorsey, *Phys. Rev.*, Vol. 25, p. 88, 1907.

TABLE I.

Observer	Glass	Chem. Comp.		Temp. interval or mean temp.	Coeff. x 10^4
Dorsey (1907)....	Crown glass tubing	Not determined		$-170°$ to $+10°$ C.	.0736
Reimerdes (1896)..	Silicate crown 0.1552	SiO_2	64.72		
		B_2O_3	2.7	37.36°	.0920
		ZnO	2.0		
		AS_2O_5	0.5	92.91°	.1004
		BaO	10.0		
		Na_2O	5.0	151.12°	.1061
		K_2O	15.0	212.34°	.1111
		Mn_2O_3	0.08		
Reimerdes (1896)..	Borosilicate crown 0.627...	SiO_2	68.24	39.0°	.0764
		B_2O_3	10.0		
		ZnO	2.0	55.1°	.0782
		AS_2O_5	0.2	94.9°	.0826
		Na_2O	10.0	151.25°	.0879
		K_2O	9.5	217.45°	.0937
		Mn_2O_3	0.06		
Reimerdes (1896)..	Borosilicate 0.802	SiO_2	70.83	34.8°	.0527
		B_2O_3	14.0		
		Al_2O_3	5.0	92.3°	.0552
		As_2O_5	0.1	148.9°	.0586
		Na_2O	10.0	212.0°	.0612
		Mn_2O_3	0.07		
Callender (1887)...	Thermometer glass	Not determined		17°	.0685
				136°	.0717
				494°	.0814
	Thermometer glass	Not determined		0 to 100°	.0334
Pulfrich (1891)....	Zinc and boric acid glass.....	ZnO	59.0		
		B_2O_3	41.0	10.35° to 92.85°	.0366
Weidmann (1891)	Borate Al. glass	B_2O_3	64.0		
		Al_2O_3	30.0	0° to 100°	.0560
		Li_2O_3	6.0		
Pulfrich (1891)....	Heavy flint	SiO_2	20.0		
		PbO	79.9		
		AS_2O_5	0.1	24.5° to 84.0°	.0935
Pulfrich (1891)....	Phosphate crown	P_2O_5	70.5		
		B_2O_3	3.0		
		K_2O	12.0	17.7° to 92.7°	.0930
		Al_2O_3	10.0		
		MgO	4.0		
		As_2O_5	0.5		
Winklemann (1891)	Soda lime glass	SiO_2	73.1		
		Na_2O	18.5		
		CaO	8.0	0° to 100°	.0968
		As_2O_3	0.3		
		Mn_2O_3	0.1		

TABLE I.—*Continued*

Observer	Glass	Chem. Comp.	Temp. interval or mean temp.	Coeff. x 10^4
Bureau of Standards (No. 18 (1919).........	Soda lime glass	SiO_2 74.0 Na_2O 16.1 CaO 7.2 MgO_2 2.7	21° to 474°	.1020
Pulfrich (1891)....	Barium crown	SiO_2 48.73 B_2O_3 3.0 BaO 29.0 ZnO 10.3 K_2O 7.5 Na_2O 1.0 Mn_2O_3 0.07 As_2O_5 0.4	18.9° to 93.1°	.0789
Bureau of Standards No. 11 (1919).........	Barium crown	SiO_2 47.6 B_2O_3 4.0 BaO 29.2 ZnO 9.9 K_2O 6.0 Na_2O 2.0 As_2O_5 1.4	23° to 499°	0.0908
Straubel (1891)...	Potash lime glass (partly hygroscopic)	SiO_2 69.5 K_2O 25.0 CaO 5.0 As_2O_5 6.3 Mn_2O_3 0.2	0° to 100°	.1016
Winklemann (1891)	Normal thermometer Jena (16 III)	SiO_2 67.3 B_2O_3 2.0 Na_2O 14.0 Al_2O_3 2.5 CaO 7.0 ZnO 7.0 Mn_2O_3 0.2	0° to 100°	.0813
Pulfrich (1893)....	Normal thermometer Jena (16 III)	SiO_2 67.3 B_2O_3 2.0 Na_2O 14.0 Al_2O_3 2.5 CaO 7.0 ZnO 7.0 Mn_2O_3 0.2	14.6° to 92.2°	.0802

TABLE I.—*Continued*

Observer	Glass	Chem. Comp.	Temp. interval or mean temp.	Coeff. x 10^4
Thiesen, Sheel, and Sell (1896)......	Normal thermometer Jena (16 III)	SiO_2 67.3 B_2O_3 2.0 Na_2O 14.0 Al_2O_3 2.5 CaO 7.0 ZnO 7.0 Mn_2O_3 0.2	0° to 100°	.0769
Henning (1907)...	Borosilicate thermometer, Jena (59 III)	SiO_2 71.95 B_2O_3 12.0 Na_2O 11.0 Al_2O_3 5.0 Mn_2O_3 0.05	$-191°$ to $+16°$	.0423
Thiesen, Sheel and Sell (1896)......	Borosilicate thermometer, Jena (59 III)	SiO_2 71.95 B_2O_3 12.0 Na_2O 11.0 Al_2O_3 5.0 Mn_2O_3 0.05	0° to 100°	.0565
Winklemann (1891)	Borosilicate thermometer, Jena (59 III)	SiO_2 71.95 B_2O_3 12.0 Na_2O 11.0 Al_2O_3 5.0 Mn_2O_3 0.05	0° to 100°	.0570
Holborn and Grüneisen (1901)....	Borosilicate thermometer, Jena (59 III)	SiO_2 71.95 B_2O_3 12.0 Na_2O 11.0 Al_2O_3 5.0 Mn_2O_3 0.05	0° to 500°	.0633
Bureau of Standards No. 31 (1919)..........	Borosilicate thermometer, Jena (59 III)	SiO_2 71.95 B_2O_3 12.0 Na_2O 11.0 Al_2O_3 5.0 Mn_2O_3 0.05	38.5° to 488.9°	.0638

With few exceptions these previous investigations show the expansion of glass to be quite regular between −191° and +500° C., so that the various observers were able to represent the coefficient of expansion by the equation $C = a+bt$. In the experiments of Callender and Holborn, however, the samples showed considerable contraction when held at 500° C., and M. So found the expansion of unannealed glass to be quite irregular in that region.

II. PURPOSE.

The investigation described in this paper was undertaken in January, 1918, in connection with the Bureau of Standards production of optical glass. The object was to determine the dimensional changes of glass in the annealing and higher temperature region, to find, if possible, the cause of the strain introduced while cooling through the annealing range, and to bring out any relation which might exist between the dilatation and the heat absorption found by Tool and Valasek [12] in their study of the annealing of glass. Our earlier observations, made on several types of optical glass (presented before the American Physical Society [13] and the Optical Society of America, December, 1918), showed a nearly linear expansion for about the first five hundred degrees. During the next one hundred degrees rise a rapid increase in the expansion occurred, and this was followed at higher temperatures by an apparent contraction. Since then, additional experiments on specimens from different kinds of plate glass, chemical glassware and glass tubing have been made. At the request of several glass manufacturers, some of their products have also been investigated. To the apparent contraction at the higher temperatures, which is evidently a spheroiding due to surface tension, we have given considerable study. A complete report of the entire investigation, to date, together with the chemical analyses of most of the glasses, is presented in this paper.

III. EXPERIMENTAL METHOD.

The Fizeau-Pulfrich method, which has been in use for a number of years at the Bureau of Standards for the measurement of small dilatations, was particularly adapted for this work. The extreme sensitiveness of the interferometer makes it possible to work with a small specimen of the material. Furthermore, the small size of this apparatus simplifies the problems of uniform heating and temperature control. The experimental method was as follows: The straight fringes produced by rays of light reflected from the two interferometer plates were observed with a Pulfrich[14] apparatus. Any elongation (ΔL) in the length (L) of the specimen which forms a separator for the plates, causes a corresponding movement of the interference fringes past a reference mark.

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[12] Tool and Valasek, *Sci. Paper of B. S.*, No. 358.

[13] Peters, *Phys. Rev.*, p. 147, Feb., 1919.

In passing from condition 1 with the plates at a distance apart equal to (L) to condition 2 with the plates at a distance equal to $L + \Delta L$, a band passes the reference mark each time the total number of wave-lengths in the path (double distance) changes by one. This holds both when varying the wave-length and varying the distance between the plates. The number of recorded passages ΔN is then obviously equal to the difference in the number of wave-lengths in the double distance under the two conditions. If λ_v represents the wave-length when a vacuum exists between the plates, then

$$\Delta N = 2\frac{L + \Delta L}{\lambda_v} - 2\frac{L}{\lambda_v}$$

and the mean coefficient of expansion

$$C = \frac{\lambda_v \Delta N}{2L\Delta t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Maintaining a vacuum at high temperatures presents serious difficulty, so in the present work the interferometer was operated in air at atmospheric pressure. The number of passages is then represented by

$$\begin{aligned} \Delta N &= 2\frac{L + \Delta L}{\lambda_2} - 2\frac{L}{\lambda_1} \\ &= 2\left(\frac{L + \Delta L}{\lambda_v} \cdot \frac{\lambda_v}{\lambda_2} - \frac{L}{\lambda_v} \cdot \frac{\lambda_v}{\lambda_1}\right) \\ &= 2\left(\frac{L + \Delta L}{\lambda_v} \cdot n_2 - \frac{L}{\lambda_v} \cdot n_1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

where λ_1 is the wave-length and $\frac{\lambda_v}{\lambda_1} = n_1$, the refractive index of air under condition 1, and likewise, λ_2 the wave-length and $\frac{\lambda_v}{\lambda_2} = n_2$ the refractive index of air under condition 2. Since in a gas the excess of its refractive index over unity is proportional to the density and the latter directly proportional to the pressure P and inversely proportional to the absolute temperature T

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[14] Pulfrich, *Zeits. f. Instk.*, Vol. 14, p. 261, 1898.

$$\frac{n_{p,t}-1}{n_0-1}=\frac{273}{T}\frac{P}{760}$$

n_0 being the refractive index at 0° C., 760 mm. From this relation

$$n_1=1+(n_0-1)\frac{273}{760}\frac{P_1}{T_1}$$

likewise,

$$n_2=1+(n_0-1)\frac{273}{760}\frac{P_2}{T_2}$$

Substituting these values in (3) and collecting, we obtain the increase in length or the dilatation.

$$\Delta L=\frac{\frac{\lambda_v\Delta N}{2}+L\,(n_0-1)\frac{273}{760}\left(\frac{P_1}{T_1}-\frac{P_2}{T_2}\right)}{1+(n_0-1)\,\frac{273\,P_2}{760\,T_2}}$$

Making the denominator of the right-hand member of this equation equal to unity introduced an error less than 0.03 per cent. in our results, since $n_0-1=.0003$ and T_2 was always greater than 273. We may then write

$$\Delta L=\frac{\lambda_v\Delta N}{2}+L\,(n_0-1)\frac{273}{760}\left(\frac{P_1}{T_1}-\frac{P_2}{T_2}\right)\quad\ldots\ldots\quad(4)$$

For the yellow radiation of helium, which was used to illuminate the interferometer, we used $\lambda_v=5877\times10^{-8}$ cm. and for (n_0-1),[15] 0.0002918. To obtain the dilatation in microns per centimetre (4) must be divided by $L\times10^{-4}$ when L is measured in centimetres. To obtain the mean coefficient (4) must be divided by $L\Delta t$, where Δt is the temperature interval (T_2-T_1). This gives,

$$C=\frac{\Delta L}{L\Delta t}=\frac{\lambda_v\Delta N}{2L\,\Delta t}+\frac{273}{760}\,\frac{(n_0-1)}{\Delta t}\left(\frac{P_1}{T_1}-\frac{P_2}{T_2}\right)\quad\ldots\quad(5)$$

IV. DESCRIPTION OF APPARATUS.

1. *The Interferometer and Specimens.*

The two plates, *A* and *B*, of the interferometer, represented in Fig. 1, were made of fused quartz, which can be heated to 1000° without serious injury to the surfaces. The upper surface of the base plate, *B*, was polished true plane, while the lower surface was left in the ground condition to avoid regular reflection from it.

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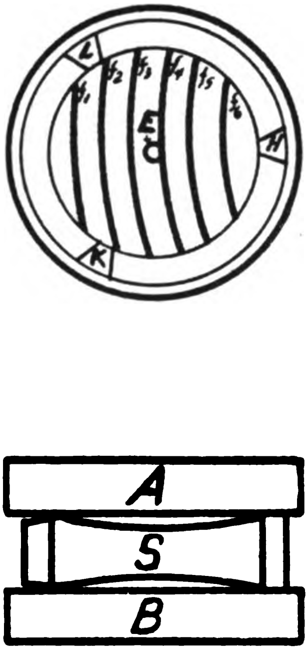
[15] Meggers and Peters, *B. S. Sci. Papers* No. 327, 1918.

Both surfaces of *A* were polished true plane and adjusted to make an angle of 20″ with each other, so that light reflected from the upper surface could be diaphragmed out of the viewing apparatus. The circle *E*, 1 mm. in diameter, ruled on the lower surface of *A*, constituted the reference mark. The separator, *S*, the test piece, consisted of a section of tubing about 1 cm. long and 3 cm. in diameter, or a ring of these dimensions cut from a block of glass, or a tripod made from three sections of glass rod. The end surfaces of the rings were ground until parallel to one another; then part of the material was cut away, leaving three triangular areas as bearing points at each end. These bearing points were adjusted to give about eight interference bands across the plates, which meant that the separation of the plates at *H* was about 2 microns greater than at *K* and *L*. A uniform expansion of the ring caused the fringe system to move toward the left, while any unequal expansion at the three points, *H*, *K*, and *L*, produced also a rotation and change in width of the fringes.

FIG. 1.

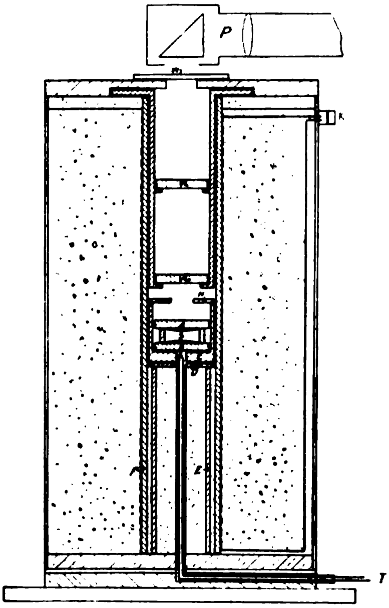


Interferometer.

2. *Furnace.*

A sectional view of the electric furnace containing the interferometer is given in Fig. 2. A porcelain tube, *F*, 5 cm in diameter and 30 cm long, wound spirally with a heating coil, *K*, is mounted vertically in a sheet-iron jacket and surrounded with insulating material. A smaller porcelain tube, *E*, which extends from the base to about 3 cm from the centre of the furnace supports the porcelain disc, *D*, the intervening space below this disc being filled with an insulating material. A porcelain cup, *C*, with a cover, *H*, acts as a container for the interferometer *A S B* and can be lowered into the furnace by a platinum wire until it rests on the disc *D*. The upper end of the furnace is closed by another porcelain tube containing two fused quartz windows, W_1 and W_2. A sheet of asbestos board with a glass window, W_3, covers the entire furnace.

FIG. 2.



Electric furnace.

The Pulfrich apparatus which contains a helium lamp for illuminating the interferometer and the optical arrangement necessary for measuring the displacement of the fringes is represented by *P*.

A small double bore porcelain tube containing a calibrated *Pt Pt–Rh* thermocouple *T* passes through the base of the furnace, the disc *D*, and the bottom of the container *C*, the thermocouple junction being adjusted so that it nearly touches the lower interferometer plate *B*. By this arrangement the interferometer can be removed from the furnace without interfering with the adjustment of the thermocouple. The heating coil *K*, which has a resistance of 17 ohms, is connected in series with an ammeter and suitable rheostat to a 110-volt battery circuit. A current of 4 ampères is sufficient to heat the furnace from room temperature to 600° or 700° C. at an average rate of 4° C. per minute.

Two thermocouples were brought in through the top of the furnace, one of them fastened in contact with the inside and the other with the outside of the ring *S*. With a current of 4 ampères heating the furnace at the rate of 4° C. per minute, these thermocouples showed that the temperature of the outside of the ring was from 8° to 10° above that of the thermocouple *T* and the temperature of the inside of the ring about 3° C. above that indicated by *T*. When the current was reduced to a magnitude which held the temperature of the furnace constant, it required about five minutes for the ring and the thermocouple *T* to come to the same constant temperature.

This difference between temperature indicated by the thermocouple and the actual temperature of the sample would have an effect upon any absolute determination of the coefficient of expansion, but we were concerned chiefly with what happened to glass in the high temperature region, so that the relatively small lag of the thermocouple was often neglected.

V. EXPERIMENTAL PROCEDURE.

1 Optical Adjustments.

The sample having been prepared in the shape of a ring or tripod as described in III, the distances between the bearing points of the two ends were carefully measured with a micrometer. When the micrometer measurements showed these to be the same, the ring was placed between the interferometer plates. If, then, the fringes were found to be distinct and of the desired width, no further adjustments were necessary, but if they were too narrow, a slight change in the distance between one set of bearing points brought them to the required width.

With these adjustments made, the interferometer was placed in the cup, *C*, and carefully lowered into the furnace. The Pulfrich apparatus was then brought into adjustment and the fractional part of the fringe width represented by the distance from the reference mark to the first fringe to the left accurately measured. In Fig. 1 this fraction would be about 0.7.

2. *Temperature Regulation and Observations.*

After being lowered into place, about twenty minutes was allowed for the sample and container to assume the temperature of the furnace. In some cases, before proceeding with the observations, the furnace containing the interferometer was heated to 100° to 200° C. and allowed to cool overnight, so that air films which might separate the plates from the bearing points would thus be driven off or at least reduced. The current from the 110-volt battery was then turned on and regulated to about 4 ampères. The temperature of the furnace which rose at the rate of 4° per minute was carried to as high a temperature as the nature of the sample would permit. The expansion of the sample due to this heating caused the interference fringes to move across the field, the number that passed the reference mark being counted. As every fifth fringe passed the mark, both the time and the reading of the thermocouple as shown by the potentiometer were recorded. With all the glasses investigated, the expansion up to about 500° C. was quite uniform and regular. But at this temperature, which varied with different glasses, the movement of the fringes became more rapid, denoting an increase in the rate of expansion of the glass. With a further rise in the temperature, the fringes continued to shift at the increased rate, then slowed down, stopped, and began to drift in the opposite direction, indicating a shortening of the sample. The heating continued for 20° to 30° beyond this point was accompanied by a rapid increase in the contraction of the sample. At this higher temperature the glass was undoubtedly in a plastic condition, in most cases fusing onto the quartz plates. The current was reduced at this point and the sample allowed to cool. When the temperature had dropped to the region where the rapid expansion had taken place, the glass hardened, broke away from the plates, thus throwing the interferometer out of adjustment and rendering further observations impossible. Pieces of quartz were often torn from the surfaces of the plates and the bearing points broken from the ring when the fracture took place.

In a few cases, observations were made at intervals of 50° to 100° C., with the temperature held constant long enough to eliminate the lag in the temperature of the thermocouple behind that of the ring. The results thus obtained were not sufficiently different from those observed under steady heating conditions to warrant losing the additional time required to follow this procedure.

VI. EXPERIMENTAL RESULTS.

1. *Reduction of Observations.*

The dilatation or change in length of the sample in microns per centimetre $\frac{\Delta L}{L}$ was obtained from the equation (4).

$$\frac{\Delta L}{L} = \frac{\lambda_v \Delta N}{2L} + \frac{273}{760} (n_0 - 1) \left(\frac{P_1}{T_1} - \frac{P_2}{T_2} \right)$$

The value of the first term was obtained by multiplying the number of fringes which passed the reference mark by the wave-length in vacuum and dividing by twice the length of the sample. The values for $\frac{273 (n_0 - 1) P}{760 \ T}$ with a 1 cm separation of the plates were computed for temperatures from 0° to 1000° C. and pressures from 730 to 760 mm., and these plotted against the temperature. The value of the last term for any temperature interval was then read directly from the curve. No correction needed to be made for the small variation of the atmospheric pressure during the experiment.

The mean coefficient of thermal expansion over the temperature interval Δt was obtained from equation (5).

2. *Values for Mean Coefficient of Expansion.*

The mean coefficient of linear expansion of the different glasses as computed from equation (5) are collected in Table II. Column 1 gives the number used throughout to identify the samples. The expansion of all the glasses, as shown in the curves that follow (Figs. 3–16), was very regular during the first 350° to 500° C. of temperature. The mean expansions in this region are given in Column 4 and the temperature interval in Column 3. Upon heating to higher temperatures, the rate of expansion changed. With well-annealed glasses, the rate increased rapidly; with unannealed, it first decreased for a time, then increased. The region in which this change occurred we shall call the *critical region* or the *annealing temperature* because it agrees almost exactly with the annealing temperature determined by Tool and Valask. Above the annealing temperature the samples expanded at the increased rate during the next 100° rise in temperature. Column 5 gives the temperature interval of most rapid expansion and Column 6 the corresponding expansion coefficients, which are from two to seven times the coefficients of Column 4. Above the upper temperature of Column 5, due to the softening of the material, a rapid decrease in the rate of expansion took place, fol-

TABLE II.

Mean Coefficient of Linear Expansion.

(1) No.	(2) Designation	(3) Temp. Interval °C.	(4) $C \times 10^4$	(5) Temp. Interval °C.	(6) $C \times 10^4$
1–2	Barium flint B. S. 145	22 to 494	0.088	519 to 550	0.331
3	Plate, American	19 to 461	.099	563 to 579	.373
4	Plate, American	20 to 508	.108	540 to 560	.401
5	Plate, American	23 to 494	.101	564 to 583	.460
6	Plate, German	21 to 496	.099	564 to 589	.477
7	Plate, French	21 to 513	.094	597 to 613	.424
8–15	Light crown, B. S. 103	24 to 422	.104	494 to 507	.548
9	Light crown, B. S. 20	22 to 426	.102	502 to 522	.555
10	Borosilicate crown, B. S. 94	22 to 498	.090	539 to 562	.393
11	Barium crown, B. S. 87	23 to 499	.090	589 to 610	.649
12	Medium flint, B. S. 110	23 to 402	.097	452 to 478	.396
13	Light flint, B. S. 188	22 to 451	.088	494 to 512	.347
14	Light flint, B. S. 33	23 to 420	.076	495 to 511	.292
15	Light crown, B. S. 103	24 to 422	.104	496 to 505	.550
16	Commercial glass	23 to 445	.107	510 to 534	.309
17	Commercial glass	22 to 452	.103		
18	Commercial glass	22 to 464	.102	523 to 552	.318
19	Commercial glass	21 to 474	.102	544 to 557	.316
20	Macbeth Evans flask	22 to 449	.069	567 to 586	.454
21	Pyrex	21 to 471	.036	552 to 571	.151
22	Schott & Genossen flask	19 to 414	.056	540 to 562	.404
23	Soda tubing	21 to 372	.120	506 to 525	.234
24	Lead tubing	21 to 338	.091	464 to 483	.236
25	Lead tubing	21 to 345	.096	457 to 477	.225
26	Fluorite tubing	22 to 364	.098	510 to 551	.284
27	Lead tubing	21 to 333	.097	430 to 469	.227
28	Vacuum tube tubing	23 to 405	.116	509 to 545	.205
29	Fusing in glass, German	23 to 383	.090	456 to 481	.283
30	Fusing in glass, Corning	22 to 376	.083	460 to 485	.258
31	Jena 59 III = to axis	38 to 522	.064	564 to 600	.437
32	Jena 59 III ⊥ to axis	22 to 491	.062	562 to 603	.452

lowed by a contraction. The region where the change from expansion to contraction occurred is represented by the maximum ordinate of the curves, and we shall call the corresponding abscissa the *softening temperature.*

3. Chemical Composition.

Table III gives the chemical composition of the different glasses, those for the samples marked with an asterisk being based upon chemical analysis, while the rest give merely the batch composition.

TABLE III.

Chemical Composition.

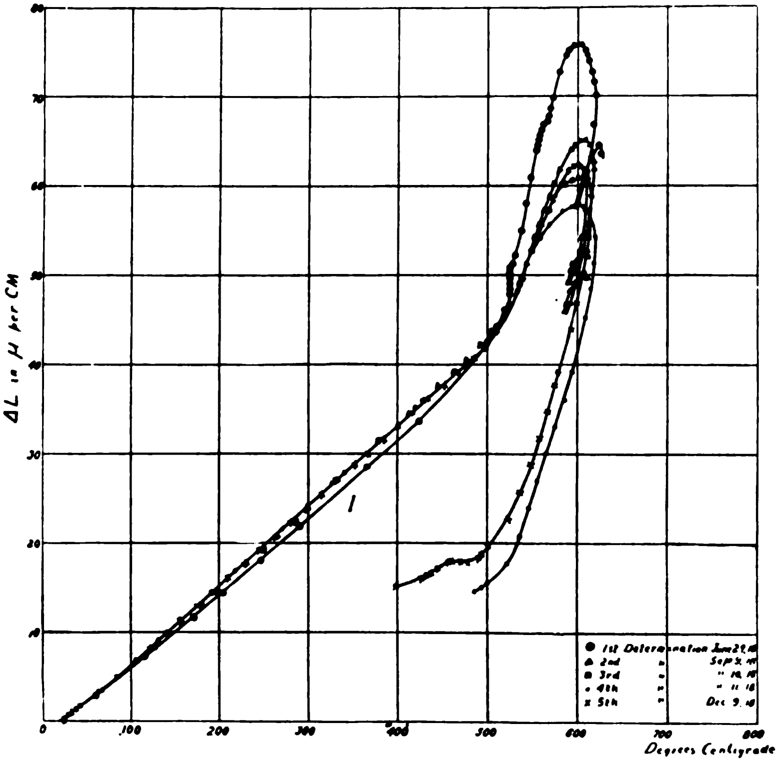
No.	SiO_2	CaO	Na_2O	As_2O_5	MgO	K_2O	Sb_2O_3	Fe_2O_3	Al_2O_3	ZnO	B_2O_3	BaO	PbO
1–2	58.8	...	1.7	...	...	8.3	...	...	...	2.5	1.7	14.3	12.7
3*	71.3	12.5	13.4	0.9	0.3	.1	...	.1					
4*	72.0	10.2	16.2	.3	...	.3	...	.1					
5*	71.5	12.4	13.7	.1	...	...	...	.1					
6*	72.5	11.5	13.8	...	...	.2	.5	.1					
7*	71.3	14.7	10.4	...	...	.4	...	.4					
8	67.0	...	12.0	.4	...	5.0	...	...	...	1.5	3.5	10.6	
9	68.6	...	12.0	.2	...	5.0	...	...	...	1.0	3.5	9.7	
10	66.5	...	9.8	.2	...	5.9	...	...	...	2.0	7.8	7.8	
11	47.6	...	2.0	1.4	...	6.0	...	...	...	9.9	4.0	29.2	
12	44.3	3.0	3.5	.2	...	5.0	...	...	...	...	...	...	44.0
13	53.9	2.0	1.0	.3	...	7.6	...	...	...	...	...	...	35.2
14	54.0	2.0	1.0	.3	...	6.0	...	...	...	...	...	...	36.7
15	67.0	...	12.0	.4	...	5.0	...	...	...	1.5	3.5	10.6	
16	74.0	5.8	17.1	...	3.1	...							
17	74.3	5.5	17.2	...	3.0	...							
18–19	74.0	7.2	16.1	...	2.7	...							
20*	73.0	.7	10.8	...	4.3	.3	.6	.4	1.0	5.6	3.6		
21*	80.5	.3	4.4	.7	.1	.2	...	.3	2.0	...	11.8		
22*	64.7	.6	7.5	.1	.2	.4	...	.3	4.2	10.9	10.9		
31*–32*	71.9	...	11.0	...	...	...	...	...	5.0	...	12.0		

4. Final Values of $\frac{\Delta L}{L}$

In the following curves (Figs. 3 to 16, inclusive) the dilatation ΔL (in microns (μ) per centimetre) of samples of glass about 1 cm. long are plotted as ordinates and the temperature (in degrees centigrade) as abscissæ. The initial temperature was very nearly 20° C. in each case.

Five separate determinations on a sample No. 1 of Barium Flint (B. S. 145) optical glass are represented in Fig. 3. In the first run the furnace was held at constant temperature for 15 minutes before each measurement was made, while in the others a steady heating was maintained throughout and the observations taken when every fifth fringe passed the reference mark. Below 510° C. the expansion was nearly linear and identical for the different determinations. Between 510° and 530° C. (the critical range), the rate of expansion increased rapidly. This rapid expansion continued up to 580° to 590° C. With further heating the rate decreased, contraction setting in at 608° C. At 620° C. the current was cut off and the sample allowed to cool. In three cases, after cooling to about 590° C., then heating again, the sample expanded up to 610°, then contracted as before.

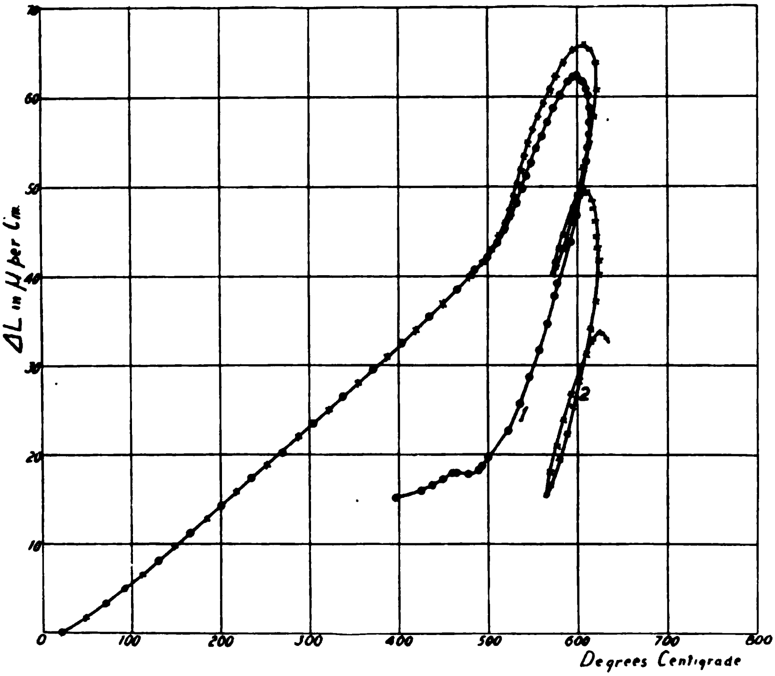
Fig. 3.



Thermal dilatation of optical glass.

In the fourth and fifth determinations, observations were made of the contraction during the cooling process. When the temperature reached the critical region, the ring suddenly broke away from the quartz plates, throwing the interferometer out of adjustment and making further measurements impossible. In the first curve for all points below 500° C., the sample stopped expanding several minutes before the thermocouple came to constant temperature. The lag of the thermocouple caused this curve then to fall to the right of the rest, which were obtained from steady heating. At 523° C. on this curve the ring continued to expand even after the thermocouple indicated a slight decrease in the temperature of the furnace. At 565° C. the behavior was again similar to that at the lower temperatures.

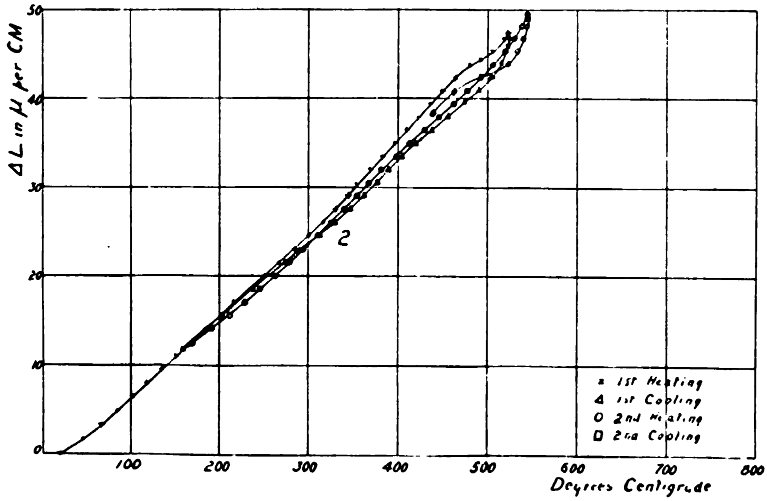
FIG. 4.



Thermal dilatation of barium flint glass, B.S. 145.

Fig. 4 shows the expansions of two specimens of Barium Flint (Nos. 1–2) which were cut from two blocks out of the same melt. Both samples show exactly the same expansion from 20° to 510° C., a rapid increase at about 515° C., then a contraction at about 600° C. No. 2, on being cooled to 570° C., then reheated, showed a contraction at approximately the same temperature as before. On being cooled a second time and a larger heating current applied, the contraction took place at a slightly increased temperature. For Fig. 5, sample No. 2 was heated to the critical temperature, then allowed to cool slowly. It returned to its original length at 160° C., the separation of the two curves being due to the lag of the thermocouple. On reheating it was taken above the critical temperature and allowed to cool relatively rapidly. The sinuous shape of the second cooling curve shows the irregular contraction in passing through the critical range. This was accompanied by an unequal contraction of different parts of the ring as indicated by the rotation of the interference fringes.

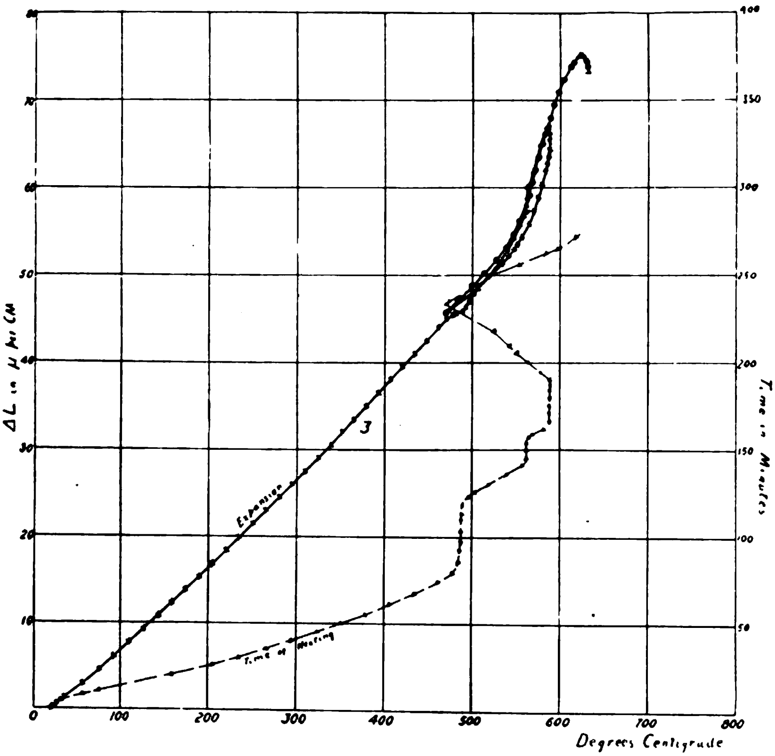
FIG. 5.



Thermal dilatation of barium flint glass, B.S. 145.

The expansion of a specimen (No. 3) of "Pittsburgh plate glass" is shown in Curve 3, Fig. 6. The curve labeled "Time of Heating," which is plotted with time in minutes as ordinates—magnitudes indicated on the right—shows the time that elapsed while the sample was being heated, cooled or held at a constant temperature. Raising the temperature at a uniform rate from 20° to 480° C. in 80 minutes caused an expansion of 45 microns. The temperature of the sample was then held constant at 488° C. for 35 minutes, during which time no expansion or contraction occurred. During the next 20 minutes, the temperature was increased to 560°, where it was again held constant for 15 minutes. Here an elongation of 3 microns was observed. During the next 10 minutes the temperature was raised to 588°, and upon being held constant for 25 minutes, the sample showed a contraction or shortening of 3 microns. The heating current was then reduced so that the sample cooled to 470° in 45 minutes. Upon

FIG. 6.

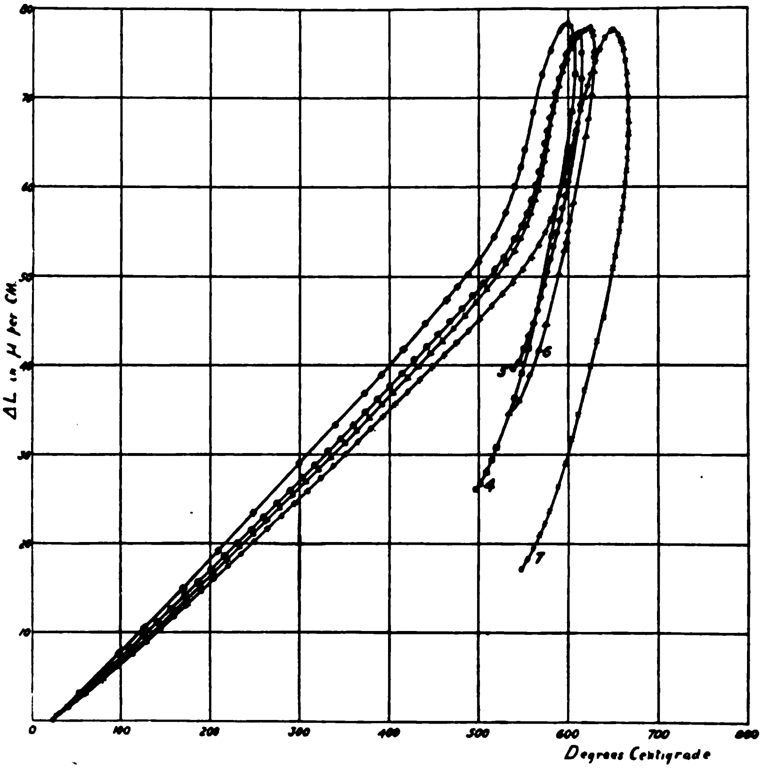


Thermal dilatation of plate glass.

increasing the current, the sample expanded over that part of the curve represented by the circles, until the temperature reached 624° C., where contraction began.

The expansions of four different kinds of plate glass, Nos. 4 to 7, are represented by the curves in Fig. 7. These samples were heated at the same rate, and all showed practically linear expansion up to 500° C. All the samples showed above the annealing region a rapid increase in expansion, which above the softening temperature was followed by a contraction. The temperatures at which these variations took place were different for the different glasses. In each case observations were made on the cooling curve until the ring broke away from the quartz plates. It should be noticed that with all four samples this rupture occurred in the critical region. These samples fused so firmly to the quartz plates that the bearing points of the ring were broken and pieces of quartz 3 mm. across were sheared from the surface of the plates.

FIG. 7.



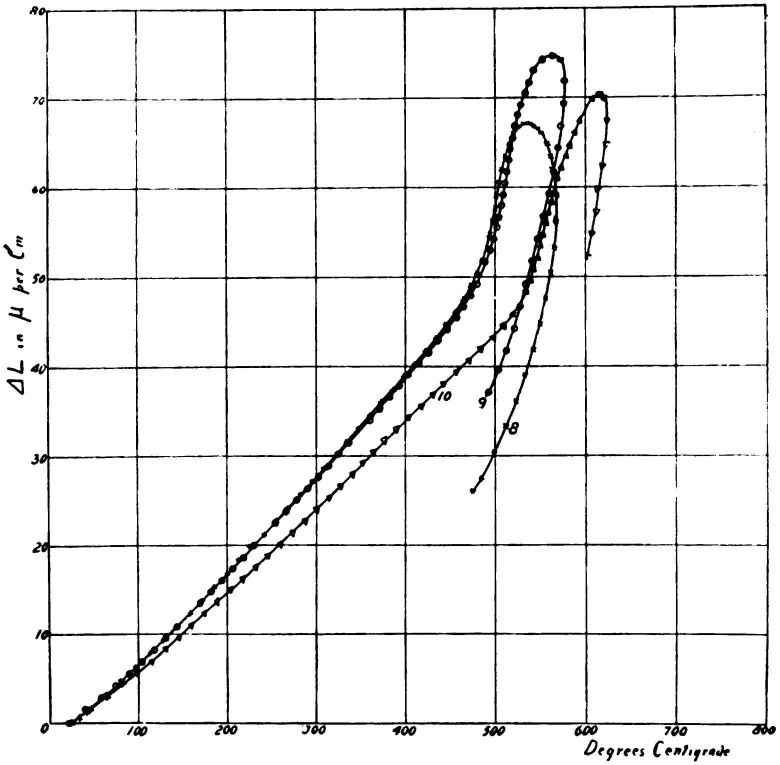
Thermal dilatation of plate glass.

By referring to Table II, it will be noticed that the samples 3, 5, and 6, which show almost the same expansion, are very nearly identical in their chemical composition. No. 4, which has a higher Na_2O and lower CaO content, has a higher rate of expansion and a lower critical temperature, while in the case of

No. 7 with a lower Na_2O and higher CaO content the reverse is true.

Figs. 8, 9, and 10 represent the expansion of nine different kinds of optical glass made at the Pittsburgh Laboratory of the Bureau of Standards. These curves show the rapid increase in the rate of expansion, followed by a contraction above the softening temperatures. The critical temperature varied from 440° C. in the case of No. 12 to 570° C. with sample No. 11. Likewise, the softening temperature varied in the same sequence from 520° C. with sample No. 12 to 660° C. with No. 11. In the case of sample No. 12, Fig. 9, the effect of the fracture in cooling was so slight as to permit observations to be taken down to 280° C. The cooling curve is very nearly parallel to the heating curve,

FIG. 8.

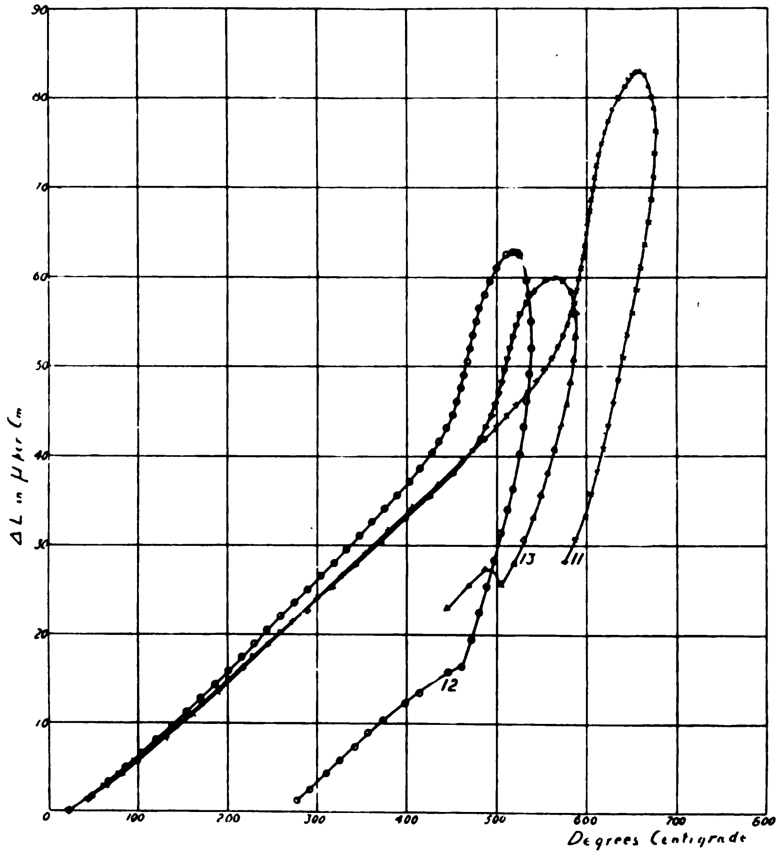


Thermal dilatation of optical glass.

and the separation of the two shows a permanent shortening of the sample.

Observations made of the expansion of four samples of commercial glass are represented by the curves in Fig. 11. These

FIG. 9.

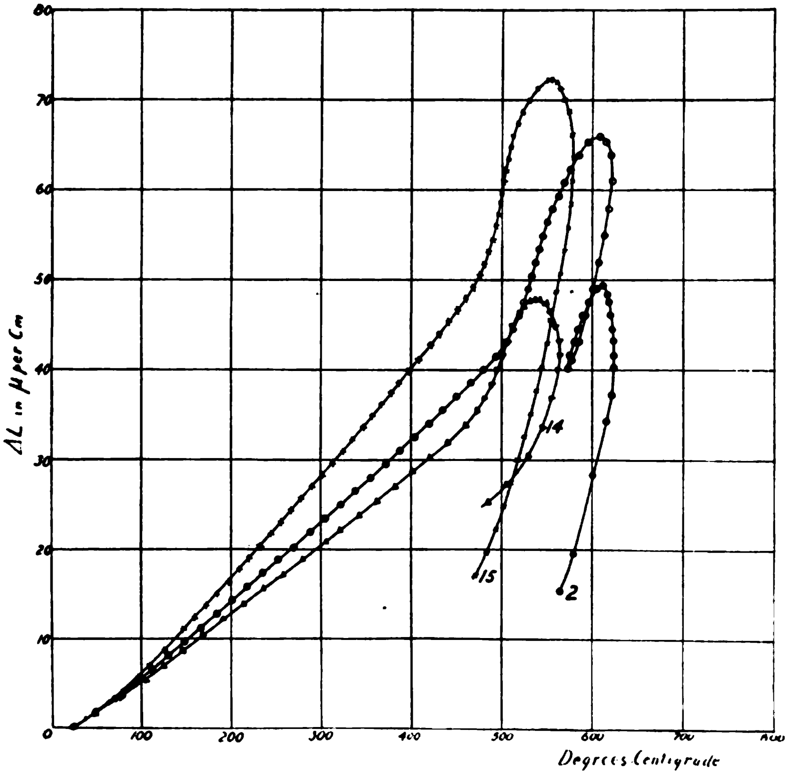


Thermal dilatation of optical glass.

show the same general characteristics as those of previous experiments. In the case of sample No. 16, the lag of the thermocouple was increased by a much higher rate of heating. Sample No. 17 was brought to a steady temperature at 530° C. and heated at a relatively slow rate during the remainder of the experiment.

Glasses 20, 21, and 22, Figs. 12 and 13, are samples of chemical glassware obtained by cutting ring sections from Florence flasks. Sample No. 21 (Pyrex) has high silica content and a low expansion, but at the higher temperatures it follows the same general characteristics of other kinds of glass. No. 20 (Macbeth Evans) showed a slight decrease in the expansion between 450° and 550°, followed by an exceptionally large and rapid increase in expansion. This decrease in the rate of expansion in the temperature region 400° to 500° C., which is doubtless due to the unannealed condition of the glass, is also in evidence in the curves of Nos. 23 and 24, but in these the increase in expansion is moderate as compared with No. 20. The first measurements on the expansion of No. 22 Jena are shown by the crosses in Fig. 13. In this case the glass was heated to 445° C., held constant at this

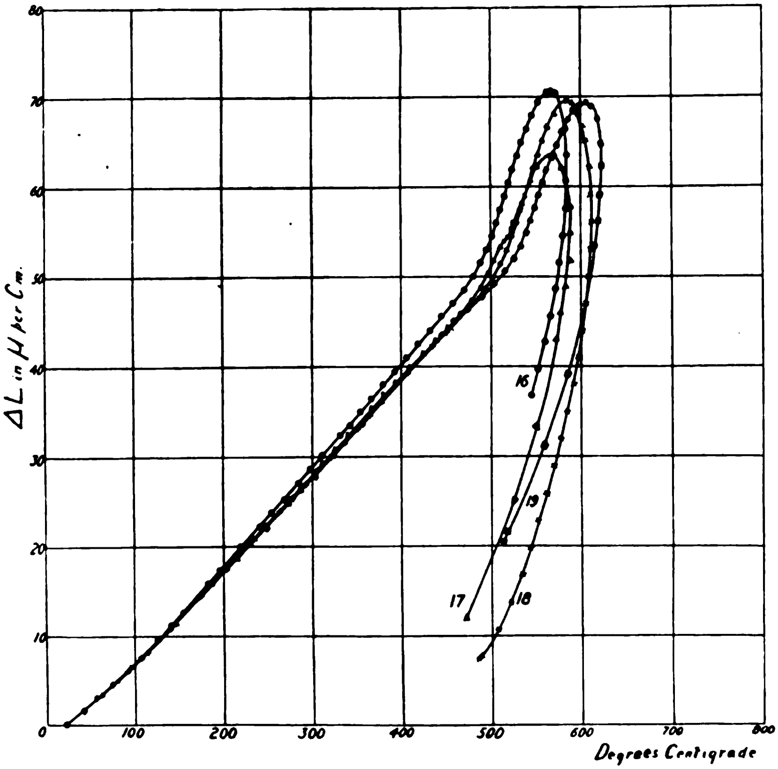
FIG. 10.



Thermal dilatation of optical glass.

point about twenty minutes, then allowed to cool. The separation between the heating and cooling curve represents the double lag of the thermocouple behind the sample. The second measurements, made the following day, are represented by the circles. Upon passing the temperature 445° C., the sample contracted slightly, then continued to expand at its previous rate. No. 23 was cut from a length of soda glass tubing and No. 24 from lead glass tubing, both had been cooled quickly in air and put under high strain, the existence of which is indicated by the decrease in their expansion rate in the critical region. With No. 24, the heating was continued approximately 70° beyond the softening point, the rise in temperature being accompanied by a rapid increase in the rate of contraction. Of those represented by Fig. 14,

FIG. 11.

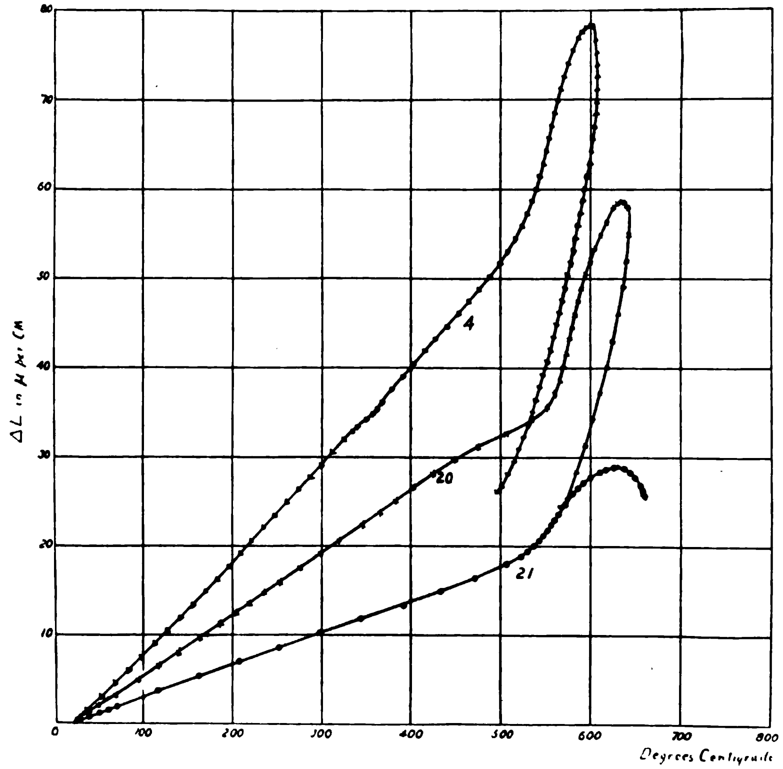


Thermal dilatation of commercial glass.

No. 25 was cut from a length of soda lime tubing of 21 per cent. lead content, No. 26 from white fluorite tubing of low lead content, and No. 27 from a potash lead tubing of 20 per cent. lead. All of these samples showed first a decrease, then an increase in the rate of expansion in the critical region.

The curves in Fig. 15 represent the thermal expansion of glass

FIG. 12.

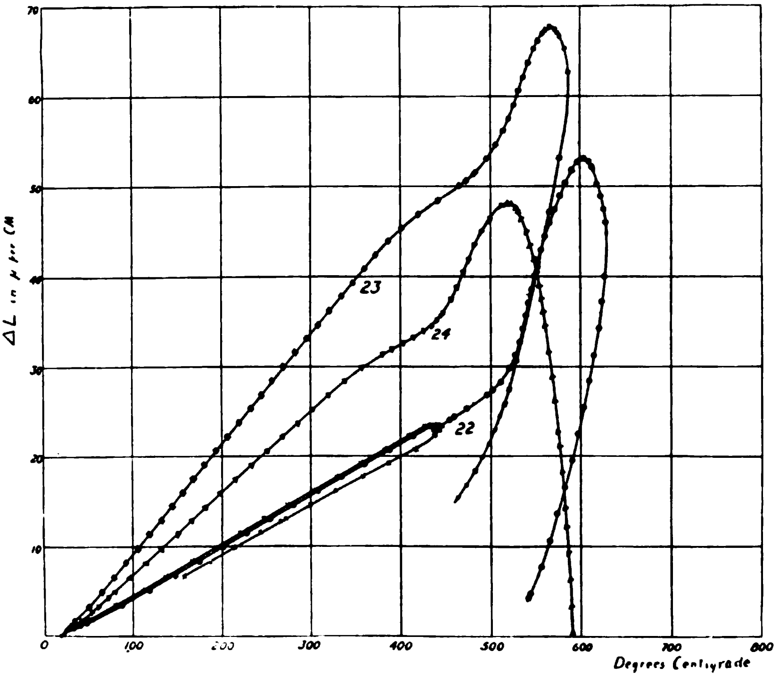


Thermal dilatation of chemical glassware.

(Nos. 28, 29, 30) recently used in making rare gas lamps at the Bureau of Standards. Many of the lamps cracked near the platinum electrodes, and it was thought that the "Corning" sealing in glass might have a different thermal expansion from the German glass previously used. Both kinds were found, however, to have almost identical thermal expansion, as shown by curves 29 and 30. The curves indicate that these were lead glasses very similar to No. 24. The expansion of the tubing used for making the lamps, however, as shown by curve 28, has a thermal expansion, annealing temperature and softening temperature very different from these which account for the breakage of the lamps.

An interesting condition is shown by the results of Fig. 16. Curve 31 represents the longitudinal, and 32 the radial expansion of sections cut from the capillary of a Jena 59^{III} thermometer,

FIG. 13.



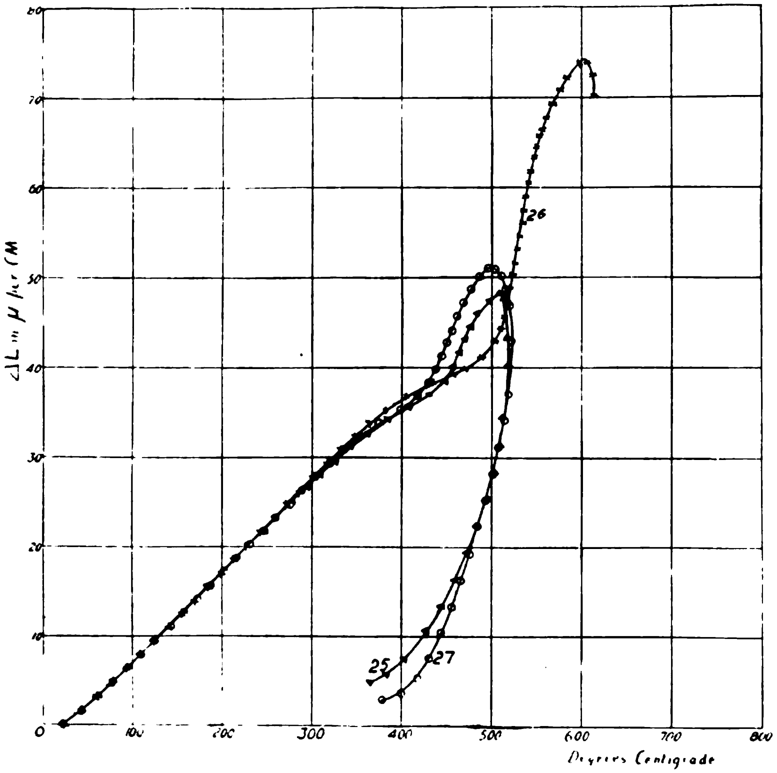
Thermal dilatation of chemical glassware and commercial glass.

graduated to read 560° C. They show that 500° C is the upper limit at which this kind of a thermometer could be used without injury. As stated before, Holborn and Grüneisen found a permanent contraction in this glass after it had been heated to 550° C. Our curves show that a definite contraction is to be expected, for 550° C. is certainly within the critical region. With both samples heated at the same rate, No. 32 continued to expand 40° beyond the softening temperature of No. 31. The permanent shortening of 20 microns in No. 31 after it was heated to 640° C. is shown by the separation of the heating and cooling curves.

5. *Dimensional Change Above the Critical Temperature.*

The measurements described in the preceding sections show that below the critical range, glass has a regular expansion and contraction, retains its dimension when held at a given temperature for a long period and returns to its original length on being carried through a temperature cycle. That is, it may be considered as a solid. Upon passing through the critical region, that is, above the first bend in the curves, it seems to soften or become viscous. When held at a constant temperature in the critical region, there is first an expansion, then a slow contraction. This took place at 525° in sample No. 1 and 560° in No. 3. When held constant at points above the critical region, the rate of contraction increases with the temperature. The highest part of

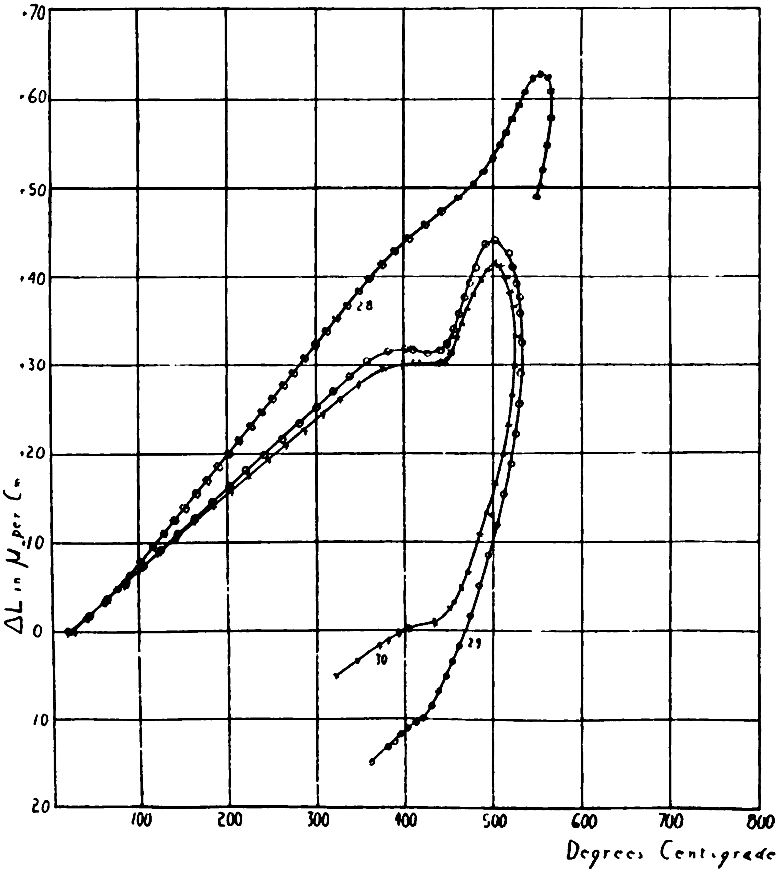
FIG. 14.



Thermal dilatation of commercial glass.

the curves which we have called the softening region gives the temperature at which the contraction, due to the softening, is just sufficient to overcome the expansion due to the heating. At higher temperatures contraction predominates, as indicated by the rapid drop in the curve.

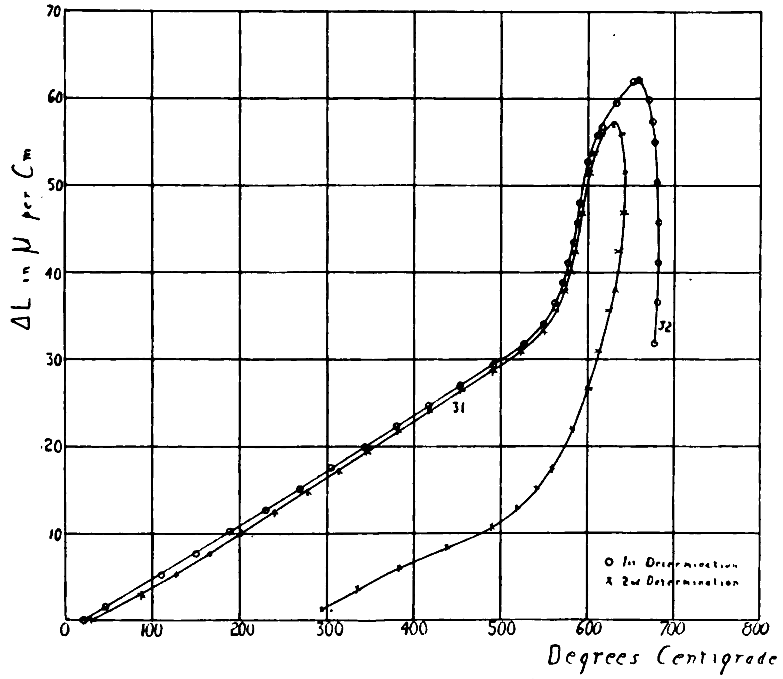
FIG. 15.



Thermal dilatation of vacuum tube glass.

To obtain some data on the actual rate of contraction above the critical temperature, the dimensional change at constant temperature of Nos. 2 and 11 was measured, and the results plotted in Fig. 17. The ascending parts of the curves represent the expansion during heating, while the descending parts show the contraction when the temperature was held constant for the indicated number of minutes. From determinations made in this way the rates of contraction of Nos. 2, 8, and 11 were computed for the different temperatures, and the results plotted in Fig. 18, with rate of contraction in microns per minute as ordinates and the temperature in degrees centigrade as abscissæ. A comparison of these curves with those representing the thermal expansion

FIG. 16.

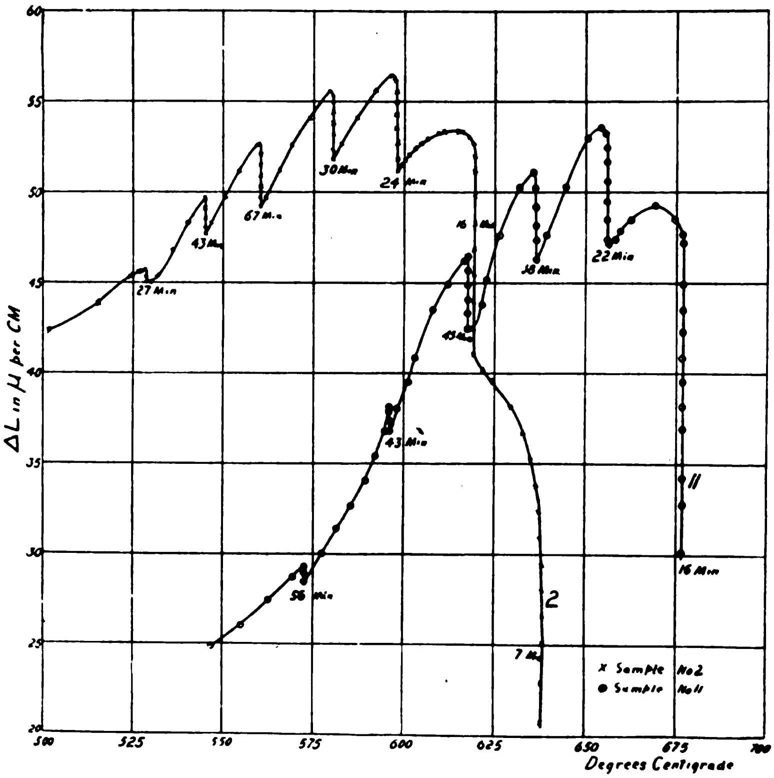


Thermal dilatation of thermometer glass.

reveals the fact that the contraction rate is very small, about 0.01μ per minute, in the critical region, that it increases slowly in the range containing the high expansion rate and rapidly beyond the temperature corresponding to the maximum of the expansion curve.

Considering the nature of this contraction we find that it is irreversible, that is, the sample undergoes a permanent decrease in length as shown by the cooling curves. This shrinking may be due to several causes. The most natural explanation is that the sample settles under its own weight, plus that of the cover plate. Since, however, the weight was only a few tenths of a gram per square mm. and since samples of different sizes and shapes did not differ as much as they should, we were led to believe that the weight was not the only cause of the contraction. To test this point further, R. L. Coleman and J. O. Eisinger, of the expansion laboratory of the Bureau of Standards, determined the expansion of a glass rod 30 cm. long and 1 cm. in diameter, by a micrometric method. For this purpose, the sample was mounted horizontally on a sheet of asbestos inside a porcelain tube and heated in the electric furnace described and used by Souder and Hidnert.[16] Their measurements were made with micrometer

FIG. 17.



Contraction of glass at constant temperature.

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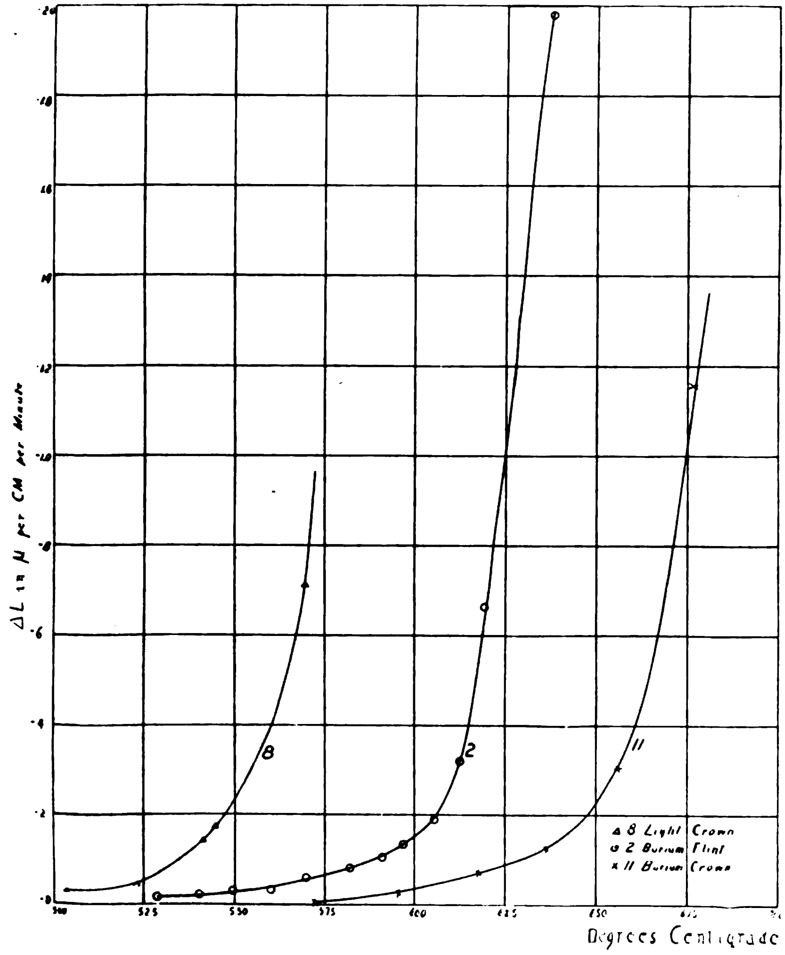
[16] Souder and Hidnert, *B. S. Sci. Papers* No. 352.

microscopes sighted upon wires hung from transverse grooves cut near the ends of the rod. In the first determination, Fig. 19, the sample showed a decrease in the rate in the critical range, found in unannealed glasses, followed by a rapid expansion rate. At the softening temperature, it stopped expanding and contracted slightly. On returning to the original temperature, notwithstanding its horizontal position, a permanent contraction of 0.6 mm. was registered. In the second determination heating the

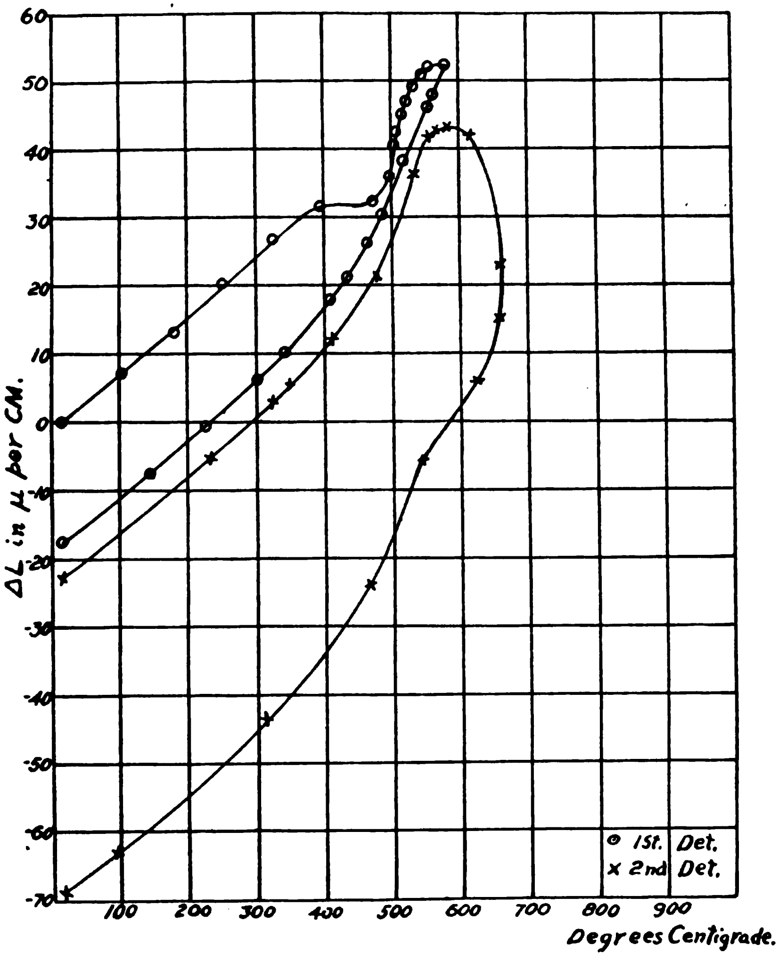
FIG. 18.



Rate of contraction of glass.

sample 100° above the softening point caused it to contract rapidly and show a permanent shortening of 1.5 mm. Thus, in this experiment, although the rod was free to expand in a horizontal direction, uninfluenced by the force of gravity, a contraction took place similar to that obtained with the interference method where the sample supported its own weight. The observed contraction must, therefore, it seems, have been due to either shrinking of the

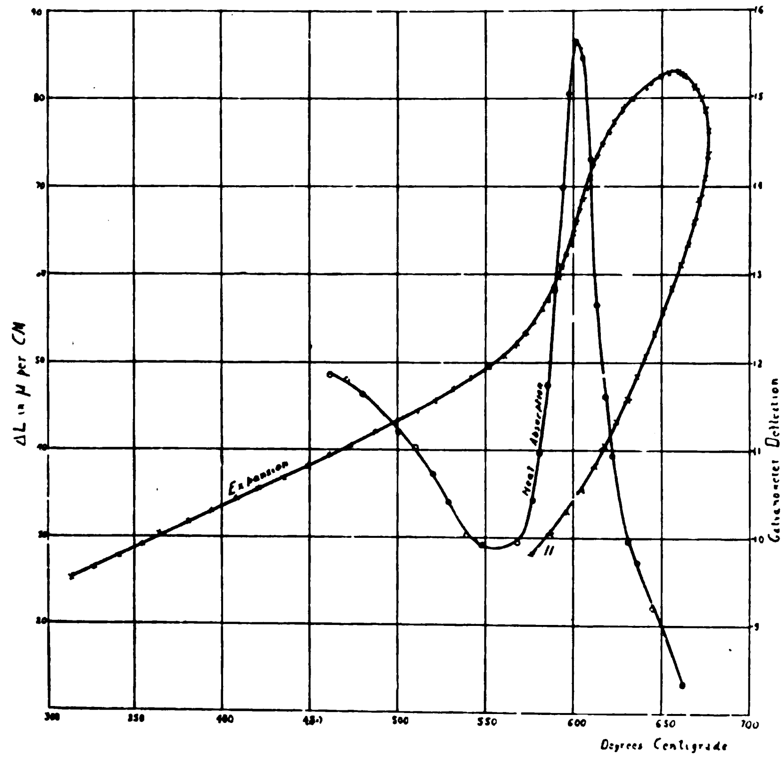
Fig. 19.



Thermal dilatation by the micrometric method.

material or spheroiding by surface tension. In the first case, there should be a decrease in volume, and in the second an increase in diameter along with the decrease in length. To test for a possible volume contraction, blocks were made from glasses Nos. 1 and 10 and their volumes, before and after heating, determined by E. L. Peffer, of the volumetric laboratory of the Bureau of Standards.

FIG. 20.



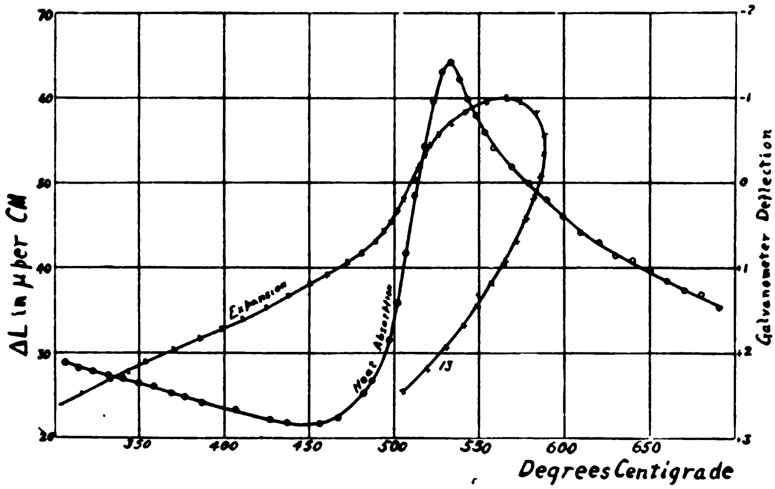
Thermal dilatation and heat absorption of barium crown glass, B. S. 87.

The results given in Table IV show that heating No. 1 for one hour at 640° C., which is 30° above its softening temperature, caused a slight increase in the volume. A second heating at 650° C. for two hours produced a small permanent decrease in the volume. Sample No. 10 showed a slight increase in volume after being held for 2.5 hours at 650° C., which is 40° above its softening temperature.

TABLE IV.

Sample	Temp.	Time	Original Vol. cc	Final Vol. cc	Original Wt. gms.	Final Wt. gms.
No. 1	640°C	1 hr.	31.554	31.614	94.185	94.182
No. 1	650°C	2 hrs.	31.614	31.604	94.182	94.182
No. 10	650°C	2.5 hrs.	28.799	28.864	75.301	75.300

FIG. 21.



Thermal dilatation and heat absorption of light flint glass. B. S. 188.

No measurable change in the weight of the samples was produced by this treatment.

These results prove that no appreciable permanent volume contraction is produced in the glass by heating it to the temperatures mentioned above, for here a volume contraction comparable with the measured linear contraction (3 per cent. in two hours) would have caused a decrease in the volume of the block of about 3 c.c. The permanent linear contraction, which Callender ascribed to a change in structure of the material, must, therefore, be due to a spheroiding of the sample. To test this last assumption, three sections 1 cm. long and 0.5 cm. in diameter were cut from the capillary of a thermometer and their longitudinal expansion, represented by curve 31, Fig. 16, measured. The radial expansion of three more sections from the same capillary is represented by curve 32. These curves show that with both sets of samples, heating at the same rate the softening temperature of No. 32 was 40° higher than for No. 31. With the longitudinal samples, the surface tension works with gravitational force, while with the radial samples it works against that force, causing the expansion at the higher temperature. Of course, when the radial samples reached their softening temperature, the surface tension was insufficient to support the weight.

6. Comparisons of Dimensional Changes and Heat Absorption.

A comparison of the heat absorption curves of Tool and Valasek with the thermal expansion curves for the same glasses (11 and 13) is readily made from Figs. 20 and 21. The ascending part of the curves marked "Heat Absorption" indicates an increase in the absorption of heat by the glass. The heat absorption commences in the critical region, increases in the range of rapid expansion and falls off in the softening region. An equally good agreement between the two effects was found for the other glasses listed in Table II. W. P. White[17] observed a difference in the specific heats of glass above and below this region, and it is probable that other properties would show a change. In order to test the effect on the refractive index, we are now preparing to measure, by an interference method, the refractive index of glass plates in the critical and higher temperature regions. The cause of these phenomena may be the melting of some of the constituents of the glass, or the melting of the compound as a whole. Whether or not we conclude that glass melts or starts to melt in the critical region depends mostly on our definition of solid and liquid. Feild and Royster[18] give melting points, for their silicates, between 1200° and 1400° C. They define the melting point as the temperature at which the fluidity becomes zero. Our measurements on the rate of softening (Figs. 17 and 18) show that the glass starts to flow at the critical region, about 500° C., the rate increasing with the temperature. These values can be expressed, as were those of Tool and Valasek obtained from the strip-bending experiment, by Tyman's equation $T_\theta = T_0 e^{\left(\frac{\theta_0 - \theta}{K}\right)}$

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[17] White, *Am. Jour. Sci.*, Vol. XLVI, Jan., 1919.

[18] Feild and Royster, *Bureau of Mines, Tech. Paper 189*, 1918.

where T_θ and T_o are the relaxation times at temperature θ and θ_o and K is a constant. Since the fluidity increases expotentially, any determination of the melting point will depend on experimental method and sensitivity of the apparatus. Instead, therefore, of speaking of a melting point, it seems better to say that glass has a softening range, which can be located by the variations in the different properties.

7. *Annealing Temperature.*

The rate of expansion of glasses Nos. 23 to 30 decreased on approaching the criticial region, and Nos. 29 and 30 show even a contraction. These samples, Nos. 23 to 30, were all taken from badly strained pieces of commercial tubing, and it is very probable that the decrease in expansion rate was due to the presence of these strains, because the well-annealed glasses Nos. 1 to 22 do not show a decrease. The same behavior was observed by M. So (*loc. cit.*) for annealed and unannealed rods 1 to 2 mm. in diameter. For unannealed glass, he found contractions in the critical region, but these disappeared after the glass had been annealed.

It seems probable that most of the strains that exist in a piece of glass are introduced when it cools through the critical region. When cooled rapidly the outer shell reaches the critical temperature first and hardens or solidifies while the inner part still contracts at the rapid rate which puts the outer shell under a high tension. When the sample is reheated the outer shell softens first and the tension of the inner part causes a contraction.

From these considerations it seems that most of the strain introduced depends upon the magnitude of difference in the expansion coefficients above and below the critical region, and the rate of cooling. Samples 11 and 22, for which the differential expansion is large, require much more careful cooling than Nos. 14 or 23, for which it is small. For real thorough annealing the glass should be held for a long time at a temperature near the middle of the critical region. For more rapid annealing the sample can be held above the critical region, then cooled very slowly through that region and more rapidly the remainder of the way.

The annealing and critical ranges of several optical glasses, as determined by Tool and Valasek, are compared in Table V with those obtained from our expansion measurements. Their values for the heat absorption range are given in Column 3, A and B, and the annealing temperature and the upper limit for annealing, *i.e.*, the highest temperature to which the glass can be heated without noticeable deformation, in Column 6.

TABLE V.

(1) No.	(2) Name	Critical Range of Heating						Annealing Range			
		(3) Heat Absorption (Tool)		(4) Rapid Expansion		(5) Critical Region		(6) Opt. Method (Tool)		(7) Expansion Method	
		A ±10°C.	B ±5°C.	A	B	A	B	Ann. Temp. ±15°C.	Upper Limit ±15°C.	Ann. Temp. ±10°C.	Upper Limit. ±10°C
1	Barium Flint No. 145.....	520	560	520	570	500	525	515	550	510	580
9	Light Crown No. 20......	495	525	500	525	460	500	480	530	480	530
10	Borosilicate Crown No. 94	515	565	535	570	515	535	525	550	525	590
11	Light Barium Crown No. 87	575	605	585	610	545	585	570	610	565	620
12	Medium Flint No. 110.....	455	485	450	680	430	450	460	510	440	500
13	Light Flint No. 188.....	485	525	490	525	460	490	485	510	475	540

Column 4 represents the range of rapid expansion corresponding to the straight part of our curves; Column 5, the critical range; and Column 7, the annealing temperature and upper limit for annealing, which were estimated from the expansion curves and the rate of softening of the glass. The values of Column 3 and 4 show the close agreement between the heat absorption region and the range of rapid expansion. The annealing temperature of Column 7 which is the mean of the critical region Column 5 agrees very well with the annealing temperature obtained by Tool and Valasek by the crossed nicol method. Their values for the upper limit are about the same as we obtain from the softening curves. Both methods, therefore, give the location of the annealing temperature and upper limit for annealing. The expansion measurements give also the coefficients of expansion above and below the critical region, which make it possible to estimate the thermal resistance of the glass and the magnitude of the strains that a given annealing process would produce.

VII. SUMMARY.

A review of the previous researches on the thermal expansion of glass shows that most of the determinations were confined to the temperature region below 100° C., except in a few cases where glasses of unknown composition were taken to 500° C. In this investigation undertaken in connection with Bureau of Standards' production of optical glass, the dimensional changes of various glasses in the annealing and higher temperature region were determined. Upon requests from several glass manufacturers, a number of their commercial products were also investigated.

The Fizeau-Pulfrich interferometer is particularly well adapted for this work, since the extreme sensitiveness of the interferometer makes it possible to use small specimens and the small size of this apparatus simplifies the problems of uniform heating and temperature control. Using straight fringes produced by the yellow radiation from a helium source the dimensional change for a given temperature interval was given by the number of fringes which passed the reference mark on the front interferometer plate. Observations were made in temperature regions between 20° and 650° C. on 32 different kinds of glass. Their dimensional changes are represented by curves which show that the glass passes through a critical expansion region, in which the expansion rate increases by two to seven times. This critical region which for any one glass does not exceed 40°, was found as low as 400° C. with Nos. 24 and 30 and as high as 575° C. with No. 11. About 75° above the critical region, the glass softens and contracts. A comparison of results shows that the heat absorption observed by Tool and Valasek occurs in the same temperature region as the critical change in the expansion.

From these determinations of the dimensional changes of glass, the following information is obtained: the thermal expansion above and below the critical temperature, the temperature for most careful and thorough annealing, the upper limit for rapid annealing and the region where careful cooling is essential.

WASHINGTON, D. C.

ATOMIC THEORY AND LOW VOLTAGE ARCS IN CÆSIUM VAPOR.*

BY

PAUL D. FOOTE and W. F. MEGGERS.

THEORETICAL.

On the basis of several possible theories of atomic structure, it is shown that the *normal* operation of an arc below ionization might result in the excitation of a single line spectrum, a single series spectrum, or a group spectrum consisting of certain lines of different series. This latter conclusion follows from an extension of Bohr's theory. Thus, if inelastic electronic-atomic impact occurred, resulting in the ejection of an electron to the pth ring, the electron in returning to the nth ring or equilibrium may produce any combination of lines presented by interorbit transitions within this range, the single line spectrum being a special case where $p = n + 1$.

A simple explanation is offered of fluorescence phenomena in vapors of the alkali metals.

A mechanism of absorption of radiation is described and the theory proposed by K. Compton that the ionization of an atom below the ionization potential may be explained by absorption of radiation arising in other atoms from electronic-atomic impact of insufficient energy to ionize is further discussed. This hypothesis suggests that vapors of the alkali metals may be so stimulated that the first and second subordinate series lines, instead of the principal series, tend to become absorption lines.

EXPERIMENTAL.

The cæsium spectrum was photographed for various accelerating voltages from λ3878 to λ9208 by use of dicyanin-stained plates. The sensitivity of the plates was investigated by density measurements of the spectrum of a black body having a known energy distribution. The general characteristics of the plates were determined and all lines of the cæsium spectrum were reduced to an absolute scale of intensity by means of density measurements made with a microphotometer and consideration of the plate sensitivity. No evidence of group or single series spectra could be obtained. Thus the ratio of intensities of the first and second lines of the principal series, both of which should appear in a single series spectrum or above 2.7 volts in a group spectrum, rapidly approaches infinity as the accelerating potential in the arc is decreased. This ratio is 350 in a 7-volt arc, 2100 at 4 volts, 10,500 at 3.4 volts and as near infinity as can be measured at 2.8 volts. Similarly the intensity ratio of either $1.5s-2p_1$ or $1.5s-2p_2$ to any other line approaches infinity at low voltage, proving for the first time the existence of a single line spectrum rather than a single series or group spectrum; in the case of cæsium the doublet λ8521 and λ8943.

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* Abstract of forthcoming Scientific Paper of Bureau of Standards.

The doublet $1.5s-2p$ is alone produced under excitation of 1.5 to 3.9 volts accelerating field. The intensity of both of these lines gradually increases approximately proportional to the total number of electrons reaching the anode until the ionization potential is reached. At this point a pronounced decrease in intensity of these two lines occurs amounting to the factor one third.

This decrease takes place at the voltage at which the complete line spectrum is produced and is readily explainable on the basis of Bohr's theory; in fact, it affords a strong argument for this theory. Thus the lines $1.5s-2p_{\frac{1}{2}}$ are the result of inelastic collision with electrons having velocities between 1.45 and 3.9 volts, but as the latter voltage is exceeded electrons, which at a slightly lower velocity would have given rise to $1.5s-2p_{\frac{1}{2}}$, now produce the complete series spectrum; and any line of the series $1.5s-mp$ where $m > 2$ is necessarily excited at the sacrifice of $1.5s-2p_{\frac{1}{2}}$.

Above a certain voltage, the intensity of any line per unit number of electrons reaching the anode attains a saturation value, in agreement with the quantum hypothesis which requires that the number of quanta radiated be proportional to the number of collisions, and hence (approximately) to the number of electrons present.

Curves are given showing the relative intensities of the prominent cæsium lines at various voltages. The ratio of intensities of the components of the first doublet of the principal series λ8521/λ8943 is constant and equal to 1.5 from 1.5 volts to 120 volts.

The cæsium arc of the type employed does not rectify alternating current of 120 volts.

Sodium and potassium occurring as an impurity of the cæsium similarly exhibited the single line or doublet spectrum $1.5s - 2p_{1,2}$ below their respective ionization potentials.

Only two types of inelastic impact between electrons and atoms of the alkali metal vapors occur, at potentials known as the resonance and ionization potentials and given by the quantum relation $h\nu = eV$ where $\nu = 1.5s - 2p_1$ and $\nu = 1.5s$ respectively.

BUREAU OF STANDARDS.

DISPERSION IN OPTICAL GLASSES. I.

BY

F. E. WRIGHT.
Geophysical Laboratory.

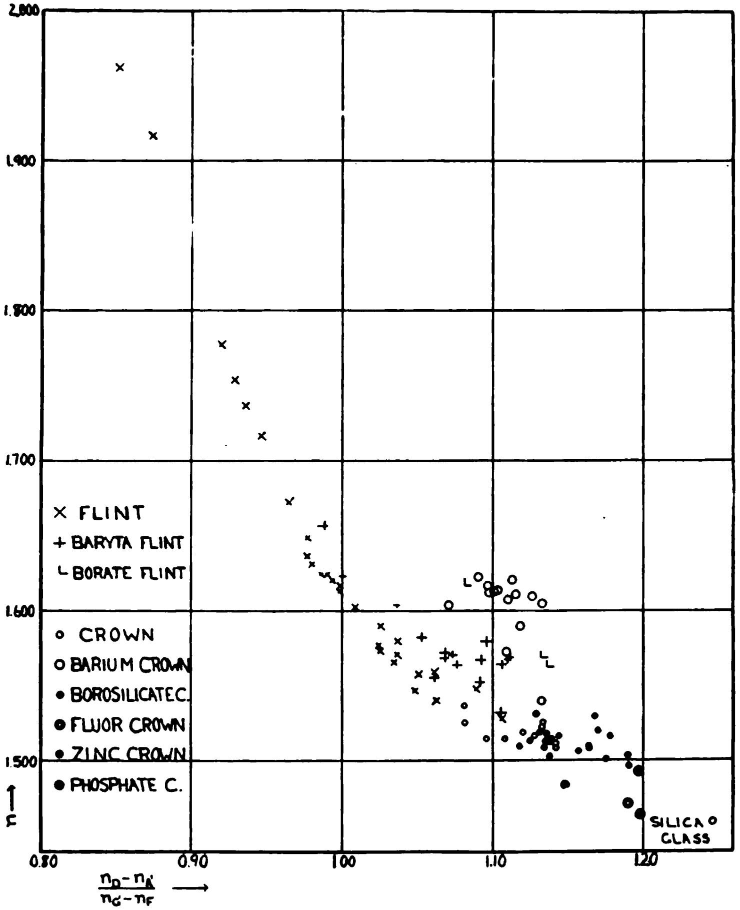
A FUNDAMENTAL requirement for optical glass is high transparency and freedom from color; this precludes the presence of an appreciable absorption band in the visible spectrum, the ultraviolet, or the near infra-red; in other words, the dispersion curve throughout the visible spectrum is restricted to that portion of the curve which is distant from an absorption band. Most optical glasses are, moreover, essentially silicate solutions in which silica plays a dominant rôle and is ordinarily, in volume at least, present in excess. This fact finds expression in the dispersion relations of optical glasses.

Following the suggestion of Abbe, it is customary for glassmakers and for the designers of optical systems to express the optical constants of a glass by stating its refractive index, n_D, for sodium light; its partial dispersions between the *A′*, *C*, *D*, *F*, and *G′* spectrum lines ($n_C - n_{A'}, n_D - n_C, n_F - n_D, n_{G'} - n_F$); its mean dispersion, ($n_F - n_C$); its partial dispersion ratios (the lengths of the different parts of the spectrum in terms of the mean dispersion); and its ν-value which is in effect the excess refractivity expressed in terms of the mean dispersion, $\left(\frac{n_D - 1}{n_F - n_C}\right)$. The reciprocal of the ν-value is called the dispersive power. The importance of the partial dispersion ratios and the ν-values arises from the fact that the degree of achromatism attainable by a combination of two glasses depends directly on the difference between the partial dispersion ratios and inversely on the difference between the ν-values.

These relations are so important that optical glasses are commonly studied from this viewpoint; changes in the characteristics from glass to glass are commonly represented by plotting the partial dispersions or the ν-value against the refractive index, n_D. This method serves to place the different types of glasses into different fields and affords an opportunity for a study of their optical characteristics.

Another method of representation (Fig. 1) is to plot the refractive index against the ratio $\frac{n_D - n_{A'}}{n_{G'} - n_F}$, which expresses the relative length of the red to that of the blue end of the spectrum. In this plot those glasses which have the same relative lengths of the spectrum but different refractive indices lie on the same ordinate;

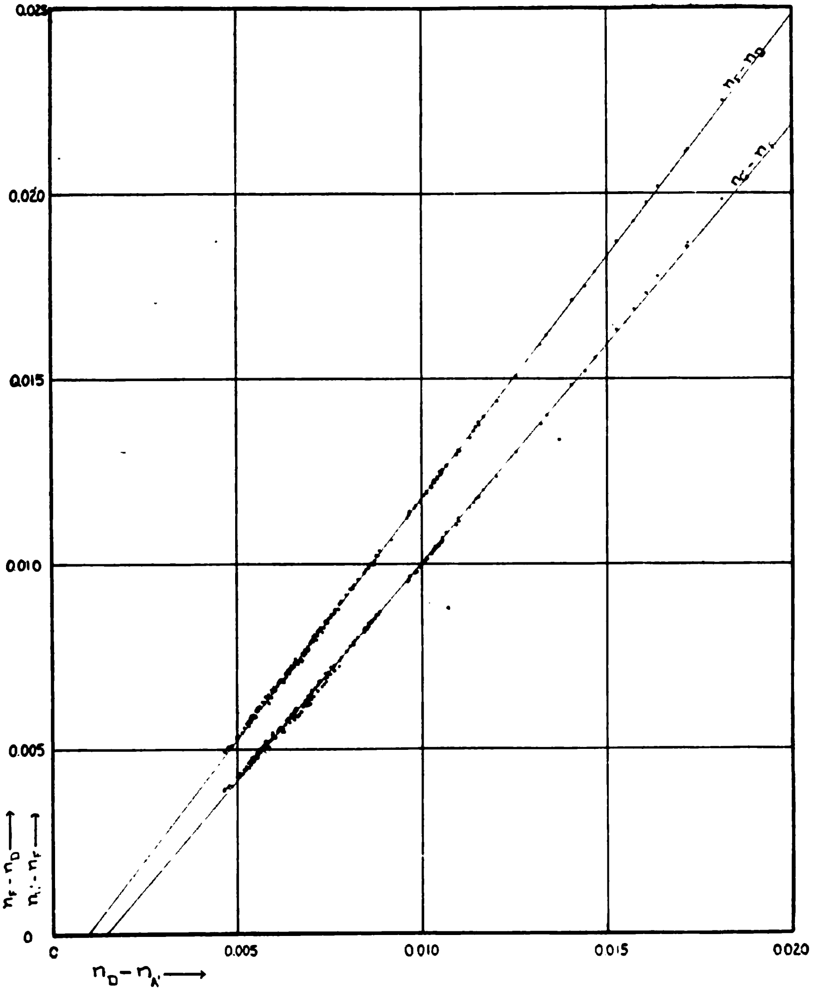
FIG. 1.



In this figure the ratio $\frac{n_D - n_{A'}}{n_{G'} - n_F}$, which expresses in effect the length of the red end of the spectrum to that of the blue end, is plotted against the refractive index n_D for a series of different types of silicate optical glasses.

it is, therefore, a simple matter to select from this diagram those glasses which have similar dispersion relations. From a knowledge of the chemical compositions it is, moreover, not difficult by means of this diagram to interpolate and to determine the approximate composition of a glass which shall have a given refractivity and dispersivity.

FIG. 2.



In this figure the partial dispersions $n_F - n_D$ and $n_{G'} - n_F$, of all silicate optical glasses listed by Parra-Mantois and by Schott, are plotted as ordinates against the partial dispersion $n_D - n_{A'}$ as abscissæ. The result in each case is a straight line.

If now we disregard the refractive index and consider only the dispersions, certain remarkable relations appear which are interesting and important both to the glassmaker and to the lens designer; incidentally from these relations new types of dispersion formulas will be developed; but their present forms are only tentative and subject to modification later.

A number of different dispersion formulas have been proposed, and express with more or less success the dispersion relations in optical glasses. Some of these are deduced from general theoretical considerations; others, especially the Hartmann dispersion formula and the new formula proposed by Nutting, are empirical equations containing two or more constants. It is not the purpose of this paper, however, to discuss these dispersion formulas, but rather to emphasize certain dispersion relations which are characteristic of all optical glasses.

The data on which the studies are based are contained in the lists of optical glasses manufactured by Parra-Mantois, of Paris, and by Schott, of Jena, as well as the 27 glasses measured with extreme precision by J. W. Gifford.[1] The total number of glasses thus considered is 316, and every type of silicate optical glass is included in the list.

In Fig. 2 the partial dispersions of the silicate glasses, n_F-n_D and $n_{G'}-n_F$, respectively, are plotted against the partial dispersion, $n_{G'}-n_{A'}$; the plot includes the entire foregoing list of 316 glasses, except the densest flint S 386 of Schott. The result in each case is a straight line; the parts of each line covered by the several types of glasses are indicated in the figure. The distance of the individual points from the straight line is ordinarily less than one unit in the fourth decimal place. Except for the dense barium crown glasses the distance is commonly only a few units in the fifth decimal place. If n_1, n_2, n_3, n_4 and n_1', n_2', n_3', n_4' are the refractive indices of two optical glasses for any four spectral lines 1, 2, 3, 4, respectively, then the straight lines in Fig. 2 prove that the following relations are valid (within one or two units in the fourth decimal place for each refractive index):

$$\frac{(n_4 - n_3) - (n_4' - n_3')}{(n_2 - n_1) - (n_2' - n_1')} = a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

or

$$n_4 - n_3 = a\left[(n_2 - n_1) + b\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the dispersion curve for one typical glass is known, then the whole series of dispersion curves are definitely fixed to the degree of accuracy indicated by Fig. 2, and the dispersion for any other optical glass can be read off directly if only one partial dispersion is known. This conclusion is significant and implies that two constants suffice to express the refractivity relations (to the fourth decimal place) of an optical glass in the visible spectrum. The slight departures from the straight line relation can be taken into account by the introduction of a third constant, as will be indicated in a later paragraph. This straight line relation imposes strict limitations on the glassmaker; once a partial dispersion of a glass is given the dispersion curve for the visible spectrum follows; the glassmaker is able as a result to change only the refractive index and to hold the dispersion relations constant or *vice versa*. It is possible to prepare a table or a graph listing, in an ascending series of values for any given partial dispersion, the corresponding partial dispersions throughout the visible spectrum. These values, however, may differ from the observed values by one or two units in the fourth decimal place. To this degree of accuracy only do the foregoing statements apply.

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[1] *Proc. Roy. Soc.*, 87, 190–191, 1912.

It is possible to determine graphically the positions of the dispersivity lines for different partial dispersions, and thus by interpolation to ascertain the course of the dispersion curve of an optical glass throughout the visible spectrum provided a single partial dispersion is given. This signifies that, having given any two refractive indices of an optical glass, the observer can write down directly its refractive index for any other part of the visible spectrum with a maximum error of one or two in the fourth decimal place.

The same result can, of course, be obtained by computation, which is not difficult because of the straight line relations between the partial dispersions.

In equation (2) the two constants *a* and *b* fix the position of any given partial dispersivity line. These constants vary with the partial dispersions under consideration and are evidently functions of the wave-lengths between which the partial dispersions are taken. Equations (1) and (2) may be written for two glasses, n_r and n_r',

$$\frac{(n_4 - n_3) - (n_4' - n_3')}{(n_2 - n_1) - (n_2' - n_1')} = \frac{f(\lambda_4) - f(\lambda_3)}{f(\lambda_2) - f(\lambda_1)} = a \quad . \quad . \quad . \quad (3)$$

$$n_4 - n_3 = \frac{f(\lambda_4) - f(\lambda_3)}{f(\lambda_2) - f(\lambda_1)} \left[(n_2 - n_1) + b \right] \quad . \quad . \quad . \quad . \quad (4)$$

in which $f(\lambda_r)$ is some function of the wave-length. From equation (3) it is evident that

$$(n_2 - n_1) - (n_2' - n_1') = C\left[f(\lambda_2) - f(\lambda_1)\right] \quad . \; . \; . \; . \; . \quad (5)$$

wherein C is a constant.

Let $n_2 = n_1 + dn$ and $n_2' = n_1' + dn$; equation (5) may then be written $dn - dn' = c[\mathrm{f}(\lambda_1 + d\lambda) - f(\lambda_1)]$ or

$$\frac{dn}{dx} - \frac{dn'}{dx} = Cf'(\lambda)$$

This relation is valid if

$$\frac{dn'}{d\lambda} = cf'(\lambda_1) + F(\lambda_1) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

wherein $F(\lambda)$ is some function of (λ) and is the same for all glasses. Therefore,

$$\begin{aligned} n_1 &= c.f(\lambda_1) + \int F(\lambda_1)d\lambda + d \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7) \\ n_2 &= c.f(\lambda_2) + \int F(\lambda_2)d\lambda + d \\ n_3 &= c.f(\lambda_3) + \int F(\lambda_3)d\lambda + d \end{aligned}$$

For convenience let $f_1 = f(\lambda_1)$ and $F_1 = \int F(\lambda_1)d\lambda$. Then by subtraction we obtain

$$\begin{aligned} n_2 - n_1 &= c(f_2 - f_1) + F_2 - F_1 \\ n_3 - n_1 &= c(f_3 - f_1) + F_3 - F_1 \end{aligned}$$

Eliminating c from these equations we find

$$\frac{n_2 - n_1}{f_2 - f_1} - \frac{n_3 - n_1}{f_3 - f_1} = \frac{F_2 - F_1}{f_2 - f_1} - \frac{F_3 - F_1}{f_3 - f_1} \quad . \; . \; . \; . \quad (8)$$

Equation (7) states that the expression for the refractive index may be considered to consist of three parts: the first, which considers only the excess refringence relations; the second, which expresses the course of the standard dispersion curve, and the third, which fixes the datum level of refringence. Inasmuch as the visible spectrum covers at most only an octave in wave-lengths, it is not difficult to find an empirical expression for $f(\lambda)$; thus the expressions are valid $f(\lambda) = e^{1/\lambda^2}$; or $\frac{1}{\lambda^2 - 0.0438}$; or $\frac{1}{(\lambda - 0.100)^2}$; or $\frac{1}{\lambda - 0.230}$; or $\frac{1}{\lambda^{2.6}}$; of these the first two are the most satisfactory. It is evident that there are many mathematical functions which represent the data well. These functions are tested by use of equation (3) applied to two different glasses. For this purpose the partial dispersions of the 316 glasses were considered in groups for each partial dispersion and the groups treated by the method of the zero sum recently proposed by N. Campbell[2] as an improvement on the method of least squares. It is significant that the values of a and b of equation (2) obtained separately from the Gifford list of glasses, the Schott list, and Parra-Mantois list differed slightly from the values obtained from the entire list. The two sets of average refractive index values obtained from the total list (158 glasses in each set) are:

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[2] *Phil. Mag.* 39, 177, 1920.

$n_{A'}$	n_C	n_D	n_F	$n_{G'}$
1.539909	1.543168	1.545958	1.552616	1.557994
1.588807	1.593565	1.597767	1.608201	1.616995

From these data the values of a and b of equation (2) are found to be for the partial dispersions $n_C - n_{A'}$, $n_D - n_{A'}$, and $n_F - n_{A'}$ plotted respectively against $n_{G'} - n_{A'}$.

	$n_C - n_{A'}$	$n_D - n_{A'}$	$n_F - n_{A'}$
a	0.14837	0.28813	0.66188
b	0.000576	0.000838	0.000737
b/a	0.003882	0.002908	0.001113

By trial, it is found from equation (8) that the following functions fit the above data:

$F = \frac{0.00854}{\lambda^{0.9}}$, or $\frac{0.00804}{\lambda}$, or $\frac{b}{\lambda - \lambda_o}$; this term is an expression for the dipersion relations in the standard dispersion curve to which the optical glasses are referred.

From these relations it is a simple matter to write down a series of two- and three-constant dispersion formulas which represent with a high degree of exactness the dispersion relations in optical glasses. A few such equations are:

$$n = ce^{1/\lambda^2} + \frac{0.008535}{\lambda^{0.9}} + d \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

$$n = \frac{c}{\lambda^2 - 0.0438} + \frac{0.008535}{\lambda^{0.9}} + d \quad . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

$$n = \frac{c}{(\lambda - .100)^2} + \frac{0.00804}{\lambda} + d \quad . \; . \; . \; . \; . \; . \; . \quad (11)$$

In case a greater degree of precision is desired, a constant may be introduced into the second term, thus:

$$n = ce^{1/\lambda^2} + \frac{b}{\lambda^{0.9}} + d$$

$$n = \frac{c}{\lambda^2 - 0.0438} + \frac{b}{\lambda} + d$$

The agreement between values computed from equation (9) and the observed values is indicated in Table I.

TABLE I.

Wave-length μ	Borosilicate Crown S6115	Calc.-Obs.	Telescope Crown S3418	Calc.-Obs.	Dense Flint O629	Calc.-Obs.
		0.0000		0.0000		0.0000
.7682	1.494581	+08	1.525231	−44	1.602071	+12
.7066	1.495965	−10	1.526730	−10	1.604699	+07
.6708	1.496902	−08	1.527775	+05	1.606572	−09
.6563	1.497318	−07	1.528248	+05	1.607418	−16
.5893	1.499597	−03	1.530848	+13	1.612137	−01
.5607	1.500773	+21	1.532211	+33	1.614712	+06
.5461	1.501472	+03	1.533016	+17	1.616196	+12
.5270	1.502434	+11	1.534128	+32	1.618368	−03
.4861	1.504898	+01	1.537010	+23	1.623982	+01
.4678	1.506221	−13	1.538584	−07	1.627065	+02
.4415	1.508394	−06	1.541201	−35	1.632327	+05
.4341	1.509071	+06	1.542020	−31	1.634648	−19
.4046	1.512262	−12	1.545794	+04	1.641979	+31

Wave-length μ	Densest Barium Crown S4704	Calc.-Obs. Three const.	Calc.-Obs. Two const.	Extra dense Flint Parra-Mantois 6548	Calc.-Obs.
		0.000	0.000		0.0000
.7682	1.603759	−001	+052	1.67918	—2
.7066	1.605638	−016	+003	———	
.6708	1.606899	+020	+020	———	
.6563	1.607501	−003	−009	1.68610	+1
.5893	1.610702	+012	−022	1.69238	0
.5607	1.612434	−002	−043	———	
.5461	1.613422	−009	−051	———	
.5270	1.614824	003	−045	———	
.4861	1.618460	−028	−059	1.70829	+3
.4678	1.620418	−013	−053	———	
.4415	1.623677	−012	+009	———	
.4341	1.624778	+010	+019	1.72206	−3
.4046	1.629511	+081	+184	———	

For the dense barium crown glasses it is advisable to use the three-constant equations. Thus, for the dense barium crown glass S 4704 of Gifford, better agreement is obtained, as indicated in the table, with the equation $n_1=0.025184e^{1/\lambda^2}+\frac{0.009372}{\lambda^{0.9}}+1.562042$.

For the computation of dispersion relations Table II has been found useful:

TABLE II.

Wave-length μ	λ^2	e^{1/λ^2}	$\frac{1}{\lambda^2 - 0.0438}$	$\frac{1}{\lambda^{0.9}}$	$\frac{0.00853}{\lambda^{0.9}}$
A' 0.7682 K	0.590131	1.184653	1.830392	1.267865	0.010821
.7066 He	.499283	1.221758	2.195472	1.366923	.011667
.6708 Li	.449973	1.248871	2.462005	1.432406	.012226
C .6563 H_α	.430730	1.261334	2.584447	1.460857	.012468
D .5893 Na	.347274	1.333704	3.295175	1.609523	.013737
.5607 Pb	.314384	1.374486	3.695710	1.683226	.014366
.5461 Hg	.298225	1.398388	3.930431	1.723672	.014712
E .5270 Fe	.277729	1.433415	4.274801	1.779797	.015191
F .4861 H_β	.236293	1.526840	5.194994	1.914023	.016336
.4678 Cd_6	.218837	1.579256	5.713078	1.981279	.016910
.4415 Cd_7	.194922	1.670328	6.617170	2.08192	.017814
G' .4341 H_γ	.188443	1.700071	6.913573	2.119186	.018087
.4046 Hg	.163701	1.842033	8.340214	2.257754	.019270

The linear relations between the partial dispersion of an optical glass are valid only for that portion of the dispersion curve which is distant from an absorption band. With the approach to an absorption band the dispersion curve departs from its even course and is no longer comparable with the other glasses. This is well illustrated in Fig. 3, in which the measurements by H. Rubens in the infra-red and by H. T. Simon in the ultra-violet[3] on a series of optical glasses are plotted in terms of partial dispersions. The different types of glasses are named on the diagram. The similarity in the course of the partial dispersions is well shown by two glasses in the list, namely, a crown of high dispersion O1151 and of refractive index $n_D = 1.52002$ and a barium crown O1143 of refractive index $n_D = 1.57422$. In Table III the partial dispersions $n_r - n_D$ are listed.

TABLE III.

Wave-length in μ	O 1151 $n_r - n_D$	O 1143 $n_r - n_D$	Wave-length in μ	O 1151 $n_r - n_D$	O 1143 $n_r - n_D$
r			r		
2.0	0.02272	0.02272	0.4861	0.00713	0.00704
1.8	.02012	.02012	.4800	.00780	.00766
1.6	.01762	.01772	.4678	.00901	.00884
1.4	.01522	.01542	.4340	.01310	.01288
1.2	.01312	.01312	.3610	.02664	.02500
1.0	.01042	.01052	.3466	.03066	.02977
0.8	.00692	.00692	.3403	.03260	.03161
0.7682	.00634	.00640	.3261	.03768	.03623
.6563	.00290	.00302	.3133	.04305	.04103
.5892	.00000	.00000	.3081	.04556	.04322
.5349	.00325	.00324	.2980	.05091	.04791
.5086	.00523	.00516			

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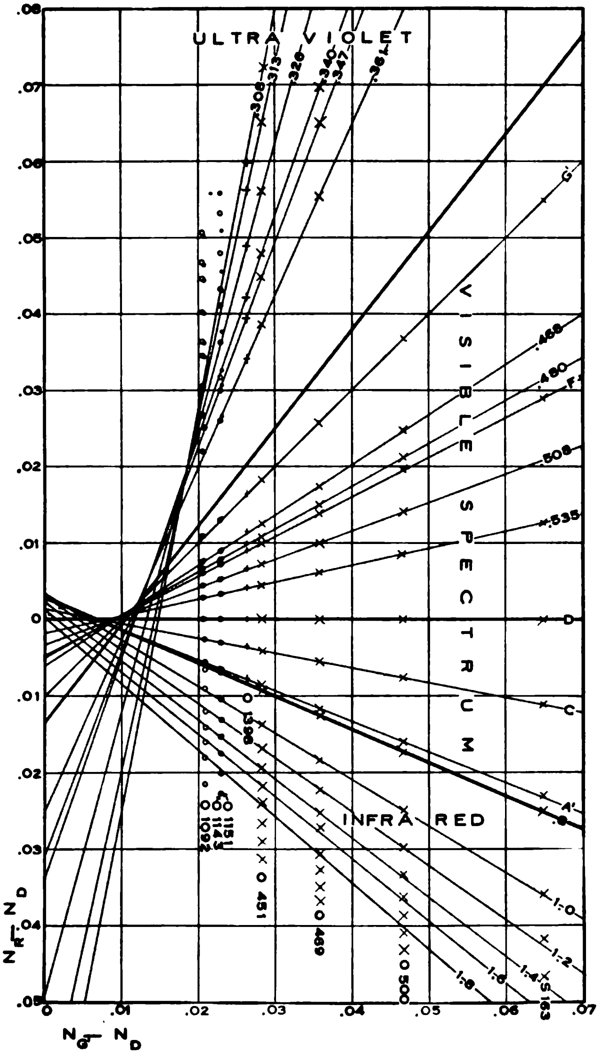
[3] *Ann. Phys. und Chem.*, N. F. 53, 555. 1894.

FIG. 3.



In this figure the partial dispersions $n_r - n_D$ between the sodium line and the following wave lengths in μ: 2.4, 2.2, 2.0, 1.8, 1.6, 1.4, 1.2, 1.0, 0.8, 0.7682, 0.6563, 0.5892, 0.5349, 0.5086, 0.4861, 0.4800, 0.4678, 0.4340, 0.3610, 0.3466, 0.3403, 0.3261, 0.3133, 0.3081, 0.2980, 0.2880, 0.2837, 0.2763, are plotted as ordinates against the partial dispersions $n_{G'} - n_D$ for a series of optical glasses measured by H. Rubens and H. T. Simon. The partial dispersions of the following Schott optical glasses are plotted on the diagram: O1092, light barium crown, (n_D=1 51698); S204, borate glass, (n_D=1.51007); O1143, dense barium crown, (n_D=1.57422); O1151, crown of high dispersion, (n_D=1.52002); O451, light flint, (n_D=1.57524); O469, dense flint, (n_D=1.64985); O500 dense flint, (n_D=1.75130); S163, extra dense flint (n_D=1.88995).

From this table it is evident that the partial dispersions of the two glasses run along fairly well together from the infra-red at 2μ to the violet of the visible spectrum. From here on into the ultra-violet the crown with high dispersion (contains evidently some lead) approaches an absorption band, and its partial dispersions rise accordingly. At 0.2980μ the difference in partial dispersions is 0.003.

The different effects of lead, barium, boron, and other glass-making oxides are most clearly shown in the infra-red and ultra-violet rather than in the visible spectrum. The maximal departure from normal partial dispersion lines in the visible spectrum caused by the presence of large amounts of barium is 2 units in the fourth decimal place.

The theoretical significance of the straight line relations between the partial dispersions of optical glasses and of colorless substances in general has not yet been worked out satisfactorily.

There are other approximately linear relations which hold for most glasses within the visible spectrum, but depart markedly from them both in the infra-red and the ultra-violet. If the squares of the frequency ($1/\lambda^2$) be plotted as abscissæ and the excess refractivities ($n-1$) or the reciprocals of the excess refractivity $\frac{1}{n-1}$ as ordinates, the dispersions of optical glasses within the visible spectrum are expressed by approximately straight lines. Another method of expressing the reciprocal relations is to plot the frequency scale on the horizontal line at unit distance from the abscissæ axis, to draw lines radiating from the origin through the points on the frequency square scale and to find the intercepts of these lines with ordinates equal to the refractive indices. The dispersion curves under these conditions are nearly straight lines.[4] It is on these relations that the new empirical dispersion formula of Nutting[5] is based. These relations will be considered in detail in Part II. of this paper.

SUMMARY.

A convenient graphical method of illustrating the relations between different types of optical glasses is to plot for each glass its refractive index, n_D, against the ratio $\frac{n_D-n_{A'}}{n_{G'}-n_F}$.

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[4] For a brief account of this method of plotting reciprocals see F. E. Wright, *J. Wash. Acad. Sci.*, 10, 185–188, 1920.

[5] Revista d'Ottica e Meccanica di Precisione, I, 54–57, 1919.

If partial dispersions alone are considered and plotted one against the other, the result in each case for a series of optical glasses is a straight line. This fact, that in a series of optical glasses the partial dispersions are related by linear functions and that these functions are the same for all glasses, proves that once a partial dispersion is given, the entire dispersion curve is fixed, irrespective of the glass type. This means that within the limits to which this statement holds, namely about one unit in the fourth decimal place, if any partial dispersion is given, all other dispersions follow automatically; in other words, a change in dispersion at one part of the dispersion curve carries with it definite changes in the curve throughout the visible spectrum. Thus a series of standard dispersion curves can be set up independent of the absolute refractive index. This means that if for any substance two refractive indices are given, the dispersion curve can be written down directly; that in case two substances of very different indices are found to have the same actual dispersion for one part of the spectrum, their dispersion curves are identical to one or two units in the fourth decimal place throughout the visible spectrum.

From these relations it is possible to build up dispersion formulas containing two, three, or more constants which represent the data in the visible spectrum with a high degree of exactness. Certain of these formulas are of such form that they are valid far into the infra-red and ultra-violet, but break down of necessity as an absorption band is approached. Certain of the dispersion formulas thus obtained are well adapted for computation purposes.

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THE "ALL OR NONE" LAW IN VISUAL RESPONSE.

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I. INTRODUCTION.

MODERN investigation of the response of neuro-muscular tissue under stimulation indicates that this response follows a so-called principle of "all or none." If a normal nerve or muscle fibre is set into action at all, it is necessarily excited to the maximal degree which is possible for it. The minimal and maximal responses are identical. This principle was demonstrated for muscular tissue before it was shown to be applicable to that of the nervous system, but the researches of Adrian and Lucas indicate quite clearly that it holds for individual nerve fibres as well as for the individual contractile elements of a muscle.[1]

At first thought this principle seems to be incompatible with the existence of delicate gradations of neuro-muscular action. The explanation of the actual occurrence of such gradations appears to lie in the fact that any muscle or "nerve" consists of a large number of individual cells which all coöperate in a single function subserved by the whole. Gradations in the intensity or power of the response can thus be attributed to variations in the *number* of individual cells which take part in the process at any time. Small power in a muscular contraction is attributed to the fact that only a few of the muscle cells are active, while the remainder are wholly inactive. A more powerful contraction would involve a larger number of active cells. Similarly in a nerve trunk which supplies such a muscle an innervation of low intensity would consist in the maximal activity of a few constituent fibres combined with the total inactivity of the remainder. A more powerful innervation would entail the action of a larger proportion of the total number of fibres. Obviously, according to this view, there must be a limiting value to the intensity of any neuromuscular process, which value is determined by the simultaneous activity of *all* of the cells of the system.

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[1] See Lucas, K., "The Conduction of the Nervous Impulse." London, 1917.

Forbes and Gregg[2] have found that this principle actually applies to the "action current" of a nerve, but that gradations in the degree of the motor response which is set off by this same nerve current are still obtainable long after the limiting value of the " action current " has been reached. They explain this fact by the assumption that although the " all or none " principle holds for each fibre, the number of *impulses* reaching the muscle along a single fibre per second can vary. The response of nerve tissue is of such a nature that the nerve current must be *pulsatory* in character, since the excitation of the nerve cell is accompanied by a condition of reduced irritability, in which it is impossible to restimulate the nerve. This condition disappears *pari passu* with the state of excitation, so that a second " all or none " response cannot be evoked until the first one has died away.

The most convincing demonstrations of the " all or none " principle have been carried out on motor or efferent nerves. It is well known that sensory nerves differ in their properties from motor nerves, but it is very doubtful whether this difference can extend to such a fundamental property of nerve activity as that which is expressed in the " all or none " principle. The results of Forbes and Gregg substantiate this doubt. However, if we assume this principle to apply to the sensory nerve we encounter grave difficulties in the explanation of certain phenomena of sensation. The facts of psycho-physiology indicate very clearly that the intensity of sensation depends directly upon some aspect of the nerve processes in the cerebral cortex. This " aspect " in turn must depend quantitatively upon the intensity of the response of the receptors or sense organs which are involved. Consequently, the intensity in question must have some quantitative representation in the sensory nerve current. In the case of sensation we have to face the added difficulty that gradations of *quality* as well as quantity or intensity, are apparently transmitted from the receptor to the central nervous system.[3]

In certain departments of sensation, at least, it is possible to account for the afferent transmission of intensity by supposing that the number of individually excited fibres in the sensory nerve varies with the intensity of the receptor activity, so that the intensity of the sensation is determined at the nerve centres by the total number of fibres which are pouring impulses into that region. This would be in harmony, for example, with many of the facts of auditory response as interpreted by the theory of Max Meyer.[4] According to Meyer's view the loudness of a tone depends upon the number of fibres of the basilary membrane which are set into action, each such fibre being responsible roughly for the excitation of a single nerve fibre of the eighth nerve. The pitch or quality of the tone would be accounted for in accordance with Rutherford's theory[5] by the frequency of the nerve impulses sent along each or all of the fibres, this frequency being equal to that of the sound vibrations. According to this interpretation, frequency of impulses would be responsible for transmission of quality to the nerve centres, while the number of individual sensory fibres which were excited would be responsible for the transmission of intensity. In the case of the cutaneous and kinesthetic sensations, it would also seem possible to account for gradations of intensity in terms of the number of individual fibres stimulated, while variations of quality seem to depend entirely upon the identity of the nerve path which is stimulated. Phenomena of gustatory—and possibly olfactory—sensation are apparently consistent with a similar interpretation if we suppose that the separate receptor cells have varying thresholds, so that more of them are excited by a strong stimulus than by a weak one.

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[2] Forbes, A., and Gregg, A., " Electrical Studies in Mammalian Reflexes. II. The Correlation Between Strength of Stimuli and Direct and Reflex Nerve Response." *Amer. Jour. of Physiol.,* 1915, Vol. 39, pp. 172–235.

[3] Similar demands would arise from a purely physiological study of the motor responses of the organism to stimuli of various intensities applied to sense organs.

When we come to consider the case of visual sensation, however, as I have pointed out in a previous paper,[6] the phenomena which we encounter are much less easily explicable in terms of the "all or none" law. In this department of sensation the number of retinal elements which are stimulated appears to determine our consciousness of the *area* of the object, and all intensities of sensation seem to be elicitable from a single rod or cone. It is true that there are some facts of visual response which suggest a connection between the intensity or luminosity of the sensation and the number of retinal elements which are excited. For instance, when the area of retinal excitation is confined to the fovea, the threshold is found to depend upon the area and the intensity in the same way, so that it is expressible in terms of the product of these two factors.[7] In this relationship the number of fibres stimulated and intensity of stimulation appear to be interchangeable. However, even at the threshold a small area of high intensity is usually discriminable from a larger area of lower intensity, so that, even here, these two aspects of the receptor process must be separately represented in the optic nerve conduction.

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[4] Meyer, Max, "Zur Theorie der Differenztöne und der Gehörsempfindungen überhaupt." *Zeitschr. f. Psychol. u. Physiol. d. Sinnesorg.*, Bd. 16, S. 1–48, esp. S. 22 ff.

[5] See Schäfer, E. A., "Text-Book of Physiology," 1900, Vol. 2, pp. 1190–1192.

[6] Troland, L. T., "The Nature of the Visual Receptor Process." This JOURNAL, 1917, Vol. 1, esp. pp. 11–12.

For intensities above that for the absolute threshold, and in general for large areas of stimulation, there appears to be no possibility of accounting for the transmission of intensity gradations in terms of the total number of optic nerve fibres which are operative. It is certain that hundreds of different gradations of this character are perceptible in such cases. Moreover, the isolated stimulation of a single retinal cone appears to be at least a theoretical possibility, since the recent analyses of Hartridge[8] have shown that with a point source the optical system of the eye is capable of concentrating about 76 per cent. of all of the light entering the pupil upon a single retinal cone, only 3 per cent. falling upon each immediately outlying cone. Histological evidence indicates that, within the fovea at least, each cone has a " private line " of nerve connection, and to explain the facts of form perception, or visual acuity, of foveal vision it seems necessary to suppose that these " private lines " are continued back to the occipital lobe of the cerebral cortex. The neural transmission of intensity in such a system—and in harmony with these facts—must obviously depend upon some property of the nerve current in the individual optic fibre.

In this situation we naturally fall back upon the idea of impulse frequency as the basis for intensity transmission. In the paper referred to above, I have suggested a mechanism of connection between the receptor cell and the nerve fibre by which various intensities of excitation of the former could be translated into corresponding nerve impulse frequencies passing along the individual optic fibres. This theory, I believe, satisfactorily explains the facts of pure luminosity vision in harmony with the "all or none" principle, and also DuBois-Reymond's law. It appears, however, to leave no plausible way of accounting for the transmission of the qualitative or chromatic aspect of the visual excitation. If we correlate intensity with the frequency of the nervous impulse, color must depend upon something different. It is generally believed that all possible qualitative variations in visual sensation are elicitable from a single retinal cone, provided the latter is located in the zone of trichromatic vision. The original "*Dreifasertheorie*" of Young and Hemholtz has been universally discarded, although it seems to the writer (*vide infra*) on somewhat inadequate grounds.

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[7] On this point cf. Parsons, J. H., " An Introduction to the Study of Color Vision," 1915, pp. 117–125.

[8] Hartridge, H., " The Chromatic Aberration and Resolving Power of the Eye." *J. of Physiol.*, 1918, Vol. 52, pp. 175–246, esp. p. 190.

The difficulties which we encounter in attempting to reconcile the facts of both achromatic and chromatic vision with the "all or none" principle are so great that we might conclude that, after all, this law is incompatible with the facts of visual response and cannot be accepted as applying to the action of the optic nerve fibres. It is well known that the optic nerve is, strictly speaking, a brain tract and not a sensory nerve in the strict sense of the term. Consequently we might feel warranted in attributing to it, properties different from those possessed by other afferent conductors. The observations and measurements to be described below, however, present new empirical data which it seems to me can only be interpreted as demonstrating the actual operation of the "all or none" law in visual response.

II. DESCRIPTION AND THEORY OF THE BLUE ARC EFFECT.

If a spot of red light surrounded by a dark field be viewed with the right eye under favorable conditions, one will see streaming off to the right of the spot two arc-shaped bands of slightly violet luminosity. These bands converge upon a portion of the visual field which measurement shows is identical in location with the blind spot. The exact form and separation of the two bands varies with the position of the stimulus spot with respect to the centre of vision; when the spot is directly fixated the bands tend to fuse together, but when the fixation point is moved toward the right they separate constantly until they finally disappear.

If the left eye is employed an exactly symmetrical appearance is observed centring at the blind spot of that eye. In binocular vision the two sets of arcs can often be seen together.

This phenomenon has been called by Mrs. Ladd-Franklin the "blue-arc effect."[9] It was apparently first described by Purkinje,[10] who seems to have observed every possible entopic phenomenon. It has been rediscovered by a number of later observers, including Zeeman,[11] Gertz,[12] Hubbard,[13] and others. It appears, however, to be still practically unknown to the majority of psychologists and physiologists in spite of the remarkable significance which it acquires when its very obvious explanation—recognized in its essentials by previous investigators—is once appreciated. It also is unfortunately true that very few quantitative studies have been made of the effect. I have, therefore, thought it very well worth while to undertake such studies of various aspects of the phenomena, one of which I am reporting in the present paper.

In another forthcoming article,[14] I shall present measurements of the topography of the blue arcs for various sizes and positions of the stimulus spot. A comparison of the map, in terms of visual angles thus determined, with the distribution of the nerve fibres in the nerve-fibre layer of the retina—in the region surrounding the fovea and between it and the optic disk—shows conclusively that the effect is attributable to the action of these nerve fibres upon neighboring portions of the retina. The individual nerve fibres of the retina are unique in consisting of bare axis cylinders without either myelin sheaths or neurilemmas.

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[9] In a forthcoming paper.

[10] Purkinje, J. E., "Beobachtungen und Versuche zur Physiologie der Sinne," 1825. Bd. 2, S. 74-78. The entire passage is quoted by Gertz (*vide infra*).

[11] Zeeman, P., "Ueber eine subjektive Erscheinung im Auge." *Zeitschr. f. Psychol. u. Physiol. d. Sinnesorg.*, Bd. 6, 233-234.

[12] Gertz, H., "Über autoptische Wahrnehmung der Sehtätigkeit der Netzhaut," Erste Abhandl. *Skand. Arch. f. Physiol.*, 1907, Bd. 19, S. 381-408. Zweite Abhandl., *ibid.*, 1909, Bd. 21, S. 315-350. Also: "Ueber entoptische Wahrnehmung des Actionsstroms der Netzhautfasern." *Zentralbl. f. Physiol.*, 1905, Bd. 19, S. 229-233.

[13] Hubbard, J. C., "A Curious Secondary Visual Phenomenon Resulting from a Stimulation of the Macular Region." *Psychol. Bull.*, 1910, Vol. 7, pp. 196-199.

[14] To be published in *Psychobiology*.

Since they are also crowded closely together, especially in the region about the fovea, they are in circumstances particularly favorable to the transfer of excitation from one to another.

The mechanism of such transfer of excitation can readily be surmised. When the fibres connected with the retinal rods or cones of a small stimulated region of the retina are excited they suffer a so-called negative variation in their electrical potential. This variation would tend to cause a flow of electric current towards the nerve fibres in question, or at any rate a change—in the sense of an increment of the current flow in this direction—in any current which might happen already to be flowing. This change in current flow would constitute, in accordance with the DuBois-Reymond law of nerve stimulation, a natural stimulus to any nervous tissue through which the lines of current flow might happen to pass. If the excitation of the nerve fibres which were primarily involved is continued without change no further secondary stimulation of other nerve fibres should occur, since the DuBois-Reymond law demands stimulation only when there is a change in the magnitude of the electric current which is acting upon the nervous tissue. The demands of this argument appear to be met by the facts, since the blue arc effect is seen only at the moment of application of the primary stimulus and lasts approximately a second. Of course, in order to insure steady stimulation of the retina it is not only necessary that the external stimulus be maintained constant, but the eye itself must not be permitted to move. In another article I shall discuss the relation of the effect to the DuBois-Reymond principle in detail.

The form of the blue arcs for various regions of primary stimulation of the retina shows clearly that the primary nervous elements involved in the transfer of excitation are the nerve fibres which connect the region in question with the entrance of the optic nerve. More doubt exists, however, with regard to the exact identity of the secondary nervous elements to which the excitation is transferred. It is unlikely that these latter elements are other nerve fibres which run along parallel with the primarily excited ones in the nerve-fibre layer of the retina, since among such fibres there are probably included some which connect with regions of the retina not lying along a continuous path between the stimulated area and the optic disk. The sensations derived from the secondary excitation will naturally be "projected" on that portion of the visual field which corresponds with the positions of the retinal receptors supplied by the nerve fibres in question, and the blue arc effect is always confined to the region of the visual field between the primary sensation and the blind spot.

There are two other possibilities regarding the identity of the elements which are secondarily excited. One is that these elements consist in the neurones of retinal layers external to the nerve-fibre layer and the axes of which are perpendicular to those of the fibres in the nerve layer. In general, a nerve fibre is much more readily stimulated by the interruption of an electrical current which travels along its axis than by that of one which is moving across its axis. This principle, therefore, would favor secondary excitation of the neurones in the external strata of the retina as compared with that of nerve fibres running parallel with those primarily excited. The final possibility is that the elements involved in the secondary excitation are actually the receptor cells of the retina themselves, although we are not certain that these cells are subject to stimulation according to the DuBois-Reymond principle. It is known, however, that they are in general much more sensitive than the simple nerve cells, and it is quite likely that such additional sensitivity would be necessary to permit one nervous element to pick up a residual effect of the activity of another such element. It should be easy to devise experiments capable of distinguishing between these three possibilities.

Since we attribute the blue arc effect to the stimulation of the retinal apparatus by an electric current, it is of interest to compare the color of these arcs with the colors obtained by the application of externally produced electrical voltages to the eye. The investigations of Helmholtz,[15] Müller,[16] and others show that if the electric current passes outwards along the line of sight the color seen is ordinarily a dark reddish-yellow, but that if it passes inward the color is a "whitish-violet." [17] This latter color is the same as that of the blue arcs, and a moment's consideration shows that the direction of the internally produced electrical current which we have assumed to be responsible for the generation of this phenomenon is also the same as in this case of the external application of electrical forces. In the latter case the visual effect continues with continued application of the voltage, but is much stronger at the "make" than thereafter. This coincidence of relations appears to offer a striking corroboration of the explanation which we have given of the effect.[18]

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[15] Helmholtz, H. von., "Handbuch der Physiologischen Optik." *3te Aufl.*, 1911, Bd. 2, S. 13–17.

[16] Müller, G. E., "Ueber die galvanischen Gesichtsempfindungen." *Zeitschr. f. Psychol. u. Physiol. d. Sinnesorg.*, 1897, Bd. 14, S. 329–374.

[17] See Helmholtz, *loc. cit.*, Bd. 2, S. 15.

If the action of the nervous elements of the retina follows the "all or none" principle, as well as DuBois-Reymond's law, we should expect to find evidence of this fact in the blue arc phenomenon. The "all or none" relationship might be inserted at one or more points of transfer, in the total propagation process which is involved. In the first place, we should expect the action currents of the primarily excited fibres to be independent in magnitude of the intensity of the radiation acting upon the retinal receptors to which they were attached. Consequently, the potential difference generated by these action currents should be independent of the receptoral stimulus in question. In the second place, we might expect the secondary excitation of other elements to be of a magnitude independent of that of the current which is set up by the potential difference just mentioned. In case the secondarily excited elements are the receptor cells themselves this expectation might not be fulfilled, since it is certain that these cells do not have an "all or none" response to radiation or to many so-called inadequate stimuli. Even in this event, however, we should still expect to find evidence of the "all or none" character of the action current in the primarily excited elements.

It is, of course, certain that a large number of individual nerve fibres are involved in the primary excitation, but it is equally certain that the individual action currents of these fibres cannot summate to any appreciable extent electrically, at least as regards their voltage characteristics. In other words, the voltages which the individual fibres severally generate cannot be conceived to be arranged in "series," but must rather be conceived as arranged in "multiple." When a very small number of fibres are involved, a reduction in the secondary excitation effect might be expected. due to "leakage," but if the number of fibres is reasonably great, the density of the current effect will not be increased by a further increase in the number of fibres involved. Moreover, when all of the fibres in a given path are in action, an absolute limit is reached, as in the case of a nerve trunk, which cannot be exceeded on any assumption regarding the manner in which the individual processes combine. It is probable that at very low intensities of primary stimulation of the retina not all of the fibres supplying the given retinal region are set into action, but that the intensity required to involve them all is still not very high above the absolute threshold.

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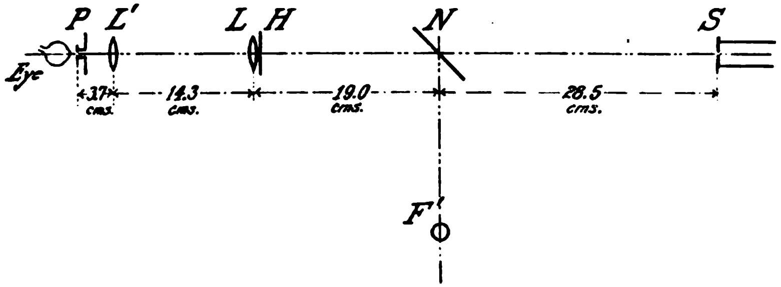
[18] I find, after writing the above, that similar arguments have been advanced by Gertz, in his very thorough and penetrating theoretical analysis of the basis of the blue arc phenomenon. *Loc. cit.*, 1909, Bd. 21, pp. 327–350.

III. MEASUREMENTS OF THE LUMINOSITY OF THE BLUE ARCS.

A cursory observation of the blue arc effect with various intensities of stimulating radiation indicates in a qualitative way that the luminosity of the arcs increases very little with increase in the intensity of the stimulus, and therefore suggests that our deductions from the " all or none " principle are correct. Gertz notes a general increase at low intensities, but offers no exact data on this point. In order to test the validity of such qualitative observations, I arranged the stimulus conditions described below, which permit a quantitative estimation of the luminosity of the arcs with various intensities of stimulation.

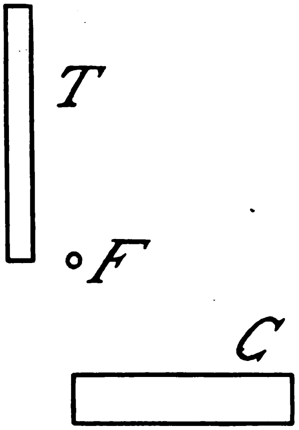
FIG. 1.



Schematic plan of apparatus.

The general arrangement of the apparatus is shown in Fig. 1. *S* is a spectrometer slit which supplies divergent radiation of the desired wave-length constitution. This radiation is caught by the dioptre lens, *L*, which converges the rays upon the 9 dioptre lens, *L'*, which in turn forms an image of the spectrometer slit at the artificial pupil, *P*. The lens, *L*, determines the plane of the external stimulus field in the experiment, and the lens, *L'*, serves to render rays from this plane substantially parallel for the observer's eye, which is placed immediately behind the artificial pupil.[19] This makes it easy for the observer to obtain a sharp image upon his retina of the stimulus field pattern, which is determined by the apertures in an opaque card placed in front of and in contact with the lens, *L*. The pattern of these apertures is shown, drawn to scale, in Fig. 2. Immediately behind the lens, *L*, is a shutter, *H*, consisting of two blades meeting at the vertical diameter of the lens but having a small opening coinciding in position with the central spot, *F*, of the stimulus field pattern. Between the lens, *L*, and the spectrometer slit, *S*, there is placed a screen, *N*, making an angle of 45° with the optical axis of the system and being of such a height as to intercept the light from the spectrometer only in the lower part of the stimulus field. This screen is illuminated by a small electric flash-lamp, *F'*, which is mounted upon a carriage running on a photometer bench at right angles to the optical axis in question. The important dimensions of the system are given in centimetres in the figure.

FIG. 2.



Pattern of stimulus field.

The functions performed by the various portions of the stimulus field pattern shown in Fig. 2 may be indicated as follows. The central spot, *F*, serves as a fixation point and is constantly filled with light from the spectrometer even when the shutter is closed. The vertical slot, *T*, is also filled with light from the spectrometer when the shutter is open, but it is dark when the latter is closed. The lower end of this slot is located on the horizontal diameter of the field which passes through the centre of the spot, *F*. The horizontal slot, *C*, is filled with light from the screen, *N*, when the shutter is open, but is dark when the shutter is closed. The left-hand end of the last-mentioned slot is located on the vertical diameter of the field, which also passes through the centre of the spot, *F*. Both halves of the shutter, *H*, open simultaneously with extremely high speed when an electric contact is made.

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[19] The distance from the artificial pupil to the cornea of the observer's eye was 8 mm.

TABLE I.

Dimensions of Stimulus Field.

Dimension (See Fig. I.)	Millimetres	Visual Angular Degrees
T, vertical	16.0	6.39
horizontal	1.5	0.60
C, vertical	2.8	1.12
horizontal	13.5	5.40
F, diameter	0.7	0.28
T to F, horizontal (neighboring edges)	2.2	0.88
C to F, vertical (neighboring edges)	7.0	2.80

The dimensions of the stimulus field pattern in millimetres and also reduced to visual angular degrees for the observer's eye are given in Table I. The angular dimensions were calculated by means of the formula,

$$\Phi = 2 \tan^{-1} \frac{mo}{2a}$$

where o is the linear size of the given field dimension, a the actual distance of the field from the eye, and m the magnification introduced by the lens, L'. This latter factor, in turn, was obtained by use of the formula,

$$m = \frac{fd}{fd - x(d - x)}$$

f being the focal length of the lens, d the distance of the field from the anterior focus of the eye, and x the distance of the lens from the same point. This magnification amounted with the given lens powers and other dimensions of the system to 1.315.

The slot, *T*, which is filled with spectral radiation of known wave-length constitution, furnishes the primary stimulation of the retina required to produce the blue arc effect. Fixation is maintained upon the spot, *F*, and for this reason only one, *viz.*, the upper, of the two arcs ordinarily seen is visible. Special tests with slot, *C*, dark, demonstrated the complete absence of the lower arc. With the given distance between *T* and *F* and for the writer's right eye, the upper arc shoots out from *T* to the right at a distance above the horizontal diameter of the field approximately equal to that of the slot, *C*, below this diameter. The slot, *C*, is seen filled with " white " (or "tungsten") light of low but controllable intensity, and the problem set for the observer is to vary the intensity of this light until it appears to be equal to that of the blue arc. In other words, the slot, *C*, furnishes a comparison field which permits the establishment of a rough photometric equation between the single blue arc and an objective standard.

The objective intensity required for this equation was found to be very low. To obtain the required low intensity it was found necessary to employ a small electric flash-lamp of the lowest obtainable candle-power on a long photometer bench and in addition to this to coat the screen, *N*, with lamp black having a reflection coefficient at the angle of reflection which was actually utilized of 0.0216. The small flash-lamp had to be operated at the low " efficiency " of 5.58 watts per candle, corresponding with a candle-power of 0.069. The lampblack surface was produced by smoking the screen, a brass plate, with an illuminating gas flame enriched by passage of the gas through benzol. This screen may be regarded, for purposes of the present investigation at least, as being non-selective in its reflecting power.

It is obvious that one of the most important conditions underlying the photometric equation which the procedure of the experiment establishes between the subjective blue arc and the white light of the slot, *C*, is the given state of adaptation of the visual apparatus. In order to have any hope of demonstrating the luminosity law of the effect we must be sure that the level of adaptation is constant. The most convenient method of maintaining such constancy is to work with complete dark adaptation, which may be assumed to represent the asymptote of the adaptation curve. For this reason the measurements to be tabulated below were made fairly late in the evening after the observer's eyes had become adapted to vision at low intensities of illumination. The work was done under dark-room conditions and all extraneous light was screened from the observer's eyes during the tests. All of the observations were made by the writer and with the right eye. The writer's vision is, so far as he is aware, normal in all respects except for a certain amount of astigmatism, which, however, is very low for his right eye and was not especially corrected in the present experiments.

The experimental procedure was as follows: Eight ranges of the spectrum were selected which roughly divide the spectrum into equal intervals from a red ending at $680\mu\mu$ and a violet ending at $420\mu\mu$. The exact ranges of wave-lengths are shown in Table II and are accurate to about $1.0\mu\mu$. Filters were employed in the orange-red and blue-violet ends of the spectrum to reduce the amount of scattered light present. It was desired to employ each of these eight spectral stimuli in the slot, *T*, of the stimulus field each at a number of different intensities. Previous experiments had shown that the threshold of the blue arc effect varies noticeably with wave-length and that its maximum value is about 5 photons.[20] Accordingly, multiples of this value increasing by powers of two, from 5 to 640 photons, were employed. It was not possible to obtain the highest of these intensities in all cases on account of limitations in the capacity of the light sources which were employed.

The intensities were established by means of flicker-photometric equations with the brightnesses of a magnesium oxide surface illuminated by a standard lamp at distances calculated to yield the required values. The area of the artificial pupil which was employed was, of course, taken into consideration in calculating the distances in question. This pupil was a square, 2.51 millimetres on the side. The reflection losses due to the four lens faces were also allowed for. It was not possible to obtain all of the desired illuminations of the magnesium oxide surface simply by varying the distance of the standard lamp from the surface, and it was therefore necessary to operate the lamp at various voltages which had previously been calibrated with respect to candle power. The flicker equations were made with a circular field 7.31 degrees in angular diameter, corresponding with a diameter of a diaphragm in the field of the lens, *L*, of 1.83 centimetres. They were made by the writer, using the same eye which was employed in the actual tests on the blue arc. The results of the flicker equations, between the various standard brightnesses thus established, were recorded in terms of the voltages required across the spectrometer lamp in order to secure the flicker equations. Owing to the general law of the phenomenon to be studied—its slow variation with respect to intensity change—it was deemed sufficient to make only two flicker equations for each stimulus of given wave-length constitution and intensity, provided the voltage readings checked within approximately 0.05 volts on a total range of 6 volts. The calibration of the spectrometer system in this way for the various levels in intensity was carried out independently of the actual observations on the blue arc effect so that fatigue of the eye due to the photometric determinations could not have any effect upon the main observations which were to be made.

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[20] A photon represents the intensity of stimulation of the retina with a stimulus field brightness of one candle per square metre and a pupillary area of one square millimetre.

On account of the large number of variations involved in these measurements it was wholly impossible to make observations representing all of the experimental settings in a single evening. The most convenient method was to go through the entire range of intensities for a single spectral region in one evening. This, of course, introduces some uncertainty regarding constancy of the various levels of adaptation which existed on successive evenings and hence for the different spectral stimuli. The influence of variations of this sort is probably responsible for some of the irregularities which appear in the results. However, our main interest in the present measurements lies in the law of the phenomenon with respect to intensity rather than to wave-length.

In general, five successive determinations of the apparent brightness of the blue arc were made for each intensity. The procedure in making these determinations was to fixate the spot, *F*, of the stimulus field and then to operate the shutter, *H*, which exposed the slots, *T* and *C*, simultaneously. A blue arc would be observed to shoot out from the side of *T* as the slot, *C*, came into sight, these two streaks of luminosity occupying symmetrical positions in the field of view. If the horizontal slot appeared brighter than the arc the small lamp illuminating the screen, *N*, was moved farther away from the screen to reduce the brightness of the slot.

or in the opposite direction if the latter appeared darker than the arc. The shutter was again closed, the point, *F*, refixated, and the experiment tried again until a satisfactory balance of luminosities was obtained. The eye was not exposed to the stimulus field any longer than necessary to make a photometric judgment, this time being approximately one second. In general, about five trials had to be made for each equation. The results of the equations were recorded in terms of the distances of the lamp from the screen, a knowledge of which would permit computation of the brightness or photon value of the slot, *C*, for the luminosity equality. The distances were in the neighborhood of one metre, and the candle power of the lamp was maintained very accurately constant by keeping the voltage across its terminals always the same.

TABLE II.

Apparent Brightness, P, of Blue Arc in Ten-thousandth Photons.

Wave-Length μ μ		Stimulus Intensity in Photons							
		5	10	20	40	80	160	320	640
683.3–700.0	P	16.79	18.0	21.0	20.10				
	A. D.	.74	1.1	1.7	.54				
646.3–660.0	P	31.2	30.22	28.8	21.5	22.76	30.28	29.90	
	A. D.	1.6	.92	2.2	1.7	.50	.93	.72	
608.8–620.0	P	16.86	16.55	18.37	19.80	20.40	18.9	22.02	31.4
	A. D.	.89	.81	.69	.28	.64	1.4	.47	1.5
571.0–580.0	P	11.68	12.22	11.70	13.20	14.94			
	A. D.	.17	.22	.20	.56	.94			
532.8–540.0	P	19.10	20.22	20.50					
	A. D.	.62	.97	.38					
494.4–500.0	P	14.92	16.7						
	A. D.	.16	1.2						
455.9–460.0	P	27.16							
	A. D.	.99							
417.4–420.0	P	25.36							
	A. D.	.95							

The first series of observations was made in five evenings ranging over a period of three weeks. The results of these observations are presented in Table II. This table shows for each stimulus condition the average (and precision measures) of the various photon values which were required in the horizontal slot to match the blue arc in luminosity for any given experimental setting. The units employed in the table are ten-thousandths (10^{-4}) of a photon. It will be observed that the intensity in question is extremely low, being only slightly above the absolute threshold for the writer, under conditions of complete darkness adaptation. The incompleteness of Table II is due to several causes. For the wave-lengths 683.3–700.0 μμ it was not possible to obtain an intensity higher than 40 photons, and for the wave-lengths 646.3–660.0 μμ the maximum intensity obtainable was 320 photons. The incompleteness of the results for wave-lengths less than 620 μμ is due to an entirely different cause, *viz.*, the difficulty of observing the effect for these wave-lengths with high intensities of stimulation. This difficulty is not due to absence or weakness of the blue arcs themselves, but rather to the fact that with these wave-lengths the entire field of view is flooded with luminosity due to light scattered by the ocular media. The scattering is probably very little greater physically for these wave-lengths than for the longer ones, but the dark-adapted peripheral retina is highly sensitive to wave-lengths less than approximately 600 μμ and relatively insensitive to wave-lengths greater than this value. This disturbing effect of general peripheral stimulation, due to scattered light acting upon the rods, becomes worse with increasing intensity and also with decreasing wave-length, so that while at 580 μμ observations can be made up to 80 photons, at 460 μμ they can scarcely be extended beyond the threshold of the effect. It is often possible at higher intensities to see the blue arcs through the general haze, but it is practically impossible to make any sort of judgment of their apparent brightness.

A study of Table II shows very clearly that the blue arc phenomenon tends to follow the "all or none" principle. Consider, for example, the results obtained for the spectral range 608.8–620.0 μμ. If we neglect the value of 31.4 obtained at an intensity of 640 photons, the increase in the apparent brightness of the arc in changing from a stimulus intensity of 5 to one of 320, involving a factor of 64, is from 16.86 to 22.02, involving a factor of only 1.31. Even this increase may be due largely to accidental variations, since the apparent brightness of the effect for 160 photons is only 18.9. The noticeable increase in the value of the effect for 640 photons is attributable to the appearance of the peripheral glare at this intensity, this glare apparently reducing the sensitiveness of the retina to the objective light which is used as a comparison. For the spectral range 646.3–660.0 μμ there is actually a general decrease in the apparent brightness of the effect with an increase in the intensity of the stimulus. This decrease is probably to be attributed to a slight increase in the sensitiveness of the retina to the comparison stimulus, *C*, due to progressive dark adaptation occurring during the course of the observations. The change, however, is very small. Similar comments will be found to apply to the luminosity values of the phenomenon obtained for other wave-length ranges.

It will be seen that the general order of magnitude of the luminosity of the blue arc remains constant not only for different intensities but also for different wave-lengths. The variations which occur between wave-lengths are probably explicable as due to two causes: (1) existence of different levels of adaptation on the several evenings during which the observations were made, and (2) differences in the disturbing influence of scattered light in the eye for the various wave-lengths.

Having determined by the above described observations that the only stimuli which it is practicable to employ over a large range of intensities are of wave-lengths greater than about 600 $\mu\mu$, a second series of observations was carried out in which conditions were somewhat more carefully controlled. For these observations a spectral stimulus constituted by wave-lengths between 642.0 and 680.0 $\mu\mu$ was selected, since experience showed that this region of the spectrum does not readily arouse the peripheral glare. The spectrometer lamp was calibrated for intensities from 5 to 640 photons as in the first series of observations. Besides being made in the evening after four or five hours of twilight vision, complete darkness adaptation was further guaranteed for this series by a period of fifteen minutes' rest of the eye in total darkness before work was commenced. A minute's rest of the eye was also permitted between individual observations. A further possible source of variations in adaptation involved in the method of the experiment, consisted in the necessity of using the eyes between experiments to read the scales of instruments and to write down the values found. The brightnesses for this work were made as low as possible, approximately 0.16 candles per square metre. In the first series of observations some slight decreases in the sensitiveness of the eye to the comparison field were noted as a result of this exposure to light. In the second series after every appreciable exposure of this nature two or three minutes' time was allowed to elapse before a new observation was made, and the regularity of the results which were obtained indicates that this factor had a negligible influence.

In order to guard against a possible effect of progressive changes in adaptation during the course of the work, two sets of observations were made. The first began with the lowest intensity, followed by progressively increasing intensities up to the highest. The second set was started with the highest intensity, followed by progressively decreasing intensities down to the lowest one. A progressive increase in adaptation during the series of increasing intensities might accidentally produce an "all or none" law for the phenomenon if the sensitivity of the retina to the objective light should happen to increase *pari passu* with an increase in the intensity of the arc. If the reverse order of intensities were employed, however, the adaptation change would tend to increase rather than to decrease the variation due to the change in the stimulus intensity.

TABLE III.

Apparent Brightness, P, of Blue Arc in Ten-thousandth Photons.

Order		Stimulus Intensity in Photons.							
		5	10	20	40	80	160	320	640
Increase.........	P A. D.	9.81 .16	9.65 .21	9.65 .21	10.28 .31	9.64 .14	9.32 .15	9.72 .18	11.25 .33
Decrease........	P A. D.	9.32 .15	9.19 .23	9.26 .11	9.21 .20	9.32 .18	9.95 .28	10.38 .45	12.61 .44

Wave-length Range of Stimulus, 642.0–680.0 $\mu\mu$

The results of the second series of observations are shown in Table III. These results leave no possible doubt that the blue arc phenomenon actually obeys the "all or none" principle. The constancy of the apparent brightness of the arc for intensities between 5 to 640 photons is remarkable, and any variations which occur can certainly be accounted for as a result of slight variation in the level of adaptation or simply the uncertainties of the general method of measurement. Indeed, considering the difficulty of making photometric matches under the conditions of the experiment the constancy of the values is surprising. It is only for the intensity of 640 photons that variations outside of the limits 9.19 to 10.38 are found, and at the intensity in question a considerable amount of peripheral glare was present.

In connection with this second series of observations the absolute threshold was determined for the light in the slot, *C*, and found to be 8.1×10^{-4} photons. This value is 0.82 times the average value (9.910×10^{-4}) of the photon intensity which matched the blue arc in this series. In another series, involving less perfect dark-adaptation, a threshold 0.68 times the apparent brightness of the blue arc was found. These determinations of the limen were made with the same fixation as that of the blue arc measurement and in the presence of the spectral stimulus. It is evident that at least under the conditions of the observations here reported, the brightness of the blue arcs is only very slightly above the threshold. These results agree in general with those of Gertz.[21]

IV. METHODS OF COMPUTATION.

The method of computing the results given in Tables II and III was as follows: Each of the individual items of these tables represents five separate determinations of the objective brightness required to match the blue arc. The original records of the conditions for these matches were in the form of distances in centimetres between the screen, *N*, and the small flash-lamp, *F′*, which was always operated at a constant voltage. The five distances thus established for any given experimental setting were first averaged by finding their arithmetic mean. The illumination of the screen for this average distance of the lamp was then calculated by means of the inverse square law from the known candle-power of the lamp. This illumination value was then converted into units of brightness through multiplication by a constant depending upon the reflection coefficient of the screen, the angle of reflection, and the general optics of the arrangement.

This constant was derived from a similar constant which had been computed for the magnesium oxide disk employed in making the flicker photometric equations. The relation between the two was established experimentally by illuminating the smoked screen, at the position in which it was used during the tests, by a lamp of constant candle-power placed fairly close to it at a known distance and then equating to it a second luminous field obtained from another light source so arranged as to permit of an equality-of-brightness comparison between the two. This secondary field was then maintained at the determined brightness and the magnesium oxide disk substituted for the smoked screen in the same plane. The source of illumination of this plane was then moved away until a photometric equation was established, this time between the magnesium oxide surface and the secondary comparison field. From the distances of the lamp from the screen and disk, respectively, the ratio of the brightnesses of these two surfaces for a constant illumination could be calculated.

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[21] *Loc. cit.*, 1909, Bd. 21, S. 326.

The original brightness value coefficient for the magnesium oxide surface was calculated on the assumption of a coefficient of reflection of the surface in question of .863, supposing also that the surface obeyed Lambert's Law. On these assumptions the brightness of the surface would be expressed by the relation

$$b = \frac{ir}{\pi}$$

where i is the illumination of the surface and r is the reflection coefficient. The photon values given in the Table could be computed from the brightness values of the disk or the screen by multiplying the latter by the area of the artificial pupil which was employed in all of the measurements, and by the transmission factor of the system. For this purpose the area of the pupil would be expressed in square millimetres and the brightness in candles per square metre. For simplicity of computation, the pupil and transmission factors were naturally combined with the constant required to reduce the illuminations to brightness values, so that the photon values were obtained by a single multiplication.

It would, of course, have been desirable theoretically to have computed the brightness or photon values for the individual distances of the lamp from the screen and then to have averaged all of these separately computed values. The results obtained by this procedure, however, would not have differed sufficiently from those actually found to have warranted the large increase in labor which would have been involved by the theoretically more exact method.

Tables II and III show under the columns marked A.D. the precision measures of each of the main values of the table. These precision measures represent the "average deviations of the averages" of the individually determined photon values. They were computed in the following customary manner. The average deviation of the individual distance readings from their own mean value was first determined. This deviation was then divided by the $\sqrt{5}$ in order to obtain the average deviation of the mean distance reading considered by itself. This latter precision measure, Δm, was then substituted in the equation,

$$\Delta p = \left(\frac{2\,\Delta m}{m}\right) p$$

where Δp is the average deviation of the mean photon values as given in the tables, p the mean photon value, and m the mean distance reading for the given case. This formula was developed from the relation between the photon and distance values by the usual method employed in calculating propagated errors. The average precision of the main values in Table III, thus computed, is 2.3 per cent., which is sufficiently low to account for the deviations from constancy of the values in question, at least of those values which were obtained under conditions of substantially constant adaptation and freedom from peripheral glare.

V. THEORETICAL CONSIDERATIONS.

The work reported in the present paper naturally requires to be supplemented by further measurements of the blue arc effect upon other subjects. In the small amount of time which is available to me for research of this character I find it more interesting to endeavor to enter new lines of investigation than to follow them up in great detail. However, I hope that others who are interested in vision and have more time at their disposal will undertake to test the results reported in this paper upon a large number of subjects.

If further analysis of the cause of the blue arc phenomenon should justify the interpretation which has been placed upon it in the present article (and by Gertz) the "all or none" characteristic of the phenomenon will have to be regarded as of fundamental importance for the theory of visual response. We shall be obliged to face courageously the problem of determining how it is possible for gradations of intensity to be transmitted to the brain centres from individual rods or cones by an activity the secondary influences of which exhibit no gradations whatsoever.

It seems almost certain that intensity must be represented in the nerve currents moving along individual fibres of the optic nerve by the total number of quantal ("all or none") pulses passing through a fixed cross-section of any one of these fibres per unit time.

No great difficulty would probably be found in reconciling the momentary character of the blue arc effect with the assumption that a constant stream of impulses is generated by the stimulus which sets off the effect in question. This effect must be produced by the action of a very large number of individual nerve fibres, and it is improbable that the pulses in all of these fibres would be exactly synchronized, so that after the first general rise in excitation of the mass of conductors no variations in their combined activities would be detectable to an outside sensitive structure. Another principle which might be involved is a possible difference in the natural periodicities of the primarily and secondarily excited tissues. The facts indicate that the retinal receptors have a much greater inertia for any change of state than have the nerve conductors. It should also be noted in this connection that the blue arc effect has a very distinct negative after-image which by itself might account on the basis of simple fatigue or adaptation for the momentary character of the effect, although I do not at present favor this as a complete explanation.

If we attribute intensity transmission to the frequency of the optic nerve impulses, however—as previously noted—there seems to be nothing left to explain the transmission of the two color characteristics, hue and saturation. Conceivably, these may be represented by some sort of "group frequency," although this seems a rather far-fetched explanation. It is difficult to imagine a mechanism in the cones or their junctions with the optic neurones which could translate the extremely high frequencies of the radiant stimulus into any kind of group frequency capable of being transmitted along a nerve fibre.

Of course, we have still much to learn with regard to the exact nature of the nerve impulse and it is possible that variations in the *form* of the pulse may occur without involving any change in its total magnitude. In this case we might endeavor to correlate transmission of the chromatic characteristics along the nerve with certain aspects of the form of the individual pulses. However, the improbability of this hypothesis leads one to wonder whether there is not some chance of being able to return to Helmholtz's original assumption that there exist in the retina three different kinds of chromatic receptors. Is the evidence which has led to the abandonment of this assumption actually convincing? The experiments of Donders and others have shown that we can perceive colors of stimulus spots so small or distant as to form images upon single retinal cones. Moreover, the accepted opinion is that a point source of white light always looks white when clearly focussed on the retina, and does not tend to appear now red, now green and now blue, as would be expected if the centre of vision were made up of a mosaic of cones, individual elements of which were responsive singly to the red, green and blue constituents of the white light.[22]

As we have already noted, the recent researches of Hartridge have indicated very clearly the possibility of concentrating upon a single retinal cone a very large proportion of all of the light reaching the eye from a point source. However, the question certainly arises as to whether it is possible to maintain sufficiently constant fixation so that the light actually remains on any given cone during a period long enough to permit a judgment upon the stimulus effect which it produces. The actual effect upon consciousness, in other words, may not be due to the instantaneous action of the retinal image upon any single element but upon the summated effects of its successive actions upon a considerable number of adjacent elements. That a process of some such nature as this is involved in color perception, if not in brightness perception, is indicated by considerations advanced by Hartridge[23] with regard to the basis of discrimination between the colors of white and yellow points of light. The eye is subject to considerable chromatic aberration and in focusing an image upon the retina it selects the yellow rays of the spectrum as the ones to be most sharply defined. Any point source of white light would therefore be represented on the retina by a nucleus of yellow, which, according to Hartridge's calculations would fall upon a single cone, surrounded by a ring of blue which would fall upon many outlying cones. A point source of yellow light, on the other hand, would be represented on the retina simply by the yellow nucleus without the blue ring. Experiment shows that we easily distinguish the difference in color between two such small sources of light, but that we do not perceive the white spot to consist of a nucleus of yellow surrounded by a ring of blue. This means that the responses of a considerable number of cones have been combined to produce an integral sensation in consciousness.

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[22] Cf., on these points, Parsons, *loc. cit.*, p. 120, and Schäfer, *loc. cit.*, p. 1112.

[23] *Loc. cit.*, pp. 198–215.

This fusion in consciousness of the results of stimulating a number of adjacent cones may be attributable either to a fluctuation of the position of the image over the retina, due to a drift of fixation, or it may result from a general functional tendency for adjacent color excitations to fuse together. Such a tendency would not of necessity demand a similar fusion of brightness excitations. If we return to the original Helmholtz assumption, the brightness values would be represented by the strength of the nerve currents started at each retinal point while chromatic characteristics would depend upon the identity or location of the stimulated elements. These two entirely different kinds of data might be treated in quite different ways by the central nervous system. The limit of visual acuity for brightness patterns appears to coincide fairly closely with the dimensions of the individual retinal elements, indicating that there is very little fusion of brightness excitations between such elements. However, it is well known that acuity for patterns determined wholly by color difference, without accompanying brightness difference, is very much lower, and this indicates that fusion of adjacent chromatic excitations actually does occur.

It is improbable that, in our quest for an explanation of the transmission of the chromatic characterstics, it will avail us anything to fall back on the overworked neurofibrillæ. supposing that each optic nerve fibre contains a number of separate conductors which carry excitations back to the brain, in accordance with the "all or none" principle, independently of one another. Recent work throws grave doubt upon the actuality of these alleged internal structures of the nerve fibres.[24] They cannot be made out in the living nerve tissue by the use of the ultra-microscope, which renders visible other internal features of the cell. It seems very probable that these fibrillæ are coagulation products or "artefacts" and are not actually present in the living cell, which is essentially a homogeneous liquid surrounded by a membrane.

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[24] Cf. Bayliss, W.: "The Principles of General Physiology," 1915, pp. 396–470.

Further experimental investigations of the questions discussed above would appear to be necessary in order to permit any reliable theoretical interpretation of the complex array of facts with which we have to deal.

VI. SUMMARY.

The present article discusses the relation to vision of the "all or none" principle in nerve action, on the basis of new experimental data. The general nature of the principle is first considered, together with the difficulties which appear when an attempt is made to apply it to the facts of visual response. An entoptic phenomenon—the blue arc effect—is then described, and evidence is presented that this phenomenon is attributable to secondary stimulation of certain retinal elements—probably the rods and cones—by the action currents of the retinal nerve fibres. Careful measurements of the luminosity values of this effect for a wide range of primary stimulus intensities show that it is practically independent of the latter, thus substantiating the view that the action currents of the retinal fibres really do follow the "all or none" principle. The conditions under which the measurements were made, and the methods of calculation, are described in detail. Representative spectral stimuli were employed, and the photometric values of the stimuli and of the effect, itself, for the various conditions of experimentation are presented in tabular form. In conclusion, some further theoretical implications of the results are considered.

1919 REPORT OF THE STANDARDS COMMITTEE ON COLORIMETRY.

THE committee has considered that its work for the first year must needs be strictly preliminary and that it is inadvisable to print a detailed technical report which would involve many questions which cannot be decided without further consideration.

In order to formulate the subject for critical consideration and provide a definite basis for discussion and further committee work, a "preliminary draft" of a report has been prepared and is being circulated among those interested, in order to obtain their criticisms and suggestions.

This preliminary draft contains about fifty typewritten pages. The table of contents is as follows:

CONTENTS:

I. INTRODUCTION.
II. NOMENCLATURE:
  1. General Terminology.
  2. Fundamental Psychologic Terms.
  3. Outline of the Methods of Practical Colorimetry.
  4. Classification of the Methods of Measurement Contributory to Colorimetry.
  5. Terms Relating to Transmission.
  6. The Physical Terms of Homo-hetero-analysis, and Their Correlation with the Attributes of Color.
III. STANDARDS:
  1. Standards of Spectral Energy Distribution.
  2. Transmission Standards.
  3. Reflection Standards.
APPENDIX I. BIBLIOGRAPHY.
APPENDIX II. USAGE OF TERMS RELATING TO THE ATTRIBUTES OF COLOR AND THEIR CORRELATIVES IN STIMULUS.
APPENDIX III. USAGE OF THE WORD "LIGHT."

Emphasis is placed on the psychological basis of color and special consideration is given to the logical correlation of psychological and physical terms. Terms relating to radiation are dealt with as forming the physical basis of colorimetry. The nomenclature of heterochromatic photometry is elaborated in detail.

The draft contains an extensive bibliography and a detailed comparison of the usages of important colorimetric terms by different authorities. An extensive tabular synopsis of the terms and symbols relating to transmissive properties of materials in current use at the National Bureau of Standards is also included.

Copies of this preliminary draft have been issued as follows, with requests for criticism and suggestions:

Bureau of Standards (C. A. SKINNER, *Chief Optical Division*).
Nela Research Laboratory (E. P. HYDE, *Director*).
Cheney Brothers (H. S. BUSBY).
Johns Hopkins University (A. H. PFUND).
DuPont de Nemours & Co. (H. S. BAILEY).
Columbia University (J. P. C. SOUTHALL).
Carnegie Geophysical Laboratory (R. B. SOSMAN).
Corning Glass Works (HENRY P. GAGE).

Copies have been retained by each member of the sub-committee. A copy has been submitted to the President to be permanently filed later with the Secretary of the Society. Two copies have been deposited in the Library of the National Bureau of Standards and one of these may be borrowed by mail by anyone caring to examine it. Members of the sub-committee will be glad to receive criticisms.

Signed: E. C. CRITTENDEN, L. A. JONES, P. G. NUTTING, L. T. TROLAND, IRWIN G. PRIEST, *Chairman.*

REVIEWS AND NOTICES.

AIRPLANE PHOTOGRAPHY. By Herbert E. Ives. 422 pages, octavo, 208 illustrations. $4.00. J. B. Lippincott Company, Philadelphia, Pa.

"Airplane photography had its birth, and passed through a period of feverish development, in the Great War. Probably to many minds it figures as a purtly military activity. Such need not be the case, for the application of aerial photography to mapping and other peace-time problems promises soon to quite overshadow its military origin. It has therefore been the writer's endeavor to treat the subject as far as possible as a problem of scientific photography, emphasizing those general principles which will apply no matter what may be the purpose of making photographs from the air. It is, of course, inevitable that whoever at the present time attempts a treatise on this newest kind of photography must draw much of his material from war-time experience. If, for this reason, the problems and illustrations of this book are predominantly military, it may be remembered that the demands of war are far more severe than those of peace; and hence the presumption is that an account of how photography has been made successful in the military plane will serve as an excellent guide to meeting the peace-time problems of the near future."

In this first paragraph of the author's preface the military origin and development of airplane photography is immediately capitalized for its enormous value in applications to peace-time problems. This is eminently fitting, since there is perhaps no other result of the Great War, except it be the spread and purification of Democracy or the end of War itself, which fills the imagination with greater possibilities in the peaceful service and uplift of mankind. Unlike poison gas, big guns, tanks, submarines, dreadnaughts, and other purely destructive devices which will either become obsolete in times of peace or will be developed in secrecy by suspicious or aggressive governments preparing for future wars, aerial photography is a military tool which readily lends itself to non-military applications of value. It is therefore extremely gratifying that this interesting story of airplane photography has been so promptly and so well written.

The author assumes "that the reader is already fairly conversant with ordinary photography. Considerable space indeed has been devoted to a discussion of the fundamentals of photography, and to scientific methods of study, test and specification. This has been done because aerial photography strains to the utmost the capacity of the photographic process, and it is necessary that the most advanced methods be understood by those who would secure the best results or contribute to future progress. No pretense is made that the book is an aerial photographic encyclopædia; it is not a manual of instructions; nor is its appeal so popular as it would be were the majority of the illustrations striking aerial photographs of war subjects. It is hoped that the middle course steered has produced a volume which will be informative and inspirational to those who are seriously interested either in the practice of aerial photography or in its development."

There is no doubt that the author's purpose has been definitely accomplished. This book is sufficiently popular to interest anyone who reads simple English, and yet contains most of the essential scientific principles and technical data which are of importance to highly trained workers and students in this field. It is so bountifully illustrated with helpful diagrams and well-selected specimens of aerial photography that it is possible to derive a large amount of information and pleasure without reading a word of text, although the latter is fully as delightful as the former. The publishers are to be congratulated on their part of the work —beautifully reproducing the illustrations and producing a text which is remarkably free from typographical errors.

Very early in the book, page 25, the author vividly describes the first flight or " joy ride " of the photographic observer to familiarize him with conditions in the air and to impress him with the appearance of the earth from the plane. These impressions are necessary to a full understanding of the problems involved in aerial photography. The most striking impression is that of the flatness of the earth, both in the sense of absence of relief and in the sense of absence of extremes of light and shade. The absence of relief is accounted for by the fact that the elevations of terrestrial objects, when viewed from ordinary flying heights, are relatively too small for the separation of the eyes to give any stereoscopic effect, but this can be overcome by photography. The absence of extremes of light and shade is in part due to the fact that natural earth surfaces present no great range of brightness, in part to the relatively small areas of the parts in shadow, but principally to the layer of atmospheric haze which lies as an illuminating veil between the observer and the earth. Because of the lack of contrast in landscapes from the air the use of photographic emulsions of considerable contrast and color sensitiveness is required.

The airplane camera is treated with completeness in a series of chapters, pp. 39–175, dealing with lenses, shutters, plate-holders and magazines, hand-held cameras, non-automatic, semi-automatic and wholly automatic plate cameras, aerial film cameras, the motive power, etc., for automatic cameras, followed by a section, pp. 179–217, on the suspension and installation of cameras in airplanes so as to eliminate the vibrations which may impair definition on the photographic plate. Only rotational motions of the camera are damaging, and a rotation of one degree per second is beyond the limits of toleration, even with exposures as short as one hundredth of a second.

American, English, French, Italian and German camera equipment is described in detail without bias, and there is a disposition to state all of the experimental facts of this development in cameras for the benefit of future improvement. It is tacitly admitted that the German apparatus in some respects excelled that produced by the Allies. This is true not only of anastigmatic lenses of large aperture ratio, which were practically a German monopoly, but also of ingenious devices, such as focal plane shutters, which move in opposite directions on successive exposures, thereby eliminating the necessity for a safety flap, and also compensating distortion in mapping work, electric heating coils in the camera to maintain the constancy of shutter operation, and photographic sensitivity at low temperatures, etc.

Sensitized materials and chemicals have exceptional importance in aerial photography on account of the distribution of light, shade and color in the aerial view, being such as to require higher speed, contrast and color sensitiveness in the photographic emulsions. The author tabulates the requirements for aerial emulsions, page 236, and lists some plates and films which have been found satisfactory. The specifications are none too rigid, but even these requirements were not satisfactorily met by the materials which were on the market before the war, and realization of this fact brought about some attempts to produce special color-sensitive emulsions for this purpose. Mention is made of the Ilford Special Rapid Panchromatic plate, which shows remarkable sensitiveness to red light. No mention is made of a remarkable orthochromatic plate produced by the Germans for aerial photography exclusively. This emulsion appears to have been dyed with erythrosin, but was treated in some way so that its contrast was much greater and its speed from 4 to 15 times that of the best orthochromatic plates used by the Allies. It may be that this extraordinary orthochromatic plate was obtained in a similar manner to that which produced the Ilford panchromatic, *viz.*, by the stimulation of the dye. Experiments of this kind have been either unattempted or fruitless in our country, but should be encouraged energetically, since the greatest hope for advancement seems to lie in that direction. Experiments in freshly staining the ordinary plates in photo-sensitizing dyes or in stimulating dyed emulsions with ammonia have shown that speed and color sensitivity can be increased, but such experiments are not entirely suited to field operations. They have already served their purpose in calling attention to the possibility of improvements in commercial emulsions, but will continue to be of great value in emergencies not equally well covered by the ready-made materials. The military requirements of aerial photography are most exacting, and their fulfillment will be of undoubted value in other applications.

Under Practical Problems and Data, the author discusses spotting, map making, oblique and stereoscopic photography, the interpretation of aerial photographs and naval aerial photography.

A chapter of special interest is that on Stereoscopic Aerial Photography, "which is one of the most striking and valuable developments." The separation of the eyes is much too small to give an appearance of relief to objects as far away as is the ground from a plane at ordinary flying heights, but stereoscopic pairs of photographs may be obtained by a method originally employed to reproduce distant mountains, clouds and planets in relief. The method is to take pictures from points separated by distances much greater than the inter-ocular separation—by meters or kilometers, instead of millimeters. In the airplane this is accomplished by making successive exposures as the plane flies over the objective, at intervals determined by the speed, the altitude and the amount of relief desired. It is shown that correct relief is obtained with a lens of 25 centimeters focal length when the interval between exposures is about a quarter of the altitude, and this shows the fallacy of the suggestion that stereoscopic pictures could be made by two cameras placed one at the extremity of each wing. Anyone who has had the pleasure of seeing and admiring the remarkable stereoscopic effects obtained by airplane photography will censure the author for not including more illustrations of this type. An oblique stereogram, Fig. 159, is not well chosen on account of gross divergences in the background.

Airplane Photography closes with chapters on the future developments in apparatus and methods, applications to technical and pictorial work and to exploration and mapping, all of which discloses the sound judgment and the practical imagination of the author. In the study of architecture, city planning, construction progress of huge projects, preparation of guide books, advertising of real estate and news events, the "birdseye" view may find extensive utilitarian value. Like the Orientals who plan their gardens to please their gods, we may be influenced to improve the beauty of our landscape gardening for the benefit of human navigators of the air.

Scientific uses for aerial views will be found in geology and archeology, where photographic maps of otherwise inaccessible regions may now be obtained. Aerial photography will no doubt find its most extensive application in exploration and mapping, in which fields surely everyone has sufficient imagination to foresee work of marvelous beauty and utility.

The present novelty of aerial views may disappear in time, but the universal appeal and educational value of pictures will always remain. Like the inspiring results of astronomical photography, the elevated birdseye views of our own earth cannot fail to charm us and make us realize the future service which may be rendered by Airplane Photography.

W. F. MEGGERS.

THE AMERICAN JOURNAL OF PHYSIOLOGICAL OPTICS.

The American Journal of Physiological Optics is the title of a new quarterly journal published by the Research Division of the American Optical

Company, of Southbridge, Mass. The editor is Charles Sheard, Ph.D., for many years Professor of Applied Optics at the Ohio State University, who is the author of several volumes on various phases of physiological optics and ocular refraction, and who is now filling the newly created position of physiological opticist in the American Optical Company.

The journal will appear during the months of January, April, July and October of each year. Each issue will carry from 75 to 100 pages of material. The mechanical make-up of this journal is all that could be desired: it is beautifully printed in large type on silk-coat finished paper.

The first issue of the journal carries a picture of Thomas Young, the father of physiological optics, as the frontispiece, and a sketch of his life by the editor. The principal papers in the issue are articles by Dr. George M. Gould on "Eyestrain in Its Relation to Occupations," Dr. W. W. Coblentz on "A Comparison of Photo-electric Cells and the Eye," and Dr. Luther C. Peter on "The Value and Limitations of Perimetric Methods of Study."

This journal is published by the American Optical Company in response to the desire expressed by many persons interested in various ways in the human eye that a journal dealing with different phases of physiological optics be established in this country. To that end each number will, in so far as possible, carry articles of a general character in addition to highly specialized papers in this realm. Various physical, physiological and psychological aspects of visual phenomena will receive attention. The journal will be of special interest to those who are professionally or scholastically interested in human vision.

Inquiries in regard to this publication should be addressed to *The American Journal of Physiological Optics,* Charles Sheard, Editor, Southbridge, Mass.

THE NEW ITALIAN OPTICAL JOURNAL.

Everyone interested in applied optics or in precision mechanics will heartily welcome the new journal *Revista D'Ottica e Meccanica di Precisione,* recently started in Florence by Doctor Occhialini. Following the organization of the Italian Optical Society, L'Associazione di Ottica e Meccanica di Precisione, on August 9, 1919, the new journal was initiated with the September-October number, and has appeared bi-monthly since. It is the official organ of that society and is published by the Laboratorio D'Ottica Practica e Meccanica di Precisione at Florence. Dr. Augusto Occhialini, editor of the journal, is head of the Department of Mechanics and Director of that research institute and secretary of the society.

The scope of the new journal is quite broad but very well covered. judging by the three numbers that have thus far appeared. It contains a number of original articles on general lens theory, lens calculation, optical glass testing, optical apparatus, photometry, radiometry, microscopy and photography, as well as a few on metrology and mechanics. There are numerous short abstracts of articles published in foreign journals, together with news items of general scientific interest and a few articles on industrial research. Some historical papers on the origin and development of certain mechanical principles are of particular interest. The avowed purpose of its founders in starting the journal is to collect and diffuse information of interest and value to the industries.

To those interested in the organization of research, the situation in Italy will prove worthy of careful study. Government testing laboratories, schools of technology, astronomical observatories and manufacturers of scientific apparatus are all closely associated, somewhat as has been attempted at various times in this country. We should have a somewhat analogous condition of affairs if the Bureau of Standards and other testing and research laboratories were to throw open their doors to students and organize regular courses of instruction for them, and if they, together with Mt. Wilson and other observatories, with Gaertner, Leeds and Northrup, Bausch & Lomb, and other manufacturers of apparatus, and such manufacturers of precision instruments as Pratt & Whitney and Warner & Swazey, were to form a national technological association and publish a journal. The problem of adequately training men for industrial research, and eventually for the industries themselves, is an acute one in this country to-day, and Italy appears to have solved it.

The officers of L'Associazione Italiana di Ottica e Meccanica di Precisione are as follows:

Presidente—Generale Eugenio Righi, Direttore del R. Laboratorio di precisione, Roma.

Vice-Presidente—Senatore Angelo Salmoiraghi, della "Filotecnica," Milano.

Consiglieri—Ing. Giuseppe Astorri, Presidente della Società "Archimede," Roma; Prof. Azeglio Bemporad, Direttore del R. Osservatorio di Napoli; Ing. Alberto Cantù, Direttore dell'Officina San Giorgio, Sestri Ponente; Commendatore Luigi Pasqualini, Direttore delle Officine Galileo, Firenze; Senatore Prof. Vito Volterra, R. Università, Roma.

Cassiere—Marchese Nello Venturi Ginori, Firenze.

Segretario—Prof. Augusto Occhialini, Direttore del Laboratorio d'ottica practica e meccanica di precisione, Firenze.

P. G. NUTTING.

THE OPTICAL CLUB OF WASHINGTON.

The Optical Club of Washington was formed October 29, 1919. Seven meetings were held during the past year at various laboratories, at which the following papers were presented:

F. E. Wright—"Crystalline Materials Used in Optics."

Paul D. Foote and F. L. Mohler—"Determination of Planck's Constant *h* from Measurements upon Electronicatomic Impact."

G. H. Peters—"Instruments and Work of the Naval Observatory."

W. J. Humphreys—"Optics of the Air."

H. H. Kimball—"Variations in the Total and Luminous Solar Radiation with Geographical Position in the United States."

Irwin G. Priest and H. J. McNicholas—"Description of Colorimetric Work at the Bureau of Standards."

A. H. Taylor—"The Measurement of Diffuse Reflection Factors and a New Absolute Refractometer.

H. E. Merwin—"Methods of Measuring Refractive Index."

The club consists of about forty members. Officers for 1920–21 are W. J. Humphreys, G. H. Peters, W. F. Meggers, and a secretary to be appointed.

PAUL D. FOOTE,
Sec., 1919–20.

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Optical Society of America
Devoted to all Branches of Optics

Vol. IV JULY, 1920 Number 4

DISPERSION IN OPTICAL GLASSES. II.

BY

F. E. WRIGHT.

CERTAIN linear relations between the partial dispersions of optical glasses are emphasized in the first paper of this series;[1] and from these relations several new two- and three-constant dispersion formulas are developed which represent the dispersion curves in the visible spectrum with a high degree of exactness. Reference is also made in that paper incidentally to certain other approximately straight-line relations which were first noted by Sellmeier[2] in 1871, and proved to be theoretically important because they indicated to him the significance of absorption bands in the ultra-violet and the infra-red on the course of dispersion in the visible spectrum. Sellmeier plotted, for a series of optical glasses, the refractive indices against the squares of the reciprocal of the wave length ($1/\lambda^2$, frequency squared) and found that the dispersion curves of many types of optical glass exhibit an inflection point in the visible spectrum, with the result that over this range the general shape of the curve approximates a straight line.

The dispersion relations in different types of optical glass are shown graphically in Figs. 1*a*, 1*b*. The optical constants of these

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[1] *J. Opt. Soc. America*, 4, 148–159, 1920.

[2] *Ann. d. Phys. und Chem.*, 143, 272, 1871.

Schott glasses were measured by H. Rubens and H. T. Simon[3] in the infra-red, the visible, and the ultra-violet. In Fig. 1*a* the refractive indices for the several wave lengths are plotted against wave lengths directly; in Fig. 1*b*, the ordinates are, as in Fig. 1*a*, the refractive indices, while the squares of the frequency, $1/\lambda^2$, are plotted as abscissæ. In both figures, and especially in Fig. 1*b*, the effects of the presence of absorption bands in the ultra-violet and in the infra-red are clearly shown.

The Sellmeier method of plotting refractive indices, n_D, against $1/\lambda^2$ was later adopted by Pulfrich[4] in his work on the change, with temperature, of the dispersion relations in optical glasses.

Under these conditions of plotting the dispersion curve may be assumed to be a straight line for the short range of the visible spectrum and the two-constant Cauchy-formula

$$n = A + B/\lambda^2 \qquad (1)$$

or

$$n - 1 = A' + B/\lambda^2$$

is the mathematical expression for this relation.

If now in these equations the reciprocal of the refractive index $\left(\frac{1}{n}\right)$ or of the excess refractivity $\left(\frac{1}{n-1}\right)$ be substituted for the refractive index or the effective refractivity, the dispersion curves become rectangular hyperbolas ($1/\lambda^2$ being considered the independent variable), which, however, do not depart much from a straight line because of the relatively short range of the visible spectrum. The result is that the equations

$$\frac{1}{n} = E + \frac{F}{\lambda^2} \qquad (3)$$

$$\frac{1}{n-1} = C + \frac{D}{\lambda^2} \qquad (4)$$

are fairly satisfactory dispersion formulas. Equation (4) was suggested recently by P. G. Nutting[5] as a substitute for the three-constant Hartman or Cornu equation

$$(n - n_0)(\lambda - \lambda_0) = C.$$

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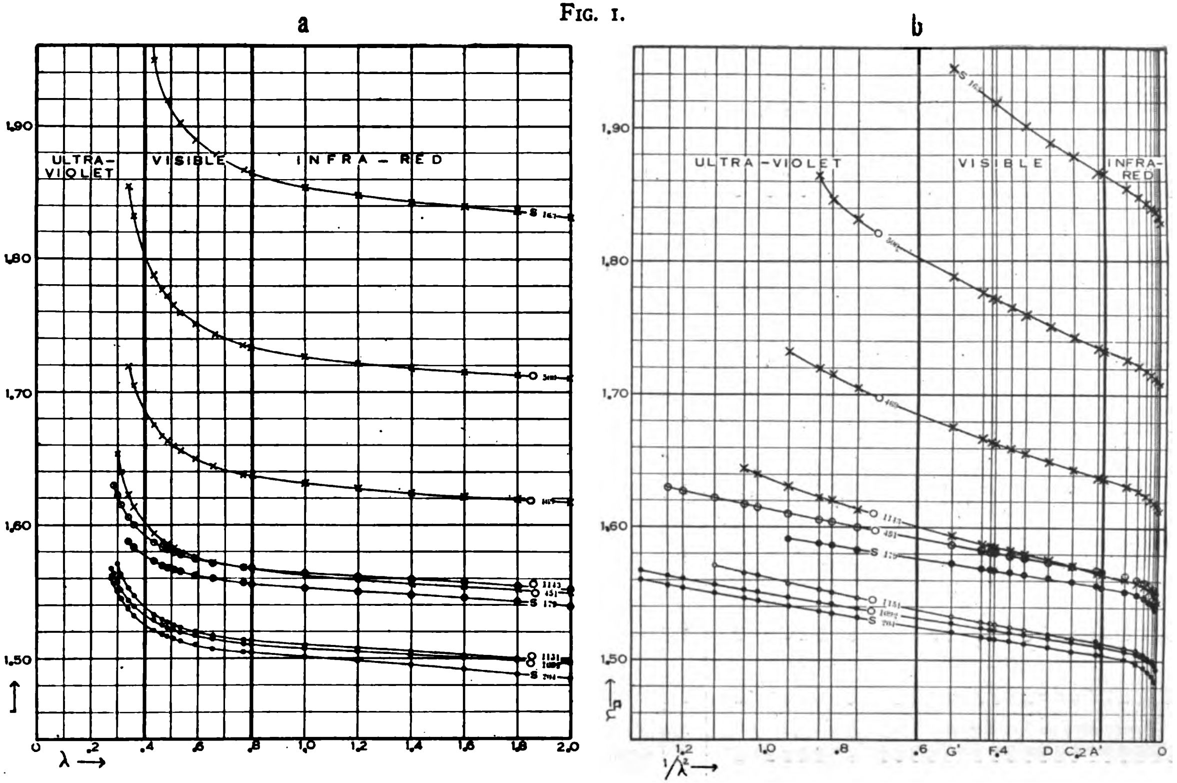
[3] *Ann. d. Phys. und Chem.*, N. F., **45**, 238, 1892; **53**, 555, 1894.

[4] *Ann. d. Phys. und Chem.*, **45**, 648, 1892. See also Hovestadt, *Jenaer Glas*, pp. 46–48. Jena, 1900.

[5] *Rivista d'Ottica e Meccanica di Precisione*, I, **54–57**, 1919.

FIG. 1.



In this figure the refractive indices for different wave-lengths in the infra-red, visible, and ultra-violet, as measured by H. Rubens and H. T. Simon on a series of Schott glasses are plotted as ordinates against the wave lengths as abscissæ.

In this figure the abscissæ scale has been changed from the direct wave-length scale (λ) of Fig. 1*a* to that of the square of the frequency ($1/\lambda^2$). The refractive indices are the ordinates. By this method of plotting, the inflection point of the dispersion curves in the visible spectrum is clearly shown as a result of the absorption band in the ultra-violet and also in the infra-red.

It is of interest, therefore, to compare the accuracy of the old two-constant Cauchy dispersion formula (equation 1) with that of the new formula of Nutting. For this purpose a number of glasses were first chosen at random from the Schott list of optical glasses and for each glass the values

$$\frac{n_C - 1}{n_{A'} - 1},\ \frac{n_D - 1}{n_{A'} - 1},\ \frac{n_F - 1}{n_{A'} - 1},\ \frac{n_{G'} - 1}{n_{A'} - 1}$$

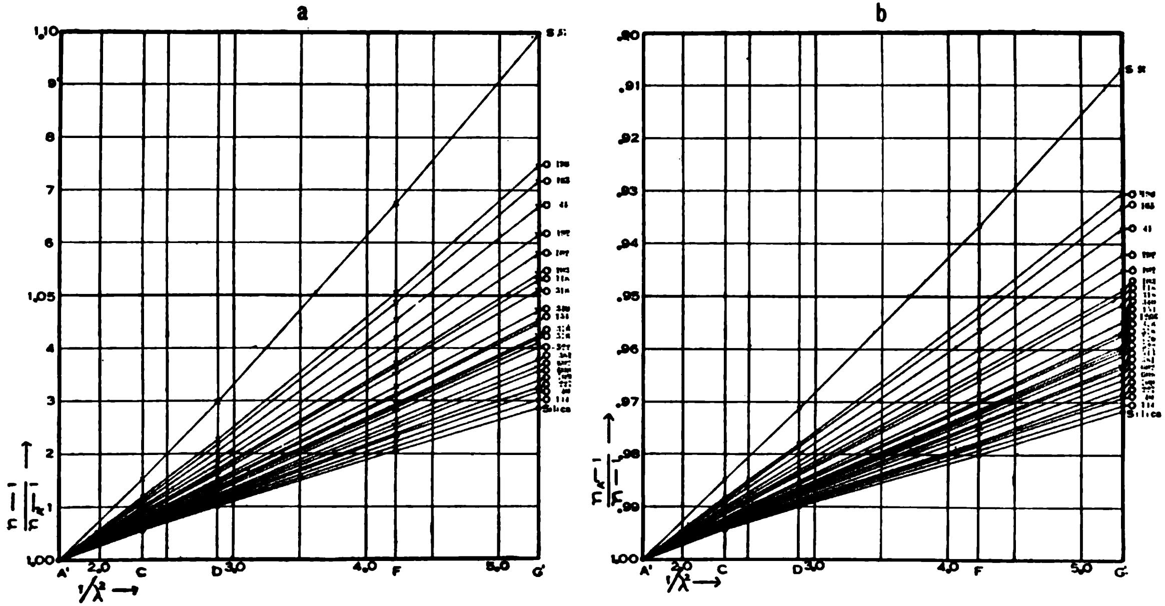
were computed; these are plotted in Fig. *2a;* the reciprocals of these values were also computed and are plotted in Fig. *2b.* In both figures the dispersion lines radiate as approximately straight lines from the *A'*-point on the abscissa axis, thus proving the approximate validity of the two different dispersion formulas, (1) and (4). It may be noted that, in order to have all dispersion curves pass through a single point, namely, the *A'*-point,

TABLE I.

In this Table the differences between the tangents of the slope angles of the chords drawn between the points A' to C, C to D, D to F, and F to G', respectively on the dispersion curve of each glass, are listed.

Type of Glass	n_D-1	$\frac{n_F-n_D}{1/\lambda^2_F-1/\lambda^2_D}$	Ia				Ib			
			n_C-n_A .00	n_D-n_C .00	n_F-n_D .000	$n_{G'}-n_F$ .00	$n_C-n_{A'}$ .00	n_D-n_C .000	n_F-n_D .000	$n_{G'}-n_F$ .00
			Borosilicate Crown.							
O 144	.5100	.00365	.083	.062	00	−002	026	20	00	−006
			Ordinary Crown.							
O 40	.5166	.00441	025	012	00	005	037	47	00	−004
O 709	.5128	.00466	030	007	00	007	044	16	00	−008
O 608	.5149	.00492	015	004	00	013	030	13	00	001
O 381	.5262	.00538	020	−011	00	017	038	0	00	004
			Barium Crown.							
O 227	.5399	.00472	026	012	00	006	039	20	00	−004
			Barium Flint.							
O 602	.5676	.00561	016	000	00	014	034	10	00	001
O 543	.5637	.00584	012	−001	00	021	032	10	00	006
O 522	.5554	.00606	006	−007	00	024	027	06	00	008
O 578	.5825	.00659	000	−007	00	029	023	07	00	010
O 1266	.6042	.00726	−005	−011	00	038	024	05	00	015
			Flint Series.							
O 726	.5398	.00599	005	−004	00	024	026	09	00	007
O 378	.5473	.00626	001	−006	00	030	023	08	00	012
O 154	.5710	.00697	−003	−009	00	039	022	07	00	017
O 340	.5774	.00735	−009	−014	00	044	020	03	00	020
O 318	.6031	.00831	−019	−023	00	055	015	−02	00	025
O 118	.6129	.00875	−030	−022	00	063	006	01	00	030
O 103	.6202	.00902	−033	−026	00	067	005	−01	00	032
O 102	.6489	.01014	−049	−034	00	084	−003	−04	00	042
O 192	.6734	.01114	−065	−044	00	098	−011	−11	00	048
O 41	.7174	.01293	−091	−065	00	122	−025	−23	00	059
O 165	.7541	.01460	−124	−082	00	150	−043	−29	00	074
O 198	.7782	.01568	−136	−096	00	170	−047	−39	00	084
O 57	.9626	.02299	−272	−184	00	314	−126	−88	00	166
			Silica Glass.							
	.4585	.00347	041	040	00	−005	67	28	00	−010

FIG. 2.



In this figure the ratios $(n_r-1)/(n_{A'}-1)$ for a series of Schott glasses are plotted as ordinates against the squares of the frequency $(1/\lambda^2)$ as abscissæ.

The reciprocal ratios $(n_{A'}-1)/(n_r-1)$ are plotted as ordinates against the same abscissæ scale $(1/\lambda^2)$. The series of glasses represented in these figures are the same as those listed in Table I and include all types of silicate glasses. These figures demonstrate clearly the feasibility of substituting in the Cauchy dispersion formula $1/(n-1)$ for $(n-1)$ and thus obtaining the Nutting formula.

each value of the excess refractivity $(n-1)$ was divided by $(n_{A'}-1)$, while in the series of reciprocals each value was multiplied by $(n_{A'}-1)$ before plotting the values in Fig. 2*a*, and 2*b*, respectively.

These relations are presented in somewhat different form in Table I, in which the procedure first proposed by Sellmeier[6] is adopted; for each dispersion curve there are listed the tangents of the slope angles of the chords drawn between certain points on the dispersion curve A' and C, C and D, D and F, F and G', respectively. Table I*a* is so arranged that for each glass the differences between the values obtained for the partial dispersions $n_C-n_{A'}$, n_D-n_C, and $n_{G'}-n_F$, respectively, are listed. Thus in the column headed n_D-n_C each value is equal to the difference

$$\frac{n_F-n_D}{1/\lambda^2_F-1/\lambda^2_D}-\frac{n_D-n_C}{1/\lambda^2_D-1/\lambda^2_C}.$$

This form of presentation enables the eye to note readily the degree of departure from the straight-line relation for which all differences would be nil.

In Table I*b* similar data on the reciprocals of the excess refractivities, as required in the Nutting formula, are listed. In this table the scale has, however, been changed in order to make it directly comparable with Table I*a*. The necessary changes in scale are readily deducible from equations (1) and (4), from which we obtain by differentiation

$$dn=B.d(1/\lambda^2)$$

$$-\frac{dn}{(n-1)^2}=D.d\,(1/\lambda^2)$$

or

$$dn=B.d(1/\lambda^2)=-D.\,(n-1)^2.d(1/\lambda^2).$$

In Table I*b*, each number is the product of $-(n_D-1)^2$ and the difference between two ratios; thus the numbers in the column headed n_C-n_A are equal to the difference for each glass,

$$-(n_D-1)^2.\left\{\frac{\dfrac{1}{n_F-1}-\dfrac{1}{n_D-1}}{\dfrac{1}{\lambda^2_F}-\dfrac{1}{\lambda^2_D}}-\frac{\dfrac{1}{n_C-1}-\dfrac{1}{n_{A'}-1}}{\dfrac{1}{\lambda^2_C}-\dfrac{1}{\lambda^2_{A'}}}\right\}$$

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[6] *Ann. d. Phys. und Chem.*, **143**, 272, 1871.

The order of magnitude of the values listed in Tables I*a* and I*b* is accordingly directly comparable, as a first approximation, and the relative degree of exact representation of the dispersion relations by the two equations (1) and (4) can be ascertained by direct inspection.

Table I demonstrates that for the crown glasses as a whole (ordinary crowns, borosilicate crowns, barium crowns) the Cauchy formula is superior to the formula proposed by Nutting, whereas for the flint series and the barium flints the Nutting formula is somewhat better; but even in this case the agreement of the values computed by the Nutting formula with the observed values is not close.

It is, of course, possible, even in cases where the calculated values depart from the observed values, to obtain, either graphically or by computation, a curve of departures or errors (residuals) and from it to ascertain the value of the refractive index for any intermediate wave length to a high degree of exactness.

To test still further the relative accuracy of equations (1) and (4), the residuals for the four glasses cited by Nutting[7] were computed for each equation. These glasses were measured with a high degree of precision by Gifford[8] and include a telescope crown *S* 3418 ($n_D = 1.530848$, $\nu = 60.6$), a dense barium crown *S* 4704 ($n_D = 1.610702$, $\nu = 55.7$), a dense flint *C* 629 ($n_D = 1.612137$, $\nu = 37.0$), and a borosilicate flint *S* 3338 ($n_D = 1.548146$, $\nu = 49.9$). For each one of the four glasses the constants of the equations (1) and (4) were computed by the method of the Zero Sum recently proposed by N. Campbell[9] as an improvement on the method of Least Squares. For this purpose twelve of the thirteen refractive indices listed by Gifford were used; the refractive indices for the wave length 0.4415μ were not used as they differ only slightly from those for wave length 0.4341μ and their exclusion facilitated the computations. The resulting equations for the several glasses are given in Table II.

The equations II of Table II are built up from the refractive indices by computing first the reciprocal of the excess refractivity for each refractive index $\left(\frac{1}{n-1}\right)$; from this list of reciprocals the constants of the equation are obtained by the Campbell method of the Zero Sum. The values $\frac{1}{n-1}$ are then computed from the equation II thus obtained and the values of the reciprocals of these in turn are computed and compared with observed values.

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[7] *Revista d'Ottica a Meccanica di Precisione I*, 56, 1919.

[8] *Proc. Roy. Soc.*, 87, 190–191, 1912.

[9] *Phil. Mag.*, 39, 177, 1920.

The differences between computed and observed values as found from the two equations I and II are listed in Table III. There is also included in this table a series of values computed for the telescope crown *S* 3418, according to the plan adopted by Nutting, namely, to compute for each glass the values *A'* and *B* or *C* and *D* of the two equations from the two refractive indices n_D and n_G. In this case the departures are slightly greater than with the better adjusted constants, but the order of magnitude of the values is the same.

TABLE II.

	I.	II.
S 3418	$n=1.517485+\frac{0.0046242}{\lambda^2}$	$\frac{1}{n-1}=1.930798-\frac{0.0162097}{\lambda^2}$
S 3418	$n=1.517593+\frac{0.0046031}{\lambda^2}$	$\frac{1}{n-1}=1.929846-\frac{0.0159981}{\lambda^2}$
S 4704	$n=1.594036+\frac{0.0057836}{\lambda^2}$	$\frac{1}{n-1}=1.681725-\frac{0.0152994}{\lambda^2}$
C 629	$n=1.586814+\frac{0.0088606}{\lambda^2}$	$\frac{1}{n-1}=1.700146-\frac{0.023137}{\lambda^2}$
S 3338	$n=1.531396+\frac{0.0058158}{\lambda^2}$	$\frac{1}{n-1}=1.879484-\frac{0.0190631}{\lambda^2}$

Table III proves definitely that in the two crown glasses the differences between computed and observed values are less for the Cauchy formula (equation 1) than for equation (4), while in the two flint glasses the departures are less for equation (4).

The foregoing different lines of approach prove definitely that in certain cases, namely the crown glasses, equation (1) furnishes values more nearly in agreement with the data of observation than does equation (4); that in the flint glasses, on the other hand, equation (4) is superior to equation (1), but that even with equation (4) the results of computation differ appreciably from the observed values and that neither equation (1) nor (4) represents the data of observation in the flint series with a high degree of precision.

The Nutting formula (equation 4) is less convenient for purposes of computation than the Cauchy formula (equation 1); and in view of the fact that it is superior to the Cauchy formula only for the flint glasses, its use for general purposes as a substitute for the Cauchy and the Hartman formulas, does not seem advisable; to substitute it for the ordinary three-constant Hartman formula, $(n - n_0)(\lambda - \lambda_0)^{1.2} = C$, does not lead to equally satisfactory results.

TABLE III.

In this Table are listed, for four Schott glasses measured by Gifford, the residuals (differences between computed and observed values) of refractive indices for different wave lengths from the Cauchy dispersion formula (Columns I) and the new formula suggested by Nutting (Columns II). In Columns I′ and II′ are listed the residuals for a straight line passing through D and G′.

Wave length	Telescope Crown *S* 3418				Dense Barium Crown *S* 4704		Dense Flint *C* 629		Borosilicate Flint *S* 3338	
	I	II	I′	II′	I	II	I	II	I	II
	.0000	.000	.000	.000	.000	.000	.00	.000	.000	.000
.7682	90	164	162	275	078	179	−0242	−002	058	172
.7066	17	048	082	195	−018	025	−0138	−033	−036	014
.6708	−13	−009	048	126	−010	−001	−0067	−043	−027	−017
.6563	−27	−032	032	097	−038	−043	−0043	057	−044	−050
.5893	−47	−093	000	000	−011	−078	0192	038	−003	−078
.5607	−17	−080	025	−001	−001	−087	0286	086	052	−047
.5461	−25	−093	012	−024	007	−084	0329	113	068	−035
.5270	−07	−063	039	−009	037	−059	0350	124	079	−029
.4861	45	−010	063	−006	052	−022	0330	149	081	−003
.4678	32	−002	043	−006	047	001	0238	123	075	024
.4415	07	031	007	−009	030	062	−0056	006	014	038
.4341	04	050	000	000	−001	062	−0214	−080	033	036
.4046	−60	125	081	023	−143	·108	−1036	−461	−271	011

SUMMARY.

In this paper, proof is given that, because of the relatively short range of the visible spectrum, the substitution in a dispersion formula of the reciprocal of the refractive index or of the excess refractivity or, by analogy, of other functions of the refractive index in place of the direct values, leads to dispersion formulas which are fairly satisfactory. Thus, if in the two-constant Cauchy formula, $n = A + B.\lambda^{-2}$ or $n - 1 = A' + B.\lambda^{-2}$, the reciprocal of the refractive index or of the effective refractivity be written: $n^{-1} = C' + D'.\lambda^{-2}$ or $(n-1)^{-1} = C + D.\lambda^{-2}$, the new equations represent rectangular hyperbolas in case λ^{-2} is considered to be the independent variable. The last equation was recently suggested by Nutting as a substitute for the Hartmann dispersion formula. A series of computations demonstrates, however, that for the crown glasses this equation is less satisfactory than the Cauchy formula, while for the flint glasses, none of the foregoing equations is especially good. The last equation is, moreover, always less satisfactory than the Cauchy formula for computation purposes; its usefulness appears, therefore, to be limited and less satisfactory for general application than the two-constant Cauchy formula.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
June, 1920.

CLASSIFICATION AND NOMENCLATURE OF OPTICAL GLASS.[1]

BY

GEORGE W. MOREY.

A STUDY of the optical glass catalogues of the various American and foreign manufacturers reveals a striking lack of uniformity in nomenclature. The names in use are the result of a gradual development, in the course of which each manufacturer has listed new glasses, as they have been developed, under names which seemed appropriate. The natural result of this haphazard method has been that in some cases the same glass is known by several different names; in other cases, glasses differing in no essential property from the usual types have received names implying the possession of special properties which the glasses do not possess in any marked degree. Furthermore, the nomenclature adopted by any one manufacturer has rarely been consistent; witness the confusion between the so-called "soft crowns," "crowns of high dispersion," "extra light flints," and "telescope flints." Indeed, there is apparently inextricable confusion between the broad general classes of the crowns and the flints.

It has been the purpose of the writer to simplify the nomenclature of optical glass by the elimination of many of the special names, retaining only names which, while indicating the position of the glass in the general scheme, leave its precise specification to be indicated by its optical properties. There are two objections which might be urged against this procedure. In the first place, it might be claimed that this means the obliteration of types chemically distinctive, such, for example, as the borate flints or zinc crowns. It should be noted in this connection, however, that such distinctions are far less real than has commonly been believed. The difference between a "zinc crown" and an ordinary crown is not one of fundamental difference in type; indeed, many ordinary crowns contain zinc oxide, introduced more for its effect on the manufacturing process than for its effect on the optical properties. Similarly, the borate flints do not differ significantly from the barium flints, to which they are in many respects inferior; they have been but little used and their manufacture has become less common. These conclusions are at variance with the views often found in the literature, but are fully substantiated by the work of Wright,[2] in which the reader will find further illustrations of the same kind.

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[1] An extract from the Report of the Subcommittee on Optical Glass, Nomenclature and Standards Committee, American Optical Society, presented at the New York meeting, February 27–28, 1920.

The second objection which might be offered to such a plan of nomenclature is that it would lead to confusion in the purchase of optical glass. Quite the contrary is the case. Glass type names are for convenience in casual reference only; purchase always should be on definite specification of the optical properties desired. This point should be strongly emphasized.

The optical constants in common use are the refractive index for sodium light, n_D, and the ν-value, the ratio of the excess refractivity, $n_D - 1$, to the mean dispersion, C to F, $\left(\frac{n_D - 1}{n_F - n_C}\right)$. It is a well-known fact that in the ordinary crown-flint series of glasses, the so-called "older" glasses, the mean dispersion may be expressed as a linear function of the refractive index,

$$(n_F - n_C) = an_D - b$$

and hence the ν-value also may be expressed as a function of the refractive index,

$$n_D = \frac{1 - b\nu}{1 - a\nu}$$

The graph of this equation is shown in Fig. 1. The constants a and b have the values 0.078984 and 0.110887, respectively. In the same figure[3] are plotted all the glasses listed in the catalogues of the Spencer Lens Optical Glass Plant, Glaswerk Schott und Genossen, and Chance Brothers and Co., Limited. It will be observed that a large proportion of the glasses lie on the curve; these are the glasses of the ordinary crown-flint series, on which the proposed system of nomenclature is based. These glasses are essentially mixtures of silica with alkalies, lime, boric acid,

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[2] "Dispersion in Optical Glasses." F. E. Wright, *J. Opt. Soc. Amer.*, 4, 148 (1920).

[3] A somewhat similar figure has been published by Montgomery. *Chem. Met. Eng.*, 21, 467 (1919).

FIG. I.

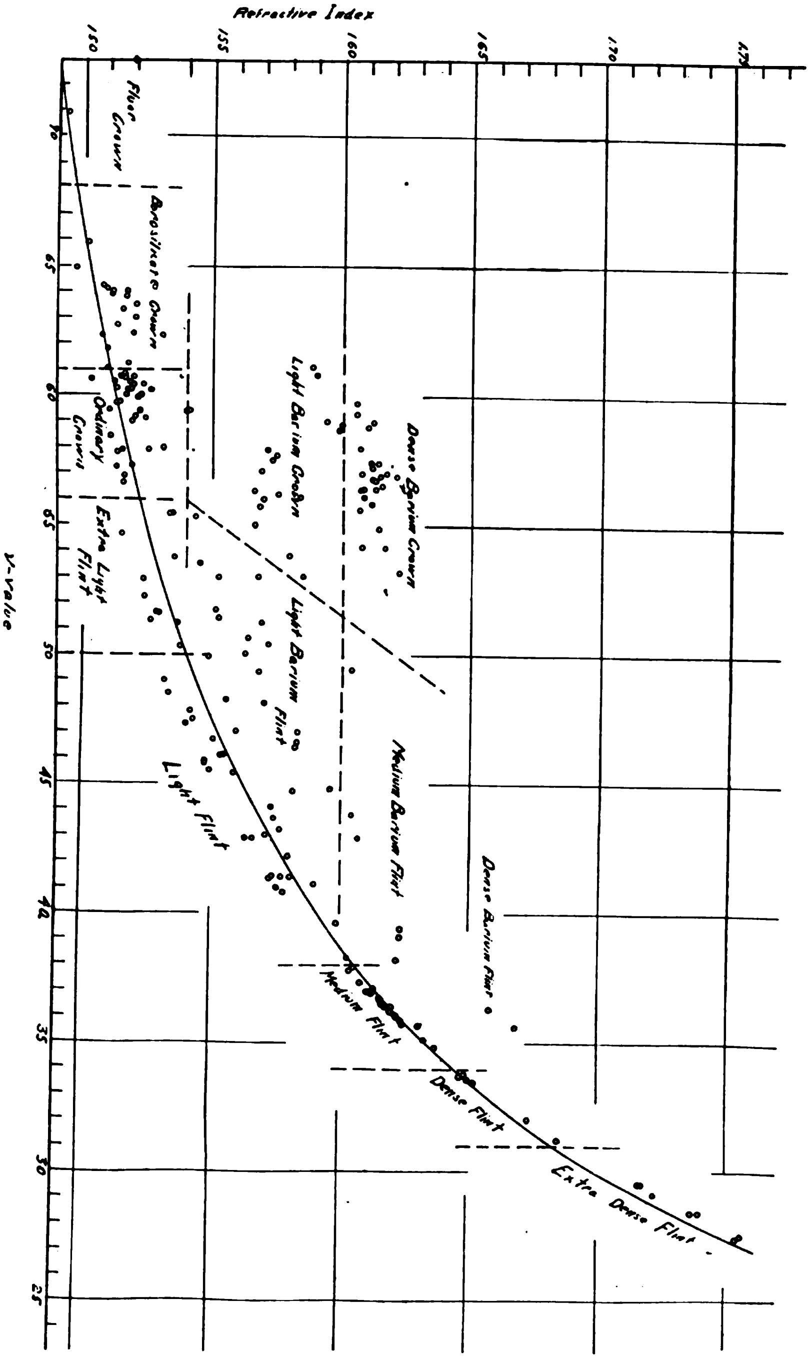

Figure showing the suggested classification of optical glasses.

and lead oxide; boric acid and lime characterizing the glasses of lower refractive index, lead oxide, those of higher index. Barium oxide is present only in small amounts, if at all. These glasses will be considered first, after which the barium glasses will be discussed.

The glasses of lowest refractive index and highest ν-value are the fluor crowns. They have a very small dispersion, and are distinctive in type from all other glasses. The lower limit of ν-value for these is 68; glasses of lower ν may contain fluorine, but it is no longer the essential constituent without the use of which glass of the property in question cannot be prepared.

The next group of glasses are the borosilicate crowns. These are characterized by a ν-value lower than the fluor crowns, but higher than the ordinary crowns. As indicated by the name, the characterizing component is boric acid, which in the lower index members of the series is present in large amounts. Their distinctive properties are conferred by boric acid, which, as is well known, is notable for its constraining effect on the spectrum, *i.e.*, for its small dispersive power. It is also present in large amounts in the fluor crowns, but with it alone we are not able to obtain the glasses with the largest ν-value. The dividing line between the borosilicate crowns and the ordinary crowns is placed at a ν-value of 61; boric acid is essential in glasses of higher ν, but, while it may be present in small amounts in glasses of lower ν-value, its presence ceases to be of paramount importance below the limit given.

The next group contains the ordinary crowns, often termed silicate crowns and hard crowns. In this group are included also the zinc crowns, which fall well on the curve.[4] The dividing line between the ordinary crowns and light barium crowns will be referred to later.

A difficult point in optical glass nomenclature is fixing the dividing line between crowns and flints. Historically, the term flint has nothing to do with the chemical composition of the glass, but is derived from the use of flints as a source of silica by the first makers of lead glass. According to present usage, however, a flint glass is one containing lead oxide, and it seems best to retain the term with this explicit significance. Accordingly, the dividing line between crowns and flints in the ordinary crown-flint series has been placed at a ν-value of 56. Lead oxide is an essential constituent of all glasses of this series, and a ν-value of 56 represents fairly closely the dividing line between glasses entirely free from, or containing but small amounts of, lead oxide, and those which cannot be produced without its aid. The same facts are clearly brought out in Fig. 1 of the paper by Wright, to which reference has already been made. Here it will be noticed that glasses of the crown-flint series having the ratio $\frac{n_D - n_{A'}}{n_{G'} - n_F}$ greater than 1.1 are typically free from lead oxide, while those having this ratio less than 1.1 all contain lead.

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[4] Compare the position of the zinc crowns in the paper by Wright (*loc. cit.*), Fig. 1.

The flints are sub-divided for convenience into the extra light flints, light flints, medium flints, dense flints and extra dense flints. The dividing line is wholly arbitrary. and represents fairly well established usage.

The first sub-group of flints, the extra light flints, with ν-values from 56–50, contains glasses listed under many different names. It includes some "crowns of high dispersion," some "soft crowns," "telescope flints" and extra light flints. All of these, however, are characterized by containing lead oxide as an essential constituent. Their resemblance to each other, and their proper position in the crown-flint series will be evident from a comparison of their optical properties; this is especially marked in the curves published by Wright,[5] to which reference has frequently been made. It is manifest that no useful purpose is served by continuing this confusion of names, all of which, while implying a higher dispersion than the crowns, also implies a contrast with one another in optical properties which is not justified by the facts.

The next group, the light flints, is one containing a far larger range of index and ν-value than the preceding. The group is one of fairly well-defined properties, special glasses used in small quantity for a wide variety of instruments, but rarely used in large quantity. With the exception of indefiniteness as to the upper and lower limits of the class, catalogues of various firms agree as to the nomenclature of these glasses. The upper limit chosen, a ν-value of 38, corresponds to a flint of index 1.605, which is usually considered as belonging to the next group.

The group of medium flints includes those glasses of the crown-flint series having refractive index from 1.60 to about 1.64, ν-values from 38 to 34. The very important flints with index 1.616 and 1.620 are included. The name chosen, medium flints, is expressive of their lead oxide content; they are commonly listed as medium or ordinary flints, though even the 1.616 flint is termed a " dense " flint by some makers.

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[5] *Loc. cit.*

The group of dense flints here includes those with ν-value from 34 to 31, index from 1.64 to 1.69, while the flints of higher lead content, index 1.70 and higher, are termed extra dense flints. This is in accord with established usage.

We will next consider glasses of totally different composition, the dense barium crowns. These are characterized by the absence of alkali, and by their large content of barium oxide, oftentimes 40 to 50 per cent.; other components are boric acid, zinc oxide, alumina and silica. Optically they have the refractive index of a medium flint, from 1.60 to 1.62 or higher, with the ν-value of a crown, usually greater than 56, though they may have a ν-value as low as 53 without the presence of lead oxide. These glasses are one of the triumphs of the application of modern science to the art of glass-making, and they have made possible the modern high-speed anastigmatic lenses so extensively used in photography to-day.

By the introduction of alkalies into the dense barium crowns we may obtain the light barium crowns, intermediate in composition and optical properties between the dense barium crowns and the ordinary crowns. Almost any desired glass intermediate between these two may be obtained, though only a few have any commercial importance. The actual boundary line between the light barium crowns and the dense barium crowns, on one hand, and the ordinary barium crowns on the other is, of necessity, somewhat arbitrary; in the boundaries selected I have been guided by the absence of alkalies in the dense barium crowns, and the great importance of barium oxide in the light barium crowns, as contrasted with its relatively slight importance, when present, in the ordinary crowns. Accordingly, the light barium crowns have been defined as including those glasses having refractive index between 1.600 and 1.540, and of lower dispersion, *i.e.*, higher ν-value, than the barium flints.

In the ordinary crown-flint series, the boundary between crowns and flints was placed at a ν-value of 56, this representing fairly well the line of demarcation between glasses in which lead oxide is an essential constituent and those containing no lead oxide. Because of the specific properties of barium oxide, this distinction cannot be made between the barium crowns and barium flints. Practically, however, the distinction is not difficult to make, and a line drawn from the point $nD = 1.54$, $\nu = 56$ and $nD = 1.62$, $\nu = 50$, will serve to separate the lead-free barium glasses from those containing lead oxide in significant amounts.

The definition of the barium flints is based on their relation to the barium crowns and the ordinary flints. From the glass-makers' point of view they are distinctive, lying between the barium crowns, on the one hand, from which they may be distinguished by their lead oxide content, and the flints of the ordinary series, on the other hand, from which they may be distinguished by the presence of barium oxide. The entire series may be considered as derived from the ordinary flints by partial replacement of lead oxide by barium oxide, and this replacement may take place to almost any degree, thus making possible a wide range in composition. Only a comparatively few of the possibilities, however, have proven commercially useful.

Sub-divisions of the barium flints into light, medium and dense have been made for convenience, the dividing lines being at the refractive indices 1.60 and 1.64, respectively; these correspond approximately to the similarly named divisions in the ordinary series.

The classification and grouping of optical glasses which has been outlined provides a systematic and logical system, in which the essential factors in composition have been emphasized in the group names, borosilicate crown, crown, flint, barium crown and barium flint. For further convenience, some of these have been further subdivided, the groups coinciding roughly with the names in common use, which have historical value and have a place in the literature.

As has already been stated, purchase of glass should be on specification of the optical properties desired. For example, if a flint of index 1.616, ν 36.6, is desired, it should not be ordered merely as "ordinary flint glass" or "dense flint glass," but as a flint, or medium flint, index 1.616, ν 36.6, plus or minus such tolerance as is reasonable and proper. Indefinite specifications are a source of confusion, and often lead to annoying delays and errors.

Another plan of nomenclature, which carries out the above principles in a far more extreme degree, is the plan adopted by the new Sendlinger Optical Works, near Berlin. They have discarded all the usual names except crown and flint, and coined the new names "barion" and "barint" to designate the barium crowns and barium flints, respectively. Glasses are designated by a compound numeral, consisting of the first three digits of the refractive index minus 1 $(n_D - 1)$, followed by the first two digits of the ν-value, the two parts being separated by a hyphen. This, however, offers but little advantage. It is not noticeably shorter than the system herein proposed, saving only one or two digits, and is less simple and more apt to lead to confusion.

SUMMARY.

The system of optical glass nomenclature proposed is based on the crown-flint series of "older" glasses, *i.e.*, barium-free glasses. The dividing line between crowns and flints is at a ν-value of 56, the approximate lower limit of the lead glasses. The crowns are divided into fluor crowns, with ν-value greater than 68, borosilicate crowns, with ν between 68 and 61, and ordinary crowns, with ν from 61 to 56. The flints are divided into extra light, ν from 56 to 50, light flints, ν from 50 to 38, medium flints, ν from 38 to 34, dense flints, ν from 34 to 31, and extra dense flints. Glasses falling outside the crown-flint series are grouped with the barium crowns and barium flints. The barium crowns have nD higher than 1.60, the light barium crowns have nD between 1.60 and 1.54. The barium flints are divided into light barium flints, nD less than 1.60, medium barium flints, index between 1.60 and 1.64, and dense barium flints.

It is emphasized that optical glass type names are for convenience in casual reference only, and do not imply a specification of optical properties. Whenever reference is made to a definite glass, as opposed to the broad general type, whether in the purchase of glass or in discussion of properties or uses, the reference should be made exact by specifying the optical properties, usually the refractive index for sodium light and the ν-value.

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SPENCER LENS OPTICAL GLASS PLANT,
HAMBURG, N. Y.

A NOTE ON THE ANNEALING OF OPTICAL GLASS.

BY

L. H. ADAMS and E. D. WILLIAMSON.

An ideal optical glass would fulfil three principal requirements. (1) the glass must be colorless and transparent; (2) it must be chemically homogeneous, that is, free from striæ, "stones" and bubbles; and (3) it must be mechanically homogeneous—free from internal stress. The last of these requirements is a subject which occupied the attention of the authors during a portion of the period of the war-time participation of the Geophysical Laboratory in the manufacture of optical glass, and is the subject which we now propose to discuss. The prevention of internal stress in glass and its removal when present is a problem which requires, for its exact solution, a determination of certain thermal, optical, and elastic constants of the different glasses. As will be apparent later, the stresses are determined by the temperature gradients and therefore a knowledge of the relations between the various factors such as temperature gradients, rate of heating, stress, and annealing temperature is required. Moreover, since stress is usually measured by the change in an optical property—birefringence—the data in the stress-birefringence ratio must also be obtained. Certain of these phases of the subject have already been covered in previous publications from this Laboratory, and the Annealing of Glass is treated in detail in a paper which is now ready for publication in another journal. It has seemed desirable, however, to summarize briefly the salient features of this subject, in so far as they pertain to optical glass, and accordingly in this note the formation of internal strain in glass is discussed, and concrete directions for the practical annealing of optical glass are presented.

The Way in Which Internal Stress Arises.—When a lump of molten glass is cooled quickly it acquires internal stresses and is said to be "strained."[1] Excessive strain cannot be tolerated in any glass, since it renders the glass liable to break when handled or heated again, while in the case of optical glass even a moderate amount of strain causes troublesome warping of finished lenses and prisms. In order to understand how glass becomes strained, it is necessary to recall that at a sufficiently high temperature glass softens and that when soft, internal stresses are relieved by viscous flow. Moreover, any solid body when heated or cooled from the surface acquires a certain temperature gradient,[2] and this temperature gradient induces temporary stresses the direction and magnitude of which depend on the temperature gradient and the dimensions of the solid.[3] Thus, for example, in a slab of glass the surfaces of which are being heated at a constant rate h (in deg. per min.), the stress F at any point is given by the equation

---

[1] It is important to remember that "stress" denotes a force and "strain" the consequent deformation. Stress usually accompanies strain, but the two words, although often interchangeable, are not synonymous.

$$F = 4.6\ h\ (a^2 - 3x^2) \tag{1}$$

in which F is in kg/cm^2 and is positive for a tension, a is the semi-thickness (in cm.) of the plate, and x is the distance from the middle. Along the middle, $x = 0$ and the equation reduces to

$$F = 4.6\ ha^2 \tag{2}$$

Thus, in a slab or plate heated at a uniform rate there will be in the middle a tension in all directions parallel to the faces of the plate, and at the surface a corresponding compression, while at a distance from the middle equal to 0.578 of the semi-thickness is a neutral zone, that is, neither tension nor compression. Fig. 1 illustrates the distribution of stress in a slab of glass 2 cm. thick heated at the constant rate of 5° C. per minute.

The corresponding formula for the stresses in a plate due to a sudden change of surface temperature is more complicated. Its general form is the infinite series:

$$F = \frac{4\ a\ (\theta_i - \theta_f)}{\pi\ C} \sum_{m=1}^{m=\infty} e^{-\frac{(2m-1)^2\pi^2\kappa t}{4\,a^2}} \left\{ \frac{2}{\pi\,(2\,m-1)^2} - \frac{(-1)^{m+1}}{2m-1} \cos\frac{(2m-1)\pi x}{2a} \right\} \tag{3}$$

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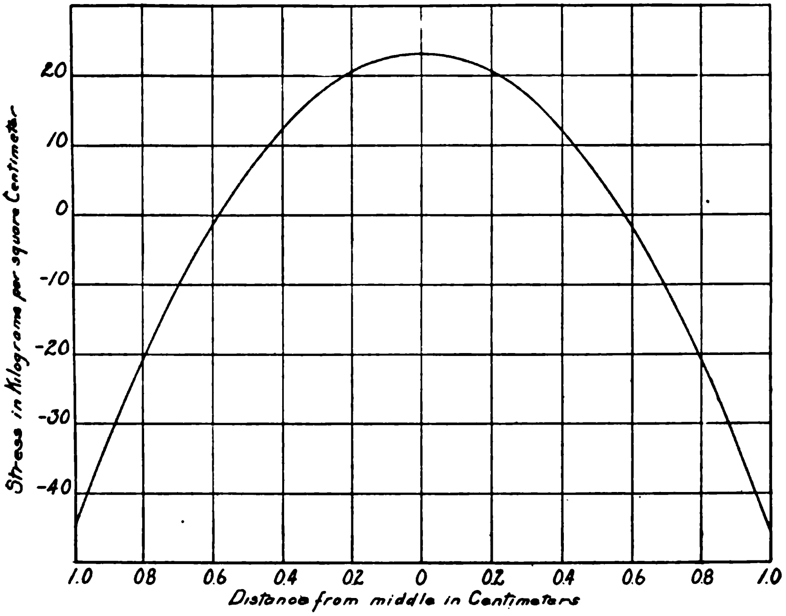
[2] E. D. Williamson and L. H. Adams. *Phys. Rev.*, 14, 99–114 (1919).

[3] For a more detailed explanation the reader is referred to the above-mentioned paper on "The Annealing of Glass."

where m is any integer, θ_i is the initial uniform temperature of the plate, θ_f is the new temperature of the surfaces, C is equal to $\frac{1}{6R} + \frac{2}{9K}$, K being the modulus of compressibility and R the modulus of rigidity, and κ the coefficient of thermal diffusivity. In Fig. 2 is shown for several values of the time, the distribution of stress in a plate of glass 2 cm. thick, initially at 0° C. and suddenly plunged into a bath at 100° C.

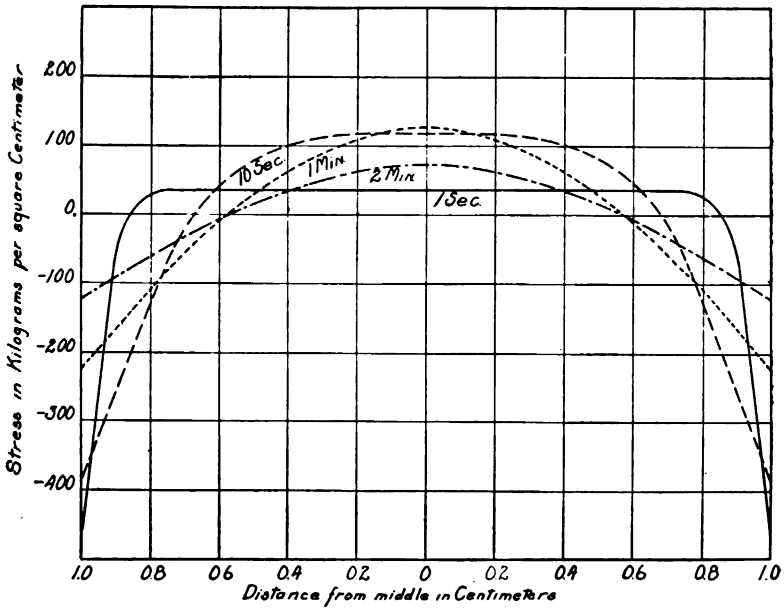
FIG. 1.



Curve showing distribution of stress in a plate of glass 2 cm. thick which is being heated at a constant rate equal to 5° C. per minute. Stresses are given in kg/cm^2 and are taken as positive for a tension and negative for a compression.

It is obvious that if a temperature gradient happens to exist in a block of glass which is unstrained, the removal of the temperature gradient will produce stresses equal and opposite in sign to those produced by introducing the same gradient. Now if we start with a lump of glass in the molten condition and suppose that it be cooled at a uniform rate, the stresses produced by the establishment of the temperature gradient disappear almost instantly, and as the cooling proceeds we have the condition of a temperature gradient without accompanying stresses. When room temperature is reached, however, the cooling stops, the temperature gradient disappears, and hence, according to the rule laid down, the removal of the gradient produces a state of permanent stress, which is exactly the same as the system of temporary stresses caused by heating the originally cold and unstrained glass at the same rate. That is to say, if the glass be in the form of a plate, the inside will be in a state of longitudinal tension and the outside in a state of longitudinal compression. The same kind of explanation holds for glass cooled at a variable rate and we may perhaps regard the stresses in unannealed glass as due to a temperature gradient (corresponding to a certain cooling rate) being " frozen in " as the glass hardens.

Fig. 2.



Set of curves showing the distribution of stress in a plate of glass 2 cm. thick which, initially at 0° C, is suddenly plunged into a bath at 100° C. The full line indicates the magnitude of the stresses throughout the glass after 1 second has elapsed; the broken line, after 10 seconds; the dotted line, after 1 minute; and the dot-dash line, after 2 minutes.

Detection of " Strain."—Glass when subjected to stress becomes birefracting. The usual method for detecting strain takes advantage of this fact, the presence or absence of strain being determined by observing the glass in polarized light between crossed nicols. A small amount of strain causes a lightening of the field;[4] a larger amount produces the higher colors of the Newton scale, and by comparing the colors with a proper chart a quantitative measure of the optical path difference is obtained. A better method consists in using a graduated quartz wedge[5] in which the path difference can be read directly in $\mu\mu$.

In any one kind of glass the birefringence is proportional to the stress; that is,

$$\Delta n = BF \tag{4}$$

where F is the stress in kg/cm^2, Δn is the birefringence, and B is a constant. The values of B for nine kinds of optical glass[6] are given in the fourth column of Table I. For most kinds of glass a uni-directional stress of 1 kg/cm^2 produces a maximum birefringence which does not differ much from about 2.8×10^{-7}, that is, about 2.8$\mu\mu$ per cm. For heavy flint glasses, however, the value of B is smaller, and for the heaviest flints is of the opposite sign.

TABLE I.

For converting strain from optical to mechanical units. The first three columns describe the glass, and the last column gives the birefringence-stress ratio.

Kind of Glass	Refractive index n_D	Constringence ν	Birefringence due to 1 kg/cm^2 B
Borosilicate Crown..........	1.516	62	2.85×10^{-7}
Ordinary Crown.............	1.523	59	2.57×10^{-7}
Light Barium Crown.........	1.574	57	2.81×10^{-7}
Heavy Barium Crown.......	1.608	57	2.15×10^{-7}
Barium Flint...............	1.606	44	3.10×10^{-7}
Light Flint.................	1.573	42	3.20×10^{-7}
Medium Flint...............	1.616	37	3.13×10^{-7}
Heavy Flint................	1.655	33	2.67×10^{-7}
Extra Heavy Flint..........	1.756	27	1.22×10^{-7}

Moreover, by determining the sign of the birefringence, that is the α and γ directions,[7] we may know the direction and sign of stress; thus having determined the α and γ directions (in any ordinary glass), we know that there is either a tension parallel to γ or a compression parallel to α (or both).

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[4] For detecting a small amount of strain, greater sensitivity is obtained by using a sensitive-tint plate similar to that ordinarily used on the petrographic microscope.

[5] See for example, F. E. Wright. The methods of petrographic microscopic research. Carnegie Inst., Washington, Publication No. 158 (1911).

[6] See L. H. Adams and E. D. Williamson. *J. Wash. Acad. Sci.*, **9**, 609–623 (1919).

[7] α is the lesser refractive index, and in a positive uniaxial crystal is for light vibrating perpendicular to the axis.

Taking the average value of B as $2.8\mu\mu$ per cm., and substituting by equation (4) in equation (2) we have

$$\Delta n = 13\ ha^2 \tag{5}$$

(in which Δn is in $\mu\mu$ per cm., h is in deg. per min. and a in cm.) for the birefringence along the middle, due to heating a plate of glass at a constant rate.

Removal of Strain—Annealing.—At a sufficiently high temperature the internal stresses relieve themselves by internal movement and thus gradually disappear. If the glass be then properly cooled it will be found to have very little internal strain. We have proposed to call glass "annealed" when the optical path difference[8] as measured along the middle of a slab is not greater than $5\mu\mu$ per cm. The "annealing temperature" is defined as the temperature at which the strain is reduced from 50 to $5\mu\mu$ in a specified time, and the "*annealing range*" is defined, also quite arbitrarily and solely for convenience in referring to this range of temperature, as that 150° interval lying immediately below the "annealing temperature" for 1 minute. Temperature changes below the annealing ranges do not readily affect the permanent strain.

Experimental Determination of Release of Strain.—In order to measure the rate at which stress disappears in various glasses and at various temperatures, rectangular plates of each glass were placed in an electric furnace which was provided with a small hole at either end so that the path-difference (and hence the strain) could be determined *in situ.* The plates were about 2 cm. thick and 8 to 10 cm. long and had the two ends ground flat and then polished.

With each kind of glass measurements were made at several temperatures—usually three—which were measured with a platinum-platinrhodium thermocouple in conjunction with a potentiometer. Glass was first strained by rapid cooling from a high temperature and the temperature of the furnace was held constant within 1° while, at frequent intervals of time, the birefringence along the middle of the slab was read by means of a graduated quartz wedge. For further details, including a diagram of the furnace and optical system, the reader is referred to the above-mentioned paper on the Annealing of Glass.

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[8] The standard of annealing proposed by F. E. Wright and adopted during the war for the inspection of optical glass was 20 $\mu\mu$ per cm. *maximum* path difference. The maximum in a slab is found at the outside surface, and is usually about twice that in the middle (where it is easier to measure the strain). Thus $20\mu\mu$ at the surface would correspond to $10\mu\mu$ in the middle. We have preferred the lower value, $5\mu\mu$, because the standard is an arbitrary one in any case, and no difficulty is encountered in annealing glass to that degree of excellence.

It was found that the release of stress at constant temperature proceeds according to the empirical equation

$$\frac{1}{\Delta n} - \frac{1}{(\Delta n)_0} = At \qquad (6)$$

in which $(\Delta n)_0$ is the original birefringence, Δn is the birefringence at the time t, and A is a constant, which we may call the "annealing constant" for a particular glass at a particular temperature, and which is a measure of the rate at which glass anneals when held at constant temperature. Since by differentiation, $-\frac{d(\Delta n)}{dt} = A(\Delta n)^2$, it is to be noted that the equation (6) is equivalent to the statement that the rate of release of stress at constant temperature is proportional to the *square* of the stress.

The dependence of the rate of annealing on the temperature θ, that is, the relation between A and θ, is expressed by another empirical equation

$$\log A = M_1\theta - M_2 \qquad (7)$$

where M_1 and M_2 are constants. The experimental results are summarized in Table II by giving the values of M_1 and M_2 for

TABLE II.

Constants, M_1 and M_2 of Equation 7, and annealing temperatures for various times.

Kind of Glass.	M_1	M_2	Annealing temperatures.							
			1 min.	5 min.	10 min.	1 hr.	5 hr.	1 day	1 week	1 mo.
Borosilicate Crown....	0.030	18.68	598	575	565	539	515	493	464	444
Ordinary Crown.......	.029	17.35	573	548	538	511	487	464	434	414
Light Barium Crown...	.032	20.10	605	583	574	549	527	506	480	461
Heavy Barium Crown..	.038	24.95	637	619	611	590	572	554	532	516
Barium Flint.........	.028	16.28	555	530	519	491	466	442	412	390
Light Flint...........	.033	15.92	460	439	429	406	385	364	338	320
Medium Flint.........	.038	18.34	463	445	437	416	398	380	358	342
Heavy Flint..........	.037	17.51	453	434	426	405	386	368	345	329
Extra Heavy Flint....	.033	15.03	433	412	403	379	358	337	312	292

each of the nine varieties of optical glass. By means of equation (7) the value of A at a given temperature can be calculated, and from A, by means of equation (6) the "annealing time" for a given temperature can be computed. Thus, recalling that the annealing time is that time in which the birefringence is reduced from 50 to 5$\mu\mu$ per cm., we have by equation (6) that the annealing time

$$t = \frac{\frac{1}{5} - \frac{1}{50}}{A} = \frac{0.18}{A}.$$

The annealing temperature for a given annealing time can be obtained in a similar manner. In making computations by equations (6) and (7) t is to be taken in minutes, Δn in $\mu\mu$ per cm. (10^7 times the true birefringence) and θ in degrees on the Centigrade scale.

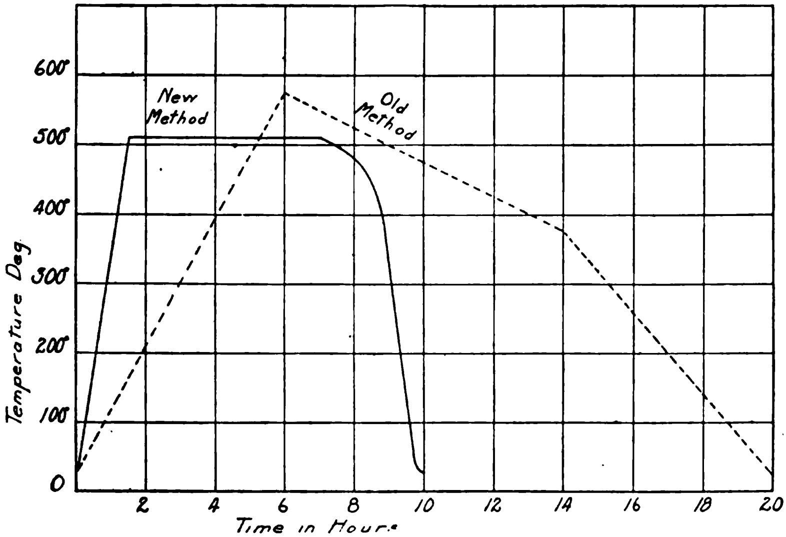
Cooling Procedure.—The strain having been removed by holding the glass for a sufficient time at the proper temperature, the next step in the practical annealing of glass consists in cooling it down to ordinary temperatures without reintroducing an excessive amount of strain. This is, from the theoretical standpoint, a much more complicated process than that of removing the stress. The usual method of annealing glass heretofore has been as follows: The glass is heated at a moderate rate to a temperature at which the strain disappears almost instantaneously (with ordinary crown glass, for example, this temperature would be about 570°), and is then cooled very slowly down to room temperature. The permanent strain acquired by the glass in this case depends on the rate of cooling, and can be calculated fairly closely by equation (5) if h is taken as the *cooling* rate (in deg. C. per minute). Thus, if a plate of glass 3 cm. thick is to be cooled from a high temperature at such a rate that along the middle not more than 5$\mu\mu$ per cm. birefringence is introduced, the cooling rate would be

$$\frac{5}{13 \times \left(\frac{3}{2}\right)^2} = 0.17 \text{ deg. per min.}$$

or about 10° per hour. Other things being equal, the cooling varies inversely as the square of the thickness.

So much for the usual method of annealing glass. A procedure which is believed to be much better is as follows: Instead of heating the glass to a rather high temperature for a short time, we hold it at a much lower temperature for a longer time. In this case the cooling rate can be much greater than with the usual method, a circumstance which compensates for the extra time spent in holding the glass at the annealing temperature. The two methods are compared graphically in Fig. 3. This improved method was used during the war in the annealing of some hundreds of thousands of pounds of glass, and the satisfactory

FIG. 3.



The broken line indicates graphically the usual method for annealing optical glass, and the full line the procedure similar to that recommended in this paper for ordinary crown glass 2 cm. thick. By this newer method the glass is heated very rapidly, is then held for some hours at a rather low annealing temperature, and is then cooled at a rapidly increasing rate.

results obtained indicated that this procedure has many advantages over the ordinary method.

In the first place, it is much easier to hold an annealing kiln at a constant temperature than to cool it at a given constant rate. The newer method requires a fairly constant temperature for some hours, but the subsequent cooling takes place in a short time and does not need careful control, so that on the whole the chances of failure, particularly when unskilled workmen are employed to regulate the burners, are less than with the older method which involves the difficult operation of cooling at a constant slow rate.

Another advantage of annealing at the lower temperature is that it minimizes the danger of overheating the glass. This is of especial importance when annealing valuable optical glass, large batches of which in several cases known to the authors have been spoiled, on account of the fact that the temperature in the annealing furnace was accidentally allowed to become about 50° too high. The sum of the time spent in annealing and the time spent in cooling is somewhat less for the second method, and by the newer method there is a saving of time spent in heating the glass up to the annealing temperature.

Annealing Schedules.—Mathematical analysis of the problem (which is too lengthy to be given here) shows that when following the newer procedure best results are obtained by annealing the glass to one-half the allowable strain (say 2.5$\mu\mu$ per cm.) and allowing the additional 2.5$\mu\mu$ to be incurred during the cooling process. The analysis also shows that for a slab 2 cm. thick a minimum time is required when the annealing temperature is such that the annealing constant, A, is about 0.0038. This corresponds to an annealing time of about one and three-quarters hours. We allow a factor of safety equal to two, and therefore anneal the glass for three and a half hours at the temperatures—for various kinds of glass—given in the second column of Table III. These temperatures are calculated from equation (7), the values of M_1 and M_2 being taken from Table II, and A being taken as 0.0038.

TABLE III.

Directions for annealing optical glass. Heat very rapidly up to annealing temperature; hold for the appropriate time, 3½ hours for plates 2 cm. thick and 12 hours for plates 4 cm. thick; and cool at an increasing rate which except at the start doubles for every 20° drop in temperature.

Kind of Glass.	Annealing temperature for plate 2 cm. thick. Initial cooling rate 24° C. per hour.	Annealing temperature for plate 4 cm. thick. Initial cooling rate 6° C. per hour.
Borosilicate Crown	542	522
Ordinary Crown	515	494
Light Barium Crown	553	534
Heavy Barium Crown	593	578
Barium Flint	495	474
Light Flint	409	391
Medium Flint	419	403
Heavy Flint	408	392
Extra Heavy Flint	382	364

As for the cooling rate, it can be shown that minimum strain is introduced by cooling in a certain specified time if the cooling rate follows the relation

$$h = \frac{h_0}{2}\left(1 + 2^{\frac{\theta_0 - \theta}{20}}\right)$$

in which h_0 is the initial cooling rate, θ_0 is the annealing temperature and θ is any temperature. Except at the beginning this is nearly equivalent to the statement that the cooling rate doubles for every 20° drop in temperature.[9] Furthermore, it so happens that the time spent in cooling is approximately equal to the time required at the annealing temperature.

In the third column of Table III are given the annealing temperatures for a slab 4 cm. thick, and at the head of the second and third columns are given the proper initial cooling rates for the two thicknesses. For the 2 cm. slab the cooling rates in the successive 10° intervals of temperature would be 24° (per hour), 29°, 36°, 46°, 60°, 79°, etc., and for the 4 cm. slab, 6°, 7°, 9°, 12°, 15°, 20°, etc. In general, doubling the thickness of the glass requires a 20° diminution in the annealing temperature, a multiplication by four of the annealing time, and a division by four of the initial cooling rate.

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[9] It is obvious that if the cooling rate be thus increased in geometrical ratio, the rate eventually becomes very great; indeed, so great that the glass would be fractured. It turns out, however, that by the time this excessive rate is reached the temperature would usually be below the annealing range, so that the cooling rate would then be without much effect on the final strain or on the total time required for cooling.

A METHOD FOR DETERMINING THE ANNEALING TEMPERATURE OF GLASS.

BY

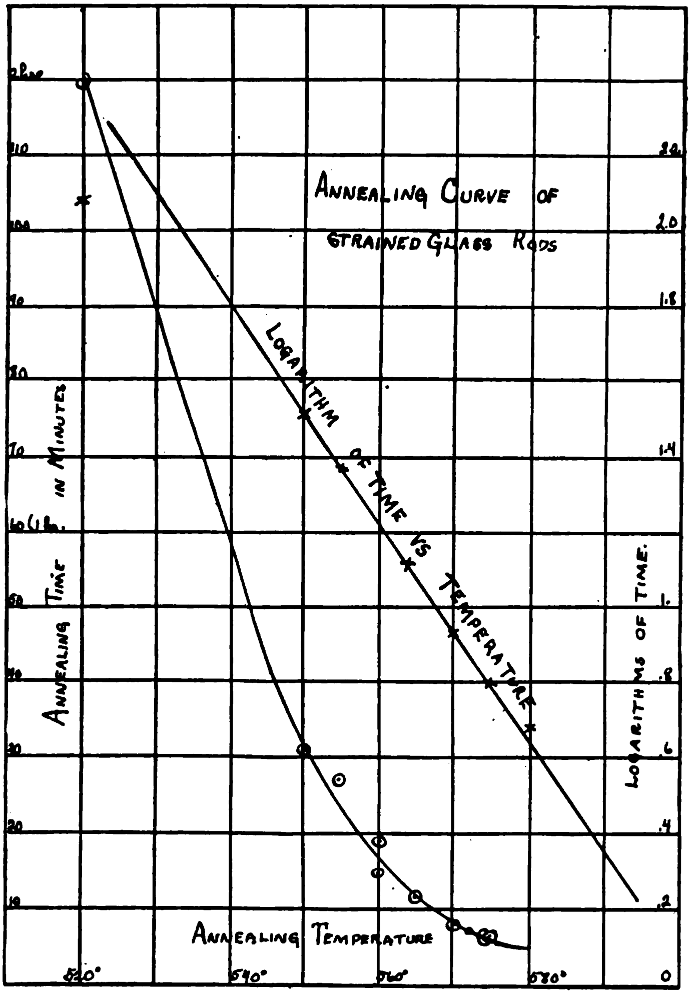
J. T. LITTLETON and E. H. ROBERTS.

ANNEALING in glass is defined as the process of relieving strains acquired in cooling. The annealing or relaxation temperature is a function of the time, but it is desirable to have this time as short as practical. After the strain has been relieved the glass should be cooled at such a rate as to prevent the acquirement of further strain. Hence in practice, the heat treatment consists of two parts: the first—annealing; the second—the cooling process.

The annealing rate of a glass may be assumed to be proportional to the viscosity of the glass, hence it would be justifiable to measure the annealing temperatures of glasses by means of a viscosity standard. Accordingly, in order to determine the most practical viscosity for annealing, the annealing temperature of a standard glass and form of ware was most carefully determined by inspection in polarized light of glass rods in an electric furnace. These results were then checked by tests on ware. Having established this temperature, it is then necessary to devise some method of measuring the viscosity of the glass at the determined temperature. All glasses which have a viscosity equal to that of the standard glass at the chosen temperature would anneal at the same rate, all dimensions being equivalent. This assumption is not strictly correct but it is sufficiently true for all practical purposes. Hence the problem is resolved into measuring the viscosity of a standard glass at its annealing point and finding the temperature of a glass at which its viscosity is equal to that of the standard glass at its annealing temperature.

The annealing temperature of a glass (to be termed the standard) was determined, as mentioned above, by observing strained glass rods in polarized light, while in an electric furnace. The cold rods were inserted in the furnace of a definite temperature and the time for the strain to be "completely" relieved was noted. This was done at a series of increasing temperatures, thus giving a curve between the temperature and time of annealing. The rods were four centimetres long and one centimetre in diameter, and were viewed along the axis of the rod. These exhibited three to six wave lengths' retardation, as was shown by the number of rings. The rate of annealing was determined by

FIG. 1.



the rate of disappearance of these rings. Using this as a measure the greater part of the strain would be relieved in a relatively small part of the time necessary to relieve the whole. For this reason the time of complete annealing does not depend a great deal upon the initial strain.

There were other interesting phenomena, such as the formation of what might be called secondary rings having very sharp definition. But since these rings were not studied in any detail, no further description of them can be given. The method given by C. V. Boys,[1] in which mechanically loaded bars were observed through a quartz wedge compensator, was also used, and while the results obtained checked those taken in the way just stated, no decided advantage was secured by this method.

The data obtained by the first method are expressed in the form of a curve (Fig. 1). This curve shows that practically all the strain disappears at 560° C. in approximately seventeen minutes. But, since at 550° C. so little strain could be noted after fifteen minutes of time, this temperature was selected as the annealing temperature of this glass. The data plotted logarithmically give, within limits, as shown by the logarithm curve, a straight line. The equation of this curve is

$$T = 10^{.0289\,(602.-t)}$$

where T is the time for "complete" annealing at the temperature t° C. In order for the glass to anneal in one second, the temperature would have to be 602° C., at which the glass would warp easily. Furthermore, at 500° C. the glass would anneal in sixteen hours. This means that at this temperature the glass was so hard that it would not acquire strain by any process of ordinary cooling. The logic of choosing this point for the strain point is given further verification in work referred to later.

Annealing at 550° C. in fifteen minutes was checked by experiments on actual ware. Hence the conclusion is that a temperature of 550° C. is a good practical annealing temperature for this glass, since a time of fifteen to twenty minutes may be considered as a desirable annealing time, and yet the glass was sufficiently hard so that no softening or warping whatever occurred. The next step was to obtain a measure of the viscosity of the standard glass at 550° C.

The drawing of Fig. 2 shows the apparatus used. The glass cane, ten centimetres long and four to five millimetres in diameter, was sealed between two larger pieces of cane of one centimetre diameter, the ends of which were held in chucks. One chuck was centred on the axis of a ball-bearing pulley wheel, the other to a universal joint. The temperature of the furnace in which the glass was heated was raised at the rate of one or two degrees per minute. This rate could probably be increased. The pulley wheel was loaded at the rim so that a shearing stress of four kilograms per square millimetre was given to the glass, the load being computed from the formula

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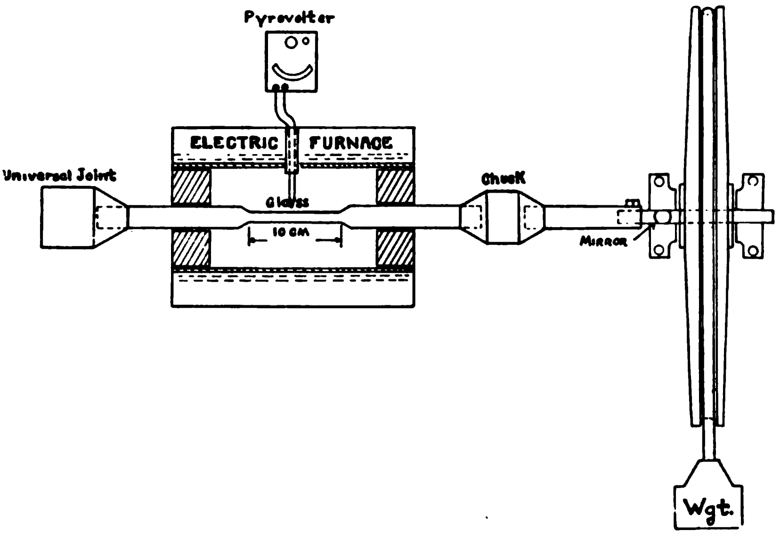
[1] C. V. Boys, *Nature*, p. 150, Oct. 26, 1916.

$$W=\frac{\pi r^3}{2R}T$$

where W is the load, T is the shearing stress, r the radius of the glass cane, and R the radius of the pulley.

FIG. 2.

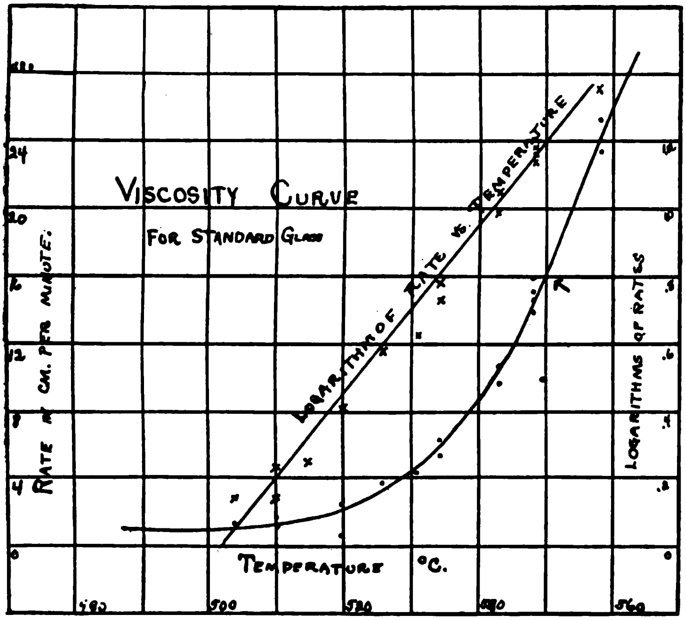


After the rod was inserted in the furnace the load was applied and the position noted of a spot of light reflected from a concave mirror attached to the axis of the wheel and focussed on a scale two metres distant. The rate of motion of this spot was taken as a measure of the fluidity of the glass. The recording of the position of this spot and the temperature of the furnace at various intervals gave values from which a viscosity curve was obtained. This determined the rate of rotation at 550° C. for standard glass to be sixteen centimetres per minute or an angular rotation of 5° 32″ per minute.

Hence to obtain the annealing temperature of any glass, the above experiment is repeated on the glass and the temperature which gives a scale deflection of sixteen centimetres per minute is the annealing temperature of the glass.

Two other very important temperatures are also obtainable from this curve. By plotting the logarithm of the fluidity against the temperature the curve is nearly a straight line. The slope of this curve is a measure of the rate of viscosity change for the glass and its intercept on the temperature axis when the rate of deflection is determined as above, using the same units and rods of the same dimensions, gives a temperature at which the glass takes on very little permanent strain when cooled at any ordinary rate. At this temperature the glass anneals in about sixteen hours. The maximum temperature to which it is safe to heat a glass may be determined by practical tests on the standard glass in the form of ware commonly used. Having determined the maximum temperature, the point on the fluidity curve at that temperature gives a measure of the maximum allowable fluidity. Therefore any glass of a fluidity equal to this standard is at

FIG. 3.



the maximum safe temperature. The curve for the standard previously given does not extend to this point.

Using these methods the following data were obtained for various Corning glasses:

TABLE I.

Glass.	Annealing Point.	Strain Point.
Borosilicates:		
1	518° C.	470° C.
2	550	503
3	524	472
4	522	467
Limes:		
1	493	441
2	472	412
3	523	472
4	514	466
Leads:		
1	419	364
2	451	382
3	436	382
4	433	353
5	429	353
6	430	369

The direct application of these data to other glassware can be safely made only when the ware to be annealed is smaller in size than the ware used as a check. If the glass be larger, such as large lenses of optical glass, it will be first necessary to establish the correct annealing conditions for that size of ware, using data obtained above as a guide, and then temperatures for any other glass having the same dimensions can be determined by the torsion experiment on a small sample.

It is practically impossible to determine the correct cooling rate for all forms of glassware made from different glasses except by actual experiment on the ware itself, therefore very little stress has been laid on this important phase of the annealing problem in this paper. Any temperature limits can serve only as a guide except when directly derived from data on glass of a certain size and shape.

Summary.—An experiment has been described which combines annealing temperatures with viscosity measurements in such a manner that the three important annealing temperatures, the maximum, the most practical, and the minimum, can be obtained from the viscosity measurements.

CORNING GLASS WORKS.

1919 REPORT OF THE STANDARDS COMMITTEE ON PHOTOMETRY AND ILLUMINATION.

BY

P. G. NUTTING, Chairman.

THE STANDARDS COMMITTEE ON PHOTOMETRY AND ILLUMINATION submits as its first report a tentative outline of photometric nomenclature, lists of fundamental units and certain fundamental definitions and data. These are based in part upon purely geometrical relations and partly upon physical relations, together with the laws of the reaction of the average human retina to radiation. The fundamental quantity is of course radiation, the fundamental unit flux density (flux per unit area) of that radiation, a unit preserved by means of certain groups of reference standards reserved for that purpose. A summary of the more useful fundamental units is given in the following table constructed to exhibit the essential relations existing between them.

Summary of Photometric Units and Nomenclature.

Nature of the Unit.	Symbols.	Derivation.	Dimensional Relations.	Def. Eqn.	Names of Units.	Relations to Standard.
Quantity.......	Q	$R \times V$	Energy × visibility	Ft	Light-Joule; Lumen-Sec.	SAt
Flux..........	F	Q/t	Power × Visibility	F	Light-Watt; Lumen	SA
Flux Density or Illumination	D	Q/At	$\frac{\text{Flux}}{\text{Area}}$	F/A	Lt. Wt./cm^2; Lumen/cm^2; Phot., M. C., Ft. C., Lux	S
Exposure......	E	Q/A	Ill. x time	Ft/A	Phot.-Sec.	St
Concentration, Intensity or Angular Density	C	$Q/\omega t$	$\frac{\text{Flux}}{\text{Solid Angle}}$	F/ω	Lt. Wt./ster.; Lumen/ster.; Candle	SA/ω
Brightness or Surface Density of Angular Density	D	$Q/\omega At$	$\frac{\text{Flux Density}}{\text{Solid Angle}}$	$F/\omega A$	Lt. Wt./cm^2/ster.; Lumen/$cm.^2$/ster.; Lambert	S/ω

This table is self-explanatory. The symbols used are unofficial; they are chosen merely for algebraic simplicity in elucidating relations between quantities. The first four units have long been in common use by illuminating engineers. The Concentration of a beam of light is its Angular Flux Density, to which the Intensity of the equivalent point source is, in a sense, equivalent. Brightness, or the Surface Density of Angular Flux Density, is a proper measure of the radiation (or light) from one unit normal area to another unit normal area and is directly applicable to what is sensed as brightness, and more generally to the field and zone pencils of a beam of light through a lens system. With a system of units as above outlined, it does not appear necessary to employ different units in specifying incident and emitted radiation.

Three systems of units based on the relations outlined above are suggested for consideration and possible adoption by the society: (1) A system of radiation units for dealing with radiant energy; (2) a rational system of photometric units for use in theoretical problems (*e.g.*, in pyrometry) involving both radiant energy and luminosity; and (3) a practical system of photometric units for dealing only with light in terms of light units and light standards.

I. RADIATION UNITS.

Nature of the Unit.	Symbol.	Dimension.	Unit.
Quantity............	$Q = Ft$	Radiant Energy	Joule.
Flux................	F	Radiant Flux	Watt.
Flux Density........	$D = dF/dS$	Flux Density	Watt per cm^2.
Exposure............	$E = Dt$	Energy Density	Joule per cm^2.
Concentration of Flux	$C = dF/dw$	Flux per unit Solid Angle	Watt per steradian.
Concentration Density	$B = dC/dS$	Flux Density per unit Solid Angle	Watt per cm^2 per steradian.

II AND III. PHOTOMETRIC UNITS.

Nature of the Unit.	Symbol.	Dimensions.	Rational System.	Practical System.
Quantity.....	Q	Luminous Energy	Light-watt-second	Lumen-second.
Flux.........	F	Luminous Flux	Light-watt	Lumen.
Flux Density.	D	Luminous Flux Density	Light-watt per cm^2	Lumen per cm^2 (Phot).
Exposure.....	E	Luminous Energy Density	Light-watt-second per cm^2	Lumen-second per cm^2 (Phot Sec.).
Intensity.....	C	Luminous Angular Density	Light-watt per ster. (pyr)	Lumen per ster. (candle).
Brightness....	B	Luminous Flux Density of Angular Density	Light-watt per cm^2 per ster. (pyr) per cm^2	Lumens per cm^2 per ster. (candle per cm^2.

The units of the Rational System are those of the corresponding radiation system multiplied by the Visibility of radiation.

The units of the Practical System are those of the Rational System multiplied by the Luminous Equivalent of radiation, in lumens per watt.

The relation between the various photometric units of the practical system are as follows:

IV. RELATIONS BETWEEN PRACTICAL UNITS.

1 lumen is emitted by a source of 0.07958 spherical candlepower.

A source of 1 spherical candlepower emits 12.57 lumens.

1 lux = 1 lumen incident per square metre = 0.0001 phot = 0.1 milliphot = 100 microphots.

1 phot = 1 lumen incident per square centimetre = 10,000 lux = 1,000 milliphots = 1,000,000 microphots.

1 milliphot = 0.001 phot = 1,000 microphots = 0.929 foot-candle.

1 foot-candle = 1 lumen incident per square foot = 1.076 milliphots = 10.76 lux.

1 lambert is the brightness of a surface emitting 1 lumen per square centimetre of projected area in the direction considered.

1 millilambert = 0.001 lambert = 1000 microlamberts.

1 lumen per square foot emitted by a perfectly diffusing surface produces a brightness of 1.076 millilamberts.

1 millilambert is the brightness of a surface emitting 0.929 lumen per square foot of projected area in the direction considered.

1 lambert = 0.3183 candle per square centimetre = 2.054 candles per square inch.

1 candle per square centimetre = 3.1416 lamberts.

1 candle per square inch = 0.487 lambert = 487 millilamberts.

Certain quantities much used in dealing with both light and radiation involve only ratios of similar quantities and are therefore expressible as pure numbers. These are the following:

V. NUMERICAL RATIO UNITS.

Emissivity = ratio of emission to that of a black body at the same temperature.

Reflectivity = ratio of reflected to incident light or radiant energy.

Absortivity = ratio of absorbed to incident light or radiant energy.

Transmissivity = ratio of transmitted to incident light or energy.

Opacity = 1/Transmissivity.

Density = $\log_{10}$ Opacity = $-\log_{10}$ Transmissivity.

Specific Density = Density per unit thickness.

Scatter = Apparent Density (specular light) — True Density.

Photovisual Relations.—The ratio of light to radiant energy, the Luminous Equivalent of Radiation, is a function of wave length determinable only by experiment. Its absolute value—

approximately 660 lumens per watt at its maximum wave length $.556\mu$—is still uncertain by at least 5 per cent. Its relative value, the Visibility of radiation, is much more accurately known. The data of the following table are averages for about 200 subjects having normal vision. The absolute visibilities given were calculated from the relative on the assumption of 667 lumens per watt or 0.0015 watt per lumen.

VI. VISIBILITY OF RADIATION.

Average Normal Eye.

	Wave Length ($\mu\mu$)	Relative to that at 556 ($\mu\mu$)	Absolute Visibility
Average Normal Eye	400	0.0004	0.27
	10	.0012	0.80
	20	.0040	2.7
	30	.0116	7.72
	40	.023	15.3
	450	0.038	25.3
	60	.060	40.0
	70	.091	60.7
	80	.139	92.6
	90	.208	139
	500	0.323	216
	10	.484	322
	20	.670	446
	30	.836	557
	40	.942	629
	550	0.993	662
	60	.996	663
	70	.952	641
	80	.870	580
	90	.757	502
	600	0.631	421
	10	.503	336
	20	.380	253
	30	.262	175
	40	.170	113
	650	0.103	68.8
	60	.059	39.3
	70	.030	20.0
	80	.016	10.7
	90	.0081	5.4
	700	0.0041	2.7
	10	.0021	1.4
	20	.0010	0.67
	30	.00052	0.35
	40	.00025	0.17
	750	0.00012	.08
	60	.00006	.04

The utility of illumination depends upon the mean brightness of the view provided by it and the variation of brightness within that field. The average brightness of the field viewed, through automatic psychophysical adjustments within the eye itself, determines (1) the absolute sensibility of the retina and hence the intensity of the brightness sensation, (2) the minimum contrasts perceptible, (3) the maximum (glaring) brightness just tolerable, and (4) pupillary diameter. Data on these quantities, based on the observations of from three to eight experienced observers, are given below for the entire range of brightnesses within which the retina is operable.

VII. SENSIBILITY AND BRIGHTNESS.

Average Brightness of Field Viewed.	Relative Sensibility of Retina.	Contrast Sensibility Relative to Maximum.	Minimum Glaring Brightness of Field.	Diameter of Pupil.
0.000001 ml.	0.77	0.017	20.1 ml.	7.4 mm.
0.00001	0.17	0.026	40.7	7.3
0.0001	0.038	0.043	89	7.2
0.001	0.0077	0.078	186	7.0
0.01	0.0018	0.22	400	6.7
0.1	0.00041	0.46	810	6.0
1.0	0.000087	0.83	1660	5.0
10.0	0.000020	0.98	3470	3.8
100	0.0000037	1.00	7250	2.7
1000	0.00000033	0.72	14450	2.1
10000	(0.000000003)	0.36	30900	2.0

Brightness values are in millilamberts. General sensibility is relative to that at the absolute threshold (about 0.000000 70 m.l.) taken as unity. The minimum glaring brightness is that just uncomfortably bright to an eye adapted to the brightness of the first column. The diameter of pupil is that for the case of both eyes open. This diameter determines not only retinal flux density but the ultimate limit of resolving power. Contrast sensibility is relative to that at its maximum at ordinary intensities. All sensibilities are referred to normal pupil and normal adaptation, zero for intensity sensibility and glare and about one minute for contrast.

These data apply for equal luminous brightnesses, to all wave lengths. For different individuals having normal vision, the variation is probably not over 5 per cent. from the mean. Brightness sensibility is proportional to the inverse $2/3$ power of the brightness of the field viewed, $B \times F^{2/3} = \text{constant}$. The minimum brightness producing a sensation of glare is 1700 × the cube root of the field brightness, $G = 1700\,F^{1/3}$. The increment in brightness represented by the contrast sensibility fraction is equal to the unadapted threshold limen. Flux density at the retina in lumens per cm.^{2} = 0.134×10^{-5} × area of pupil in mm.^{2} × brightness of field viewed in millilamberts.

1919 REPORT OF THE STANDARDS COMMITTEE ON LENSES AND OPTICAL INSTRUMENTS.

BY

C. W. FREDERICK, Chairman.

THIS first report of the Standards Committee on Lenses and Optical Instruments deals with single lens combinations only. It covers the classification of the various types of lenses, the limits of performance specifications, standards of performance and tolerances. Subsequent committees will doubtless find further revision necessary, and many other portions of this field remain to be covered.

I. NOMENCLATURE AND CLASSIFICATION.

Photographic Objectives.

Objectives	Aperture Ratio	Focal Lengths	Field Angle
Cinematograph	F/1.9, F/3.5, F/4.5	2, 4, 6″	7½, 15, 20°
Portrait	F/4.5, F/5.6	6–12″	12–20°
Speed Ordinary	F/5.6, F/6.3	4–12″	25–30°
Ordinary	F/6.3, F/8	4–12″	25–35°
Process	F/8 Down	8–24″	27–30°
3-Color Process	Same, but achromatized for red, green and blue.		
Wide Angle	F/8 Down	3–6″	40–55°
Meniscus	F/14 Down	3–6″	15–25°
Rapid Rectilinear	F/8 Down	3–8″	20–25°
Telephoto	F/8 Down	8–20″	10–20°
Airplane	F/4.5–F/10	10, 24, 48″	15–25°
Astro. Refractor	F/6.3–F/32	24–72″	5– 7°
Astro. Reflector	F/4.5–F/10	36–100″	2– 5°

Telescope Objectives (Visual).

Reading telescopes and collimators	F/6 –F/12	4–20″	1–3°
Position observing telescopes	F/10–F/18	20–40″	1–2°
Ordinary observing telescopes	F/15–F/20	36–720″	1–3°

Microscope Objectives.

	Numerical Aperture	E. F. L.	Working Distance
Ordinary 48	0.08	48 mm.	53 mm.
32	0.10	32	38
16	0.25	16	7
8	0.50	8	1.6
4	0.6–0.9	4	0.3–0.6
Oil Immn. 1.9		1.9	0.13–0.15

Oculars, Numerous Types	5–50 mm. E. F. L.	1–3° field
Projection Lenses, Numerous Types ..	2–18″ E. F. L.	1–15° field
Condensers F/0.5–F/1	1–6″	....
Magnifiers F/2–F/3	5–25 mm.	1–3° field

II. LIMITS OF PERFORMANCE SPECIFICATIONS.

The definition given by any lens operating with zone and field pencils of but small to moderate aperture (1 :8 or 7° in extreme angle) should approach theoretical resolving power, *i.e.*, the diffusion in the image of any line or point source equal to ratio of focal length to aperture of lens times wave length of light used (diffusion $\delta/\lambda = F/D$). Thus for an F/10 lens using green light the diffusion in the image should not be much over 0.005 mm. The objectives of telescopes for visual observation fall in this class. Lenses of wider aperture (F/2 – F/4.5) may be designed to give from 1/5 to 1/2 their theoretical resolving power over a limited field (5 – 20°). Wide angle lenses with small apertures may also give from 1/5 to 1/2 their theoretical resolving power over a wide field.

In photographic objectives with which negatives and prints are to be made to be viewed by the eye the definition required is not greater than the resolving power of the eye, which is half a minute of arc or 0.033 mm. at 10 inches. In motion picture film, the lower limit of tolerance in diffusion is such as corresponds to 3 mm. on the screen, or 0.03 mm. in the projected negative. The screen tolerance is half the resolving power of the eye at 35 ft. In oculars the limiting tolerance is again the resolving power of the eye, $t = f/7500 = 0.003$ mm. for an ocular of 25 mm. focal length f. If the oculars give stigmatic zone pencils, the eye can accommodate to considerable departure from a plane image. The permissible departure is $f^2/$ accommodation or $f^2/1000$ for 1 diopter.

III. TOLERANCES IN LENS TYPES.

	Confusion Tolerances Lower Bogey Upper	Stigmatic Tolerances Normal to Focal Plane (Color, Zone and Field)	Distortion.
Process lenses........	0.01 0.02 0.04 mm.	0.16 0.32 0.64 at $F/16$	0.0002 f
Cinematograph lenses	0.02 0.03 0.05	0.06 0.10 0.15	0.0005 f
Landscape lenses.....	0.03 0.05 0.10	Bogey 0.4 at $F/8$	0.0005 f
Portrait lenses.......	0.10 0.20 0.30	Bogey 1.0 mm.	0.0010 f
Telescope objectives..	Theoretical resolving	power $\delta = 2f\lambda/D$ $\delta V =$	f^2/D^2
Micro objectives.....	½ to ⅘ theoretical re	solving power.	
Oculars.............	$\delta = f/7500$	$\delta V = f^2/A$	
Condensers..........	Uniform illumination	required, not definition.	

Other Standards.—In single lens combinations there remain to be specified various tolerances in definition as affected by wave length, zone angle and field angle, individual Seidel aberrations, standard screw threads, tables of standard glass types and of glass combinations. In combinations of lenses in complete optical instruments such as telescopes, microscopes, field glasses, periscopes, sextants, telemeters, sights, transits, spectrometers, refractometers, spectrophotometers, goniometers, polarimeters, etc., types remain to be classified, limits of performance specified in resolving power, sensibility range, field, etc. It is recommended that such specifications be undertaken by subsequent committees.

THE STANDARDS COMMITTEES OF THE OPTICAL SOCIETY OF AMERICA.

BY

P. G. NUTTING.

One of the more important specific objectives of the Optical Society is the development of Technical Optics. Perhaps the greatest need in that field is for broad and authoritative works of reference, outlining the best practice as regards instruments and methods and giving the best standard data available. Private enterprise has failed to provide such aside from a few scattered books of varied character. It is the obvious function of a technical society, such as ours, to foster the preparation of reference material of a uniform high quality covering the entire field covered by the society. The direct method of attack is through a series of carefully selected standards committees. Such committees, with personnel gradually changing from year to year and publishing reports from time to time, may be expected in five to ten years to accumulate the material desired.

This work was initiated during 1919 by the appointment of a general committee on Nomenclature and Standards organized in nine sub-committees reporting at the annual meeting. While these reports were necessarily of a preliminary nature and fragmentary in character, they were an indication of what could be accomplished and of the earnest support to be expected of the Society. The Executive Council has accordingly made plans to continue and extend this work through the Standards Committees listed below. Each committee will be expected to cover at least the ground indicated; the personnel of these committees is in the hands of President Richtmyer.

1. *Colorimetry.*—Nomenclature. Instruments and Methods. Standard Data on Hue, Saturation and Brightness Sensibility and Scales for the average normal Human Eye. Spectral Energy of Standard White. Standard Transformation Data between Monochromatic, Trichromatic, Rotary Dispersion and Spectrophotometric Specifications. Reference Standards of Color. Progress during 1920. Bibliography.

2. *Lenses and Optical Instruments.*—Classification. Nomenclature. Standards of Performance. Tolerance. Precision and

Factory Testing Instruments and Methods. Standard Screw Threads. Lens Materials. Lens Theory. Methods of Computation, Notation and Tables of Data. Special Instruments, Standards of Performance and Tolerances. Progress during 1920. Bibliography.

3. *Optical Glass.*—Nomenclature of Glass Types, Specification and Listing of Technical Glasses. Standard Specifications of Quality, Tolerance, Testing Instruments and Methods. Progress during 1920. Bibliography.

4. *Photographic Materials.*—Classification of Standard Types of Plates, Films and Papers. Standards and Tolerances in Speed, Contrast, Latitude, Development Factor. Standard Developers, Fixing Baths, Toning Solutions, Sensitizing Baths. Progress during 1920. Bibliography.

5. *Photometry and Illumination.*—Nomenclature. Fundamental Relations, Photometric Units. Instruments and Methods for Photometry, Microphotometry, Comparison Photometry, Illuminometry. Theory of Sensibility. Standards and Tolerances in Illumination. Standard Visibility Data, Standard Contrast Sensibility Data, Standard Adaptation Data. Sources of Illumination, Diffusing Media, Diffusion Photometry. Progress during 1920. Bibliography.

6. *Polarimetry.*—Standard Instruments and Methods. Reference Standards and Tolerances. Theory of Sensibility and Precision. Precision and Tolerances in Technical Testing. Progress during 1920. Bibliography.

7. *Projection Apparatus.*—Nomenclature, Projection Lenses and Instruments. Standards and Tolerances. Illumination Standards and Tolerances. Definition, Field and Flicker Standards and Tolerances. Progress during 1920. Bibliography.

8. *Pyrometry.*—Instruments and Methods. Nomenclature. Radiation Constants and Formulas. Standard Data on Nonblack Radiation and Temperature. Data on Standard Light Filters. Progress during 1920. Bibliography.

9. *Reflectometry.*—Nomenclature. Instruments and Methods. Reflecting Power Standards. Progress. Bibliography.

10. *Refractometry.*—Standard Instruments and Methods. Standards of Precision and Technical Testing. Air Correction Tables and Formulas. Standard Wave Lengths. Comparison Standards. Birefractometric Instruments and Methods. Progress. Bibliography.

11. *Spectacle Lenses.*—Nomenclature. Units. Standard Types. Standard Specifications and Tolerances. Standard Materials. Standard Dimensions and Forms. Instruments and Methods for Precision and for Technical Testing. Progress during 1920. Bibliography.

12. *Spectrophotometry.*—Standard Instruments and Methods. Nomenclature. Classification of Instruments and their Applications. Approved Standard Sources. Specification of Data. Fundamental Relations and Constants in Density and Concentration. Tolerances in Wave Length and in Transmission and Reflection. Progress during 1920. Bibliography.

13. *Spectroradiometry.*—Standard Radiometers and Radiometric Methods. Spectral Absorption of Standard Lens and Prism Materials. Standard Data on Spectral Emission, Absorption and Reflection of Specific Materials. General Theory. Formulas and Constants of Radiation. Progress during 1920. Bibliography.

14. *Visual Sensitometry.*—Approved Instruments and Methods. Nomenclature. Units, Formulas, Constants and Standard Data on (1) The Visibility of Radiation, (2) Intensity and Contrast Sensibility, (3) Chromatic Sensibility, (4) Rates of Adaptation, and (5) Absolute Sensibility. Progress during 1920. Bibliography.

15. *Wave Lengths.*—Approved Standard Instruments and Methods. Tables of Standard Data and Probable Errors. Air Correction Tables and Formulas. Primary Standards. Comparison Standards. Progress during 1920. Bibliography.

From this list and outline of proposed activities, the scope and depth contemplated may be judged. It is planned to build up from year to year the material for a standard reference text in each sub-field, avoiding controversial matter and matter of little consequence beyond individual preference. Such texts should cover not only nomenclature, fundamental theory and standard data, but standard instruments and methods, sensibility, reference standards and a full bibliography of each subject.

RECENTLY ELECTED MEMBERS OF THE OPTICAL SOCIETY ARE AS FOLLOWS:

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A FOCAL-LENGTH EQUATION.

BY

C. W. WOODWORTH.

No general equation for calculating the focal length of a lens system appears to have been published. The mathematical equivalent of the author's graphic method of calculating paraxial ray courses * may be stated in the form of such an equation as follows:

$$j_m = j_{m-1} + i_m\left(1 - \sum_{k=1}^{k=m-1} \frac{j_k d_k}{n_{k+1}}\right)$$

in which

i = surface constant $= \frac{n' - n}{r}$ = n/ focal length;
j = lens constant = n/ focal length;
d = separation between the surfaces; and
n = index of refraction.

The focal length of a lens system, stated in another way, is equal to the sum of the lens constants plus the sum of the correction factors for the successive separations.

The following is a calculation of the telescopic objective given in Southall's "Principles and Methods of Geometrical Optics," pp. 319–321:

		n'	=	1.5146	$\Sigma \frac{jd}{n}$
i_1	.0086053512	r_1	=	59.8	
$i_2\ \Sigma$	−.0000324321	d_1	=	1.	.0056815999
i_2	.0057082640	r_2	=	−90.15	
j_2	.0142811831	d_2	=	.013	.0001856554
$i_3\ \Sigma$	.0000424008	n''	=	1.6121	.0058672553
i_3	−.0072266824	r_3	=	−84.7	
j_3	.0070969015	d_3	=	1.	.0044022713
$i_4\ \Sigma$	.0000153317				.0102695266
i_4	−.0014929268	r_4	=	410.	
j_4	.0056193064				

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* *Proc. Calif. Acad. Sci.*, Vol. i, p. 192.

The procedure is (1) arrange the data as in column two; (2) determine the lens constant for each value of r; (3) determine first number in column three $=\frac{j_1 d_1}{n_2}$; (4) determine first correction by multiplying the previous number by i_2; (5) determine lens constant of the first lens by addition; (6) determine the second number in column three $=\frac{j_2 d_2}{n_3}$; (7) determine second correction by adding the two numbers in column three and multiplying by i_3; (8) determine constant for first three surfaces by adding the constants of the three surfaces and the corrections for the first two separations; (9) determine the number corresponding with the third separation $=\frac{j_3 d_3}{n_4}$; (10) determine the third correction by multiplying the sum of the three numbers of column three to i_4; and, finally, (11) determine the constant for the whole system by adding the four surface constants and the three correction numbers.

Because of the alternate addition and multiplication, machine calculation will be very much more rapid than the use of logarithms.

The focal length of the lens is $n(=1)/j_4 = 177.9579060$ the distance from the last lens vertex to the principal plane, is the sum of the corrections divided by the lens constant $\Sigma/j_4 = 1.8275835$ and the difference between these numbers the focal distance from the last lens vertex $= 176.1303625$.

The same lens calculated in the opposite direction gives different correction values corresponding with the different relative location of the other principal plane with respect to the plane of the other outer lens surface, but gives, of course, precisely the same final value to the lens constant of the system and its focal length.

The use of this equation accomplishes a considerable saving in the time required for a calculation, since it eliminates some of the non-essential steps in the ordinary methods.

UNIVERSITY OF CALIFORNIA.

NOTE ON PROFESSOR WOODWORTH'S FORMULA FOR FOCAL LENGTH.

BY

JAMES P. C. SOUTHALL.

THE formula for the refracting power of a centered optical system given by Professor Woodworth in his paper, entitled "A Focal-length Equation," which is published in this number of THE JOURNAL OF THE OPTICAL SOCIETY,* is extremely compact and convenient and lends itself readily to numerical computations, as he shows. So far as I am aware, this formula has never been published before in precisely this form, although it is difficult to see how it could have escaped notice. It may be derived in various ways, but perhaps most easily by using the ordinary formulæ for the combination of two optical systems, as given, for example, in Chapter XI of my text-book on "Mirrors, Prisms and Lenses." Using a notation similar to that employed in that book (and in conformity with the notation recommended in the first issue of THE JOURNAL OF THE OPTICAL SOCIETY), let

$$F_k = (n_{k+1} - n_k) R_k$$

denote the refracting power of the kth surface whose curvature is denoted by R_k and which separates the two media of indices n_k and n_{k+1}. Moreover, let $F_{a,b}$ denote the refracting power of the entire system of spherical refracting surfaces which is comprised between the media of indices n_a and n_b where the subscripts a, b denote here two integers; concerning which it is agreed that in case $a > b$, we must put $F_{a,b} = 0$, and if $a = b$ then $F_{a,a} = F_a$. Let the vertex of the kth surface be designated by A_k, and put $d_k = A_k A_{k+1}$ and $c_k = d_k / n_{k+1}$ where c denotes therefore the reduced thickness of the medium as measured along the axis of the system. And, finally, let $H'_{1,k}$ designate the second principal point of the system composed of the first k surfaces whose refracting power is equal to $F_{1,k}$; and put

$$n_{k+1} \cdot x_k = H'_{1,k} A_{k+1}.$$

Then evidently we can write the following recurrent formula:

$$F_{1,k} = F_{1,k-1} + F_k (1 - x_{k-1} \cdot F_{1,k-1}). \qquad (1)$$

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* Preceding article.

It is easy to see that

$$x_k \cdot F_{1,k} = x_{k-1} \cdot F_{1,k-1} + c_k \cdot F_{1,k},$$

and therefore that

$$x_{k-1} \cdot F_{1,k-1} = \sum_{p=1}^{p=k-1} c_p . F_{1,p}\,; \tag{2}$$

which in combination with (1) above gives immediately Professor Woodworth's equation expressed, however, in terms of my symbols, namely:

$$F_{1,k} = F_{1,k-1} + F_k \left\{ 1 - \sum_{p=1}^{p=k-1} c_p \cdot F_{1,p} \right\}, \tag{3}$$

where k must be given in succession all integral values from k=2 to k=k. Professor Woodworth's symbols j_k and i_k are equivalent to $F_{1,k}$ and F_k as here used.

It is surprising that a relation which is at once so convenient and obvious seems never to have been given in any of the books. Practically it is equivalent to two formulæ published by T. Smith in a paper entitled "Note on Optical Imagery" in *The Proceedings of the Physical Society of London*, xxv (1912–1913), pp. 239–244. Thus, if equation (1) is differentiated with respect to F_k, the partial derivative of $F_{1,k}$ will be found to be the term contained within the brackets; that is,

$$\frac{\partial F_{1,k}}{\partial F_k} = 1 - x_{k-1} - F_{1,k-1}; \tag{4}$$

and hence formula (1) may be written:

$$F_{1,k} = F_{1,k-1} + F_k \, \frac{\partial F_{1,k}}{\partial F_k}. \tag{5}$$

And, moreover, since

$$1 - x_{k-1} \cdot F_{1,k-1} = 1 - x_{k-2} \cdot F_{1,k-2} - c_{k-1} \cdot F_{1,k-1},$$

we may write also:

$$\frac{\partial F_{1,k}}{\partial F_k} = \frac{\partial F_{1,k-1}}{\partial F_{k-1}} - c_{k-1} . F_{1,k-1}. \tag{6}$$

Equations (5) and (6) are the formulæ of T. Smith referred to above. Comparing (5) with (3), we see that

$$\frac{\partial F_{1,k}}{\partial F_k} = 1 - \sum_{p=1}^{p=k-1} c_p \cdot F_{1,p}, \tag{7}$$

and equation (6) follows at once from (7). This shows how (5) and (6) are together equivalent to (3).

If the total number of surfaces is denoted by m, then we may write also:

$$\left.\begin{aligned} F_{k,m} &= F_{k+1,m} + F_k \frac{\partial F_{k,m}}{\partial F_k} \\ \frac{\partial F_{k,m}}{\partial F_k} &= \frac{\partial F_{k+1,m}}{\partial F_{k+1}} - c_k . F_{k+1,m}. \end{aligned}\right\} \qquad (8)$$

These latter formulæ may be obtained from (5) and (6) by imagining the light to traverse the system from the other end, so that the light is incident first on the mth surface. All that is necessary is to change the order of the double subscripts, and write subscript m in place of subscript 1 and subscript (k + 1) in place of subscript (k − 1), leaving other subscripts and everything else unchanged.

If the focal points of the system are designated by F, F′, it is easy to show also that

$$A_1F = -\frac{1}{F_{1,m}} \cdot \frac{\partial F_{1,m}}{\partial F_1}, \quad A_mF' = \frac{1}{F_{1,m}} \cdot \frac{\partial F_{1,m}}{\partial F_m}. \qquad (9)$$

By way of illustration of the application of formulæ (5) and (6), a calculation is given below of the famous portrait objective of Petzval with data given by von Rohr as follows:

(Distances expressed in mm.)

Radii:

$$r_1 = +52.9;\ r_2 = -41.4;\ r_3 = +436.2;\ r_4 = +194.8;\ r_5 = +36.8;$$
$$r_6 = +45.5;\ r_7 = -149.5.$$

Thicknesses:

$$d_1 = +5.8;\ d_2 = +1.5;\ d_3 = +46.6;\ d_4 = +2.2;\ d_5 = +0.7;\ d_6 = +3.6.$$

Indices of Refraction (n_D):

$n_1 = n_4 = n_6 = n_8 = 1$ (air); $n_2 = n_7 = 1.517$; $n_3 = n_5 = 1.575$.

The values of F_k and c_k must be calculated first, and then the work may be arranged in tabular form as follows, according to T. Smith's scheme:

	$\frac{\partial F_{1,\kappa}}{\partial F_\kappa}$	$F_\kappa \cdot \frac{\partial F_{1,\kappa}}{\partial F_\kappa}$	$F_{1,\kappa}$	$-c_\kappa \cdot F_{1,\kappa}$
k = 1	+1.0000000	+0.0097732	+0.0097732	−0.0373661
k = 2	+0.9626339	−0.0013486	+0.0084245	−0.0080234
k = 3	+0.9546105	−0.0012584	+0.0071661	−0.3339430
k = 4	+0.6206675	+0.0034054	+0.0105716	−0.0147666
k = 5	+0.6059009	−0.0094672	+0.0011044	−0.0007730
k = 6	+0.6051279	+0.0068759	+0.0079802	−0.0189379
k = 7	+0.5861900	+0.0020272	+0.0100074	

Working through the system from the other end, we obtain also:

	$\frac{\partial F_{\kappa,7}}{\partial F_\kappa}$	$F_\kappa \cdot \frac{\partial F_{\kappa,7}}{\partial F_\kappa}$	$F_{\kappa,7}$	$-c_{\kappa-1} \cdot F_{\kappa,7}$
k=7	+1.0000000	+0.0034582	+0.0034582	−0.0082067
k=6	+0.9917933	+0.0112694	+0.0147276	−0.0103093
k=5	+0.9814840	−0.0153357	−0.0006081	+0.0008494
k=4	+0.9823334	+0.0053897	+0.0047816	−0.2228230
k=3	+0.7595105	−0.0010012	+0.0037804	−0.0036004
k=2	+0.7559101	−0.0010590	+0.0027214	−0.0104049
k=1	+0.7455052	+0.0072859	+0.0100074	

The two results are in complete agreement. We find for the focal length: $f = n_1/F_{1,7} = +99.927$ mm. or 100 mm. Moreover, the positions of the focal points are:

$$A_1F = -(1/F_{1,7})\,(\partial F_{1,7}/\partial F_1) = -74.496 \text{ mm.};$$

$$A_7F' = (1/F_{1,7})\,(\partial F_{1,7}/\partial F_7) = +58.576 \text{ mm.}$$

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY,
July 1, 1920.

SOME GENERAL CHARACTERISTICS OF SPECTRO-PHOTO-ELECTRICAL CONDUCTION IN SOLIDS.*

BY

W. W. COBLENTZ.

I. INTRODUCTION.

A SUBSTANCE which changes in electrical conductivity when exposed to light is said to be photo-electrically sensitive. It is found that most of these photosensitive substances have a low electrical conductivity when they are kept in the dark. Hence, when connected in a series with a galvanometer and an electric battery, a small " dark " current flows through the circuit. On exposure to light, the photosensitive substance usually increases greatly in electrical conductivity. The reaction, or response, is said to be " photopositive." The most commonly known substance showing this type of reaction is selenium.

On a few occasions experimenters have observed that, on closing the circuit, the galvanometer deflection was less (*i.e.*, the resistance was higher) when the substance was exposed to light than when it was kept in the dark. Or, on a closed circuit, the galvanometer deflection moved in a negative direction from its zero position, when the substance was exposed to light, thus indicating an apparent rise in resistance which was higher than that which obtains in the dark. This type of response is said to be " photonegative," and is of such rare occurrence that no attempt is made to generalize the phenomenon.

Although much work has been done on the photo-electrical properties of selenium, not until within the past four years was a systematic search made to find solid substances which exhibit photo-electrical properties.[1]

In the meantime a number of spectro-photo-electrical investigations, made principally at this Bureau, on various substances, have produced results which can be used to formulate a few general characteristics of spectro-electrical sensitivity in solids.

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* Published by permission of Director, Bureau of Standards.

[1] Case, *Phys. Rev.* (2), 9, p. 305; April 1917. See also papers by the writer *Proc. Phil. Soc. Wash.*, Feb 1917; *Jour. Wash. Acad. Sci.*, 7, p. 525; 1917, and B. S. Bulletin 14, p. 591; 1918.

It is found that numerous solid substances which have a low electrical conductivity in the dark have a high electrical conductivity when exposed to light, selenium being the most widely-known example. This photosensitive property is a function of the temperature of the substance; also of the intensity and of the wave length of the thermal radiation stimulus.

II. SELECTIVITY OF REACTION TO WAVE LENGTH OF RADIATION STIMULUS.

Using an equal energy stimulus, it is found that the photo-electrical response of substances, in general, is fairly uniform throughout the visible spectrum, terminating in a band or bands of high photo-electrical sensitivity in the extreme red[2] or near infra-red spectrum at 1.2μ ($\mu = 0.001$ mm.). Molybdenite is unique in having three bands of spectro-photo-electrical sensitivity which extends to 2μ. No substance is known which reacts photo-electrically to radiation of wave length greater than 3μ.

It is of interest to compare the distribution of energy in the visible spectrum of the Nernst glower as registered by means of a selenium cell and by a thermopile. The former was used in connection with a d'Arsonval galvanometer and the latter with a Thomson galvanometer. Fig. 1, Curve *A*, gives the spectral energy distribution obtained with a bismuth-silver thermopile (a non-selective radiometer), while Curve *B* gives the response of a selenium cell when similarly exposed (for 10 seconds) in different parts of the spectrum. After exposure to the low intensities, in the blue-violet, it required 20 to 30 seconds for the galvanometer reading to return to zero; and after exposure to the highest intensities it required over 2 minutes for the cell to return to its initial resistance. The curve obtained with the selenium cell is entirely erroneous as regards the actual spectral energy distribution.

Curve *C* gives[3] the response that is obtained by exposing parts of the spectrum using a high intensity, while Curve *D* represents the response at one-sixteenth the intensity used to obtain Curve *C*.

III. SELECTIVITY OF REACTION TIME.

It is well known that the photo-electric response of selenium lags considerably more for infra-red rays than for visual rays.

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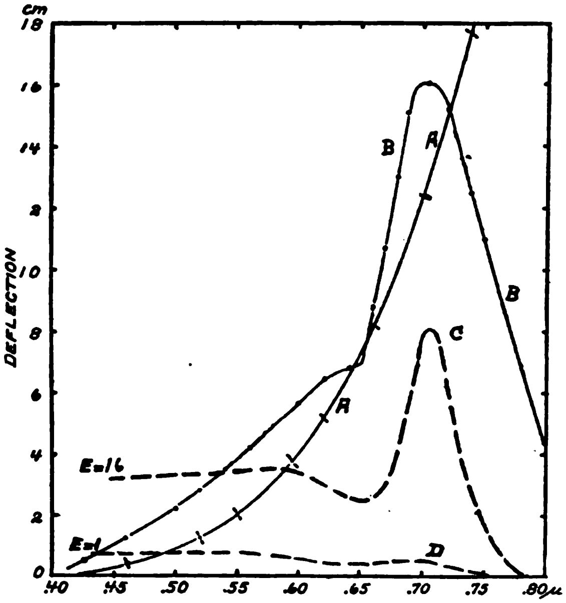
[2] Cuprous oxide appears to have its maximum sensitivity in the ultra-violet; Pfund, *Phys. Rev.* 7, p. 289; 1916.

[3] Pfund, *Phys. Rev.*, 34, p. 370; 1912.

Molybdenite and Case's thalofide cell are much quicker than selenium in their photo-electric response (change in electrical conductivity) when exposed to radiation, irrespective of wave-length.

In the visible spectrum the photo-electrical reaction attains equilibrium within a few seconds after exposure. On the other hand, it requires from one to several minutes' exposure to infra-red rays in order to obtain the maximum photo-electrical action.

FIG. 1.



Distribution of energy in the spectrum of a Nernst glower as observed with a non-selective radiometer, *A*, and with a selenium cell, *B*. Curves *C* and *D* (from Pfund) give the responses of a selenium cell in an equal energy spectrum.

This is shown in Fig. 2, which illustrates the photo-electrical behavior of molybdenite. Decreasing the temperature appears to shorten the time required for attaining the maximum photo-electrical response for all wave lengths.[4]

In making photo-electrical observations it is important to expose the substance for a sufficient time to obtain the maximum reaction. This is illustrated in Curve *A*, Fig. 3, which gives the spectro-photo-electrical sensitivity of molybdenite when exposed for 10 seconds, while Curve *B* gives the corresponding curve for unlimited exposure; that is, exposure until there is no further increase in the galvanometer deflection.

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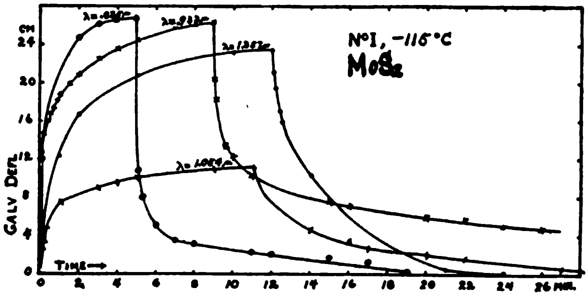
[4] *Bul. Bur. Stds.* 15, p. 132; 1919.

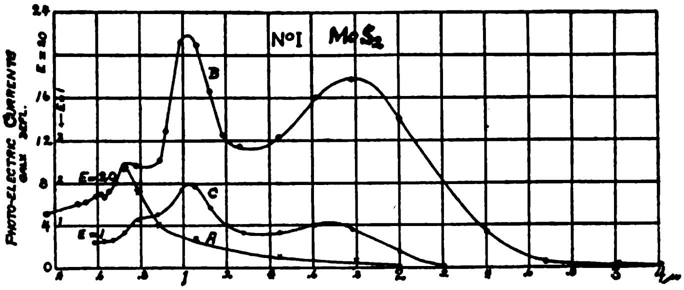
At $\lambda = 0.76\mu$ the difference in photo-electric current (galvanometer deflection) for limited exposure and for unlimited exposure (which at this point was about 30 seconds) was very small. On the other hand, beyond $\lambda = 1.4\mu$ the response is so slow that, on a 10-second exposure, the photo-electric sensitivity appears to be inappreciable.

FIG. 2.



Variation of photo-electric current with time of exposure to radiation.

FIG. 3.



Effect of intensity of radiation upon spectral photo-electric sensitivity.

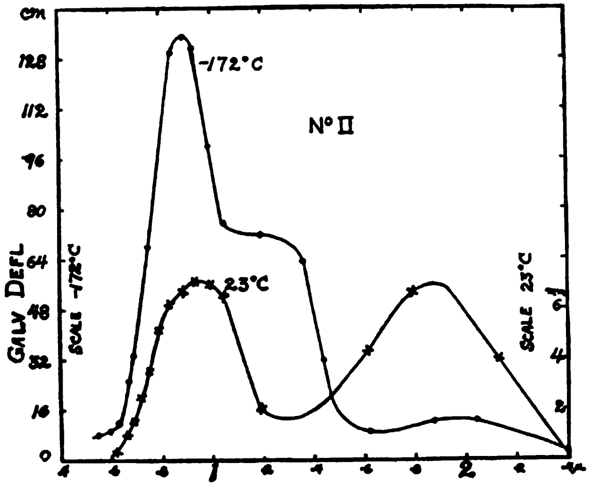
IV. SELECTIVITY OF REACTION TO INTENSITY OF RADIATION STIMULUS.

Using an equal energy spectrum, and different intensities, it is found that the resulting change in electrical conductivity or reaction is selective with respect to the wave length of the radiation stimulus.

As illustrated in Curves *C* and *D*, Fig. 1, and Curves *B* and *C* in Fig. 3, the effect of increasing the radiation stimulus (intensity E = 1 = 16 = 20) appears to produce a greater response in the long wave lengths than in the short wave lengths, with a resultant shift of the maximum of the spectral photo-electric sensitivity curve towards the long wave lengths.[5]

There does not appear to be a simple law governing the variation in the photo-electric response with variation in intensity of the radiation stimulus.

FIG. 4.



Effect of temperature upon spectral photo-electric sensitivity of molybdenite.

V. SELECTIVITY OF REACTION TO TEMPERATURE OF SUBSTANCE.

The temperature of the substance has a very marked effect upon the spectro-photo-electrical reaction of a substance. Reducing the temperature to a low degree, by immersion in liquid air, the substance is practically an electrical insulator. Nevertheless, on exposure to radiation of certain wave lengths, the electrical conductivity is greatly increased.

A typical example is that of molybdenite, illustrated in Fig. 4.

These curves show that, as the temperature decreases, the radiations of short wave lengths produce a greater change in electrical conductivity than do the long wave lengths. As a result, the maximum of the sensitivity curve is shifted towards the short wave lengths. Furthermore, the maxima are generally very much sharper at low temperatures as found by Elliot[6] for selenium, and by the writer[7] for silver sulphide.

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[5] See further data on silver sulphide, *Bul. Bur. Stds.* 15, p. 208; 1919.

VI. SUMMARY.

In the foregoing pages attention is called to the fact that numerous solid substances, which have a low electrical conductivity in the dark, have a high electrical conductivity when exposed to light.

A study of some of these substances shows that their photo-electrical reactions have the following characteristics:

1. The photo-electrical response, to an equal energy stimulus, is fairly uniform throughout the visible spectrum, terminating in a band or bands of high photo-electrical sensitivity in the extreme red or near infra-red spectrum.

2. The time required to attain equilibrium of the photo-electrical reaction is selective to wave length. For visible radiation the photo-electrical response is complete in a few seconds; whereas, it requires an exposure of one to several minutes to infra-red rays in order to induce the maximum photo-electrical response.

3. The effect of increasing the intensity of the radiation stimulus is to produce a greater increase in the photo-electric response to the long wave lengths than to the short wave lengths, with a resultant shift of the maximum of the spectral photo-electric sensitivity curve towards the long wave lengths.

4. Decreasing the temperature of the substance greatly increases the intrinsic photo-electrical sensitivity throughout the whole spectrum and shifts the maximum sensitivity towards the short wave lengths.

WASHINGTON, D. C.
June 25, 1920

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[6] Elliot, *Phys. Rev.* (2) 5, p. 59; 1915.

[7] *Bul. Bur. Stds.* 15, p. 242; 1919.

ON THE RELATION BETWEEN PHOTOGRAPHIC DENSITY, LIGHT INTENSITY, AND EXPOSURE TIME.*

BY

FRANK E. ROSS.

It is a matter of experience that the degree of blackening of the photographic plate depends upon the intensity I, the wavelength λ, and the time of action t, of radiation falling upon it. This effect, which, from its analogy with a somewhat similar phenomenon taking place in vision, may be called "photographic sensation," is measurable physically as transparency T, or opacity O, where $O = T^{-1}$. It has been found convenient in photography to use a derived quantity called "density," defined by

$$D = \log_{10} O. \tag{1}$$

There are several reasons making density a desirable measure of photographic sensation. It is found, for instance, that a series of equidistant densities, or densities increasing in arithmetical progression, produce visually a sensation or appearance of *equal steps;* in other words, visual response is, within certain limits, a linear function of density (Fechners' Law). Again, it is found experimentally, that within wide limits the relation between mass (M) of reduced silver per unit area (cm.^{2}) and density is linear, of the form

$$D = c\,M. \tag{2}$$

c is called the photometric constant. Its value is approximately 0.10 milligrams for ordinary photographic negatives. It is assumed that the viewing illumination in measuring density is completely diffuse.

It can be assumed as a matter of experience that

$$D = F\,(I, t, \ldots), \tag{3}$$

or, density is a function of intensity, time of action, and other variables or parameters. These parameters are so numerous that it is impracticable even to specify them. For the present the reciprocity law will be assumed to hold, namely, that I and t occur in (3) only in a product form.

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*Communication No. 93 from the Research Laboratory of the Eastman Kodak Company.

Neglecting all formulæ purporting to take account of the phenomenon of "reversal," four specific formulæ of the form (3) have been proposed: (a) by Abney [1] in 1889; (b) by Hurter and Driffield [2] in 1890; (c) by Elder [3] in 1893; (d) by Channon [4] in 1906. Abney's formula is purely empirical. The formulæ of Hurter and Driffield, of Elder, and of Channon are based upon differential equations specifying the velocity of the light action, or the velocity of "infection" of the silver grains, and are to that extent to be considered as theoretical equations.

The formula of Abney is

$$T = e^{-\mu\left(\log\frac{It}{i}\right)^2}, \tag{4}$$

or, in terms of density,

$$D = \mu' \log^2 \frac{It}{i}. \tag{4a}$$

An example is given in Figure 1 of the character of the agreement to be expected between Abney's formula (4) and observed curves. In the diagram, transparency is plotted against the logarithm of the exposure *It*. The full curves are of a commercial rapid plate, developed for one and three minutes respectively. The dotted curves are computed from Abney's formula, the constants μ and i being so chosen as to produce the closest possible agreement with the measured curves. The values of the constants are:

	μ	$\log i$	γ
1m. development................	1.59	0.179	0.91
3m. development................	2.81	0.176	1.24

The quantity γ in the last column refers to the measured curves, and will be explained presently. It will be noticed that i is independent of, while μ varies with, development time. i can be taken as a measure of the speed of a plate, while μ is a measure of the contrast value. Agreement of observed and computed curves is fairly good from transparency unity to $T=0.30$ $(D=0.52)$ in the case of one minute development, and as far as $T=0.15$ $(D=0.82)$ for three minutes development. The formula has no value for lower transparencies, corresponding to higher densities. In fact, Abney's formula can be considered to give a very good representation of the relation between transparency or density, and exposure, for the first step in the process, which covers the appearance of the image and its development in strength up to the point where rapid action begins. For certain character of work making use of weak or so-called under-exposures, Abney's formula applies more accurately than any other which has been proposed. There does not appear, however, to be any theoretical basis for it, at least none has been proposed. It fails to take account of the very obvious fact that there is a limit to the density which can be reached depending upon the amount of silver in the emulsion, the relation being given by equation 2 above.

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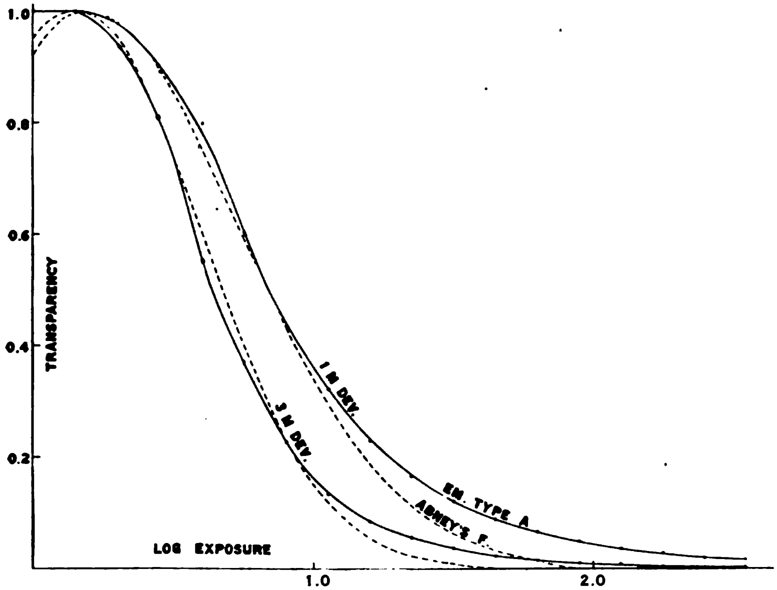
[1] *Journ. Camera Club*, April, 1889, and April, 1893.

[2] *Journ. Soc. Chem. Ind.*, May, 1890.

[3] *Journ. Camera Club*, April, 1893.

[4] *Photographic Journ.*, June, 1906.

FIG. 1.



Comparison of observed characteristic curves with Abney's formula.

On account of its simplicity, Elder's formula will be considered next, although preceded in point of time by the more complicated Hurter and Driffield formula. Elder's equation is

$$D = D_m\left(1 - e^{-\kappa It}\right), \tag{5}$$

which is derived by integration of the differential equation

$$dn = \kappa I(N - n)dt, \tag{5a}$$

where D_m is the maximum density obtainable, and is a function of the amount of silver in the emulsion; N is the total number of grains per cm.^{2}, to which D_m is proportional; n is the number of grains changed to latent image in the time $t = 0$ to $t = t$; I is light intensity; κ is a parameter depending principally upon grain-sensitivity and wave-length of light, and is a measure of the speed of the emulsion.

Equation (5a) asserts that the number of grains changed to latent image in any small interval of time is proportional to the number of grains not yet acted upon. It is a well-known formula for the velocity of chemical reaction. In making use of this equation one is not necessarily committed to a chemical theory of the latent image, for the photo-electronic theory of the formation of the latent image leads to a similar equation. For a very suggestive account of the photo-electric theory the reader is referred to Allen's "Photo-electricity."

Comparison of formula (5) with observed curves is made in Fig. 2. Photographic density D (diffuse) is the ordinate, logarithm of the exposure log E is abscissa. Curves made in this way from measures of density on a plate which has been given a suitably graded series of exposures[5] are called "Characteristic Curves," or H. and D. curves, so named after Hurter and Driffield, who first proposed their use. They are not, however, to be confused with the curves resulting from the Hurter and Driffield formula.

It is seen that the parameter D_m in equation 5 controls the height of the theoretical curves (Elder) in Fig. 2, while the parameter κ controls the position of the curve along the exposure axis, and is accordingly a measure of the speed of the emulsion. D_m and were chosen in each case to agree with the maximum density and average position along the exposure axis of the observed curves, thus giving the closest possible agreement. It will be seen that the agreement is not at all good, but is better in the case of the emulsion of Type B than in that of Type A.

These emulsion types need a few words of explanation. A characteristic curve consists of three parts: (1) A portion of low density and slow change in density, called the "toe" of the curve;

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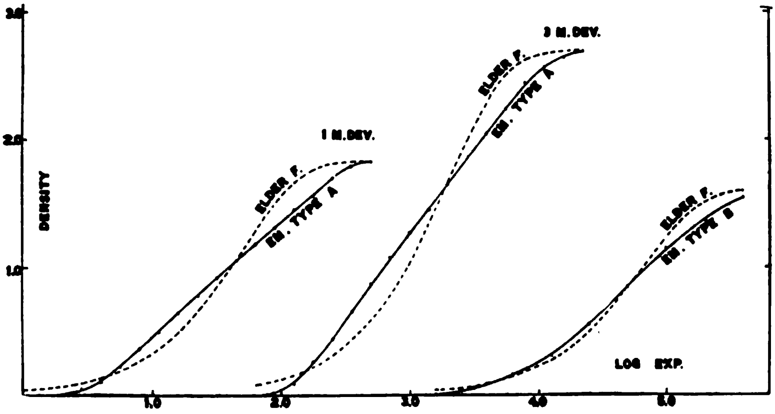
[5] Instruments in which such exposures are made are called "Sensitometers." For a description, see paper by L. A. Jones, "A New Non-Intermittent Sensitometer." *J. Frank. Inst.*, 1920, p. 303.

(2) a straight, or nearly straight portion comprised between the regions of low and high density, in which the *change* of density reaches a maximum, called the straight-line portion. The maximum slope is called gamma (γ) and is an important constant in an emulsion, as it determines the contrast or power to reproduce shades in lighting. It is the factor controlling sensitivity in photographic photometry. Mathematically, gamma is given by

$$\gamma = \left(\frac{dD}{d \log E}\right)_{\text{Max.}}; \tag{6}$$

(3) the third portion of the characteristic curve is called the shoulder, and lies above and beyond the straight-line portion. It is a region of high density and slow change of density. Briefly, the toe is the region of under-exposure, the straight-line portion is the region of correct exposure, and the shoulder is the region of over-exposure.

FIG. 2.



Comparison of observed characteristic curves with Elder's formula.

An emulsion of Type A is one having a very short "toe." Type B is one having an extended toe. Similar differences are to be found in the shoulder, but generally they are not of such importance. These differences in the toe in emulsions of various types are of importance in the subjects of photographic resolving power and sharpness.

If equation 5a is a complete description of the formation of the latent image, assuming regular and proportional development, computed and observed curves in Fig. 2 ought to coincide. As already remarked, this is found not to be the case. Only two parameters are at our disposal in equation 5. A third parameter appears to be necessary. At the time the equation was proposed, the question was raised whether it should not be modified to take account of the thickness of the emulsion. Elder[6] advanced what purported to be a proof that the equation as it stands virtually takes account of film thickness, it being only necessary to multiply the light intensity in equation 5a by the factor $\frac{a^s - 1}{a \log e}$, where a is the absorption coefficient of the film and s the thickness. Accordingly the form of equation 5 should be unaltered if his demonstration is correct. The fallacy in the proof lies in the statement, " . . . the action, other things being equal, is proportional to the intensity of the light acting . . ." It will be shown in a later paragraph that the thickness of the emulsion profoundly alters the characteristic curve and introduces a third parameter necessary to fit observation with theory. It can be concluded that the Elder equation fails in general to explain the characteristics of emulsions.

The Hurter and Driffield equation will now be considered. The differential equation upon which it is based is (*ibid.*, p. 465):

$$dx = \frac{I}{b}(1-r)\left[e^{-\kappa x} - e^{-\kappa \alpha}\right] dt, \tag{7}$$

where dx is the number of grains per cm.^{2} changed to latent image in the time-interval dt; b the energy necessary to change one grain; I the light intensity; r the reflection coefficient; α the total number of grains per cm.^{2} in the emulsion; κ is a parameter to be considered later. The integral equation of (7) is easily found. Combining parameters and constants and making use of equation 2, there results

$$D = \rho \log_{10}\left[O - (O-1)e^{-qIt}\right], \tag{8}$$

where $O = e^{-\kappa\alpha}$. There are three parameters in (8), one more than in equation 5, so that it can be expected to give closer agreement with observed curves than is possible in the case of the Elder equation.

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[6] *J. Camera Club*, July, 1893, p. 166.

Equation (8) will now be applied to the two emulsion types A and B used in previous comparisons. In fitting equation 8 to plotted curves, the parameters O and ρ are determined by the following formulæ:

$$\left.\begin{aligned} \rho \log_{10} O &= D_m, \\ \rho &= \frac{\gamma}{1 - e^{-\log o}}\,. \end{aligned}\right\} \tag{9}$$

The parameter q is determined by making the mid-point of the straight-line portion a common point of the observed and computed curves. The constants and parameters found in this way are as follows:

	Em. A	Em. B
D_m	2.70	1.60
γ	1.34	1.00
O	40.46	10.72
ρ	1.675	1.555
q	.00127	.00235

The curves (H. D. F.) in Fig. 3 were computed from the above numbers and plotted in conjunction with the observed curves. In the case of Em. A, disagreement in the toe and shoulder is very marked. With Em. B agreement is much better. Comparing with Fig. 2, it is seen that the Hurter and Driffield formula conforms to actual emulsions considerably better than the Elder formula.

It will be worth while to examine the assumptions underlying the Hurter and Driffield formula. The differential equation (7) is based on energy relations. The reasoning is as follows: If I is the intensity of the light incident on the plate, corrected for reflection loss, and T the transparency of the emulsion, the amount of energy absorbed by the plate in unit time is obviously

$$E_a = I\,(1 - T_a), \tag{10}$$

it being assumed that there are a grains of silver halide per cm.^{2} in the emulsion. The subscripts in (10) and following formulæ are simply descriptive of the particular mass of grains considered.

Let the number of grains changed to latent image after a time t be denoted by x. Call the transparency of this group of grains T_x. The energy absorbed by this group in unit time is said to be

$$E_x = I\,(1 - T_x). \tag{11}$$

Let the amount of energy necessary to change one grain of silver bromide to latent image be denoted by b. It is claimed that the number of grains dx so changed during the time dt will be given by the relation

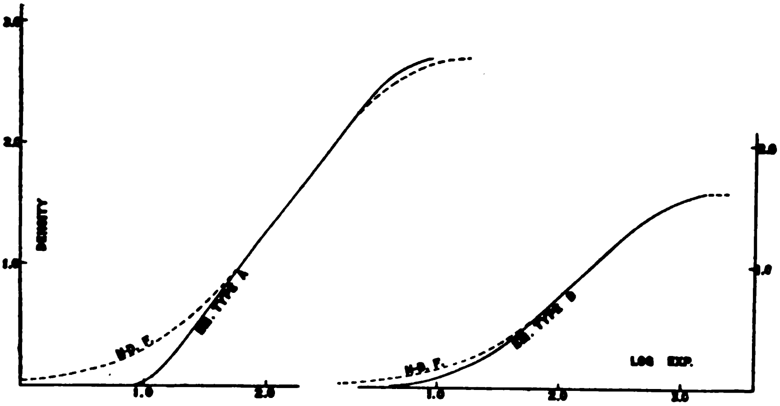
$$b.dx = \text{amount of energy absorbed by the } \textit{unchanged} \text{ silver grains;} \qquad (12)$$

from which

$$b.dx = (E_a - E_x)dt. \qquad (12a)$$

The truth of equation (12) can not be admitted. It is not clear how the energy absorbed by the group of grains $a-x$ can be applied to the sub-group dx.

FIG. 3.



Comparison of observed characteristic curves with Hurter and Driffield's formula.

Continuing, the next step is the substitution of the following values in (12a):

$$T_a = e^{-\kappa a}, \qquad (13)$$

$$T_x = e^{-\kappa x}. \qquad (13a)$$

In writing equation (13a) serious error is made. Equation 13 is undoubtedly valid, at least as an approximate equation. If (13a) is true, it is equally true with regard to the group of grains $a-x$, *i.e.*,

$$T_{a-x} = e^{-\kappa(a-x)}; \qquad (14)$$

and equally with (11)

$$E_{a-x} = I(1 - T_{a-x}). \qquad (15)$$

We have, however,

$$E_a = E_x + E_{a-x}, \tag{16}$$

or,

$$I(1-T_a) = I(1-T_x) + I(1-T_{a-x});$$

substituting the assumed values of T there results

$$e^{-\kappa a} - e^{-\kappa x} = e^{-\kappa a} e^{-\kappa x} - 1,$$

which is true only for $x = o$. The assumption of Hurter and Driffield that the transparency of the x-group of grains is given by $e^{-\kappa x}$ is accordingly incorrect, for it leads to a mathematical inconsistency. Their equations (7) and (8) in consequence must be considered purely empirical.

Channon's Formula: Channon follows Elder in using the mass-action equation, but takes into account emulsion thickness. The equation to which he is led is not directly integrable. By expansion in series his solution takes the form

$$D = \gamma \left\{ \phi(aE) - \phi(aE\vartheta) \right\} \tag{17}$$

where γ, a, and ϑ are parameters, E is exposure, and ϕ a functional form as follows:

$$\phi(n) = n - \frac{n^2}{2\underline{|2}} + \frac{n^3}{3\underline{|3}} - \frac{n^4}{4\underline{|4}} + \ . \ . \ .$$

Agreement with observed curves is of the same order as the H. and D. formula. Theoretically, it is well grounded, but fails to take account of varying sensitivity of grain.

Development of a New Formula.—On account of the general applicability of the mass-action equation to certain classes of chemical and physical problems, it seems worth while to apply it to the present problem in a manner more circumspect and cautious than has hitherto been done. It will be assumed that the law applies only to the case of a single layer of grains, which therefore individually receive equal exposures. Actual emulsions are composed of grains lying in layers many layers thick and very often of grains of varying sensitiveness. Obviously many formulæ can be constructed, depending upon particular assumptions of the following classes:

a. Relative frequency of grains of varying size.
b. Relative sensitivity of grains of varying size.
c. Variation of grain-sensitivity with light-intensity and wave-length for each class of grains.

d. Ratio of inert to active grains, depending on grain size.

e. Number of layers of silver halide.

f. Opacity of the emulsion and its variation with wavelength.

This is not an exhaustive enumeration of the factors which must be considered in developing the theory of the subject. It would be unprofitable in the absence of data on the controlling factors enumerated above to develop a formula on any but the simplest assumptions. These are:

1. All the grains in an emulsion divisible into n groups, each group as a whole obeying the mass-action law.

2. The silver mass the same for each group.

3. The sensitivity factor ( ) of the groups arrangable in geometrical progression. This takes account of the factors of: (a) true sensitivity differences, (b) apparent sensitivity differences, due to the thickness of the emulsion. This equivalence is possible on account of the reciprocal equivalence of κ and I in formula (5a).

With these assumptions, the equation connecting density with exposure can easily be derived. Calling d_m the maximum density (for infinite exposure); κ the sensitivity factor; r the common ratio of the sensitivities of the various groups, the equation becomes

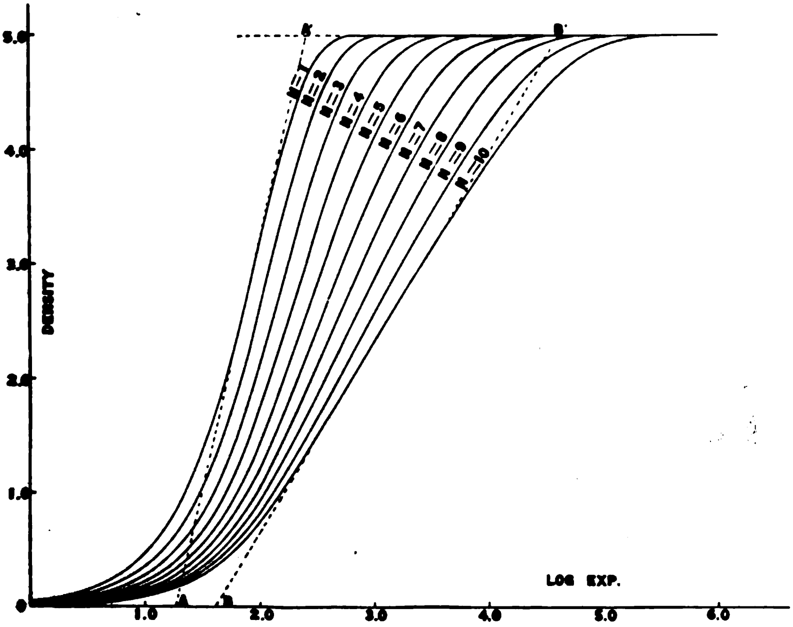
$$D = d_m \left[1 - \frac{1}{n} \sum_{s=0}^{s=n-1} e^{-\kappa r^s It} \right]. \qquad (18)$$

In order to test formula (18), it will be necessary to give values to the parameters r and n. If the values $r = \frac{1}{2}$ and $n = 1, 2, 3 \ldots 10$ are assigned, the field of practical emulsions should be well covered. For, neglecting real sensitivity differences, the opacities, including real sensitivity differences, of our ideal emulsions would in this case vary from unity to $2^9 = 512$, a variation more than sufficient to include all emulsions.

Fig. 4 shows a series of curves calculated from equation 18, for $r = \frac{1}{2}$, and $d_m = 5$, for successive values of $n = 1$ to $n = 10$. The curves for any other value of d_m can be obtained from these curves by multiplying the ordinates by $\frac{1}{5}$ d_m. It will be noticed that for increasing values of n the contrast or gamma becomes less and the straight-line portion longer. The " toe " diminishes with *n*, a characteristic not to be found in the case of the Hurter and Driffield equation.

In photographic literature the "latitude" of an emulsion is defined as the antilogarithm of the projection of the straight-line portion of the characteristic curve on the log-exposure axis. It will be convenient to define a term "range," of similar properties to latitude, in order to meet the objection that a mathematical curve has no true latitude. By "range" is meant the antilogarithm of the projection of the line AA′ or BB′ (Fig. 4) on the exposure-axis. It can be taken as a measure of the reproducing power of an emulsion and is accordingly a useful and important function. By measurements made on Fig. 4 it is found that the range has the following values:

FIG. 4.

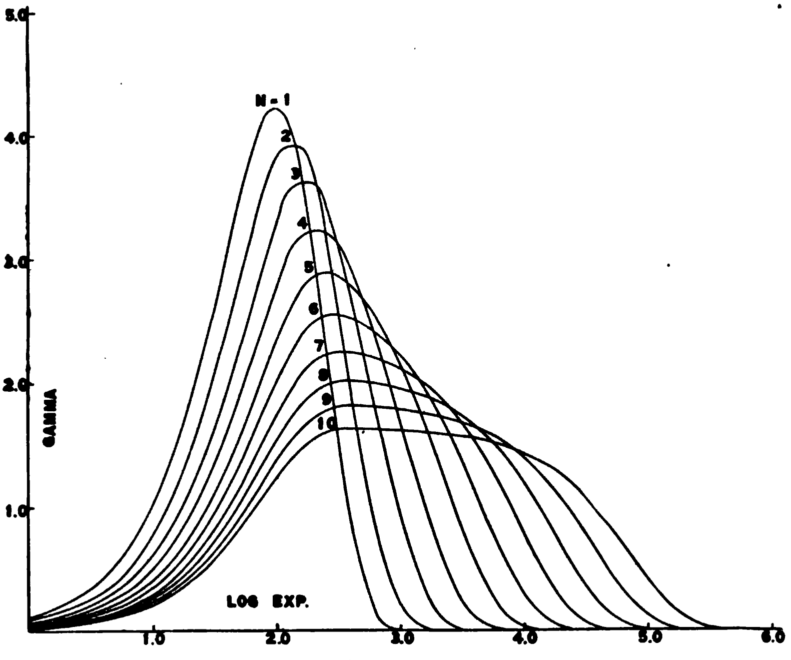


Characteristic curves from new formula.

n	*Range*	*n*	*Range*
1	15	6	90
2	17	7	160
3	25	8	290
4	35	9	530
5	55	10	1000

Fig. 5 shows the first-derivative curves for the same series of emulsions, obtained by differentiating equation (18) with respect to the logarithm of the exposure. The maximum ordinate of each curve is the gamma of the corresponding emulsion. From the broadening out of the curves with increasing *n* it is seen how the range increases, and gamma decreases, with emulsion thickness. The relation between gamma and *n* can be empirically derived from this series of curves. However, gamma derived from Fig. 5 will be found slightly greater than if obtained from the straight-line portion of the curves in Fig. 4. In comparing theory with observed curves, it will be better to use values obtained from Fig. 4. They have accordingly been scaled off as accurately as possible, plotted, and a smooth curve drawn. The result, an exponential curve, is shown in Fig. 6 reduced to $d_m = 1$.

FIG. 5.



Slope curves from new formula.

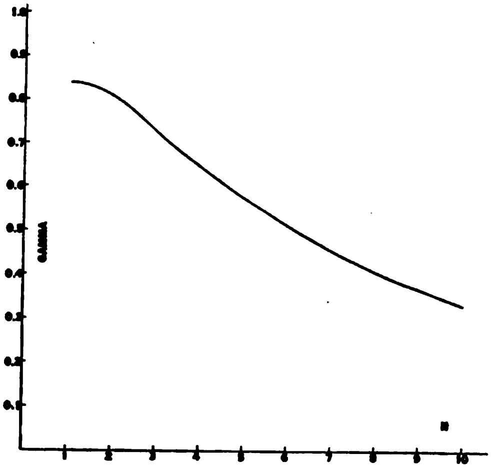
It remains to apply formula (18) to the emulsion types A and B, which have been used in the previous comparisons. This can be easily done with the help of Fig. 6. From the characteristic curves of A and B, γ and d_m, are known. With the ratio as ordinate, n is obtained directly from the figure. We have:

	γ	d_m	γ/d_m	n
Em. A..................	1.34	2.70	0.496	6
Em. B..................	1.00	1.60	0.625	4

Scale off the ordinates of the curves in Fig. 4 for $n = 6$ and for $n = 4$. Multiply each value by $d_m/5$. The result is the theoretical characteristic curves, which are given in Fig. 7. Agreement with the observed curves is seen to be very good in the case of Type B.

It is quite evident that there is an appreciable secondary action depending on the type of emulsion, which depresses the initial portion of the curve to a greater or less extent, depending upon the emulsion. Translated in terms of light action, this means that there is an inertia period—possibly a chemical change of some sort produced by the light is necessary before the normal action is effective. The other alternative is that it is a developer effect. There is evidence that the latent image forms at numerous points of the silver halide grain, which increase in number and size with exposure. To explain the observed depression of the curve it is only necessary to assume that a grain does not fully develop until the latent image connected with it has reached a certain phase. The effect must vary with the emulsion. It is

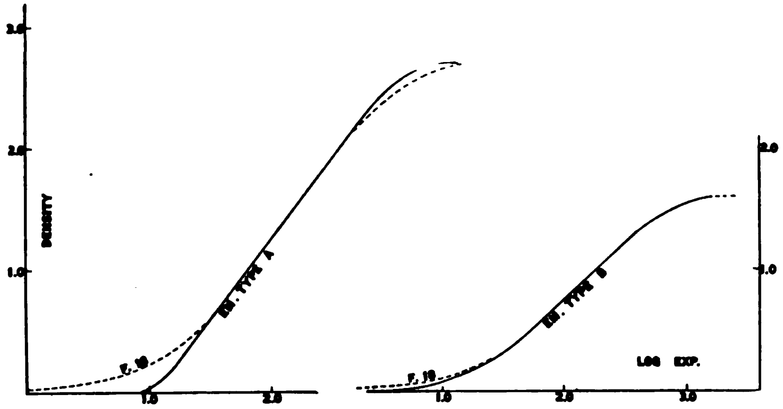
FIG. 6.



Variation of Gamma with n.

necessary to state here that the observed characteristic curves shown in this paper have all been obtained from a time-scale, or *t* variable. Curves resulting from an intensity-scale are not at hand. They should coincide with the time-scale curves if the reciprocity law holds. If Schwarzschild's law holds, they will differ in the density-ordinate only by a common factor p, and must be accordingly of the same form, as will be shown. It is necessary to collect further data on the subject, however, before definite conclusions can be reached.

FIG. 7.



Comparison of observed characteristic curves with new formula.

THE RECIPROCITY LAW.

The early experimenters found that the same photographic effect could be obtained by reciprocal variations of light-intensity and exposure-time. The relation, therefore, assumed the comparatively simple form

$$D = F(E);\ E = It.$$

It will be useful to assume that density D can be expressed as a function F of a single variable U and then find from experimental data the relation between U and the variables I and t. With this assumption

$$D = F(U), \qquad (21)$$
$$U = \phi(I;\ t).$$

Differentiating,

$$\delta D = \frac{dF}{dU}\left[\frac{\partial\phi}{\partial I}\,\delta I + \frac{\partial\phi}{\partial t}\,\delta t\right];$$

or, expressed in logarithmic increments,

$$\delta D = \frac{dF}{dU}\left[I\,\frac{\partial\phi}{\partial I}\,\delta(\log I) + t\,\frac{\partial\phi}{\partial t}\,\delta\,(\log t)\right]. \qquad (22)$$

Assume that a pair of equal densities D_0 are produced by two pairs of values of the variables I and t; *i.e.*,

$$D_0 = F(U_0),$$
$$U_0 = \phi(I_1;\, t_1) = \phi(I_2;\, t_2).$$

What is the condition that δD in (22) shall depend only on U or on ϕ? It is clearly

$$\left.\begin{aligned} t\frac{\partial\phi}{\partial t} &= \psi(\phi), \\ I\frac{\partial\phi}{\partial I} &= \chi(\phi). \end{aligned}\right\} \qquad (23)$$

If a power series be assumed to satisfy (23) it will be found to reduce to a single term

$$\phi = I^{m}_{m}.t^{n} \equiv I.t^{p}, \qquad (24)$$

or alternatively

$$\phi = \log\,(I^{u}t^{v})\;\cdot \qquad (24a)$$

It is important to state the physical meaning of equations (23). If these equations hold, the variables I and t do not occur explicitly in the coefficients in the right-hand member of equation 22. In the case assumed, of two equal densities D_0 produced by the same value of U but different values of I and t, the increments δD which they undergo from equal increments of log I or of log *t* accordingly must be identical. In order that this proposition may extend to finite values of density change, it is further necessary that $t^n\dfrac{\partial^n\phi}{\partial t^n}$ and $I^n\dfrac{\partial^n\phi}{\partial I^n}$ shall be functions of ϕ. Formulæ (24) and (24a) are seen to satisfy this criterion. The interpretation is as follows:

Suppose a series of densities D_0, D_1 . . . D_m are produced by a series of exposures in which either the time or the intensity increases in geometrical progression (logarithmically), *i.e.*,

$$\begin{array}{lccc} \text{time or intensity} \ldots\ldots\ldots\ldots & \left\{\begin{matrix} t \\ I \end{matrix}\right. & \begin{matrix} 2t \\ 2I \end{matrix} & \left.\begin{matrix} 4t \\ 4I \end{matrix}\right\} \\ \text{density produced} \ldots\ldots\ldots\ldots & D_0 & D_1 & D_2 \end{array}$$

Suppose another series produced in the same way, with this difference, that the initial density D_0' is produced by an entirely different combination of I and t than obtains in the first series. We have then

$$\begin{array}{lccc} \text{time or intensity} \ldots\ldots\ldots\ldots & \left\{\begin{matrix} t' \\ I' \end{matrix}\right. & \begin{matrix} 2t' \\ 2I' \end{matrix} & \left.\begin{matrix} 4t' \\ 4I' \end{matrix}\right\} \\ \text{density produced} \ldots\ldots\ldots\ldots & D_0' & D_1' & D_2' \end{array}$$

The theorum just proved asserts that if $D_0 = D_0'$, we must have

$$\left.\begin{matrix} D_1 = D_1' \\ D_2 = D_2' \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{matrix}\right\} \tag{25}$$

Or, in words, a series of densities produced by logarithmic variations of I or t is independent of how the initial density in the series is produced. Expressed otherwise, if two series of densities are produced in this way, if any one member of one series coincides in density with any one member of the other series, then the two series will coincide throughout.

To what extent are equations (23) and (24) realized in practice? The earlier experimenters found the very simple expression

$$\phi = I.t, \tag{26}$$

which satisfies (23). Schwarzschild and others have since found

$$\phi = I.t^p, \tag{27}$$

which also satisfies (23). On the other hand, E. Kron[7] has found

$$\phi = t.I.10^{-a\sqrt{\left(\log \frac{I}{I_0}\right)^2 + 1}}, \tag{28}$$

in which a and I_0 are parameters depending upon the emulsion. Applying (23) to this equation gives

$$t\frac{\partial\phi}{\partial t} = \phi,$$

$$I\frac{\partial\phi}{\partial I} = \phi\left\{1 - \frac{a}{\text{Mod}} \log\frac{I}{I_0}\left[\left(\log\frac{I}{I_0}\right)^2 + 1\right]^{-\frac{1}{2}}\right\}. \tag{28a}$$

---

[7] "Ueber das Schwarzungsgesetz photographischen Platten" Eder's Jahrbuch, 1914, p. 6.

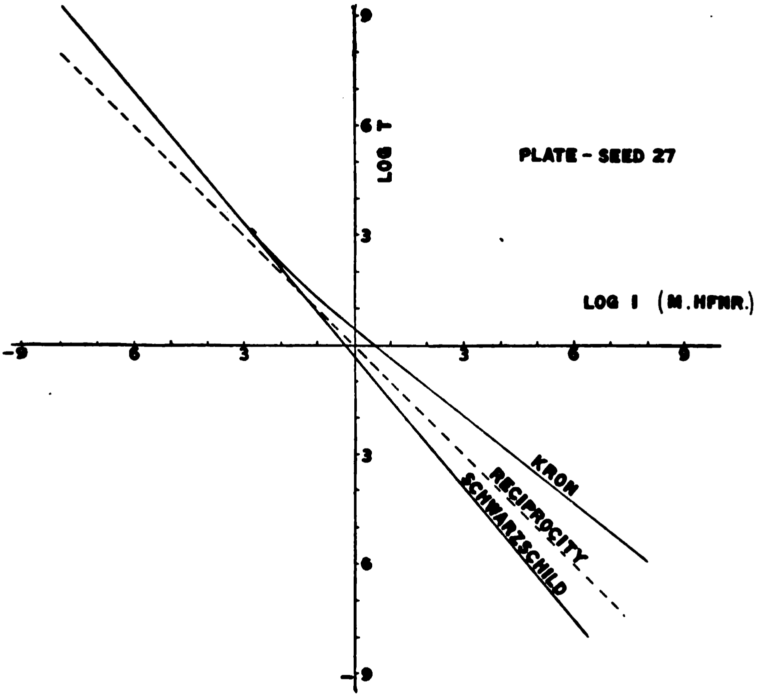
Accordingly only the first of (23) is satisfied. This means that according to Kron's experimental results the time-scale holds, but not the intensity-scale. In other words, the two series of densities considered above

$$D_0 \quad D_1 \quad D_2,$$

and

$$D_0' \quad D_1' \quad D_2',$$

FIG. 8.



Curves of equal blackening.

are equal for time-variations, but not for intensity-variations. This is a departure from results so far obtained and should be confirmed by numerous experiments.

The relation between log I and log t to produce equal blackening in the respective cases of the reciprocity law, of Schwarzschild's, and of Kron's law holding is plotted in Fig. 8 for Seed

27 plates, for which the parameters a and log I_0 are given by Kron. They are:

$$a=0.21;\ I_0=.013 \text{ meter-Hefners.}$$

Kron shows, which can easily be verified, that his law becomes identical with Schwarzschild's for low intensities, the relation between the parameters being

$$p=\frac{1}{1+a}.$$

From the figure this is seen to be true. Study of the curves is instructive. If Kron's results are correct, the reciprocity law holds with considerable exactness between the intensities of one meter-Hefner, and .001 meter-Hefners, it breaking down only for higher or lower intensities. Furthermore, for all higher intensities, the reciprocity law holds with more exactness than Schwarzschild's.

If Schwarzschild's law is correct, the reciprocity law does not hold for any value of the intensity, the error being the same in percentage amount for all intensities. A practical example is as follows: Suppose in a given case that a certain blackening is produced, and that an equal blackening is desired when the intensity is increased a hundred-fold, the exposure-time necessary will be 2.4 times the value computed from the reciprocity law, assuming $p=0.83$ as in the figure.

Only by systematic and careful experimentation can the conflicting results so far obtained by different investigators be explained and reconciled. The factors upon which the exact laws of photographic photometry depend are probably the following:

a. Character and homogeneity of the emulsion.
b. Emulsion thickness.
c. Wave length of light.
d. Development factors.

If it is true that these factors are of importance, and theoretical consideration indicates that they are, it is not surprising that concordant results have so far not been obtained.

So long as the reciprocity law of photographic action was supposed to hold, the character of the fundamental process of latent image formation appeared to be simple. Since energy absorbed is the product of intensity and time of action ($E=It$), the formation of the latent image appeared to be only a function of the energy absorbed by the silver grains. In other words, exposure should be merely a matter of energy absorption. It was from this standpoint that Hurter and Driffield developed their equation (8) of photographic action.

There are two elegant methods available for determining the exponent p in equation 24 as follows: (1) measurements of density; (2) measurement of diameters of star images. A discussion of the second method will be contained in a following paper. It is based upon considerations too extensive to be treated here. The first method is based upon a simple formula, which can be derived as follows: Assume

$$D = F(U);\ U = It^{p}.$$

Differentiation gives

$$\frac{\partial D}{\partial \log I} = I\frac{dF}{dU}\frac{\partial U}{\partial I} = U\frac{dF}{dU}, \tag{29}$$

and

$$\frac{\partial D}{\partial \log t} = t\frac{dF}{dU}\frac{\partial U}{\partial t} = pU\frac{dF}{dU}; \tag{30}$$

from (29) and (30),

$$\frac{\partial D}{\partial \log t} = p\frac{\partial D}{\partial \log I}; \tag{31}$$

or since

$$\frac{\partial D}{\partial \log t} = \gamma_t;\ \frac{\partial D}{\partial \log I} = \gamma_I; \tag{32}$$

we have

$$\gamma_t = p\gamma_I.$$

Accordingly p is the ratio of the gammas of the characteristic curves obtained by increasing the exposure by time-increments and by intensity-increments, respectively. Since it is possible to measure gamma with considerable precision, an accurate value of p should be obtainable.

If Kron's law holds the characteristic curves obtained by intensity variations (intensity-scale) will vary, being functions of the intensity (see equation 28a). Study of characteristic curves obtained under varying conditions as to time and intensity therefore will give a direct indication of the law relating photographic effect to exposure-time and intensity.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.,
May 14, 1920.

THE CEMENTED TELESCOPE OBJECTIVE OF BARIUM CROWN AND FLINT.

BY

I. C. GARDNER.

HARTING[1] showed that the most satisfactory pair of glasses available for a telescope objective of the two component cemented type is a barium crown, N_D 1.57 to 1.59 and an ordinary flint lying between 1.60 and 1.66. Glasses within this range offer the possibility of correcting simultaneously for chromatic and spherical aberration and the sine condition. When the war made necessary the manufacture of optical glass in this country the number of types to be manufactured was, for obvious reasons, made the smallest possible consistent with the demands to be satisfied. Five types were produced, and of these one was a barium crown, N_D 1.57, and a second was a flint, N_D 1.61 to 1.62. These two types were used in the numerous short focus, large aperture telescope objectives required for field glasses and other military optical instruments. Harting's paper had already standardized, to a considerable degree, the types of glass used in telescope objectives, and as outlined above, the emergency created by the war furthered the standardization. In view of this, and the fact that these glasses are now produced, having substantially similar characteristics, by the different American manufacturers, it is thought worth while to publish the following data and graphical representations which were obtained preliminary to more general and extended calculations and which show the possible combinations of values of the three aberrations which may be obtained by the use of barium crown and flint.

The following three glasses have been selected as typical and used throughout the computations:

	N_D	$N_D - N_C$	$N_F - N_D$	ν
Barium Crown No. 1	1.5740	.00299	.00718	56.4
Barium Crown No. 2	1.5740	.00286	.00713	57.5
Flint	1.6160	.00476	.01202	36.7

It is convenient to consider a two-parameter series of lenses in which one radius, say that of the cemented surface, is held constant and the two remaining radii varied. The refracting powers of the two outer surfaces, as defined by the following equations, will be selected as suitable parameters:

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[1] Harting, Zeits. Instrumentenk. 18, p. 357, (1898).

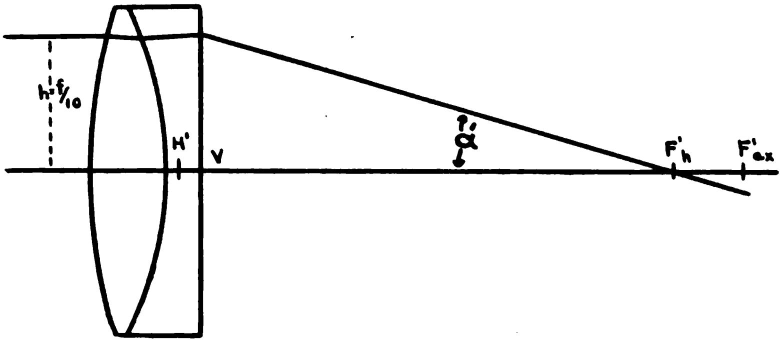
$$\varphi = \frac{n_1 - 1}{r_1} \tag{1}$$

$$\psi = \frac{1 - n_2}{r_3}$$

The index of the crown component (placed facing the parallel light) is n_1, that of the flint n_2.

The linear aberrations may be measured in terms of the focal length. There is then no sacrifice of generality if we consider only those lenses included in this doubly infinite array with a constant second radius. For any lens not included explicitly in this series may be considered as corresponding to the member of the series to which it is geometrically similar and the two have linear aberrations which are identical when measured in terms of the focal lengths.

FIG. 1.



In Fig. 1, H′ is the second principle point of the objective, F'_{ax} the focus for paraxial rays and F'_h the point at which the axis is cut by a ray parallel to the axis before refraction and incident at a point distant one-tenth of the focal length from the centre of the lens. Then $VF'_{ax} - VF'_h$ is the linear spherical aberration. If f is the equivalent focal length for paraxial rays, $f - \frac{h}{\sin \alpha'}$ is the departure from the sine condition. The chromatic difference of equivalent focal lengths is $f_C - f_F$ where the subscripts denote that the equivalent focal lengths are to be taken for the C and F spectrum lines. All of the above differences have been so selected that they are positive for an uncorrected converging lens.

We may measure φ and ψ along two coördinate axes as abscissæ and ordinates, respectively. Then any point φ_0, ψ_0 corresponds to a lens having its radii defined by the equations

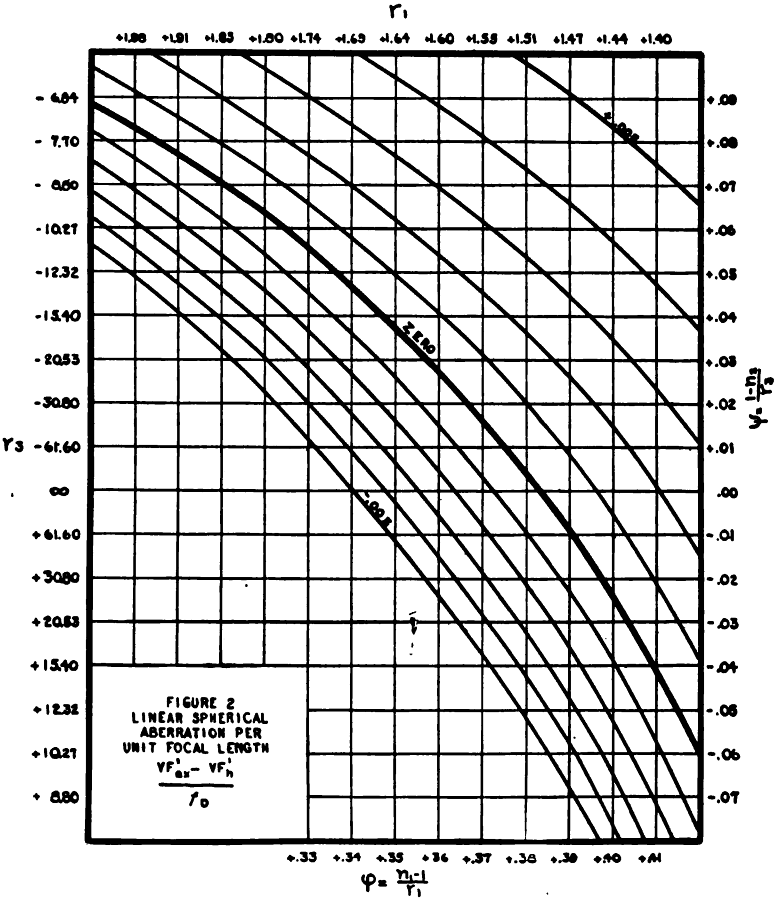


FIGURE 2
LINEAR SPHERICAL ABERRATION PER UNIT FOCAL LENGTH
$\frac{VF'_{ax} - VF'_h}{f_0}$

$$r_1 = \frac{n_1 - 1}{\varphi_0}$$

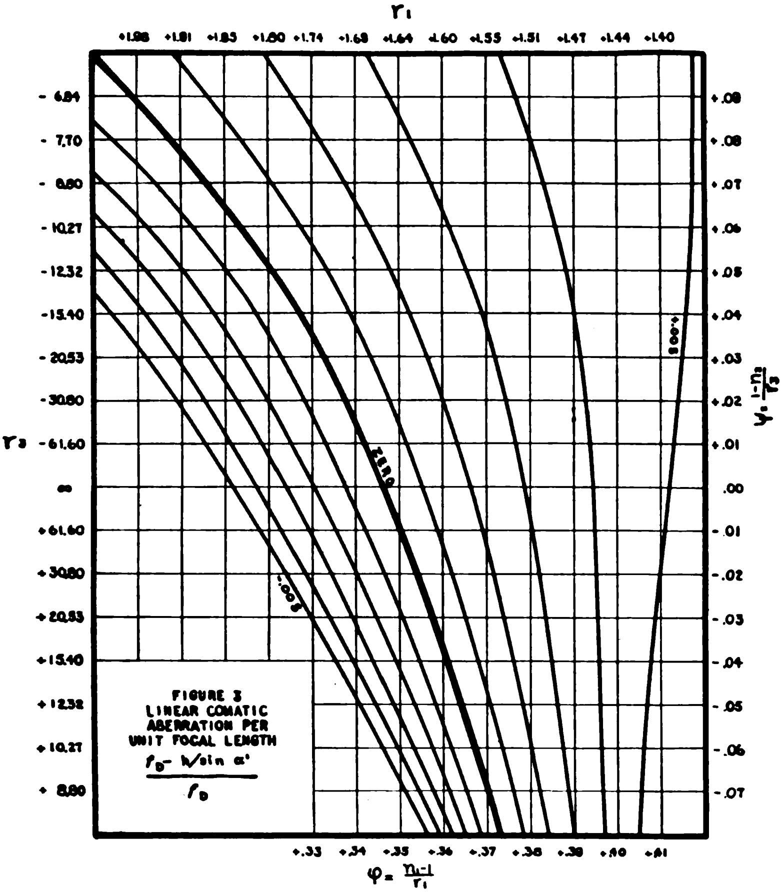
$$r_2 = -1 \qquad (2)$$

$$r_3 = \frac{1 - n_2}{\psi_0}$$

where -1 is an arbitrary constant value selected for the second radius. Upon this system of coördinates contour lines may be drawn connecting points for which any aberration is constant.

In Fig. 2 and Fig. 3 contour lines have been drawn, showing, respectively, values of the functions

$$\frac{VF'_{ax}-VF'_{h}}{f_D}$$



and

$$\frac{f_D-h/\sin\alpha'}{f_D}$$

Figs. 4 and 5 show contour lines for

$$\frac{f_{C.}-f_{F.}}{f_D}$$

when the barium crown numbers 1 and 2, respectively, are combined with the flint.

For conciseness these three functions will be referred to as the linear spherical, comatic and chromatic aberrations per unit focal length. In all the figures the values of φ and ψ are indicated along two sides of the diagram and the corresponding values of r_1 and r_3 along the two opposite sides. As previously noted, the value of r_2 for all points is -1. The values of the aberrations upon which the contours are based have been determined by trigonometric tracing of the rays. The thicknesses have not been neglected but are four and two thirty-fifths of the

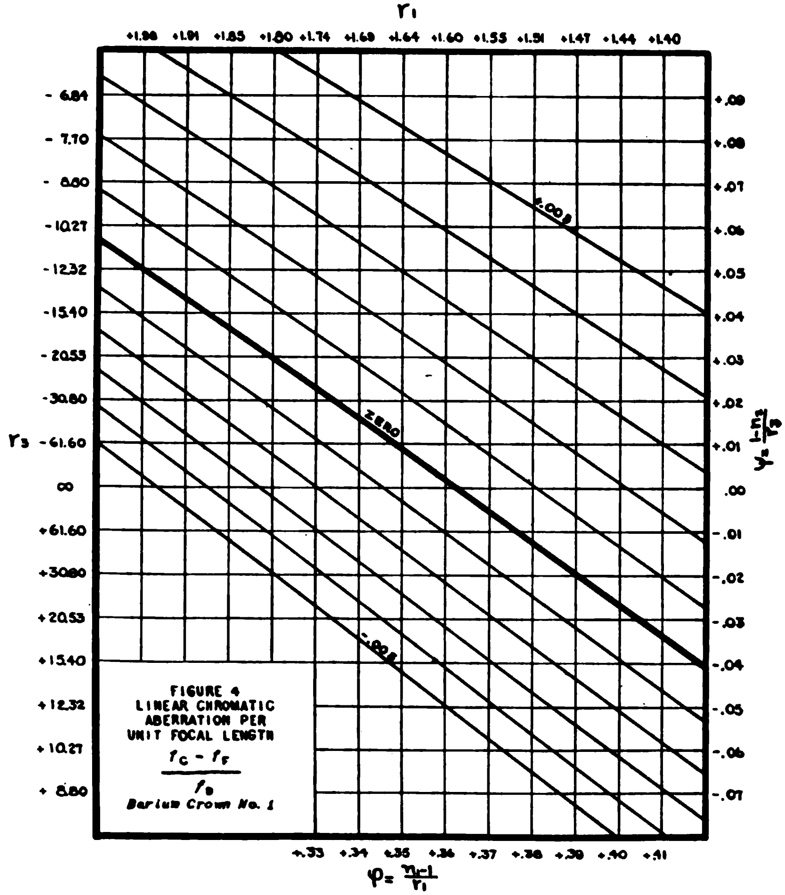


FIGURE 4
LINEAR CHROMATIC
ABERRATION PER
UNIT FOCAL LENGTH

$$\frac{f_C - f_F}{f_D}$$

Barium Crown No. 1

radius of curvature of the second surface for the crown and flint components, respectively. These thicknesses are large and permit the large apertures to be secured which are required for the objectives of low-power telescopes.

The uses which may be made of such a set of contours are self-evident. If desired values of the three aberrations per unit focal length are given, it is possible to determine whether the particular glasses here selected enable the conditions to be met or the most suitable set of compromise values may be selected. The three radii may be read directly from the diagram and multiplied by a suitable factor of proportionality in order to secure the desired focal length.

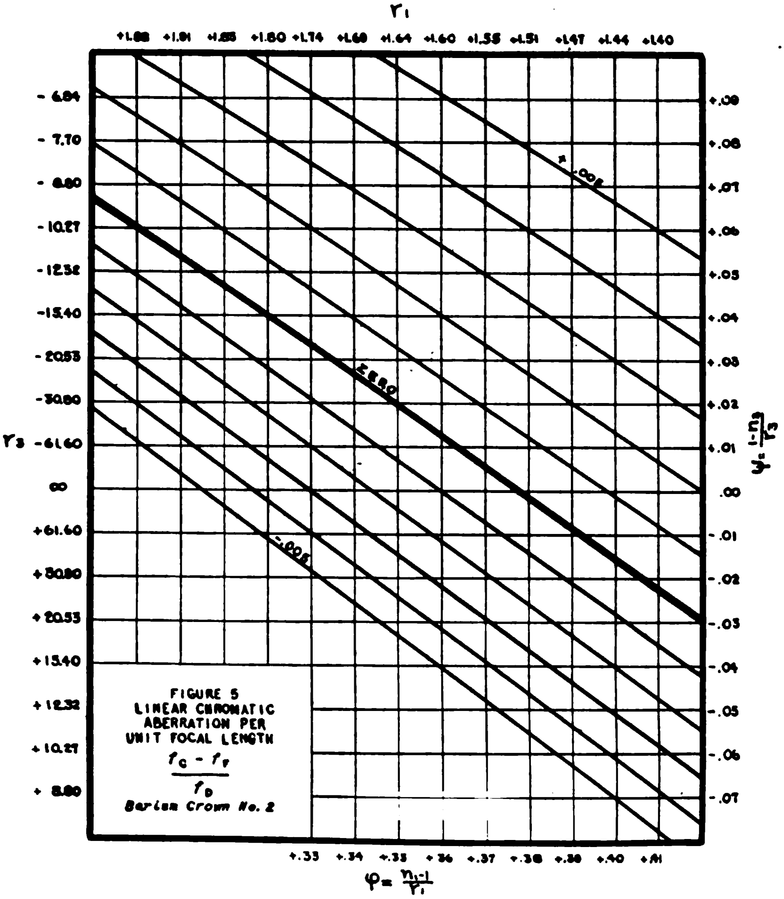


FIGURE 5
LINEAR CHROMATIC
ABERRATION PER
UNIT FOCAL LENGTH

$$\frac{f_C - f_F}{f_D}$$

Barium Crown No. 2

It should be observed that while the contours are based upon the use of the particular glasses, the constants of which are given above, the radii may be approximately determined for any barium crown and flint for which the difference of index of refraction is .042 by use of the parameters φ and ψ and equations 2. In such an application it is well to check by trigonometric computation the values thus obtained, after which the corrections to be applied may be determined by considering the slopes of the surfaces represented by the contours.

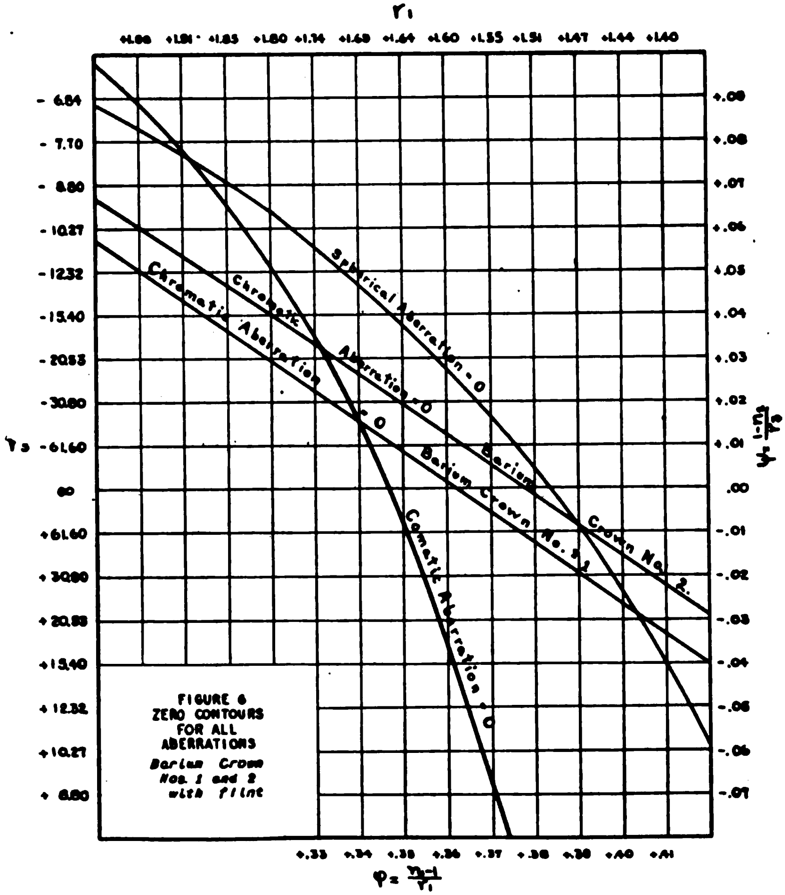


FIGURE 6
ZERO CONTOURS
FOR ALL
ABERRATIONS
Barium Crown
Nos. 1 and 2
with flint

The tolerances which may be permitted in the manufacture of an objective can be readily determined. Consider, for example, the objective having no spherical and no comatic aberration, $\varphi = +.3015$, $\psi = +.076$. If a variation of $\pm .0005$ from the normal value of the aberrations is allowed, an examination of the contours shows that the tolerances will not be exceeded if $\varphi = +.3015, \pm .003$ and $\psi = +.076 \pm .003$. Expressing this in terms of the radius of curvature, for a lens of unit focal length $r_1 = +.641 \pm .006$, $r_2 = -.336$, $r_3 = -2.72 \pm .10$. In a case such as here selected the allowable variation in focal length might well determine tolerances closer than those given above. A comparison of Fig. 4 and Fig. 5 shows that if the flint remain unchanged, a change in the value of ν of ± 0.5 produces a change in the chromatic aberration of ± 0.0005.

In Fig. 6 the zero contours for all aberrations have been superposed. Two triangles are formed (having two sides in common), one of which characterizes the results obtainable with barium crown No. 1 and flint, the other barium crown No. 2 and flint. If for either pair of glasses, all three aberrations could be made zero simultaneously, the three contours would pass through a point and the corresponding triangle would vanish. The size of the triangle is indicative of the degree to which the three corrections may be made to vanish simultaneously.

The lenses corresponding to the vertices of the two triangles are interesting as in each case two aberrations vanish. The characteristics of these lenses are given in the following table:

	Barium crown 1 and 2	Barium crown, number 1		Barium crown, number 2		Ordinary crown
	No spherical or comatic aberration	No spherical or chromatic aberration	No comatic or chromatic aberration	No spherical or chromatic aberration	No spherical or chromatic aberration	No spherical or comatic aberration
φ	+0.3015	+0.404	+0.340	+0.390	+0.3315	
ψ	+ .076	− .029	+ .015	− .009	+ .0325	
r_1	+ .641	+ .4765	+ .529	+ .501	+ .557	+ .615
r_2	− .336	− .335	− .324	− .340	− .322	− .397
r_3	−2.72	+7.13	−13.97	+23.35	−6.11	−1.44
Spherical aberration	− .0001	+ .0002	− .0035	− .0001	− .0020	− .0002
Comatic aberration	− .0000	+ .0048	− .0001	+ .0038	+ .0002	+ .0001
Chromatic aberration	+[1] .0026 +[2] .0017	.0000	− .0001	− .0001	.0000	+ .0047

[1] Barium crown No. 1.

[2] Barium crown No. 2.

The values of φ and ψ, as read from the contours and the values of the radii after multiplication by a suitable constant to give unit focal length, are given. The computations were made with five-place tables and the values of the aberrations may be in error by ± .0003. For purposes of comparison corresponding data have been given for a lens of ordinary crown (N_D 1.514, $N_D - N_C$ .00249, $N_F - N_D$ .00599, ν 60.6) combined with the flint and corrected for spherical aberration and the sine condition.

A comparison of the values of the aberrations for the different lenses is significant. As is to be expected, the barium crown having the larger value of ν, combined with the flint, is the most favorable for the elimination of all aberrations simultaneously. A consideration of the lenses corrected for spherical and comatic aberration and constructed from the two barium crowns shows that if ν were increased to approximately 59.5 all the aberrations for the aperture here considered would vanish. The advantage to be gained might, however, be illusory. The definition of chromatic aberration is, to a degree, arbitrary. If the usual curves showing the zonal aberrations for each of the colors is plotted, it will be seen that when there is a slight amount of under-correction for achromatism at the axis the correction for an outer zone may be very satisfactory. This is due to the fact that when a cemented doublet is corrected for spherical aberration for the D spectrum line, it is overcorrected for the F spectrum line. The large amount of undercorrection in the lens of ordinary crown shows the disadvantage of this type of glass as compared with the barium crown in an objective of large aperture.

Summary: A system of contours has been constructed which is helpful in the preliminary design of a telescope objective for which the desired values of the aberrations are determined.

The characteristics of some typical lenses are tabulated.

TECHNICAL STAFF,
ORDNANCE DEPARTMENT, U. S. A.
14th, July, 1920.

NOTE ON THE PATH OF A RAY THROUGH A PRISM IN A PRINCIPAL SECTION.

BY

JAMES P. C. SOUTHALL.

Mr. T. Smith has recently published a trigonometrical formula for the calculation of the angle of incidence of a ray which traverses a prism in a principal section and emerges under a prescribed angle of deviation.* My impression is that several years ago I saw a geometrical solution of this problem perhaps in a number of the London *Optician,* but I have not been able to obtain a file of this journal and therefore cannot verify the reference. However, the geometrical construction is quite obvious from a consideration of the accompanying diagram which shows the ordinary familiar method of tracing the path of a ray through a prism. The present problem may be stated as follows: Being given the refracting angle (β) of a prism made of glass of index of refraction n, to determine the direction (α) of an incident ray which shall traverse a principal section of the prism and emerge with a prescribed deviation (ϵ). Around a point V as centre describe the arcs of two concentric circles, the radius of one being n times that of the other; and construct the central angle JVG equal to the given angle of deviation (ϵ), so that the straight line joining the two points designated here by G and J shall be a chord of the inner circle. Describe the segment of the circle JHG which has this same chord GJ and which contains the given refracting angle (β) as inscribed angle; and let the two points of intersection of this segment with the outer of the two concentric circles be designated by H and H′. Connect H by straight lines with G and J, and let E and F designate any convenient points on the prolongations of HG and HJ beyond G and J, respectively. Then $\angle EGV = \alpha$ will be the required angle of incidence at the first face of the prism of the ray which emerges with the prescribed deviation (ϵ). If HE and HF are drawn normal to the two faces of the prism, then the straight lines VG and VJ will be parallel to the incident and emergent rays, respectively, and the straight line VH will be parallel to the path of the ray

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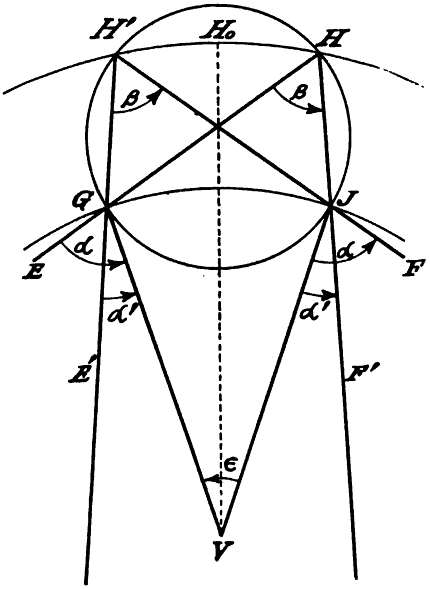
*T. Smith: "The incident angles corresponding to given deviation produced by a prism." *Transactions of the Optical Society,* xxi (1919–1920), 49, 50.

inside the prism. The angle of emergence at the second face is $\angle VJF = \alpha'$.

Evidently, in general the problem has two solutions, the other one being obtained in exactly similar fashion by connecting H′ by straight lines with G and J; and in this case it is easy to see that the angle of incidence at the first face, namely $\alpha \angle E'GV$, is equal to the angle of emergence (α') of the other ray at the second face; as indeed might easily have been inferred by reversing the path of this latter ray. Evidently, also, the two values of α are connected by the relation:

FIG. 1.



$$\alpha + \alpha' = \beta + \epsilon.$$

Of course, in case the segment of the circle does not intersect the outer of the two concentric circles, there will be no ray which can go through the prism with the required deviation. Analytically, the conditions may be expressed as follows:

If $\tan\frac{\beta}{2} < \dfrac{\sin\frac{\epsilon}{2}}{n - \cos\frac{\epsilon}{2}}$, the two points designated by H, H′ will be real and different (as shown in the figure), and the problem will have two solutions as above. If $\tan \frac{\beta}{2} = \frac{\sin \frac{\epsilon}{2}}{n - \cos \frac{\epsilon}{2}}$, the two points designated by H, H′ will coincide with each other in the point H_0 on the bisector of the angle JVG; and in this remarkable case there will be only one solution, namely, the ray which traverses the prism symmetrically so that the angles of incidence and emergence are equal ($\alpha = \alpha'$) and $\beta = 2\alpha = \epsilon_0$ where $\epsilon = \epsilon_0$ denotes now the angle of minimum deviation of the prism. And, finally, when $\tan \frac{\beta}{2} > \frac{\sin \frac{\epsilon}{2}}{n - \cos \frac{\epsilon}{2}}$, the points H, H′ are imaginary, and in this case the problem has no solution.

The trigonometrical calculation of the angle α may be derived easily from the relation:

$$\sin(\beta+\epsilon-\alpha) + \sin\alpha . \cos\beta = \sin\beta\sqrt{n^2 - \sin^2\alpha},$$

which can be obtained by a consideration of the diagram. The solution of this equation is:

$$\tan\alpha = \frac{A+B}{C},$$

where

$$A = \sin(\beta+\epsilon)\left\{\cos\beta - \cos(\beta+\epsilon)\right\},$$

$$B = \sin\beta\sqrt{n^2\left\{2 - n^2 . \sin^2\beta - 2\cos\beta . \cos(\beta+\epsilon)\right\} - \sin^2(\beta+\epsilon)},$$

$$C = (n^2 - 1)\sin^2\beta - \left\{\cos\beta - \cos(\beta+\epsilon)\right\}^2 .$$

This formula is different from the one given by Mr. T. Smith in the paper above mentioned. It is possible that it may be arranged in a form more convenient for logarithmic computation. In case of a prism made of glass of index $n = 1.5$ whose refracting angle is $\beta = 60°$, I find, by means of this formula, the following values of the angle α corresponding to $\epsilon = 40°$:

$$\begin{array}{rlll} \alpha = & 63° & 27' & 27.7'' \\ \alpha' = & 36° & 32' & 32.3'' \\ \hline \alpha+\alpha' = & 100° & 0' & 0.0'' = \beta+\epsilon. \end{array}$$

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY,
July 1, 1920.

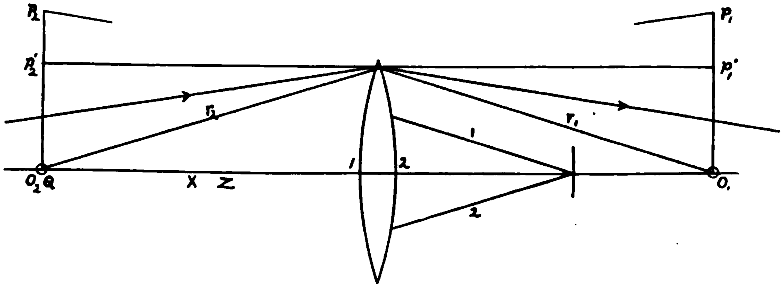
THE TREATMENT OF CERTAIN SPECIAL CASES IN LENS CALCULATION.

BY

C. W. WOODWORTH.

CERTAIN sources of error in lens calculation cannot be avoided. For example, there is no method of accurately determining a distant focus. The index of refraction of glass is usually known with great accuracy, but the remaining physical data are likely to be doubtful beyond three or four places. Assuming that all the physical data are known to five places and that the distant focus is ten thousand or upwards, it is not possible to determine more than a single digit with certainty, while if the focus was not above one hundred, there would be four certain digits.

FIG. 1.



Graphic calculation of a biconvex lens; X optical axis, Z graph axis, Q graph centre, O_1O_2 centres of curvature of surfaces 1 and 2, r_1r_2 radii to points of refraction, p_1p_2 intercepts of incident and refracted ray on reference planes, p'_1 p'_2 intercepts of segment of ray within the glass, 1 and 2 lens surfaces and the corresponding graphs.

Fig. 1 illustrates such a case after refraction at the first surface when the ray becomes nearly parallel with the axis. The complete calculation of this ray requires nine steps and may be accomplished by the following equations:

First Surface

$$(1) \quad \sin \alpha = \frac{o \sin \theta}{r}$$

$$(2) \quad \sin \alpha' = \frac{\sin \alpha}{n}$$

$$(3) \quad \theta' = \theta + \alpha' - \alpha$$

$$(4) \quad o' = \frac{r \sin \alpha'}{\sin \theta}$$

Second Surface

(5) $o' = o' + d$

(6) $\sin \alpha' = \dfrac{o' \sin \theta}{r}$

(7) $\sin \alpha = n \sin \alpha'$

(8) $\theta = \theta + \alpha - \alpha'$

(9) $o = \dfrac{r \sin \alpha}{\sin \theta}$

The physical data are

radii	10
separation of surfaces	1
index of glass	1.5
focus of incident ray	28.125
slope of incident ray $\sin \theta =$	.16

Assuming all of this was known to five significant figures, the calculation would proceed.

			Sines			Arcs			Foci
First Surface		s	.16000		θ	.16068			28.125
	(1)	a	.45000		α	.46536			
	(2)	a'	.30000		α'	.30452			
		s'	.00014	(3)	θ'	.00014	(4)	o'	21428.
Second Surface	(6)	a'	.30026		α'	.30479	(5)	o'	21447.
	(7)	a	.45039		α	.46577			
		s	.16042	(8)	θ	.16112	(9)	o	28.076

In the above since θ' has only two significant figures, o' can have only one certain figure, and in fact, with a variation of less than .000005 in the value of $\sin \theta'$ the value of o' will be found to vary between 20,000 and 22,000.

The same difficulty occurs where a different method of calculation is used. The most different method is the form which the writer has recently devised for expressing the focus of a lens system as follows:

(10)
$$\frac{1}{o_m} = \frac{1}{o_1} + \sum_{k=1}^{k=m} \frac{\sin \varphi_k}{r_k} (\cot \alpha'_k - \cot \alpha_k)$$

and it will be evident that when $\frac{1}{o_m} = .00042$ and $\frac{1}{o_1} = .03556$, as in the above case, the quantity indicated by the summation must be negative and of almost the same magnitude; that is, .03514 giving precisely the same situation, since if these latter figures are not known beyond five places, there can be but two significant figures in the first, of which but one can be certain.

Fortunately it is rarely required to determine the magnitude of a distant focus and still more fortunate that in a calculation like that given above the uncertainty of the value of o' does not prevent the final o value having four certain figures. This is because that while both θ' and o′ are uncertain, their product is accurate.

The difference between a′ of the first and second surfaces amounting to .00026 corresponds to the difference of 19 between the centres. This quantity is no more accurate than o′ but the error is in the fifth digit. If this quantity was much larger, the error might reach the fourth digit.

The inaccuracy of a distant focus is a very serious matter in the case where the next surface requires a different type of equation, which is true, if it is a flat surface. Such a case occurs in the calculation of a Huygenian eyepiece, in which the extreme oblique ray may pass from the field lens to the eye lens along a course nearly parallel with the axis, and the calculation becomes and remains completely untrustworthy instead of immediately regaining its accuracy as in the preceding example.

It is necessary therefore to substitute another method of calculation for such cases and no such method has heretofore been available. A method of calculation is now proposed which entirely avoids such errors. It is based on the determination of the intercepts on a series of reference lines made by the successive segments of the ray.

The following calculation of an edge ray of violet light through the thirty-six-inch refracting telescope at the Lick Observatory illustrates the precise conformity of the proposed method with the one commonly used. The indices for this light are crown glass 1.508721, flint glass 1.618812. The remaining optical data are given in the following calculations of the distances between the centres of curvature or reference planes:

1 r = 259.52
s = 1.96
257.56
2 r = 259.52
d = 517.08

r = 259.52
s = 8
267.52
3 r = 239.59
d = 27.93

r = 239.59
s = .93
240.52
4 r = 40,000.
d = 39,759.48

		Foci		Ray Intercepts		Angles	Sines	Cosines Tangents
1	o	∞	p	18.	α	.0694145	.0693588	
	o′	509.3270(5)	p′	11.93390	α'	.0459881	.0459719	
	d	517.08	dt′	12.11555	θ'	.0234264	.0234243	.9997256 .0234307
2	o′	1,026.407(8)	p′	24.04945	α'	.0927765(6)	.0926435(6)	
	o	512.1781(7)	p	36.36525	α	.1402323(4)	.1397732(3)	
	d	27.93	dt	1.98306	θ	.0708822	.0708229	.9974890 .0710011
3	o	484.2481(7)	p	34.38219	α	.1436372	.1431439	
	o′	1,342.121(12)	p′	21.18846	α'	.0885409	.0884253	
	d	39,759.48	dt′	627.69086	θ'	.0157859	.0157853	.9998754 .0157872
4	o′	41,101.60	p′	648.87932	α'	.0162207	.0162200	
	o	40,673.37(9)	p	1,050.63428	α	.0262601	.0262571	
	u	673.4 ± 1	u	673.4 ± 1	θ	.0258253	.0258224	.9996666 .0258310

In making the above calculation the ray-intercept method was first used. The equations differ from those used above as follows:

Instead of (1) and (6) use

$$\sin \alpha = \frac{p \cos \theta}{r} \tag{11}$$

instead of (4) and (9) use

$$p = r \frac{\sin \alpha}{\cos \theta} \tag{12}$$

and instead of (5) use

$$p = p + dt \tag{13}$$

and finally determine the focus by

$$o = p / \text{tang}\, \theta \tag{14}$$

and

$$u = r - o \tag{15}$$

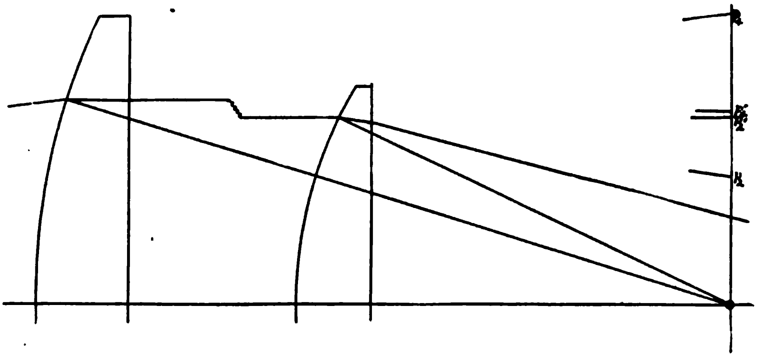
This last equation applying to both methods.

After calculating the above (but not including column one) according to the intercept method, the lens was then recalculated by the former equations and all angular differences indicated by giving in parentheses the final digit when different. In no case was the difference greater than .0000001. This recalculation involves the calculation of the foci given in column one. After obtaining these figures the foci were then recalculated from the intercepts by equation (14), and as before the differences indicated in parentheses and the variation in each case is not larger than a figure in the seventh place.

The intercept method thus affords a most excellent checking method against the focus method of calculation. It is more than this, because it will detect and avoid the error referred to above, such as may occur, for instance, with what I have called the comatic ray in an eyepiece calculation, as shown in Fig. 2.

It will only be necessary in illustrating its superiority in such a case to trace the ray from the time it leaves surface two till it reaches surface four, calculating the ray from the eye downward through the lenses. The error showing in the value found for sin α_4.

FIG. 2.



Lenses of a Huygenian eyepiece so drawn that the two curved surfaces have a common centre. p_2 intercept of the ray in the eye lens. p'_1 intercept of the ray passing from eye lens to field lens. p'_4 intercept of the ray in the field lens, and p_4 intercept of the ray below the field lens.

The intercept p_2 is 6.6 mm. The radius of surface two is 15 mm., of surface four 24 mm. The glass index for both lenses is 1.5085. The value of the angle θ is .00500 corresponding with sin α_2 .44. The distance from the reference line of surface two to surface three is 46.8 mm. From surface three to the reference line of surface four is 20.8 mm.

The calculation by the focus method proceeds

o_2 1320.0
d_2 46.8

o_3 1366.8 θ_3 .00500
o'_3 2061.8 θ_4 .00331
d_3 20.8

o'_4 2041.0 sin α'_4 .28148 ± 40

By the intercept method we have

p_2	6.6	θ_3	.00500	
p_3	6.834	θ_4	.00331	cos .99999
p_4	6.7652	sin α'_4	.28188 ± 1	

The angle θ in the preceding calculations is so small that arc sine and tangent are equal for more than the number of decimal places considered and therefore equation (2) becomes $\theta_4 = \theta_3/n$. In both calculations the plane surface three is used as a reference plane, since evidently the centre of curvature at infinity cannot be employed.

These two calculations show a difference of four in the fourth place, the latter being the correct value of this sine. A difference of .000005 in θ_3 would make this same amount of difference in the fourth place by the focus method and only one in the fifth place by the intercept method as indicated by the ± signs. The latter method is thus shown to be about forty times more accurate than the former method in this case where the θ angle is .005. If the angle were still smaller, the difference of accuracy would very rapidly increase.

Another source of error is presented in the calculation given above of the Lick Observatory telescope. This error is due to the very great radius length of the fourth surface. The calculation given above indicated an error in the fourth place, even though the seven-place functions were employed. The error is actually twice as great as here indicated, as will be shown below.

The following method enables one to entirely avoid this source of error increasing the accuracy of the final result by two digits. The plan is to modify the intercept method by substituting at the last refraction a plane tangent to the vertex of the lens in place of the reference plane through the centre of curvature. This requires the development of a new equation. In Fig. 3 one will see that

$$(16)\quad h''+h'+p'=r\tan(\alpha'-\theta')$$

and also that

$$(17)\quad h''=h'\,\frac{\tan(\alpha'-\theta')}{\tan\theta'}$$

Substitute this value in equation (16) and we may write the combined equation in the form

$$(18)\quad h'=\frac{r\tan(\alpha-\theta')-p}{1-\dfrac{\tan(\alpha'-\theta')}{\tan\theta'}}$$

and since, as likewise shown in the figure,

$$(19)\qquad h = h' \frac{\tan\theta}{\tan\theta'}$$

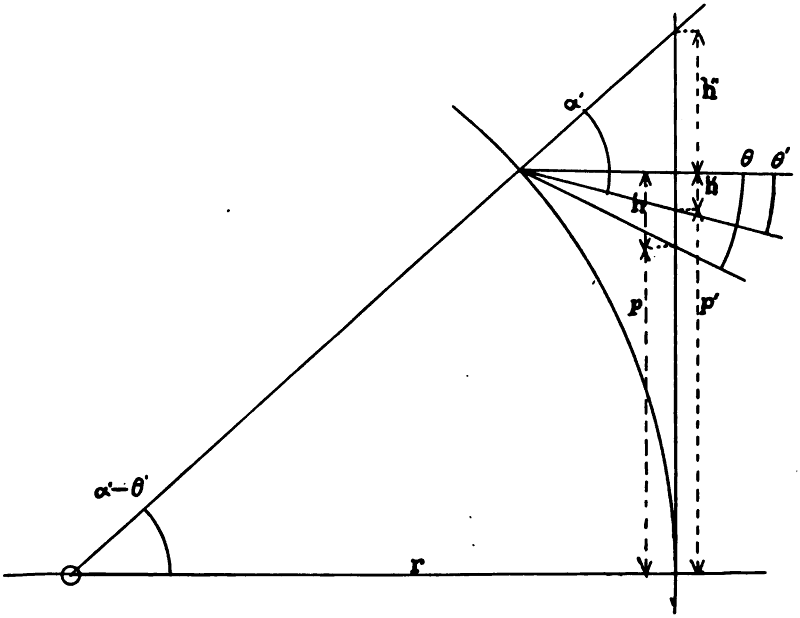
and

$$(20)\qquad p = p' + h' - h$$

we may combine the last three equations and write the final equation in the form

$$(21)\qquad p = p' - \frac{\tan\theta - \tan\theta'}{\tan\theta' - \tan(\alpha' - \theta')} \left\{ r\tan(\alpha' - \theta') - p \right\}$$

FIG. 3.



Ray intercepts on a plane tangent to the lens vertex α, α', θ and θ' the angles made by the incident and refracted rays with the radius at the point of refraction and with the optical axis, p p' the intercepts on the vertex plane of the incident and refracted rays, h, h' and h'' the difference between the height of the incident point from the optical axis and these intercepts and that of the radius.

In order to apply this equation to the calculation of the fourth surface, we must substitute a different value of d_3, because of the new position of the reference plane. This new value is 240.52, and therefore

p'_3	21.18846
dt	3.79714
p'_4	17.39132

Stated numerically, equation (21) is

$$\begin{aligned} p &= 17.39132 \frac{-.025809-.0157872}{.0157872-.0004348}(40000\times .0004348-17.39132) \\ &= 17.39132 \quad \frac{.0100438}{.005743}\,.00065 \\ &= 17.39132-.00114 \\ &= 17.39018 \end{aligned}$$

for which the value of u is 673.229 ± 1 instead of 673.4, as previously determined.

The determination of the amount of the error in this and in the former calculation is obtained by augmenting and decreasing the last value of tan θ (or of sin θ) .00000005. and represents only the error in the last operation.

In all the above calculations angles are given in radians and the natural functions are employed, the arcs and functions being determined by the use of the well-known trigonometric series.

The intercept method of lens calculations applies to three conditions, the ordinary one of a curved surface with short radii, the case of a flat surface with an infinite radius, and also the intermediate case of the lens of slight curvature and long radius, and it permits the use of a lesser number of decimal places since the practice of using a large number is in order to secure sufficiently accurate results, notwithstanding such sources of errors as are pointed out above. The new method can be generally used as a substitute for all prevailing methods, including paraxial rays as well as rays affected by spherical aberrations. It is as satisfactory in all cases, is more accurate in some cases, as shown above, and has the additional advantage of corresponding term by term with the steps of the author's graphic methods of lens calculation.

UNIVERSITY OF CALIFORNIA.

NOTE ON CALCULATION OF PATH OF RAY THROUGH A SYMMETRICAL OPTICAL INSTRUMENT.

BY

JAMES P. C. SOUTHALL.

If the radius (r) of a spherical refracting surface is measured along the optical axis from the vertex A to the centre C, that is, if $r = \text{AC}$, the refracting power will be

$$F = (\text{n}' - \text{n})R, \tag{1}$$

where $R = 1/r$ denotes the curvature of the surface and n, n′ denote the indices of refraction of the two media in the order in which the light traverses them.

Let α, α' denote the angles of incidence, and refraction of a ray of light which is refracted at this surface, and which crosses the straight line AC before and after refraction at points designated by L and L′ and at slopes denoted by θ and θ', respectively; so that (taking account of the signs of these angles) we have the well-known invariant relation:

$$\alpha' - \theta' = \alpha - \theta. \tag{2}$$

And, finally, let the abscissæ of the points L, L′ measured along the optical axis from the centre C as origin be denoted by c, c' ($c = \text{CL}$, $c' = \text{CL}'$); and put $C = \text{n}/c$, $C' = \text{n}'/c'$. With this notation we derive immediately the following geometrical relations:

$$\frac{C \cdot \sin\alpha}{\text{n} \cdot \sin\theta} = \frac{C' \cdot \sin\alpha'}{\text{n}' \cdot \sin\theta'} = -R; \tag{3}$$

and since the ray is subject to the optical law of refraction, *viz.*,

$$\text{n}' \cdot \sin\alpha' = \text{n} \cdot \sin\alpha, \tag{4}$$

we find by imposing this condition that the ray-coördinates are connected with each other by the following equation:

$$\frac{\text{n}'^2}{C'} \sin\theta' = \frac{\text{n}^2}{C} \sin\theta. \tag{5}$$

So far there is nothing at all novel about these formulæ except perhaps the mode of writing them. Now let us introduce by way of abbreviation the symbol E to denote a function of the angles α, α', θ and θ' which is defined as follows:

$$\text{E} = (\sin\theta' - \sin\alpha') - (\sin\theta - \sin\alpha). \tag{6}$$

Evidently with the aid of (1), (3) and (4) we can write:

$$\frac{\sin\theta'}{\sin\theta} = 1 + \frac{E}{\sin\theta} + \frac{n}{n'} \cdot \frac{F}{C},$$

so that from (5) we derive the formula:

$$\frac{n}{n'} C' = \frac{n'}{n} C\left(1 + \frac{E}{\sin\theta}\right) + F; \qquad (7)$$

which may be written also as follows:

$$\frac{C'}{n'^2} = \frac{C}{n^2} + \frac{F}{nn'} - \frac{R}{n\cdot\sin\alpha}E. \qquad (7a)$$

The function E may be put in a form more convenient for logarithmic computation; for, since according to (2),

$$\alpha + \alpha' + \theta + \theta' = 2(\alpha' + \theta), \quad \alpha - \alpha' + \theta - \theta' = 2(\alpha - \alpha'),$$

obviously,

$$E = 4\sin\frac{\alpha-\theta}{2}\cdot\sin\frac{\alpha'+\theta}{2}\cdot\sin\frac{\alpha'-\alpha}{2}. \qquad (8)$$

Obviously, also, in the case of a paraxial ray for which all the angles are so small that their second and higher powers may be neglected, this function E vanishes entirely; and if the symbols B, B' are employed instead of C, C' to indicate that the ray is a paraxial ray, formula (7) becomes:

$$\frac{n}{n'}B' = \frac{n'}{n}B + F. \qquad (9)$$

The trigonometrical expressions which occur in the formulæ (3) and (5) suggest immediately the central perpendiculars let fall on the incident ray and the corresponding refracted ray which have been frequently employed in systems of formulæ for lens-calculation as well as the perpendiculars let fall from the vertex of the surface. For if p, p' denote the lengths of the perpendiculars dropped from the centre C on the ray before and after refraction, respectively, then

$$p = \frac{\sin\alpha}{R} = -\frac{n\cdot\sin\theta}{C},\ p' = \frac{\sin\alpha'}{R} = -\frac{n'\cdot\sin\theta'}{C'},\ n'\cdot p' = n\cdot p. \qquad (10)$$

Thus, formula (7a) may be expressed in terms of p as follows:

$$\frac{C'}{n'^2} = \frac{C}{n^2} + \frac{F}{n\cdot n'} - \frac{E}{n\cdot p}.$$

And therefore for an edge-ray and a paraxial ray refracted at the kth surface of a centred system of spherical refracting surfaces we have the following recurrent formulæ:

$$\left.\begin{aligned}\frac{C'_k}{n_{k+1}^2} &= \frac{C_k}{n_k^2} + \frac{F_k}{n_k \cdot n_{k+1}} - \frac{E_k}{n_k \cdot p_k},\\ \frac{B'_k}{n_{k+1}^2} &= \frac{B_k}{n_k^2} + \frac{F_k}{n_k \cdot n_{k+1}}.\end{aligned}\right\} \quad (11)$$

Now if these two equations are subtracted from each other and the result cleared of fractions, taking account of the relation $n_k . p_k = n_{k+1} . p_k'$, and if, finally, we put

$$C - B = \delta C, \quad C' - B' = \delta C',$$

we shall obtain the result:

$$p_k'^2 . \delta C_k' = p_k^2 . \delta C_k - n_k . p_k . E_k ; \qquad (12)$$

which enables us to find the spherical aberration after refraction at the kth surface in case we know its value before refraction.* In passing to the next succeeding surface we may write:

$$\delta C_{k+1} = \left(\frac{C_{k+1}}{C_k'}\right)^2 \delta C_k', \text{ approximately;}$$

and since it follows from (10) that

$$p_{k+1} . C_{k+1} = p_k' . C_k', \qquad (13)$$

the last result is equivalent to the equation:

$$p_{k+1}^2 \cdot \delta C_k = p_k'^2 . \delta C_k', \text{ approximately.} \qquad (14)$$

Consequently, if we give k in succession all integral values from k = 1 to k = m, where m denotes the total number of refracting surfaces, the recurrent formulæ (12) and (14) taken together

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* It may be of interest to see how this formula (12) may be derived very simply starting from the abscissa-equation for a paraxial ray as given by formula (9). With the aid of (1) this latter formula may be put in an invariant form as follows:

$$\frac{1}{n'}\left(\frac{B'}{n'} + R\right) = \frac{1}{n}\left(\frac{B}{n} + R\right).$$

Now according to (10):

$$\frac{B}{n} + R = -\frac{\delta C}{n} + \frac{C}{n} + R = -\frac{\delta C}{n} + \frac{\sin \alpha - \sin \theta}{p};$$

and since for the refracted ray a precisely similar expression may be obtained by priming all the letters except R, the following equation may be written immediately:

$$\frac{\delta C'}{n'^2} - \frac{\sin \alpha' - \sin \theta'}{n' p'} = \frac{\delta C}{n^2} - \frac{\sin \alpha - \sin \theta}{n p}.$$

which is obviously capable of being transformed into (12).

yield the following equation for calculating the spherical aberration:

$$p_m'^2 \cdot \delta C_m' = p_1^2 \cdot \delta C_1 - \sum_{k=1}^{k=m} n_k \cdot p_k \cdot E_k \,. \tag{15}$$

Now if the incident paraxial ray and marginal ray cross the axis at the same point L_1, we shall have $\delta C_1 = 0$, as is always the case for a monocentric bundle of incident rays emanating from a point on the optical axis. Moreover, if the points where the paraxial ray and the edge-ray cross the optical axis in the last medium are designated by M_{m+1} and L_{m+1}, respectively, then the linear measure of the spherical aberration along the axis will be $\delta c_m' = M_{m+1}L_{m+1}$; and since

$$\delta c'_m = -n_{m+1} \frac{\delta C_m'}{C_m'^2} \text{, approximately,} \tag{16}$$

we obtain finally the following formula for calculating the spherical aberration:

$$\delta c_m' = \frac{n_{m+1}}{(p_m' . C_m')^2} \sum_{k=1}^{k=m} n_k \cdot p_k \cdot E_k, \quad (\delta c_1 = 0). \tag{17}$$

Summing up these results, we have the following system of formulæ:

$$\left.\begin{aligned} & p_k = -\frac{n_k \cdot \sin\theta_k}{C_k}, \; p_k' = \frac{n_k}{n_{k+1}} p_k, \\ & \sin\alpha_k = p_k \cdot R_k, \; \sin\alpha_k' = p_k' \cdot R_k, \\ & \theta_{k+1} = \theta_k + \alpha_k' - \alpha_k, \\ & C_k' = -\frac{n_{k+1} \cdot \sin\theta_{k+1}}{p_k'}, \\ & n_k \cdot p_k \cdot E_k = (n_k \cdot p_k)^2 \left(\frac{F_k}{n_k \cdot n_{k+1}} + \frac{C_k}{n_k^2} - \frac{C_k'}{n_{k+1}^2} \right). \end{aligned}\right\} \tag{18}$$

And since

$$a_k = d_k + r_{k+1} - r_k \,, \tag{19}$$

where $d_k = A_kA_{k+1}$ and $a_k = C_kC_{k+1}$, we pass from the kth to the (k + 1)th surface by means of the formula:

$$C_{k+1} = \frac{n_{k+1} . C_k'}{n_{k+1} - a_k . C_k'} \,. \tag{20}$$

Or, without computing the values of C and C', instead of (20), we may employ the following formula:

$$p_{k+1} = p_k' + a_k \cdot \sin\theta_{k+1} \,, \tag{21}$$

and calculate E_k by means of either of the two following expressions, the first one being convenient for machine calculation, and the second being adapted to logarithmic work:

$$E_k = \sin \theta_{k+1} - \sin \alpha_k' + \sin \alpha_k - \sin \theta_k; \qquad (22)$$

$$E_k = 4 \sin \frac{\alpha_k - \theta_k}{2} \sin \frac{\alpha_k' + \theta_k}{2} \sin \frac{\alpha_k' - \alpha_k}{2}. \qquad (23)$$

As a numerical illustration we may compute the object-glass of a telescope composed of a triple cemented lens of two kinds of glass, crown and flint, with following data:

	k=1	k=2	k=3	k=4
Radii: r_k	53.4759	34.6075	−41.7573	−171.2457
Thicknesses: d_k	0.4994	0.9988	0.4994	

Indices of refraction (for Fraunhofer D-line): $n_1 = n_5 = 1$ (air),

$n_2 = n_4 = 1.61358$ (flint), $n_3 = 1.51806$ (crown).

The so-called edge-ray is supposed to be initially parallel to the axis ($\theta_1 = 0$, $C_1 = 0$) at a distance from it equal to 6 ($p_1 = 6$). The focal length of the lens will be found to be 100.

The following values were obtained by logarithmic computation according to formulæ (18), (19) and (20):

	k=1	k=2	k=3	k=4
lg F_k	8.0597131	7.4409235−	7.3593612−	7.5542512
lg C_k		8.1858497	7.8511258	7.5849386
lg C_k'	8.2692065	8.0398055	7.7444548	7.5684442
lg p_k	0.7781513	0.6537176	0.8688988	1.0019135
lg $n_k . p_k . E_k$	7.0024979−	6.6982407	7.2368742	7.0815801−

(A minus sign appended to a logarithm indicates that the number itself is negative.) Thus we find by formula (17) that

$$\delta c_4' = 0.0034, \text{ approximately.}$$

Following is same calculation made on a machine which will be found to be considerably quicker. The C's do not have to be calculated, but only the p's. We use here the formulæ in (18) which do not involve C or C', formula (21) and formula (22). In formula (17) we put $\sin \theta_{m+1} = p_m' . C_m'$.

	Angles	Sines	Central perpendiculars
θ_1	0° 0′ 0.00″	0.0000000	$p_1 =$ 6.00000
α_1	6° 26′ 31.74″	0.1122000	
α_1'	3° 59′ 14.16″	0.0695348	
θ_2	− 2° 27′ 17.58″	−0.0428327	$p_1' =$ 3.71844
α_2	7° 28′ 48.21″	0.1301811	$p_2 =$ 4.50523
α_2'	7° 57′ 13.17″	0.1383721	
θ_3	− 1° 58′ 52.62″	−0.0345730	$p_2' =$ 4.78871
α_3	−10° 11′ 58.76″	−0.1770788	$p_3 =$ 7.39434
α_3'	− 9° 35′ 23.93″	−0.1665963	
θ_4	− 1° 22′ 17.79″	−0.0239368	$p_3' =$ 6.95662
α_4	− 3° 21′ 45.21″	−0.0586540	$p_4 =$ 10.04413
α_4'	− 5° 25′ 50.77″	−0.0946429	
θ_5	− 3° 26′ 23.35″	−0.0600001	

Here we find:

	k=1	k=2	k=3	k=4
E_k	−0.0001675	0.0000687	0.0001537	−0.0000744
$n_k.p_k.E_k$	−0.00100500	0.00049942	0.00172529	−0.00120580

and hence $\Sigma\, npE = 0.00001391$, which divided by $\sin^2\theta_5$ gives $c_4' = F'L_5 = 0.003863$, approximately. (Here F′ designates the focal point of the object-glass.)

Exact calculation of the two rays (paraxial-ray and edge-ray) with a calculating machine gives for the edge-ray:

$$C_4L_5 = 270.1185,$$

and for the paraxial-ray:

$$C_4F' = 270.1167;$$

so that the correct value of the spherical aberration is:

$$\delta c_4' = 0.0018,$$

perhaps as accurately as it can be computed. And while the approximate result is seen to be about twice as great as this value, nevertheless the two results are in substantial agreement, at least, so far as the position of the first significant figure is concerned, and both show that the system is practically free from spherical aberration. The approximate method here is at least as good and reliable as any other approximate method that has been devised. But the chief interest in the whole matter is rather theoretical than practical. After all it is nearly as easy to calculate the paraxial-ray, and indeed this calculation generally has to be performed anyhow.

DEPARTMENT OF PHYSICS,
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NEW YORK, N. Y.
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A NEW METHOD FOR THE MEASUREMENT OF PHOTOGRAPHIC FILTER FACTORS.*

BY

RAYMOND DAVIS.

ORTHOCHROMATIC and panchromatic plates are almost always used with photographic filters (colored screens of glass or gelatine) to limit the color and intensity of the light falling on the photographic plates. Since the use of a filter decreases the intensity of certain colors on the plate, the photographic exposure through a filter will be greater than that required for the same plate used without a filter.

A filter factor is the ratio of the exposure required with a given filter to the exposure without the filter; hence, to obtain the proper time of exposure when using the filter, the time of exposure without the filter should be multiplied by the filter factor.

Heretofore, so far as the author knows, filter factors have been determined by either of two methods. First, the method of trial and error, and second, the measurement of plate speeds without and with the given filter in some form of sensitometer. In the course of a study of the plates and films made in the United States, it was desired to measure the filter factors of all orthochromatic and panchromatic emulsions for a number of different filters.

A device occurred to the author for measuring the factors with the same accuracy as a careful speed measurement and yet with very much less effort.

The apparatus is constructed as follows: Light from the two sides of a standard metal filament lamp L, is reflected by similar reflectors M_1, M_2, M_3, M_4, and by similar prisms, P_1 and P_2, so that the two beams fall side by side on the photographic plate E, the filter factors of which are to be determined. In front of the plate is a simple shutter. The source of light L is movable along the line joining M_1 and M_3. At F, between M_3 and M_4, carriers to hold the filters employed may be inserted.

The method of procedure is as follows: The lamp initially at the position *a,* will give two beams of equal intensity at a photographic plate, if the distances traversed by the two beams are the same and if identical optical conditions of reflection and absorption obtain for the various media through which the beams pass. If a filter is inserted in one path, the intensity of that beam will be decreased, and in order to make the beams of the same intensity it will be necessary to shift the lamp toward the filter, thus shortening that path and increasing the other. The ratio by which the filter decreases the intensity of the light is equal to the square of the length of the path which contains the filter divided by the square of the length of the path without the filter.

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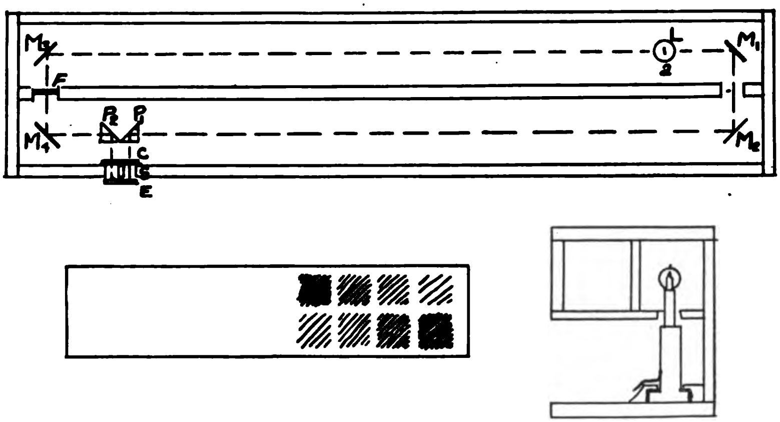
* Published by permission of Director, Bureau of Standards.

In practice the point of balance is obtained by making a series of exposures on the plate with the lamp at arbitrary distances from the centre position *a,* choosing these distances so that some are on one side of the point of balance and some on the other. The plate is developed and fixed and the density differences measured on a polarization photometer. These differences are plotted as positive or negative according to whether the setting made the lamp distance too large or too small, and the setting for a balance is found by interpolation. The distance of the new point of balance from the position of equilibrium without a filter, together with the distance from the lamp to the plate by both paths in

FIG. 1.



Schematic arrangement of apparatus. Upper: Plan of filter factor apparatus. Left: Specimen of test plate. Right: Vertical section.

its position of equilibrium without a filter, enable the law of inverse squares to be applied and the filter factors calculated.

The apparatus as constructed has each path about one metre long when the lamp is in the zero position, and the lamp has a movement of about 69 centimetres so that the range of factors which can be measured is from one to thirty. The filters used must be larger than ½ inch square. The strips of the photographic plate tested are 1⅛ inches x 5 inches and two filter factors may be measured conveniently on each. The exposure time is about half a second, since the intensity of the lamp used with its compensating filter is 2¾ C.P.

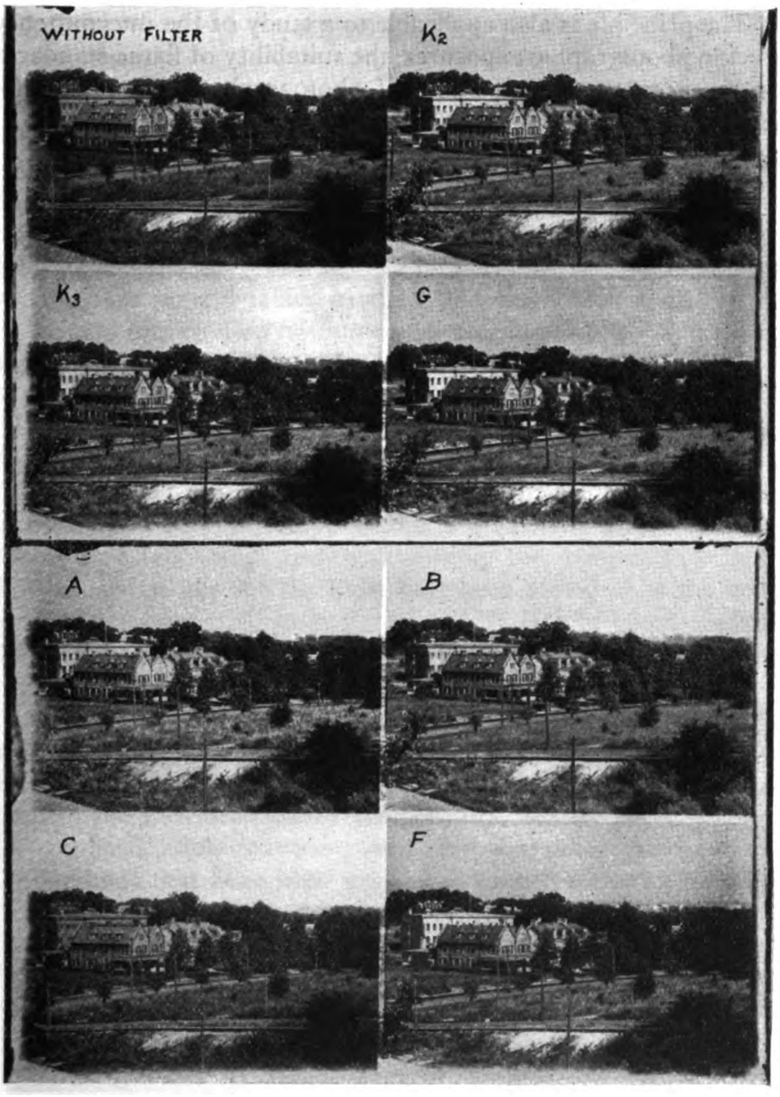
In order to get filter factors which can be used out of doors, it is necessary to modify the distribution of spectral energy of the metal filament lamp by placing in both beams a suitable screen. It is also necessary to burn the lamp at a constant and specified current so as to keep constant its spectral distribution of energy. The combination of lamp and filter is identical with that used as the standard source of the Bureau of Standards sensitometer. How successful the apparatus described has proven is shown in Fig. 2. The exposures were proportional to the filter factors as measured, the two plates on which the eight exposures were made being developed together for the same length of time, and printed together on one sheet of paper. The Wratten filters and their factors on the plate used (Wratten M) were as follows:

Without filter		1
K_1	"	4
K_2	"	4.5
G	"	5.5
A	"	18
B	"	14
C	"	12
F	"	24

In conclusion, attention may be called to other possible applications of the principle of this apparatus, which is essentially a photographic photometer.

If the plate-holder is made moveable, between two light sources, it would be easy by means of narrow banded filters, to measure the relative candle power of two sources for any part of the spectrum; to color match two lamps through variation of the current in one or both of them; to compare electric standards with flame standards, to study the effect of humidity and atmos-

FIG. 2.



Photograph to test filter factors.

pheric pressure since this effect is probably greater in the blue, and the photographic plate therefore better adapted than the eye for observing it.

The principle is also applicable to a study of the intermittency effect in photographic exposures, the suitability of flame standards for sensitometry, comparisons of photographic with visual efficiency and other problems of a similar character.

WASHINGTON, D. C.
July, 29, 1920.

1919 REPORT OF STANDARDS COMMITTEE ON PYROMETRY.

BY

W. E. FORSYTHE, Chairman.

In the report of the Standards Committee for 1919, Doctor Nutting called attention to the need of collecting reference material in such a form that it would be useful to as many persons as possible. Following out that suggestion, this report will contain several tables of data that have been found useful in the field of pyrometry. Some of these data have been computed and some of them have been taken from various papers that have already been published. They all depend upon the values of the radiation constants included in this paper, which best represent all the experimental facts.

In the past when an experimenter desired to do some work in the region of high temperatures, a great part of his time was spent in establishing and maintaining the temperature scale. The scale that has for the most part been aimed at is the centigrade thermodynamic scale, as defined by Kelvin about the middle of the last century. There has been considerable work done in the field of radiation and pyrometry in the last few years, so that now methods and constants are available that had to be worked out by each experimenter in the past. Even at the present time the experimenter in this region must spend considerable time in the measurement of temperature, but by making use of the standard points that have been very well determined and also the methods that have been worked out much time can be saved.

The summary of the data included in this report are:

1. The temperature scale and fixed points, including methods for establishing and maintaining the high temperature scale, and recommended methods for calibrating and intercomparing technical pyrometers.
2. Brightness of black body at various temperatures and related data.
3. Spectral energies of the black body at various temperatures.
4. The constants of the black-body laws.
5. The emissive powers of various metals.

TEMPERATURE SCALE AND FIXED POINTS.

One of the most important single items in pyrometry and radiation is the temperature scale. For this reason the first table of data is a summary of the more important fixed points. The temperature scale is defined in terms of the fundamental interval, *i.e.,* the interval of temperature from melting ice to boiling water under standard conditions. On the standard scale the interval is divided up into 100 degrees. The magnitude of the degree will in general depend upon the method used in measuring the temperature in this interval. It has therefore been decided to define the temperature in terms of a constant-volume hydrogen thermometer under specified conditions.

The standard scale that is aimed at for all temperatures is Kelvin's thermodynamic scale which does not depend upon the properties of any substance. This is the temperature scale that would be obtained with a gas thermometer using a perfect gas. The temperature scale obtained by a real gas can be corrected to the thermodynamic scale if the departure of the gas from a perfect gas is known. A very good discussion of this is given by Buckingham in the Bulletin of the Bureau of Standards, vol. 3, page 237. In his paper it is shown that the value that best fits all the experimental data for the ice point on the thermodynamic scale is 273.13° K. The corrections to the different gas thermometers are also given. These corrections have been worked out for temperatures up to 1000° C., and by extrapolation they have been extended to 2000° K., and are given in Table I. These corrections were calculated for an initial pressure

TABLE I.

Corrections to Constant-Volume and Constant-Pressure Scale for Nitrogen (All Corrections to be Added Algebraically. Pressure = 1000 mm. Hg.).

Temperature in degrees K.	Constant volume	Constant pressure
173°	−0.06	+0.34
273	0	0.0
600	+0.10	.30
800	.24	.67
1000	.42	1.08
1200	.64	1.54
1400	.86	1.98
1600*	1.1	2.5
1800*	1.4	3.0
2000*	1.7	3.5

* Extrapolated values.

on the gas of 1000 mm. of mercury. It was shown by Buckingham that "the thermodynamic correction of the centigrade constant-pressure scale at the given temperature is very nearly proportional to the constant pressure at which the gas is kept" and "that the thermodynamic correction of the centigrade constant-volume scale is approximately proportional to the initial pressure at the ice point."

These two propositions are very useful, since after finding the corrections for any pressure for a gas thermometer of either type, it is possible to calculate very approximately the corrections for the same gas at any other pressure.

The region of the gas thermometer is limited by the container for the gas. Day and Sosman[1] carried the gas thermometer up to the melting-point of palladium several years ago and to date there has been indication of but a very small error at this high point, *i.e.*, an error of about twice what they set as their limit of error. However, for most work, the region of the gas thermometer would be considered to end at about 1000° C. (or 1273° K.).[2] Above this, temperature is measured by means of one of the radiation laws. In whatever manner the temperature is measured in practice, it is the thermodynamic scale that is aimed at.

In order to secure a uniform temperature scale there has been a proposed international agreement by representatives of the various standardizing laboratories in which standard values and methods have been agreed upon. According to this proposed international agreement the temperature scale has been divided up into intervals according to the method used in measuring the temperature in the different intervals. The first interval is from −40° C. to 450° C. and the temperature is to be measured in this interval by means of a platinum resistance thermometer calibrated at the temperature of melting ice (0° C.), the temperature of saturated steam (100° C.) and the temperature of sulphur vapor (444.6° C.), all under standard atmospheric pressure. Points on the temperature scale between the fixed points are found by interpolation, using the relation between the resistance and temperature expressed by one of the Callender formulæ.

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[1] *Am. Jour. Sci.*, xxix, p. 93, 1910.

[2] In many laboratories temperatures are all given on the absolute centigrade scale and marked °K (Kelvin), that is they are given as degrees centigrade +273.1°.

$$(1)\ Pt=\frac{Rt-R_0}{R_{100}-R_0}100, \qquad (2)\ t-Pt=\delta\left(\frac{t}{100}-1\right)\frac{t}{100},$$

where t is the temperature, R the resistance, and δ a constant, Pt the platinum temperature.

Other fixed points in this interval that have been found useful in calibration are given in the first part of Table II.

The second interval agreed upon extends from 450° C. to 1100° C. The temperature in this interval is generally defined as measured by a standard platinum platinum-rhodium thermocouple that has been calibrated at certain standard fixed points. The thermocouple is generally calibrated at the freezing points of zinc (or cadmium), antimony and copper. The relation between E.M.F. of the thermocouple and temperature of the fixed points that is then used to calculate intermediate points is expressed by the following equation:

$$(3) \qquad e=a+bt+ct^2$$

the constants being obtained from the calibrations at the three fixed points. This connects the scales thus defined for the two intervals so that the temperature scale is continuous and still there is no uncertainty due to the use of different methods to measure the standard temperature in the same region. Other standard fixed points that are useful in calibration are given in the second part of Table II. The values given in Table II are supposed to be on the thermodynamic scale and are adopted as the values that best represent all the experimental data. In calibrating a platinum platinum-rhodium thermocouple after finding the E.M.F. of the fixed points, there is some difficulty in the interpolation for different points between two fixed points. Sosman[3] has outlined a very good method for doing this. The E.M.F. of a standard thermocouple at the fixed points has been experimentally determined. Now, if the differences between these standard values and the values found for the couple being calibrated are obtained and these differences plotted against the temperatures of the fixed points, a curve will be obtained from which the E.M.F. of any temperature can be obtained, or from which the temperature for any E.M.F. can be readily found.

No attempt has been made as yet to fix upon an international standard scale above 1100° C. (or 1373° K.). However, most experimenters that work in this region base their results upon certain of the radiation laws with particular values of the constants of these laws and certain fixed points. The radiation laws all apply to the black body and thus the temperature of a non-black body cannot be directly determined from the measurements of the energy radiated. When it is necessary to measure the temperature of non-black bodies from the intensity of the energy radiated, correction must be made for the emissive power. This correction for the emissive power, together with an explanation of this term, is more fully discussed below.

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[3] Sosman, *Am. Jour. of Sci.*, xxx, 1, 1910.

The melting point of gold, which is now quite generally accepted as 1336° K., is a very convenient standard fixed point for the calibration of pyrometers that are based upon the radiation laws. Another satisfactory point for the calibration of pyrometers is the palladium point. The value of this point is somewhat in doubt. Day and Sosman, in their extended work with the nitrogen thermometer, found 1823° K. as the value of this melting point. According to some recent work[4] this value seems too low if the value 14,350 is used for the constant c_2 of Wien's equation. It seems, therefore, that it should be raised about 5° C. or 1828° K.

The temperature scale in the region above 1336° K. is generally determined by means of an optical pyrometer with a calibration that is based upon Wien's equation.

$$J_\lambda = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \tag{4}$$

By comparing the brightnesses of a black body at two temperatures and applying this equation, the following formula is obtained:

$$\log R = \frac{c_2 \log e}{\lambda}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{5}$$

where R is the ratio of the brightnesses, λ the wave-length used, T_1 and T_2 the two temperatures, and $c_2 = 14{,}350\mu$ deg. ($\mu = 0.001$ mm.). Thus, if R is measured and one temperature known, the other can be calculated.

Some of the better-known temperatures in this region are given in the third part of Table II.

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[4] *Astrophys. Jour.*, li, 244, 1920. Hoffmann und Meissner, *Ann. d. Phys.* **60**, 201, 1919.

ESTABLISHING THE TEMPERATURE SCALE.

The temperature of the fixed points below 1100° C. are now pretty well known. As was pointed out above, certain values have been decided upon by agreement for the standard fixed points below 1100° C. These, of course, represent the best experimental data available and are probably as accurate as can be obtained without a very great amount of additional work. Until better are available therefore, the values given in this part of Table II should be used. The values in the region above this represent the best available data and are now very well known. After the temperatures of the fixed points have been determined by one of the standard methods, *i.e.*, in terms of the gas thermometer, corrected to the thermodynamic scale, in the gas thermometer region, or by the optical pyrometer or total radiation pyrometer in the region above this, there comes the difficulty of maintaining this scale both from the standpoint of the standardizing laboratory and the man using the temperature scale in the practical field. In the region of the platinum-resistance thermometer and the thermocouple, standards of either can be obtained from the standardizing laboratories and these used for checking up the regular instruments. It is not very difficult to actually check up a resistance thermometer at any one of the standard points in the region –40° C. to 450° C. It is a little more difficult to check

TABLE II.

Standard Fixed Points.

PART 1.—Interval—40° C. to 450° C.	Degrees.
Freezing point of mercury	–38.88° C.
Freezing point of tin	231.84
Freezing point of cadmium	320.9
Freezing point of zinc	419.4
Boiling point of naphthalene	217.96+0.058 (p—760)
Boiling point of benzophenone	305.9 +0.063 (p—760)
Boiling point of sulphur	444.6
PART 2.—Interval 450° C. to 1100° C.	
Freezing point of antimony	630. ° C.
Freezing point of aluminum	658.7
Freezing point of silver	960.5
Freezing point of sodium chloride	801.
Freezing point of gold	1063.
Freezing point of copper, free from oxide	1083.
PART 3.—Interval above 1100° C. (1373° K.)	
Melting point of palladium	1828° K.
Melting point of platinum	2034
Melting point of molybdenum	2810
Melting point of tungsten	3675

up the thermocouple in the region 450° C. to 1100° C. Most of the standard fixed points in this region are given by melting points of metals that must be melted in such a way as to avoid oxidization. This requires a neutral atmosphere, or that the sample be covered with some flux that will protect it. In the region above 1100° C. (*i.e.*, 1373° K.) there are two difficulties. The first is the necessity of reaching the high temperature in such a way that it can be controlled and measured and in addition there is the second difficulty of getting and maintaining the black-body conditions. This makes it very difficult for the man in the works to make a check on his pyrometer by a direct comparison with the black body at the temperature of the standard fixed points. It is necessary, therefore, to have a secondary method that is convenient and accurate enough for practical purposes. Such a method is described below.

MAINTAINING THE HIGH-TEMPERATURE SCALE.

As pointed out above, the high-temperature scale above 1300° K. is based upon the brightness of a black body at the temperature of the melting point of some one or more selected metals. Owing to their convenience, both as to their freedom from oxidization and as to the temperature of their melting points, gold and palladium are generally chosen. Both of these metals can be successfully melted in a platinum-wound black-body furnace. This is a very great convenience because the whole operation can be carried out in the open air, thus requiring no special precautions in maintaining either a vacuum furnace or a neutral atmosphere within the furnace. The black body at a particular temperature is the standard, whether the scale is determined by means of a total radiation or by means of an optical pyrometer. Most experimenters determine their temperatures in this region by means of an optical pyrometer.

For calibration at the palladium point the thermocouple of platinum, platinum-rhodium is not very reliable in that it will not remain constant, as it is used inside the platinum-wound furnace. This necessitates that each time a calibration is made the point be found by a melting-point determination. Any one who has used a black body for such purposes will know that this requires a great amount of work. Thus, while in the standardizing laboratory the scale should be maintained and checked by actual measurements from the black body at the standard melting point, some other method is necessary in practice. Much time can be saved if a tungsten lamp with a filament of suitable size is standardized so as to have the same brightness for a particular part of the filament, when observed with the optical pyrometer, as the standard black-body furnace for one or more definite temperatures. Such lamps with single-loop wire filaments have been made, calibrated and used in this laboratory for this purpose for several years. The bend in the wire is made somewhat sharp and the centre of this loop used as a point of reference. This method has recently been employed by a number of research laboratories in this country using lamps calibrated at several currents for temperatures between 1200° K. and 2000° K.

For the highest accuracy the tungsten lamp that is to be used for calibration purposes should be standardized with an optical pyrometer, using a monochromatic screen .(usually red glass), which is the same as that on the pyrometer to be compared, or corrections should be made for the difference. If the effective wave-lengths are known, this correction can easily be made.[5] For practical purposes, however, if similar red glasses are used the error will be quite small.

Tungsten filaments have been found to depart very markedly from Lambert's cosine law in their radiation.[6] To avoid error due to this cause, care must always be taken in determining the temperature of circular filaments to measure the brightness of the central part of the filament. For this reason, the pyrometer filament should always be parallel to the background filament. This, of course, requires that the pyrometer filament be much smaller than the image of the background filament used as a standard.

A ribbon tungsten filament with some mark, or a pointer to indicate the exact spot sighted upon, is sometimes used as a standard background against which to calibrate a pyrometer filament. The ribbon filament does not require as much magnification as the wire filament. A commercial form of the disappearing-filament pyrometer which has a small magnification can be calibrated, using such a ribbon filament, providing that either the filament is about 3 mm. wide or an extra lens is used between the filament and the pyrometer, so as to give a large image of the filament, this extra lens being of such size that it will not limit the cone of rays that enters the objective lens of the pyrometer. If a lens is used corrections must be made for its transmission, or the background lamp must be calibrated by means of the standard pyrometer with the lens before the lamp. The transmission of a lens is measured in much the same manner as is the transmission of a piece of absorbing or other glass. The method generally employed is to form an image with the lens whose transmission is being measured and then, taking care to see that the lens is not limiting the beam that enters the objective lens, to measure the brightness of this image and compare this brightness with that of the standard source as observed directly.

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[5] *Gen. Elec. Rev.*, 20, p. 749, 1917.

[6] Worthing, *Astrophys. Jour.*, 36, p. 345, 1912.

A ribbon-filament lamp and lens combination may be used to calibrate a Wanner-type of optical pyrometer. In this case, the image of the filament is to be focused on the slit of the pyrometer and readings made by obtaining the relation between the scale of the pyrometer and the temperature of the ribbon filament corrected of course for the transmission of the lens.

Several industrial plants have recently had such ribbon-filament lamps made and have either used them without or with an extra lens. Such a lamp or lamp and lens combination can be calibrated by a standard laboratory or they can be calibrated with the calibrated optical pyrometer before it has been used, and in this way any change in the calibration of the optical pyrometer can be detected.

With such lamps properly aged and properly calibrated, any one may maintain his own temperature scale for years, if the calibration does not extend higher than that of the palladium point and the standard lamp is not accidentally heated to a higher temperature.

An intercomparison of the temperature scales of the Bureau of Standards, the Research Laboratory at Schenectady, the Physical Laboratory of the University of Wisconsin, and Nela Research Laboratory was made by means of measurements made on a number of tungsten lamps. The temperatures of several tungsten lamps were very carefully measured in this Laboratory for different currents and sent to three other laboratories where the temperatures were measured for the same currents. The lamps were then returned to this Laboratory for a second check on the temperatures. This gave an intercomparison of the temperature scales that are in use in the different laboratories.

In each of the laboratories the temperatures were measured by means of a disappearing-filament optical pyrometer, using red glass as the monochromatic screen. As the different laboratories used red glasses having slightly different effective wave-lengths, a small correction was necessary to reduce the temperatures to the same wave-length. Such a correction would have been unnecessary if the sources whose temperatures were measured had been black bodies. A summary of the results, using the data on but three of the six lamps, is given in Table III. The data obtained on the other three lamps are about as those presented in the table, as can be seen by a reference to the original paper.

TABLE III.

Results of Intercomparison of Temperature Scales.

$c_2 = 14350 \mu$ deg. $\lambda = 0.665 \mu$ Melting point of Au = 1336° K. (Pd = 1828° K.)

Lamp	Nela Research Laboratory (10-17-16)	Research Laboratory of General Electric Company (Schenectady)	Nela Research Laboratory (11-28-16)	Bureau of Standards	Nela Research Laboratory (4-2-17)
T—16B	....	....	1429	1431	1427
T—30C	1813° K.	1813	1813	1814	1813
T—30C	....	2307	2304	2302	2303
T—30C	2756	2752	2752	2762	2752

	Nela Research Laboratory (3-27-16)	Physical Laboratory University of Wisconsin		Nela Research Laboratory (7-14-17)
		C. E. M.	G. R. G.	
T—17—C	1810	1813	1816	1810
	2193	2197	2202	2196
	2490	2506	2516	2497

Of the lamps used in the intercomparison all, except one, had flat filaments about 3 cm. long, and about 1½ mm. wide. The exact point at which it was desired to have the temperature measured was indicated either by a pointer, a notch in the supporting lead, or a small notch in the filament itself. Three of the lamps were gas-filled, and the other two were of the vacuum type. The other lamp, a gas-filled lamp, had a 20-mil. (0.5 mm.) filament in the shape of a hairpin loop. As the loop was rather sharp, the exact point at which it was desired to have the temperature measured was easily indicated. This Laboratory is planning to send similar lamps to several other laboratories for comparisons in the near future.

An inspection of Table III will show that the results by the Bureau of Standards, Research Laboratory of the General Electric Co., Schenectady, and Nela Research Laboratory, agree very well for the entire range studied. The results obtained in the Physical Laboratory of the University of Wisconsin do not agree quite so well, but even here the differences are quite small.

The above results show two things: first, that the temperature scales in use in the four laboratories that made the intercomparison are in very good agreement, and second, that the temperature can be satisfactorily measured from such lamps.

Six years ago a tungsten lamp with a single hairpin loop (ten-mil. filament) was standardized in this Laboratory so as to have the same brightness as observed with the pyrometer as the black body at the temperature of melting palladium, and has been used ever since as one of two standards of temperature. This lamp has also been used as a brightness standard against which to calibrate pyrometer filaments for brightness. It has not been operated at a higher temperature than that corresponding to the brightness of a black body at the palladium point. When it was first calibrated, six years ago, the current required for this brightness was 4.087 amperes. A recent recalibration of this lamp indicated that the current should be 4.095. The other standard has been in use for about four years and the recent calibration showed no change in its current for this brightness. The change in current noted in the first lamp corresponds to a little over one degree centigrade. Thus, if the temperature scale had been maintained for the six years by this one lamp alone, the error would amount to about 1° C. This for most work would be negligible.

PRECAUTIONS IN USING OPTICAL PYROMETERS.

The Disappearing-filament Type.—In working with an optical pyrometer of the disappearing-filament type, experience has shown that several precautions are necessary in order to avoid errors. If an optical pyrometer of this type is constructed without a limiting diaphragm between the objective lens and the pyrometer lamp, an error will be made if the position of the objective lens is changed, even though there is a limiting diaphragm between the pyrometer lamp and the eyepiece. In other words, the current required through the pyrometer filament for an apparent brightness match with a particular source is a function of the angle that the cone of rays from the objective lens makes at the pyrometer filament. This has been found to be due to light from the source being diffracted[7] around the pyrometer filament.

It has often been assumed by different experimenters that the pyrometer filament and the background source were at the same brightness (excepting for small differences due to lens absorption, etc.), when there was an apparent brightness match between them. This is not the case since part of the light that appears to come from the pyrometer filament comes from the source being investigated by being diffracted by the filament. These errors can be avoided by keeping the cone of rays from the objective lens to the pyrometer filament as well as the cone of rays entering the eyepiece, fixed for any particular set of measurements. This can readily be done by having fixed limiting diaphragms between both the objective lens and the pyrometer filament, and between the pyrometer filament and the eyepiece, the latter diaphragm being the smaller. It has been shown[8] that conditions can be so arranged that the pyrometer filament will be as much as 1.6 times as bright as the source being investigated, and still as observed through the eyepiece there will be an apparent brightness match. On the other hand, by increasing the size of the cone of rays that reaches the pyrometer filament to a sufficiently large value it will be found that there will be an apparent brightness match when the pyrometer filament is only about 0.9 as bright as the source. The former condition would probably not be met with in practice since the cone of rays is rather small, being about such as would be subtended by an opening 0.2 mm. in diameter at a distance of 50 cm. In the latter case the cone is about such as would be subtended by an opening 1 cm. in diameter at a distance of 50 cm.

To get a perfect brightness match the different parts of the pyrometer should be so lined up that there is approximately axial symmetry, and the resolving power of the eyepiece used should not be so great as to prevent the practical disappearance of the pyrometer filament against the background image. This image should be large in comparison with the pyrometer filament for reasons given below.

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[7] *Phys. Rev.*, N. S., 4 p. 163, 1914.

[8] *Loc. cit.*

If a rotating sector is used to cut down the apparent intensity of the background, care must be taken as to the location[9] of the sector. There is a very marked difference in the results of temperature measurements, depending upon whether the sector is located near the objective lens or as near as possible to the pyrometer lamp. There is also a difference depending upon the relative positions of the openings in the sector and the source, providing the source is a lamp filament. If a sector of small transmission is mounted near the lens and so placed that the openings of the sector are parallel to the axis of the background filament when the sector is passing across the centre of the lens, the definition will be very bad, while if the openings of the sector are turned through 90° so that they are perpendicular to the axis of the filament, the definition will be quite good, but not so good as if the sector is located near the pyrometer lamp. When the rotating sector is located near the pyrometer lamp, the definition is good and practically independent of the relative position of the opening of the sector. If a very large source is used no such effect is noted. Using a pyrometer calibrated against such a large background and thus independent of the position of the sector to measure the brightness temperature of a small tungsten filament, large variations in temperature were found when different sectors were used near the objective lens. No such differences were found when the sector was located near the pyrometer filament.

In Table IV are given results of a test showing the effect of the position of the sector. A fifteen-mil (0.381 mm.) tungsten lamp, operated at a brightness temperature of about 2275° K., was used as a background, and readings were made on the current through a 2.5-mil tungsten pyrometer filament, for an apparent brightness match with a sector having two one-degree openings. From the table it can be seen that the position of a sector of this size can cause an error of about 14° K. for this condition if care is not taken as to its location.

If in place of a rotating sector an absorbing screen is used to cut down the apparent intensity, care must be used, due to the fact that absorbing screens are not neutral-tint, and thus their transmission depends to some extent upon the temperature of the source being investigated. This change in transmission may

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[9] Hyde, Cady and Forsythe, *Astrophys. Jour.* 42, p. 303, 1915.

TABLE IV.

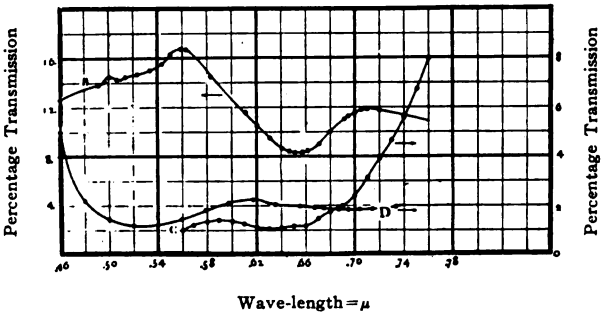
Errors in Temperature Measurements Due to Improper Location of Sector.

	Position of a 2-degree sector			
	Near lens		Near pyrometer lamp	
	Opening of sector parallel to background filament	Opening of sector perpendicular to background filament	Opening of sector parallel to background filament	Opening of sector perpendicular to background filament
Current in amperes through pyrometer filament for brightness match.......	0.3332	0.3354	0.3357	0.3357
Apparent relative brightness....................	.939	.995	1.000	1.000
Temperature of background for these readings.......	2263° K.	2275° K.	2277° K.	2277° K.

be enough to cause a considerable error. The transmission of an absorbing screen also depends upon its own temperature.

Another point that should be noted [10] is that when two absorbing glasses are used together, the transmission of the two used together is not, in general, the product of their separate transmissions. This is due to the fact that the spectral transmission of absorbing screens is not in general independent of the wave-length.

FIG. 1.



Spectral transmission of various absorbing glasses. Curve *B*, Jena absorbing glass 1.5 mm. thick. Curve *C*, Noviweld obtained from Corning Glass Works; shade about 6. Curve *D*, Leeds & Northrup absorbing glass made of purple and green glass.

In Fig. 1 are shown the spectral transmissions of a piece of noviweld and of a piece of a Jena absorbing glass. Either one of these glasses is nearly enough neutral tint for use with the red glasses whose transmission curves are shown by curves B, C, and D in Fig. 3. The noviweld absorbing glass and all the samples of Corning high-transmission red glasses were obtained from Mr. H. P. Gage, of the Corning Glass Works, Corning, New York. This absorbing glass is made in different shades with transmissions, when used in connection with red glass, ranging from less than one per cent. to several per cent.

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[10] Foote, *Bur. Stds. Bull.*, Vol. 12, p. 500, 1916.

If a red glass is used in the eyepiece, the ratio of the brightness of the source as observed through both the red glass and the black glass, to the brightness of the same source as observed through the red glass alone, is called the total transmission for the particular temperature. Without a red glass, using the entire visible spectrum, it is generally very hard to make such measurements owing to the color differences introduced by even the best absorbing glasses, but with a good red glass in the eyepiece, such transmission measurements can be made easily.

The total transmission of the absorbing glass when used with a red glass can be calculated for any black body distribution by the following formula taken from Preston's Theory of Light:

$$T_B = \frac{\int_0^\infty J(\lambda T) V_\lambda t'_R t'_B d\lambda}{\int_0^\infty J(\lambda T) V_\lambda t'_R d\lambda} \tag{6}$$

where $J_\lambda \, d\lambda$ = black-body energy for interval λ to $\lambda + d\lambda$. V_λ = visibility, t'_R and t'_B = spectral transmission of the red and absorbing glasses, respectively. It is very evident that if the spectral transmission of the absorbing glass is different for different wavelengths, the total transmission will be a function of the temperature of the source under investigation.

In Fig. 2 is shown as a function of the temperature of the source, the total transmission for red light of the absorbing glasses whose spectral transmission is given by curves B and C, Fig. 1. The measured points were determined by the author, and the calculated values were obtained by means of equation (6) by making use of an average visibility curve[11] for this spectral region. Transmission values were also calculated, using the writer's visibility curve. Values thus obtained, using the two different visibility curves, differ from each other by only a small fraction of a per cent.

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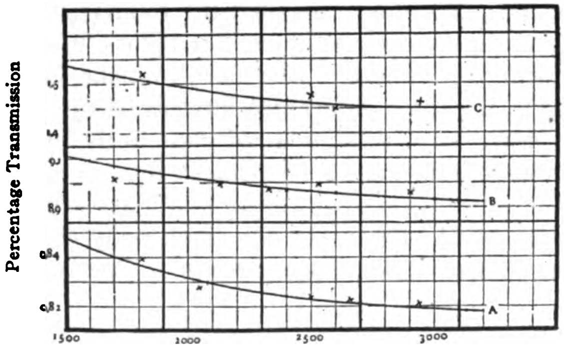
[11] *Astrophys. Jour.*, **48**, p. 87, 1918.

The Effective Wave-length of the Monochromatic Screen.—An optical pyrometer can be calibrated and so used as to make unnecessary a knowledge of the extent to which the screen is monochromatic. To do this requires a black-body furnace that can be operated at various temperatures up to the highest temperature for which the pyrometer is to be used. However, if any attempt is made to use Wien's equation to extend the temperature scale, either above or below that of the standard furnace by the use of rotating sector discs or absorbing glasses, that is, to find the temperature of a black body whose brightness, as seen through the pyrometer, is, say, ten times (assuming a sector or absorbing-glass transmission of one-tenth), that of a black body whose temperature can be measured directly, a knowledge of what wave-length to use is necessary. A knowledge of the wave-length that must be used may also be necessary if the pyrometer is used to measure the temperature of non-black bodies. The wave-length which must be used is called the "effective wave-length."[12] In using the pyrometer, it is the integral luminosities through the red glass that are compared, and for this reason the effective

FIG. 2.



Temperature in degrees *K*

Total transmission of absorbing glasses, as a function of temperature when used with red glass No. 4512—5.8 mm. thick. Curve *A*, two pieces Jena absorbing glass. *B*, one piece Jena absorbing glass. *C*, Noviweld glass from Corning Glass Works. Curves drawn through points calculated from equation $T_B = \dfrac{\int_0^\infty J_\lambda V_\lambda T_R T'_B \, d\lambda.}{\int_0^\infty J_\lambda V_\lambda T_R \, d\lambda}$ Crosses represent values of transmission obtained with optical pyrometer.

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[12] *Astrophys. Jour.*, **42**, p. 294, 1915.

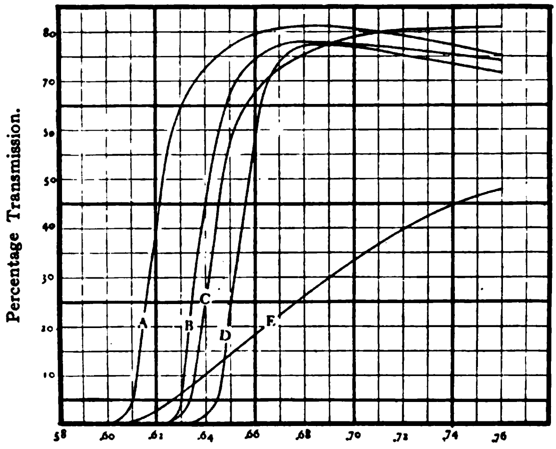
wave-length of the red glass screen for a certain temperature interval has been defined as the wave-length such that, for the definite temperature interval for a black body, the ratio of the radiation intensities for this wave-length shall equal the ratio of the integral luminosities through the screen used.

Knowing the spectral transmission of the red glass, it is possible to calculate the effective wave-length λ_e for any temperature interval by means of the following equation:

$$\left[\frac{J(\lambda T_1)}{J(\lambda T_2)}\right]_{\lambda e} = \frac{\int_0^{\infty} J(\lambda T_1) V_\lambda t'_R d\lambda}{\int_0^{\infty} J(\lambda T_2) V_\lambda t'_R d\lambda} \tag{7}$$

These integrals can be computed by the step-by-step method with

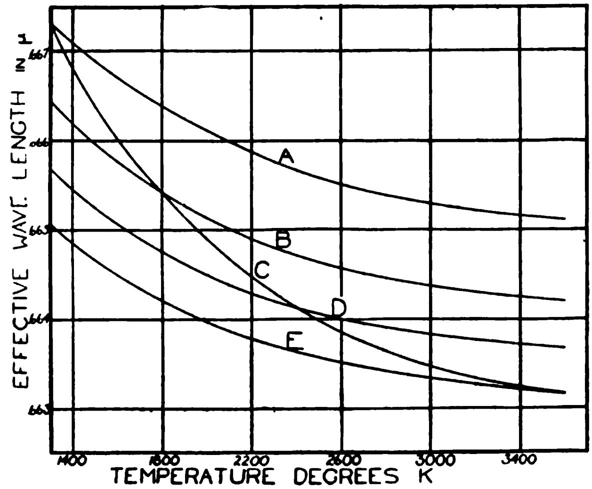
FIG. 3.



Spectral transmission of various red glasses. Curve *C* for Jena red No. 4512, 2.93 mm. thick. Curve *E* for Jena red No. 2745, 3.2 mm. thick. Curve *A* for Corning high transmission red, marked 150 per cent., 5 mm. thick. Curve *B* for Corning high transmission red, marked 50 per cent., 5 mm. thick. Curve *D* for Corning high transmission red, marked 28 per cent., 6 mm. thick.

sufficient accuracy for this purpose. Using equation (7) the effective wave-length was calculated for the red glass whose spectral transmission is shown by curve C, Fig. 3, for a number of temperature intervals and plotted as shown in Fig. 4. By connecting the points where the curve for the effective wave-length from any particular temperature crosses the same temperature ordinate, a curve is obtained (curve C, Fig. 4) that gives the limiting effective wave-length for any particular temperature. To show how these curves may be used, the effective wave-length for a couple of temperature intervals will be found. The effective wave-length between 1800° K. and 2900° K. is given by the ordinate of the point where the 1800° K. curve crosses the 2900° K. ordinate; that is, it is 0.6644μ. For the range between 2100° K. and 2900° K. the effective wave-length is likewise given by the point where the 2100° K. curve would cross the 2900° K. ordinate. However, in this case the 2100° K. curve is not drawn, but will have to be imagined as drawn parallel to the 1800° K., one point of its position being determined by where the curve crosses the 2100° K. ordinate. The effective wave-length for this interval is 0.6641μ. It can be seen from the figure that the effective wave-length for any temperature interval is given quite closely by the mean of the limiting effective wave-length for the two temperatures.

FIG. 4.



Effective wave-length for Jena red glass No. 4512. Spectral transmission shown in curve *C*, Fig. 3.

Curve *A*, effective wave-length from 1300 °K to other temperatures.
Curve *B*, effective wave-length from 1800 to other temperatures.
Curve *D*, effective wave-length from 2400 to other temperatures.
Curve *E*, effective wave-length from 3600 to other temperatures.
Curve *C*, limiting effective wave-lengths.

In Table V are given the errors that would result due to

TABLE V.

Changes in Temperature of 2400°, and 3000° K. extrapolated from 1800° K. as Initial Temperature, Using Wien's Equation, Due to Various Changes.

$c_2 = 14350\mu$ deg.
$\lambda = 0.665\mu$

Variation leading to error	Percentage change			Actual change		
	1800° K.	2400° K.	3000° K.	1800° K.	2400° K.	3000° K.
One per cent. change in initial temperature........	1%	1.3%	1.7%	18°	32°	50° K.
3° K. change in initial temperature................	..	....	....	3°	5°	8°
Using a λ that is one per cent. in error.................	..	0.3	0.7	....	8°	20°
0.001μ error in λ............	..	0.05	0.10	....	1.2°	3°
If in extrapolating the λ_e of red glass between 1300° and 1800° K. is used. See Fig. 4....................	..	0.1	0.3	....	2.4°	7.5°
Calibrating pyrometer filament against tungsten lamp as background that was standardized with a red glass different from the one used in the pyrometer being calibrated. Suppose λ_e to change from 0.665μ to 0.650μ................	..	....	....	3.5°	....	....
One per cent. error in value used for transmission of sector or absorbing glass...	..	0.11	0.14	....	2.8°	4.2°
One per cent. variation in the current through 2½-mil. pyrometer filament...	0.5	0.7	0.8	9°	16°	25°

various changes. From this table it can be seen what error in temperature would result in an error in the effective wave-length.

The Wanner Pyrometer.—In the Wanner optical pyrometer the beams of light from the comparison source and the source studied are so arranged that, by the use of a polarizing device, the two beams, as viewed through the eyepiece, are polarized in a plane at right angles to each other. By rotating another nicol located in the eyepiece, the two sources can be made to appear the same in brightness. When this form of pyrometer is used there is always the danger of error due to polarized light from the source. If readings are taken with the pyrometer in two planes at right angles to each other, the chance for error due to this cause will be greatly lessened. There must be a very large source in order to fill the entering cone of the pyrometer and the instrument must be pointed directly at the source being studied. It is sometimes difficult to be sure that these two conditions are fulfilled. For good working conditions, the different parts of the instrument must be very carefully lined up. If this is not the case there will be either an overlapping of the two fields, or there will be a dark line between them.

All of the possibilities of errors due to absorbing screens discussed under the disappearing-filament type, also apply to this pyrometer.

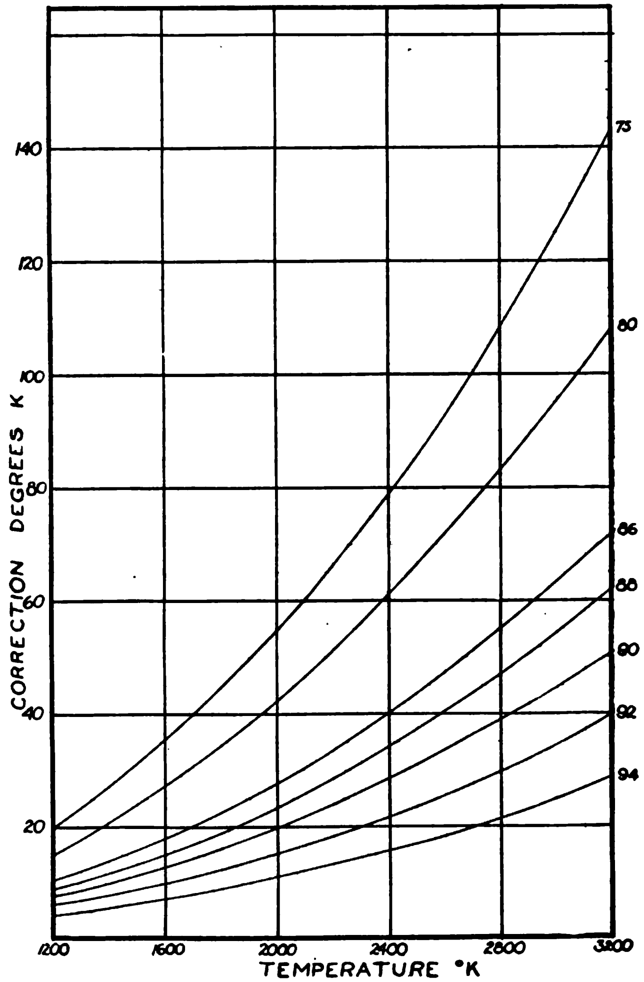
If a red glass is used to obtain monochromatic light, the same care concerning the red glass is necessary, as was pointed out above. If the monochromatic radiation is obtained by the use of a direct-vision prism, the slits must not be too wide (as expressed in wave-length interval transmitted), or the same, or even greater error, will result. The comparison lamp must be kept in position and stray light kept from the different paths.

The Le Chatelier Pyrometer.—In this form of optical pyrometer the comparison source is off at one side and the comparison is made between this and the source being studied by means of a photometer cube or a mirror. This pyrometer is made up in several forms. They all, in general, require a rather large source for observation, and it is also sometimes difficult to be sure that the instrument is pointed directly at the source.

The same care is required concerning the absorbing screen and the monochromatic screen, as was pointed out under the heading of the disappearing-filament pyrometer.

Corrections for the Transmission of Window.—When measuring temperatures with an optical pyrometer it is often necessary to make observations through a glass or other window, or through the lamp bulb. This, of course, makes necessary a correction to the temperature, which correction depends upon the transmission of the glass or other window and upon the temperature measured. This correction can be calculated by the use of Wien's equation and the transmission of the window. In Fig. 5 are shown the corrections for various transmissions. From these curves the correction for any transmission from about 100 per cent. to 75 per cent. and for a temperature range from 1200° K. to 3200° K. can readily be found. These corrections apply to wave-length 0.665μ. If a different effective wave-length is used, corrections can easily be made. From Wien's equation it can be shown that the percentage variation in this correction is equal to the percentage variation in the wave-length used, *i.e.*, if a wave-length one per cent. greater than 0.665μ is used the correction will be one per cent. greater than the one given in the curves.

FIG. 5.



Corrections to be added for various transmissions of windows for different observed temperatures. Each curve shows the corrections for the per cent. transmission marked at the end of the curve in the right margin.

The transmission of a clean lamp bulb for red radiation has been found to be 90 per cent. or slightly more. If the glass is blackened, it may be lowered to 88 per cent. or even lower, depending upon the amount of the blackening. A mica window was measured and the transmission found to be about 75 per cent.

BRIGHTNESS OF A BLACK BODY.

The black body at some particular temperature has often been suggested as the standard of intensity of light. There have been several determinations of this brightness for different temperatures. However, it is very difficult to compare such data for two reasons. In the first place, different temperature scales have been used and there is not always enough data included to determine just what the temperature scale was. The second difficulty is due to the fact that different observers have, in general, different visibility curves and obtain different values for the brightness when there is a color difference between the sources being compared. Since in general nothing is known concerning the visibility characteristics of the different observers, it is almost impossible to correct for this error. The experimental values[13] of brightness given in Table VI are on the temperature scale given above, and in order to avoid possible errors arising from individual idiosyncrasies of vision, the candlepower scale on which the values given in the table were based was that given for

TABLE VI.

Brightness of Black Body at Various Temperatures.

Temperature degrees K.	Brightness in candles per cm.^{2}
1000	0.00251*
1200	.0132*
1400	.231*
1600	2.03*
1800	11.3
2000	44.3
2200	137.6
2400	354.0
2600	785.0
3000	2892.0*
3600	11880.0*
5000	87300.0*

* Calculated values.

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[13] Hyde, Forsythe and Cady, *Phys. Rev.* II, 13, p. 45, 1919.

a tungsten lamp by Middlekauff and Skogland.[14] The procedure of calibration consisted in transferring the photometer and comparison lamp to a standard photometer bench, and substituting a tungsten vacuum lamp for the black body. The comparison lamp was then brought successively to the various voltages used against the black body and readings made against the tungsten lamp. In cases of large color difference blue screens calibrated at the Bureau of Standards were used. The relative candlepowers of the standard tungsten lamp were computed from the data of Middlekauff and Skogland, and hence the candlepowers of the comparison lamp, and so the brightnesses of the black body were obtained on this same scale. The candlepower scale of the Bureau of Standards was established originally by eight observers. Very fortunately the average visibility of these eight is not markedly different from that of the average of 125 observers as determined by Coblentz. At any rate these values are given on a scale of candlepower that is known and does not depend upon observation of a single observer whose visibility curve is unknown. In the table the values of the brightness are given in candles per square centimeter at various temperatures. The values between 1700° K. and 2600° K. are observed values, and the values above and below these limits are calculated from the distribution of energy in a black body as given by Wien's equation, the visibility of radiation, and the mechanical equivalent of light. This required an evaluation of the following definite integral:

$$B_0=\frac{1}{\pi M}\int_0^\infty J(\lambda T)V_\lambda d\lambda \tag{8}$$

where M is the mechanical equivalent of light, B_0 the brightness, and the other terms as defined above. This integral was evaluated by the step-by-step method, using steps where $d\lambda$ was 0.005μ. In some other work it has been shown that with steps of this size the error in relative values was negligible. The value of the mechanical equivalent of light, M, used is 0.00150 watts per lumen. The values of the other constants are given in the summary below. The mechanical equivalent of light used here is the one adopted by the Illuminating Engineering Society. It is the value found from the experimental value of the brightness given

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[14] *Bull. Bur. of Stds.*, 11, p. 483, 1915.

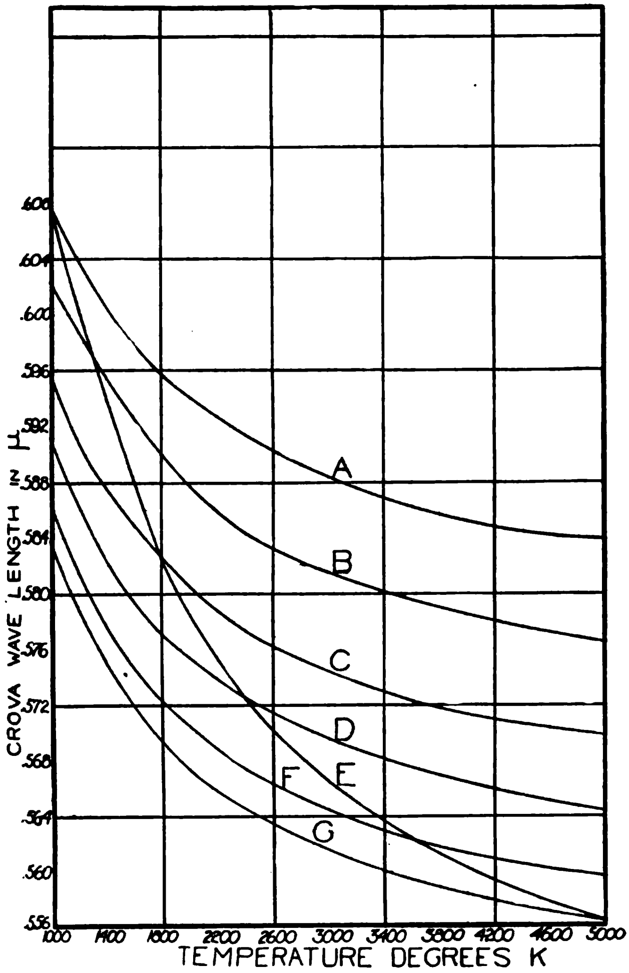
TABLE VII.

Investigators	Recommended values of the mechanical equivalent of light in watts per lumen	Visibility data	Method
P. G. Nutting...	0.00120 ± 0.00005	Nutting	Measurements of total radiant flux and total luminous flux from acetylene. Coblentz energy curve for acetylene used.
H. E. Ives.......	0.00160 ± 0.00003	....	(1) Measurement of total radiant flux and total luminous flux from a monochromatic source.
		Ives – Kingsbury curve from equation Ives – Kingsbury	(2) Measurement of radiant flux through luminosity filter and integral luminous flux direct from the source.
	Secondary value 0.00154	....	(3) Measurement of brightness of a platinum-wedge black body at the melting point of platinum taken as 2037° K.
W. W. Coblentz	0.00162 ± 0.00005	Coblentz and Emerson	Several methods using observations by himself and others. (1) Computation using preliminary data on brightness of a black body by Hyde, Forsythe and Cady. (2) Direct measurement on monochromatic radiation by Coblentz, Ives and Kingsbury. (3) Measurements on incandescent lamps by Coblentz and Emerson.
Hyde, Forsythe and Cady	0.00150 ± 0.00005	Hyde, Forsythe and Cady	Direct measurement of brightness of carbon and platinum tube black bodies.

in Table VI. In Table VII are given the recent determinations of the mechanical equivalent of light, together with a statement of the method used.

Crova Wave-length.—Crova and others have shown that the relative brightness of a black body at different temperatures varied as the brightness of some particular wave-length interval near the centre of the visible spectrum. This wave-length is not constant as was first thought, but varies in such a way that the

FIG. 6.



Crova wave-length.

Curve *A*, Crova wave-length from 1000 to other temperatures.
Curve *B*, Crova wave-length from 1300 to other temperatures.
Curve *C*, Crova wave-length from 1800 to other temperatures.
Curve *D*, Crova wave-length from 2400 to other temperatures.
Curve *F*, Crova wave-length from 3600 to other temperatures.
Curve *G*, Crova wave-length from 5000 to other temperatures.
Curve *E*, limiting Crova wave-length.

Crova wave-length moves towards a shorter wave-length for higher temperatures.

The Crova wave-length was calculated by means of the following formula:

$$\left[\frac{J(\lambda T_1)}{J(\lambda T_2)}\right]_{\lambda_c} = \frac{\int_0^\infty J(\lambda T_1)V_\lambda d_\lambda}{\int_0^\infty J(\lambda T_2)V_\lambda d_\lambda} \tag{9}$$

and the results plotted as shown in Fig. 6. This integral can be calculated by the step-by-step method with sufficient accuracy for this purpose. Curve A is the Crova wave-length for 1000° K. to other temperatures as shown; curve B for 1300° K. to other temperatures as shown, etc.; curve E is the limiting Crova wave-length for any particular temperature.

The Distribution of Energy in the Visible Spectrum of a Black Body.—When making such calculations as those referred to above with reference to the brightness of a black body and the Crova wave-length, use is made of the distribution of energy in the visible spectrum of the black body as calculated from Wien's equation. Such values have been calculated and are given in Table VIII. These values were calculated, taking c_1 as unity, c_2 as 14,350μ deg., and the wave-length in μ. Thus, to reduce the values to ergs per square centimeter per second per μ, they must be multiplied by the value of c_1 (3.72×10^{-12}). The values[15] as they are printed thus are relative intensities and are to be used for relative calculations. If the absolute values are desired, the above corrections must be made.

All the values given in Table VIII, excepting the last column, were calculated by means of Wien's equation. It has been shown that this equation does not fit the observed facts for large values of the product λT. Planck's equation

$$J_\lambda = \frac{c_1\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} \tag{10}$$

has been shown to fit the experimental data quite well as far as experiments have gone. It is quite likely that Planck's equation would better represent the experimental values at very high

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[15] If above values for the spectral distribution are reduced to actual energies care must be taken to introduce the value for $d\lambda$. Wien's equation, as generally used, gives the value of the intensity rather than the actual energy. To reduce to energy the value of $d\lambda$ must be introduced.

TABLE VIII.

Relative Energy of a Black Body at Various Temperatures and Various Wave-lengths.

λ	1000 $\times 10^{-16}$	1200 $\times 10^{-13}$	1400 $\times 10^{-12}$	1600 $\times 10^{-11}$	1800 $\times 10^{-10}$	2000 $\times 10^{-9}$	2200 $\times 10^{-8}$	2400 $\times 10^{-8}$	2600 $\times 10^{-7}$	3000 $\times 10^{-7}$	3600 $\times 10^{-6}$	5000 $\times 10^{-5}$	5000 $\times 10^{-5}$
0.40	26	101	727	1787	2158	1584	809	3148	994	6256	4612	7465	7482
.41	54	186	1200	2728	3101	2168	1064	4005	1230	7403	5056	7872	7878
.42	111	330	1926	4073	4368	2915	1378	5026	1502	8663	5782	8244	8252
.43	214	568	3024	5949	6037	3856	1757	6220	1813	10038	6407	8591	8601
.44	416	954	4642	8519	8204	5022	2212	7608	2163	11520	7051	8911	8923
.45	766	1559	6946	11976	10965	6449	2747	9194	2555	13111	7724	9208	9222
.46	1375	2490	10210	1655	1444	8171	3373	10995	2989	14798	8373	9475	9492
.47	2396	3388	14740	2251	1875	10226	4096	13020	3464	16580	9041	9716	9737
.48	4091	5947	2089	3014	2403	12650	4923	1528	3982	18450	9712	9933	9957
.49	6776	8933	2913	3981	3042	1548	5860	1777	4544	19975	10378	10123	10151
.50	1099	13169	4003	5189	3808	1876	6914	2050	5143	22410	11039	10286	10319
.51	1746	1902	5417	6682	4714	2251	8087	2348	5784	24480	11688	10429	10466
.52	2723	2709	7234	8502	5778	2677	9382	2669	6464	26614	12329	10544	10586
.53	4174	3800	9545	10706	7016	3158	10810	3015	7180	28780	12954	10639	10686
.54	6267	5255	12430	13340	8441	3695	12370	3383	7930	30980	13563	10712	10764
.55	9273	7171	16020	16460	10072	4292	14050	3775	8712	33205	14148	10762	10820
.56	13490	9666	2033	20120	11925	4952	1587	4190	9524	35444	14715	10797	10862
.57	1935	12870	2577	2439	14006	5674	1782	4624	10364	37675	15260	10813	10882
.58	2741	16940	32214	2934	16347	6464	1990	5079	11234	39920	15782	10811	10887
.59	3826	2205	3990	3499	18943	7318	2211	5553	12112	42058	16280	10793	10877
.60	5275	2842	4901	4146	21820	8240	2443	6046	13012	44356	16752	10763	10853
.61	7184	3627	5967	4876	2498	9230	2688	6553	13936	46542	17197	10716	10815
.62	9684	4588	7215	5699	2843	10285	2945	7077	14868	48702	17618	10660	10767
.63	12915	5753	8660	6618	3218	11410	3212	7612	15808	50804	18010	10590	10702
.64	17041	7153	10321	7640	3625	12600	3491	8162	16742	5288	18332	10510	10632
.65	22261	8824	12213	8772	4064	13850	3779	8720	17691	5490	18713	10420	10547
.66	28820	10802	14375	10016	4533	15170	4076	9285	18642	5699	19024	10323	10457
.67	36979	13129	16814	11382	5036	16550	4382	9861	19590	5877	19313	10218	10362
.68	47060	15859	19555	12870	5571	1799	4696	10441	2054	6064	19573	10101	10252
.69	59398	19008	22617	14480	6138	1949	5016	11029	2148	6240	19809	9985	10141
.70	74405	22667	26019	16230	6736	2104	5342	11614	2240	6409	20022	9861	10024
.71	92512	26860	29792	18100	7367	2265	5674	12198	2321	6574	20207	9732	9903
.72	11419	31650	33951	20120	8030	2430	6012	12786	2422	6732	20368	9600	9776
.73	14005	37068	38513	22270	8720	2600	6352	13372	2512	6882	20511	9462	9652
.74	17063	43217	43486	24560	9443	2773	6696	13957	2598	7023	20627	9323	9514
.75	20663	50130	48910	27000	10195	2951	7042	14535	2684	7160	20725	9179	9377
.76	24874	57856	54809	29570	10971	3133	7391	15110	2767	7287	20828	9035	9239

All of the values in this table were calculated by means of Wien's equation with the exception of the last column, headed "5000×10^{-5}" and that is calculated by means of Planck's equation.

temperature if such observations could be made. In Table IX are given the correction factors to be used to convert the values obtained by calculation, using Wien's equation to values that would be given by Planck's equation for different values of the product λT.

TABLE IX.

Ratio Between Planck's and Wien's Equations for Different Values of λT.

λT	E_P/E_W
2000	1.0008
2500	1.003
3000	1.008
4000	1.028
5000	1.056

In the last column of Table VIII are given the values for the relative intensity for 5000° K. corrected to Planck's equation by means of these factors. Using Planck's instead of Wien's equation in calculating the brightness of the black body gives an increase in the calculated brightness of about 0.7 per cent. for a temperature of 5000° K., and an increase of less than 0.1 per cent. for the temperature of 3600° K. The error would of course be negligible for lower temperatures.

RADIATION CONSTANTS.

When referring to the different radiation laws mention was made of different constants. There has been a great deal of work done on these radiation constants. A very good summary of all this work is given by Coblentz.[16] Taking account of all the work and the accuracy of the different determinations, the best value for c_2 of the Wien-Planck equation is 14,350μ deg. Some later work seems to indicate that this value is slightly too high. However, until more data are available it does not seem necessary to make a change in the value. The value of the constant δ of the Stefan-Boltzmann total radiation laws which best fits all the experimental data is

$$5.7 \times 10^{-12} \text{ watts cm.}^2 \text{ deg.}^{-4}$$

Using these values for c_2 and δ the value of c_1, the Wien-Planck equation is found to be

$$3.72 \times 10^{-12} \text{ watts cm.}^2$$

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[16] *Bull. of Bur. of Stds.*, xiii, p. 459, 1916.

EMISSIVE POWERS OF VARIOUS RADIATORS.

All solid bodies when heated send out radiation and the intensity of this radiation has been found to depend upon the substance used and upon the temperature. It also depends upon the condition of the surface, that is, whether the surface is rough or smooth. The words rough and smooth are used in a relative sense, for a surface that is rough for one wave-length interval may well be rated as smooth for another. Since the intensity of the radiation in the various wave-lengths depends upon the temperature and upon the surface studied, the actual temperature can be calculated if the relation between the intensity of the radiation and temperature is known for this surface. As mentioned above, this relation between temperature and intensity of radiation for the black body is represented by Planck's equation. For values of the product λT below 3000 this equation may be reduced to Wien's form which is much easier to use in calculation. All other bodies that owe their brightness to temperature alone when at a particular temperature send out less radiation than does a black body at the same temperature. Thus, equation 4, if applied to other than a black body, would be written

$$J_\lambda = \epsilon c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \tag{11}$$

where ϵ (less than unity) is the spectral emissive power of the substance under consideration. ϵ will depend upon the wave-length, the conditions of the surface and upon the temperature. In Table X are given the emissive powers of some of the metals used as standards.

From the definition of a black body and Kirchhoff's law it follows that all other bodies that owe their brightness to thermal causes alone are less bright when at a particular temperature than a black body at the same temperature. Thus, if the temperature of any hot non-black body is calculated from a measurement of its brightness, as though it were a black body, values that are lower than the true temperature will be obtained. The temperature obtained, however, is the temperature that a black body must have in order to have the same brightness for the particular wave-length interval as the body being investigated. As the brightness thus measured corresponds to some particular wave-length interval the temperature obtained corresponds to that particular wave-length interval. The difference between the true

TABLE X.

Emissive Powers of Various Metals.

Metal	Temperature in °K.	Emissive power
Silver	1200	0.044
Gold	1300	0.14
Copper	1200	0.100
	1300	0.114
Platinum	1000	0.36
	1400	0.32
	1800	0.29
Palladium	1000	0.36
	1400	0.33
	1500	0.31
Iridium	2000	0.30
Nickel	1400	0.36
	1700	0.36
Iron	1300	0.379
	1500	0.375
	1800	0.360
Molybdenum [1]	1300	0.56
	2600	0.62
Tungsten [2]	1200	0.458
	1600	0.449
	2000	0.439
	2400	0 429
	2800	0.419
	3400	0.405
	3675	0.398

The values for the different emissive powers given in the above table are taken from a table by Burgess and Waltenberg (*Bull. Bur. of Stds.*, 11, p. 591, 1915) excepting as noted.

(1) Mendenhall and Forsythe, *Astrophys. Jour.*, 38, p. 380, 1913.

(2) Worthing, *Phys. Rev.*, N. S., X, p. 377, 1917.

temperature and the temperature thus obtained varies from a few degrees for such a substance as untreated carbon to more than 200° C. for such a metal as polished platinum at its melting point.

This temperature of non-black bodies has been called the black-body temperature, the apparent temperature and the brightness temperature. Arguments might be made in favor of any of these names for this temperature. It is the apparent temperature or the black-body temperature in that it is the temperature that is obtained directly from brightness measurements by assuming that the body behaves like a black body. However, the color temperature, as will be defined below, is also an apparent temperature, or a black-body temperature, although based upon entirely different principles. In this Laboratory, because it is the brightness that is measured, this temperature is called the brightness temperature and is referred to a particular wave-length. There is, to be sure, another temperature that might be called the brightness temperature; that is, a temperature based upon the intensity or brightness of the total light radiated. However, both of these temperatures are based on the same principle. In using any one of these names, one must, if accuracy is desired, specify the wave-length to which the temperature belongs, as any non-black body has as many different brightness temperatures as there are wave-lengths to be considered. This brightness temperature, of course, varies for different wave-lengths. If a red glass is used as a monochromatic screen, the wave-length used varies with the temperature. It has been shown that the brightness temperature should be ascribed to the effective wave-length of the screen used for black-body radiation for the temperature interval from T equal to the brightness temperature to T_c, the color temperature of the source being investigated.

The color temperature of a particular source has been defined as the temperature of a black body, which has the same distribution of energy in the visible spectrum as the source under consideration. It has been found experimentally that most metals when heated radiate in such a manner that they can be color-matched against a black body. These color matches are very easily and accurately made with an ordinary photometer. It is to be noted that when two bodies have the same color temperature, it is not necessary that they shall have the same brightness for any particular wave-length interval.

To calculate the brightness temperature from readings on the brightness of a non-black radiator, equation 2 is used when the true temperature T is replaced with the brightness temperature S. From this equation and equation 3 the following relation can be derived:

$$\frac{1}{T} - \frac{1}{S} = \frac{\lambda_1 \log \epsilon}{c_2 \log e} \tag{12}$$

In Table XI are given values of T – S for different values of S and different values of ε. From the table the correction for any emissive power from $\varepsilon = 0.10$ to 1.00 and from any temperature from 1200° K. to 3600° K. can be found. These corrections have been calculated for $\lambda = 0.665\mu$. If a different wave-length is used corrections can be made for the change in wave-length. The same relation holds here as was mentioned above in connection with the correction for the transmission of a window. It was shown there that the correction varied exactly as the variation

TABLE XI.

Corrections to Add to Brightness Temperature Readings for Different Emissive Powers.

Emissive power	Pyrometer using red light, wave-length, $\lambda = 0.665\mu$ at observed temperatures, degree Kelvin of—															
	1000	1100	1200	1300	1400	1500	1600	1700	1800	2000	2200	2400	2600	2800	3000	3600
0.10	119	146	176	209	246	286	329	377	428	542	675	826	998	1193	1413	2246
.20	81	98	118	140	163	189	217	247	279	351	432	523	626	739	865	1322
.30	59	72	86	102	119	137	157	178	201	251	308	371	441	518	603	905
.40	44	54	64	76	89	102	117	132	149	186	227	272	323	378	438	650
.50	33	40	48	57	66	76	87	98	110	137	167	201	237	277	320	471
.60	24	29	35	41	48	55	63	71	80	99	121	145	170	199	299	336
.70	17	20	24	29	33	38	43	49	55	68	83	99	117	136	157	228
.80	10	13	15	18	21	24	27	30	34	42	51	61	72	84	96	139
.90	5	6	7	8	10	11	13	14	16	20	24	28	33	39	45	65
1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

in the wave-length. Thus a one per cent. variation in the wave-length causes a one per cent. variation in the correction. The correction for the transmission of the window through which observations are taken is calculated in exactly the same manner as the correction for the emissive power. From this it is evident that the correction given in the curves in Fig. 5 will give the correction for an emissive power equal to the transmission given in that set of curves.

The Report of the Pyrometry Committee for this year has been made in the form of a summary of existing methods and data. It is planned for next year that the report shall contain current data. It is probable, also, that an attempt will be made to agree upon some definitions and nomenclature.

NELA RESEARCH LABORATORY,
    CLEVELAND, OHIO.
        July, 1920.

A COÖPERATIVE COLLEGE COURSE IN ILLUMINATING ENGINEERING.

BY

F. E. CADY.

THE proximity of an industrial concern, engaged in the manufacture of electric incandescent lamps and including engineering and physical research departments, to an institution furnishing collegiate technical training, has made possible in Cleveland a coöperative arrangement whereby the institution, with the aid of the industry, has been able to offer a course in illuminating engineering which, since its initiation in 1916, has proved of considerable interest and value.

Shortly after the formation of what are now known as Nela Research Laboratories, Dr. Charles S. Howe, President of Case School of Applied Science, took up with Dr. Edward P. Hyde, the Director of the Laboratories, the question of assisting in outlining and establishing, as a part of the curriculum of the electrical engineering department of the school, a course in illuminating engineering. It was not until the fall of 1916, however, that a comprehensive plan was finally evolved and put into execution, whereby the National Lamp Works of General Electric Company agreed to assist the college with the service of its men and apparatus. Under this arrangement Dr. Hyde laid out the subjects for the lectures and in coöperation with Professor H. B. Dates, head of the electrical engineering department of Case School, has since supervised their presentation, together with accompanying laboratory work.

Three lectures a week, running through two terms, has permitted a choice of subjects which practically covers the whole field of illuminating engineering, as that term is now understood. The theoretical side is presented in the first semester when the student is given a thorough groundwork in the physics of light production, in photometry and in the fundamental principles of illumination, together with light, shade and color. The second term is devoted to the study of the production and characteristics of light sources and illuminating engineering as practiced. In this semester the first lectures are on the manufacture, characteristics and testing of incandescent lamps, arc lamps, vapor lamps and gas. These are followed by a discussion of glass manufacture and the characteristics and design of globes, shades and reflectors. After showing the relation of illuminating engineering and architecture, specific cases of illumination are presented, starting with general daylight illumination and continuing with the lighting of interiors, such as theatres, museums, churches, schools, libraries, stores, homes, and exteriors, such as streets, parks, etc. A number of lectures are devoted to the subject of projection in all its phases, covering the principles and apparatus, such as searchlights, auto headlights, moving-picture projection, and exterior flood-lighting. Industrial lighting is discussed, together with lighting codes and the complicated subject of rates. It was originally planned to include in the course lectures on the physiology and psychology of vision, but these have not as yet been given. It is expected to provide for them in the near future. A lecture period is assigned to most of the above subjects given in the second term, but some require three or more lectures.

The personnel of the lecturing staff is made up of three men from Nela Research Laboratories, five from the engineering staff of the National Lamp Works, one from the National Carbon Company, one from the Welsbach Company, one from the Cooper-Hewitt Company, one from the Cleveland Illuminating Company, and the architect of the city schools.

When the course was first started each lecturer was requested to prepare a rather full outline of the topics to be discussed and copies of these outlines were placed in the hands of each student. Since then several of the lecturers have written out their material, and arrangements have been made to have all the subjects written in such form that they may ultimately be incorporated in a text-book.

The laboratory work occupies fifteen three-hour periods and is given during the first semester. Twelve experiments have been prepared which are designed to give practice in the use of photometric equipment, familiarity with methods of measurement and practical experience in the use of both instruments and methods. Among the instruments employed are the Lummer-Brodhun contrast photometer, the Ulbricht sphere, different types of flicker photometer, the Sharp-Millar, the Macbeth, and the Beckstein illuminometers as well as such auxilliary apparatus as a mirror rotator and a universal rotator for measuring distribution curves, and a set of rotating sectored disks for use in connection with the measurement of sources of high intensity. Three of the experiments, which include a study of the characteristics of light sources, are used by the students in solving an assigned problem involving the most economical type and efficiency of lamp to use for a certain installation. Another group of experiments is centred around an illumination survey of the actual lighting conditions in certain rooms. Two members of the staff of Nela Research Laboratories alternate in attendance at the laboratory periods to supervise the experiments and give any assistance that may be desired.

NELA RESEARCH LABORATORIES,
June, 1920.

THE ABSORPTION OF HEAT IN GLASS.*

BY

A. Q. TOOL and C. G. EICHLIN.

INTRODUCTION.

At the annual meeting of the Optical Society, held at Baltimore, December, 1918, a report was given concerning certain phenomena to be observed when glass is subjected to temperatures covering that range usually involved in any process of annealing. Since then more detailed results obtained in a study of these phenomena have been reported in a Bureau of Standards Publication.[1] One of the subjects discussed in these reports was the marked increase in the rate of heat absorption, occurring at a temperature slightly above the usual annealing temperature, while the glass is being heated at a fairly uniform rate. The results obtained for the region in which this absorption occurs were in fair agreement with those published by M. So.[2] This heat absorption was of such a nature that it appeared to be the result of some peculiar transformation in the glass. Since this effect is observed slightly above the usual annealing temperature, it has been used in this laboratory as an aid in determining the upper limit for the temperature region in which glass should be annealed.

Since the publication of the Bureau Scientific Paper mentioned above, a further investigation of this subject has been made, and considerable data have been collected. From these it appears that the treatment to which the glass has been previously subjected affects this heat absorption to a considerable extent. For instance, annealing seems to produce a marked increase in the magnitude of the effect. If this be true, it would appear that this effect is intimately involved in the process of annealing, and should, accordingly, be thoroughly investigated. Furthermore, a careful investigation of the whole subject should be made, because it is sometimes stated in publications dealing with glass [3] that such a heat absorption in glass does not exist. Consequently, it appeared to be of interest to report on the results obtained from a recent series of experiments on a larger number of glasses than were investigated in the previous tests.

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* Published by permission of Director, Bureau of Standards.

[1] A. Q. Tool and J. Valasek, *B. S. Sci. Paper* No. 358, Jan., 1920.

[2] M. So, *Proc., Tokyo Math. and Phys. Soc.* (2), 9, p. 426, 1918.

[3] W. Rosenhain, *Glass Manufacture*, p. 2, 1919.

Other interesting changes have been observed in the same region, *e.g.*, the increased heat absorption is accompanied by a marked increase in the expansivity, as has been shown by C. G. Peters[4] and C. H. Cragoe. It is also quite likely that the absorption is coincident with a marked change in the specific heat, indications of which have been observed by W. P. White.[5] Furthermore, the change in the temperature coefficient of the refractive index, such as observed by J. O. Reed,[6] is also probably related to this effect. It is possible, further, that the thermal conductivity, and perhaps other physical properties, suffer rather abrupt changes in this region. So far only the change in the expansion has been definitely related to the heat absorption. Further investigation will be required to show whether the other effects occur in exactly the same temperature region, and whether they are so definitely related to the heat absorption as the change in the expansivity.

METHOD.

The observations made in the present series of experiments were confined entirely to the heat absorption. The object was to observe the absorption effect on a number of glasses, using the same definite procedure throughout. This plan furnished more consistent observations on any given sample of glass, a better means of comparing similar effects in different glasses, and greater likelihood of detecting any changes produced in a particular glass by subjecting it to different treatments.

In determining the region of heat absorption the usual differential thermocouple method was used, *i.e.*, the difference in temperature between the glass and a neutral body was observed while both, packed closely together, were being heated at practically the same uniform rate. In this method any endothermic effect occurring in the glass cools it relatively to the neutral body. That is, if the glass while heating is at a slightly higher temperature than the neutral body, the difference will decrease when this effect occurs, while if it is at a lower temperature, the difference will increase. In case this effect is both of considerable magnitude and occurs in a comparatively narrow temperature range, it is easily detected and the changes in the difference in temperatures may be determined by measuring the differential e.m.f.s between the junctions in the glass and the neutral body.

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[4] C. G. Peters, Meeting Opt. Soc., Baltimore, 1918. C. G. Peters and C. H. Cragoe, JOUR. OPT. SOC., 4, p. 105, May, 1920.

[5] W. P. White, *Am. J. Sci.* (4), 47, p. 44, 1919.

[6] J. O. Reed, *Wied. Ann. d. Phys. u. Chem.*, 65, p. 707, 1898.

For measuring the differential e.m.f.s, a Pt. Rh.–Pt.–Pt. Rh. differential thermocouple was used. Ordinarily these e.m.f.s were not measured, but their variations were observed by noting the deflections of a sensitive galvanometer in series with the differential thermocouple, as diagrammatically illustrated in Fig. 1. At room temperature a temperature difference of 1° C. produced a deflection of approximately 1.5 cms., at 300° C. of 2.1 cms., at 600° C. of 2.3 cms. In most of the experiments this temperature difference, between the glass and the neutral body, seldom exceeded 5° or 6° C. The deflections of the galvanometer were noted every half minute in the critical ranges, while at other points they were observed less frequently. These deflections were then plotted as ordinates against the temperatures of the glass as abscissæ (see Fig. 3), these temperatures being obtained by determining, by means of a potentiometer, the e.m.f.s between the Pt. Rh. wire of the glass junction and an additional Pt. wire leading to the same junction. The temperatures of the glass were observed

Fig. 1.

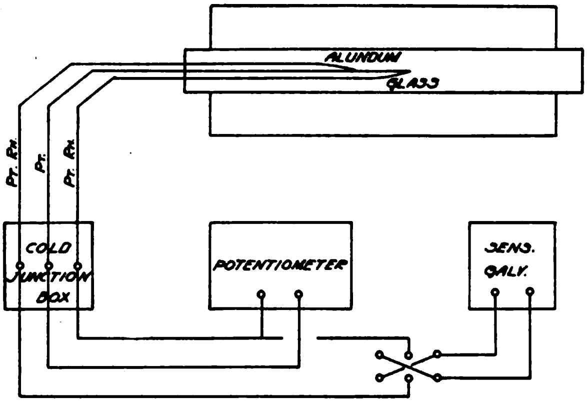
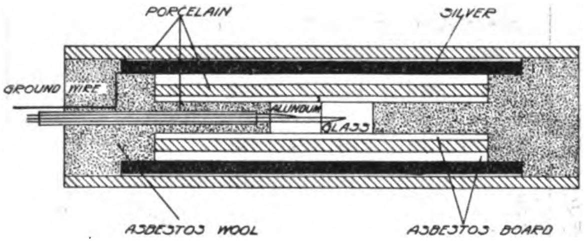


Diagram of apparatus.

at ten-minute intervals, points between being obtained by interpolation. Measurements involving time were subject to a possible maximum error of about five seconds. There was also the usual error involved in the question as to whether the thermocouple took up the actual temperature of the glass at any instant. If such an error was present, it was presumably the same for all cases where the same rate of heating was used.

Throughout this investigation the mode of packing the glass and the neutral body was the same. (See Fig. 2.) Powdered glass was packed closely about one junction of the differential thermocouple and 120-mesh alundum about the other. These, in close contact, were enclosed in a small porcelain tube, lined with burnt asbestos board and plugged with burnt asbestos wool.

FIG. 2.



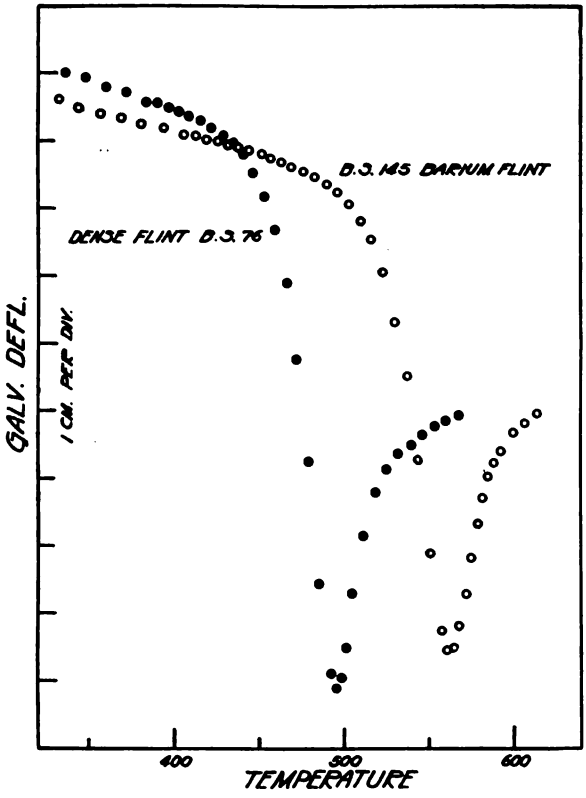
Mode of packing.

This tube was then placed in a heavy asbestos-lined silver tube. This in turn was packed with burnt asbestos in a closely-fitting porcelain tube which fitted loosely in the tube of the electric furnace.

The glass was powdered in order to facilitate its packing and unpacking. Powdering was especially advantageous, since it made it possible to bring the glass and the junction in close contact, and in certain cases to remove the thermocouple without injury. The quantity of glass used averaged about 1.7 gms. The small porcelain tube, in which the glass and the neutral body were packed, was about 2.5 cms. in length and 3.0 cms. in diameter. Burnt asbestos was used for lining and plugs because it was very convenient for packing and unpacking, and because it appeared to be free from any peculiar thermal properties in the region of the temperatures involved in this investigation. Furthermore, in this region no reactions between the asbestos and the glass, or between the alundum and the glass, were noticed. The purpose of the silver tube was to equalize, as nearly as possible, the heating of the glass and the neutral body. This tube had a wall thickness of 0.25 cms., a length of 10.2 cms., and a diameter of 2.3 cms.

The heating rate of the platinum-wound electric tube furnace could be controlled quite easily. In most cases an approximately uniform rate of about 6° C. per minute was employed. The heating current was either direct or alternating, as desired. When direct current was used, it was found necessary, owing to leakage, to ground the silver tube, particularly when temperatures much above 600° C. were used. A battery was used whenever a particularly uniform rate of heating was desirable.

FIG. 3.

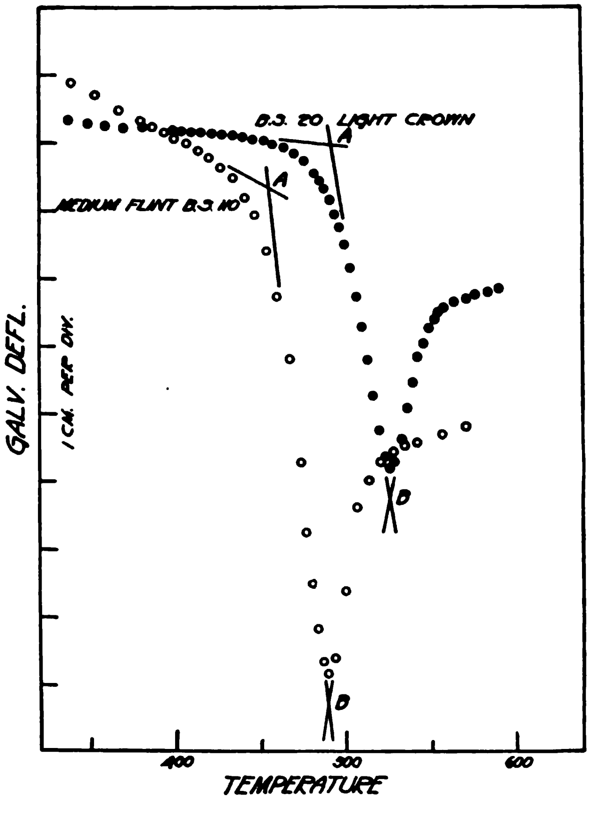


Curves showing heat absorption.

DISCUSSION OF EXPERIMENTAL RESULTS.

The accompanying table contains all the data obtained between March 22 and June 7 of this year on the glasses discussed in this article, giving the temperatures obtained for the beginning (A) and the maximum (B) of the cooling effect, when the method shown in Fig. 4 for the determination of these points is used. The data include redeterminations of these points for a number of the optical glasses which were investigated as reported in the previously mentioned Scientific Paper.[7] Determinations on certain other glasses whose expansivity had been measured by

Fig. 4.



Curves showing heat absorption.

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[7] *Loc. cit.*

Table

Name	Source	Date	Mass of glass in grams	Rate of heating in degrees per minute	Critical range in glass		Remarks
					A ±5° C.	B ±5° C.	
Dense flint	B. S. 76	....	....	....	460*	490*	
Dense flint	B. S. 76	Mch. 31	2.6	6.4	460	494	See Fig. 3.
Medium flint	B. S. 110	....	....	....	455*	485*	
Medium flint	B. S. 110	Mch. 31	2.8	6.6	457	489	See Fig. 4.
Medium flint	B. S. 110	Apr. 29	2.2	5.8	452	492	2nd heating.[1]
Medium flint	B. S. 110	Apr. 29	1.8	5.7	459	490	Ann.
Light flint	B. S. 188	....	....	....	485*	525*	
Light flint	B. S. 188	Apr. 2	1.9	7.0	489	527	
Light flint	B. S. 188	Apr. 28	1.8	6.4	490	527	See Fig. 6—M. T. 790°.
Light flint	B. S. 188	Apr. 28	1.8	6.2	487	527	2nd heating.
Light flint	B. S. 188	Mch. 31	2.2	6.7	504	532	Ann.
Light flint	B. S. 188	Apr. 27	1.8	6.0	504	532	Ann.—See Fig. 6—M. T. 780°.
Light flint	B. S. 188	Apr. 27	1.8	6.2	485	525	Ann.—2nd heating.
Light flint	B. S. 188	May 6	2.0	6.2	488	534	Ch. 600°—See Fig. 6.
Light flint	B. S. 188	May 7	1.5	6.7	487	534	Ch. 600°.
Light flint	B. S. 188	May 17	1.8	5.9	485	530	Ch. 900°.
Light flint	B. S. 188	May 17	1.8	5.7	487	531	Ch. 900°.
Barium flint	B. S. 145	....	....	....	520*	560*	
Barium flint	B. S. 145	Apr. 1	2.5	7.0	520	562	See Fig. 3.
Light crown	B. S. 20	....	....	....	495*	525*	
Light crown	B. S. 20	Apr. 1	1.8	6.5	495	526	
Light crown	B. S. 20	May 4	1.1	6.4	492	526	See Fig. 4.
Light crown	B. S. 20	Apr. 30	1.3	6.5	506	534	Ann.—M. T. 675°.
Light crown	B. S. 20	Apr. 30	1.3	6.3	495	529	Ann.—2nd heating.
Boro. crown	B. S. 94	....	....	....	515*	565*	
Boro. crown	B. S. 94	Apr. 1	1.9	6.3	538	572	
Boro. crown	B. S. 94	Apr. 2	1.5	6.3	534	570	
Boro. crown	B. S. 94	May 8	1.8	6.2	538	568	See Fig. 7.
Boro. crown	B. S. 94	May 4	1.2	6.1	540	570	See Fig. 7—Ann.—M. T. 790°.
Boro. crown	B. S. 94	May 5	1.2	6.3	528	568	Ann.—2nd heating.
Boro. crown	B. S. 94	Apr. 7	1.5	6.2	533	576	Ch. 950°—M. T. 710°.

Boro. crown	B. S. 94	Apr. 8	1.5	6.5	534	573	Ch. 950°—2nd. heating.
Boro. crown	B. S. 94	May 8	1.4	6.5	530	574	Ch. 950°—M. T. 680°.
Boro. crown	B. S. 94	May 10	1.4	6.6	532	571	Ch. 950°—2nd heating—reground.
Boro. crown	B. S. 94	May 7	1.6	6.5	528	576	Ch. 950°—See Fig. 7.
Light barium crown	B. S. 87	....	....	....	575*	605*	
Light barium crown	B. S. 87	Apr. 2	1.8	6.4	581	620	
Light barium crown	B. S. 87	May 5	1.7	6.5	586	619	See Fig. 5—M. T. 830°.
Light barium crown	B. S. 87	May 6	1.7	6.6	582	617	2nd heating.
Heavy barium crown	B. S. 124	....	....	....	575*	630*	
Heavy barium crown	B. S. 124	Apr. 3	1.8	6.3	612	642	
Heavy barium crown	B. S. 124	Apr. 13	1.9	6.0	610	641	See Fig. 5—M. T. 915°.
Heavy barium crown	B. S. 124	Apr. 13	1.9	6.5	566	604	See Fig. 5—2nd heating.
Plate	Standard	Mch. 27	1.8	6.2	529	564	See No. 4 (Peters & Cragoe).[2]
Plate	Frieden	Mch. 26	2.2	6.6	554	586	See No. 6 (Peters & Cragoe).
Plate	Pittsburg	Mch. 30	1.7	6.0	556	588	See No. 5 (Peters & Cragoe).
Plate	Pittsburg	Mch. 27	1.4	6.2	558	593	See No. 5 (Peters & Cragoe).
Plate	Le Personne	Mch. 24	1.8	6.0	584	614	See No. 7 (Peters & Cragoe).
Therm. Stem	....	Mch. 29	2.0	6.1	436	474	100° C. scale (H. J. Green).
Therm. Stem	....	Mch. 23	1.7	6.0	460	494	320° C. scale (Tycos).
Therm. Stem	....	Mch. 24	2.6	6.0	457	494	320° C. scale (Tycos).
Therm. Stem	....	Mch. 26	1.7	6.0	558	598	510° C. scale (Tycos).
Therm. Stem	....	Mch. 30	2.0	6.2	558	595	510° C. scale (Tycos).
Therm. Stem	Jena 59 III	Mch. 25	0.9	5.9	578	609	500° C. scale (Arno Haak).
Therm. Stem	Jena 59 III	Mch. 25	1.9	6.1	574	610	500° C. scale (Arno Haak).
Therm. Stem	Jena 59 III	Mch. 27	1.6	6.8	568	608	560° C. scale (W. Niehls).[3]
Therm. Stem	Jena 59 III	Mch. 30	1.6	6.4	568	606	560° C. scale (W. Niehls).
Therm. Stem	Corning Norm.	Apr. 10	1.3	6.1	536	576	Unico-Beckmann.
Therm. Stem	Corning Norm.	Apr. 14	1.3	6.4	548	584	Unico-Beckmann—M. T. 655°.
Therm. Stem	Corning Norm.	Apr. 15	1.3	6.0	539	571	Unico-Beckmann—2nd heating.
Therm. Stem	Corning Norm.	Apr. 16	1.1	6.3	540	576	Unico-Beckmann—M. T. 825°.
Therm. Stem	Corning Norm.	Apr. 17	1.1	6.5	547	579	Unico-Beckmann—2nd heating.
Therm. Stem	Jena 16 III	Mch. 22	2.4	4.6	542	574	200° C. scale[4]—M. T. 715°.
Therm. Stem	Jena 16 III	Mch. 23	2.4	5.6	542	575	200° C.—2nd heating.
Therm. Stem	Jena 16 III	Mch. 29	2.2	6.4	542	578	200° C.
Therm. Stem	Jena 16 III	Apr. 5	1.5	6.6	541	577	200° C.—See Fig. 8—M. T. 700°
Therm. Stem	Jena 16 III	Apr. 5	1.5	6.3	544	579	200° C.—2nd heating—M. T. 690°.

TABLE—*Continued.*

Name	Source	Date	Mass of glass in grams	Rate of heating in degrees per minute	Critical range in glass		Remarks
					A ±5° C.	B ±5° C.	
Therm. Stem..............	Jena 16 III	Apr. 6	1.5	10.0	547	583	200° C.—3rd heating—See Fig. 8—M. T. 735°.
Therm. Stem..............	Jena 16 III	Apr. 6	1.5	2.8	537	568	200° C.—4th heating—See Fig. 8—M. T. 655°.
Therm. Stem..............	Jena 16 III	Apr. 7	1.5	7.1	546	582	200° C.—5th heating.
Tubing....................	Jena 16 III	Mch. 29	1.7	6.3	544	585	
Pyrex.....................	Tubing	....	....	....	520*	670*	
Pyrex.....................	Beaker	Apr. 8	1.4	6.7	528	671	
Quartz....................	Cryst.	June 7	1.9	6.4	571	576	See Fig. 9.

* Results given in B. S. Sci. Paper No. 358.

[1] First heating failure—M. T. 505°.

[2] *Loc. cit.*

[3] Peters and Cragoe, Nos. 31 and 32.

[4] Seibert and Kuhn.

Ann.—Annealed at 415° C. for 10 days.

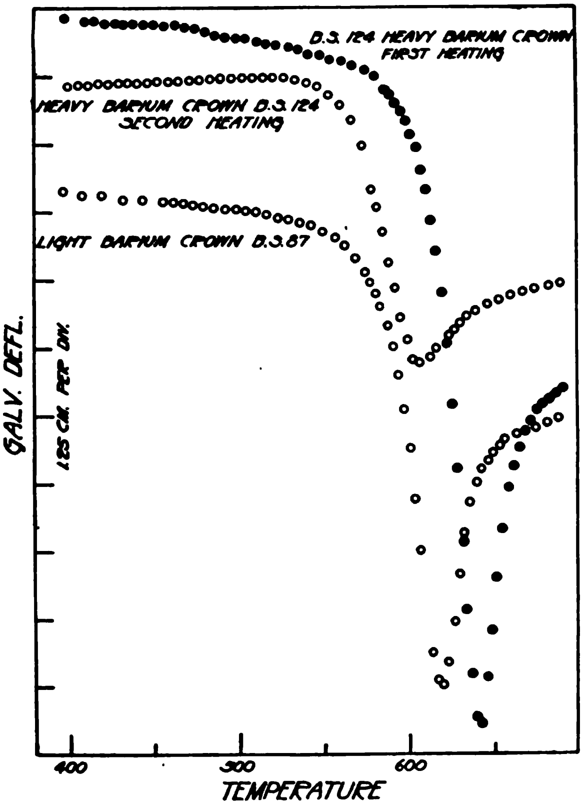
Ch.—Chilled by dropping into water from temperature indicated.

M. T.—Maximum temperature reached.

C. G. Peters and C. H. Cragoe [8] are also included, as well as a few of the glasses often used in thermometer bulbs and stems.

With few exceptions the agreement between the redeterminations and the previous results on the optical glasses was well within experimental error. The discrepancies observed, especially in the cases of the two barium glasses examined, can probably be explained on the basis of the treatment given them before the tests recorded in the previous reports had been made, because it has been found that heating to relatively high temperatures, previous to a test, often lowers the region of the absorption

FIG. 5.



Curves showing heat absorption.

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[8] *Loc. cit.*

effect for comparatively unstable glasses. In the case of the borosilicate crown, the piece of glass from which the samples used in this series of tests were taken, was not the same previously used. This may account, in part at least, for the discrepancies disclosed in this case.

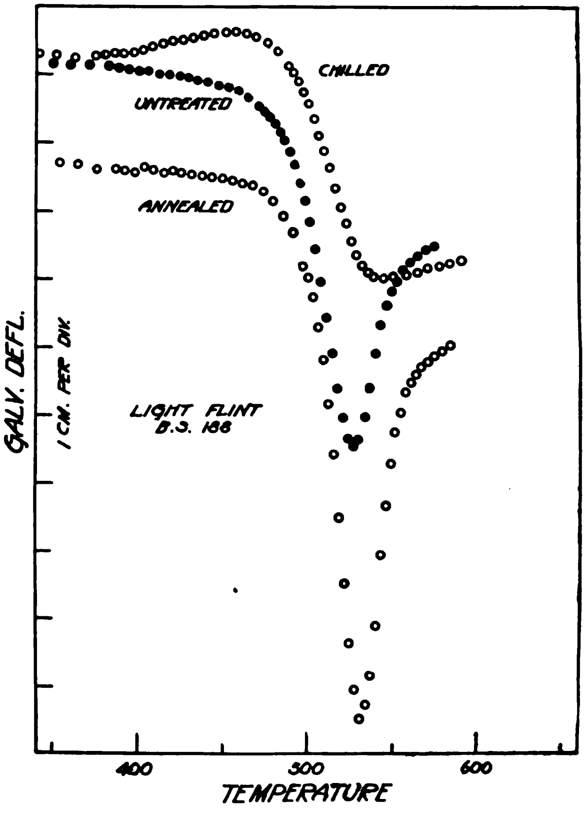
In addition to the temperatures "A" and " B " the table contains the date of the test, the mass of the glass used, the rate of heating, and remarks concerning the source and any special treatment (such as annealing or chilling) to which the glass had been subjected for experimental purposes. Where no such notation is made, the glass had received no treatment other than that usually involved in its manufacture. Whenever the same charge of glass was tested more than once, without repowdering, the succeeding tests are denoted by second, third, etc., heatings. In such cases, the maximum temperature reached during the previous heating is also noted.

From a comparison of the results gained through a repetition of tests on fresh samples of a glass, it is possible to determine easily the variations to be expected and the accuracy to be obtained in locating this heat absorption region.

Whenever a glass receives only one heating and its temperature is not carried more than 50° C. above the " B " point, and furthermore, if its cooling is reasonably rapid, it is, after the test, still practically in the original powdered form. Besides, if it is a fairly stable glass, the "A" and " B " points for a second heating are shifted very little. Higher maximum temperatures, very slow rates of heating or cooling, or repeated heatings cause the glass particles to fuse into a compact mass, and sometimes, if the glass is unstable, produce considerable changes both in the magnitude of the effect and in the values obtained for "A" and " B." In general, then, when the glass was heated more than once, the effects, as observed in the first heating were not greatly changed in succeeding tests, provided none of the previous heatings had been carried higher than the above-mentioned limit. Such changes as are noticeable consist in sharper returns from the " B " point. (See curves b and c, Fig. 8.) If the temperature is carried to a much higher value on the first heating, the endothermic effect is usually much less pronounced in the succeeding tests. Very often, also, the "A" and " B " points are displaced, usually to lower temperatures. Glasses that show a considerable tendency to crystallization, especially on the surface, display the greatest shifting of these points. (See Fig. 5.)

Chilling the glass through this region of the heat absorption effect, or annealing it at certain temperatures, also produces marked changes in the character of the curves obtained, as shown in Figs. 6 and 7. In the case of light flint (Fig. 6) the effects, due to chilling, are not so marked as in borosilicate crown (Fig. 7), especially with respect to the rise in the curve (apparently characteristic of chilled glasses) preceding the drop, due to the heat absorption. On the other hand, the effect produced by annealing is more marked in the case of light flint (Fig. 6) than

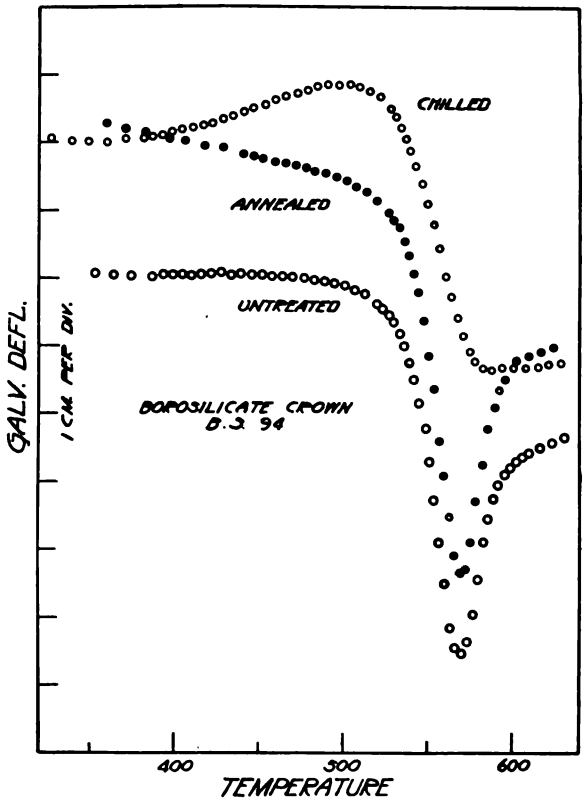
Fig. 6.



Curves showing effects of chilling and annealing on heat absorption.

in borosilicate crown (Fig. 7), probably because the borosilicate crown was annealed at a temperature much lower in relation to the beginning (A) of the heat absorption. Results have been obtained which indicate that annealing for a definite time at temperatures considerably below, or somewhat above "A," produces relatively little or no appreciable change in the form of the curve, but that annealing at some temperature between these extremes will produce a maximum effect. (This question is being further investigated.)

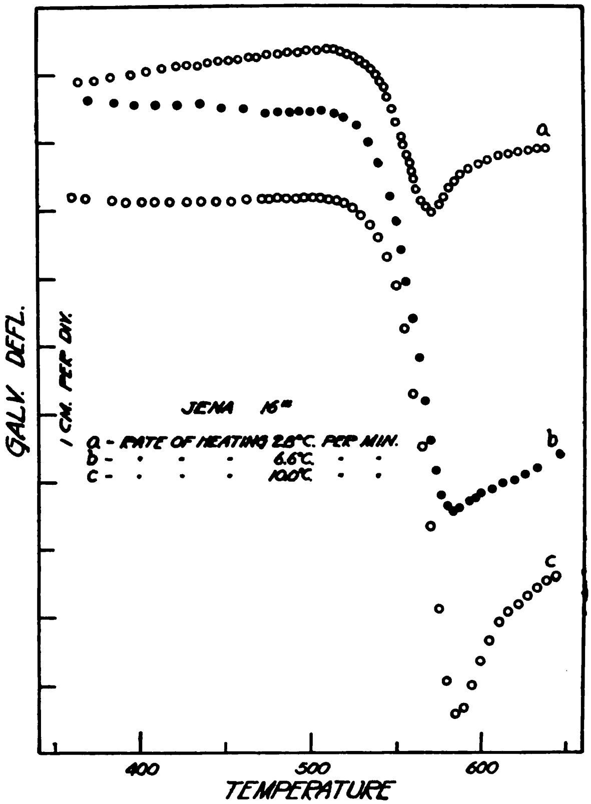
FIG. 7.



Curves showing effects of chilling and annealing on heat absorption.

In Fig. 8 three curves are given to show the effects on the position of the "A" and " B " points, produced by changing the rate of heating. It is evident from these curves that a high rate of heating shifts these points to somewhat higher temperatures, while a slow rate shifts them to lower temperatures. Furthermore, the magnitude of the effect decreases considerably as the rate diminishes. It seems improbable, from observations taken by this method on crystalline media, that the shift was due entirely to the failure of the thermocouple junctions to take up the actual temperatures of the glass and the neutral body. This shift in the "A" and " B " points, being small, was not observed in the earlier work, because the curves then obtained were not so well defined as these in the present tests. Consequently, any shifting

FIG. 8.

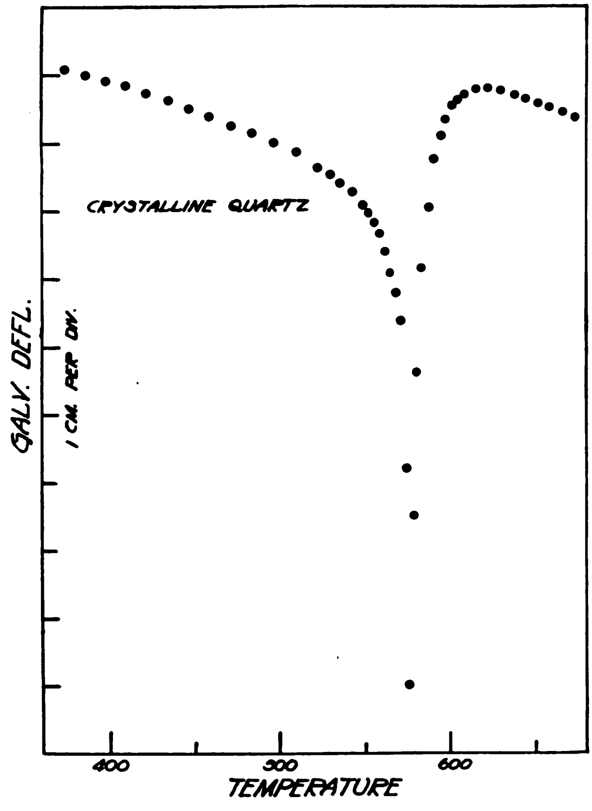


Curves showing effects of change of rate on heat absorption.

in these points, as then observed, was well within the error involved in their determination.

The curves here given, some of which have just been discussed, are chosen as representative of those obtained in the immediate vicinity of the heat absorption. Some show a marked

Fig. 9.



Curve showing heat absorption in quartz.

but uniform slope in the beginning. This is due to the fact that it is practically impossible to obtain, in any case, a horizontal approach to the region of the heat absorption, because of the difficulty in properly adjusting the position of the thermocouple junctions and in controlling the heat flow in the furnace.

The curve in Fig. 9, for crystalline quartz, is added to show the type of curve that is obtained with this apparatus and method for a transformation in a purely crystalline medium. The difference between the temperatures "A" and "B," determined from this curve for the purpose of comparison, shows what is to be expected when there is little or no lag in the endothermic effect, *i.e.*, where the difference is probably due almost entirely to the temperature gradient in the quartz.

GENERAL DISCUSSION.

It is evident from the results given in the table and shown in the curves that a glass, when heated, shows a heat absorption which increases sharply within a definite temperature range. Throughout the approach to this range, in the case of all glasses so far tested, while in their normal condition, the temperature difference between the neutral body and the glass either remains constant, or shows only a gradual uniform increase or decrease—so far as can be determined from the curves. This would indicate that these temperature differences are due only to the unsymmetrical situation of the junctions, as regards the heat flow in the furnace. It would also indicate that the amount of heat absorbed by the glass at temperatures below "A," indicated on the curves shown, depends only on its specific heat, and that at these low temperatures no marked transformations occur. The relative cooling at "A," characteristic of all glasses so far tested, proceeds with such rapidity that this effect takes on a character very similar to the endothermic effect observed in crystalline media (see Fig. 9), when some transformation takes place which involves a change of state or crystal form.

POSSIBLE CAUSES OF HEAT EFFECTS.

It does not necessarily follow from this fact that glass, or any of its components, has a crystalline structure, and that this structure disappears, or changes form, when the endothermic effect occurs. For there are other possibilities that can be conceived, such, for instance, as the rearrangement or breaking down of certain more or less heterogeneous molecular aggregates which are peculiar to the glass at low temperatures. It is also possible that there may be a certain separation of some of the components on cooling and the dissolving of these on heating.

In this connection there are several facts that seem to differentiate the effects observed in glass from those observed for transformations in pure crystals. For example, this endothermic effect covers relatively a broader range and is considerably shifted towards higher temperatures by an increase in the rate of heating. (See Fig. 8.) Moreover, as was shown in the Scientific Paper previously mentioned, the corresponding exothermic effect on cooling, though marked, is much less distinct, and also shifted more towards lower temperatures, than in the case of those observed in pure crystals.

The question, however, as to whether the structures are crystalline or of some other nature must, so far as the present article is concerned, remain undecided. In this connection, however, it is of interest to consider some of the phenomena which have been recently discussed by Sir Herbert Jackson.[9] According to his discussion it is usually considered that truly vitreous bodies do not fluoresce when subjected to Röntgen rays, ultra-violet light, or cathode rays. Practically all glasses, however, unless chilled very rapidly, do show such fluorescence very distinctly. Such chilled glasses, too, when reheated to a temperature where they begin to deform slightly are again capable of fluorescing. This may be taken to indicate that glass normally cooled actually does possess a crystalline structure. Indeed, in a polycomponent medium, such as glass, only a part of these components need have such a structure. In this respect it would be interesting to determine, if possible, by means of X-rays, whether such structures actually do exist in glass, although a negative result might not conclusively disprove their presence.

If these observed endothermic and exothermic effects in glass are due to the formation of crystalline structures, or some other molecular aggregates peculiar to the glass, it might appear that the breadth of the range and the shifting of the effect with change in the rate of heating, previously mentioned, are due to the lag produced by the high viscosity of the glass. On the other hand, the shifting and breadth of these effects may be due to a superheating or a supercooling, such as is often observed in "slow-melting compounds."[10] It is also possible that the presence of a number of polymorphic crystalline structures, or aggregates, having slightly different melting points, results in a broadening of such an endothermic effect. Some glasses, for example, pyrex, appear, when studied in this way, to have two absorption effects so close together that it is difficult to separate them. In fact, unless care is taken in the observations, the effects appear as one broad region of heat absorption. In any case it would seem reasonable, in view of the phenomena observed where crystallization occurs in undercooled liquids, to assume that the formation of these structures, or aggregates, which produce these effects, begins, on cooling, near the temperature " B "; further, that the tendency for this formation increases continually below this, as the undercooling proceeds, but that the actual power of formation,[11] which was zero above " B," soon reaches a maximum, and again returns practically to zero after still lower temperatures are reached. If such structures, or aggregates, have not been allowed to form while the glass was cooling, it also appears that, on reheating, they will, on reaching a temperature somewhat below "A," begin to form at a rate which increases rapidly as the region of the endothermic effect is approached.

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[9] Jackson, *J. Roy. Soc. of Arts,* **68**, p. 134, 1920.

[10] A. L. Day and R. B. Sosman, *Am. J. Sci.* (4), 31, p. 341, 1911.

EFFECTS PRODUCED BY CHILLING AND ANNEALING.

In view of these possibilities it would appear that the causes of this endothermic effect may have a far-reaching significance in the problem of annealing. Consequently, it seemed wise to continue the investigation of this endothermic effect and to begin an investigation of the variations produced by annealing and chilling. From the preceding discussion it seems probable that rapid cooling, or even the rates usually employed in annealing, do not allow the formation of the molecular aggregates, whether of a crystalline configuration, or of any other character, which produce the endothermic effect on heating. Preliminary results from investigations of this possibility are given in this paper.

Only two of the glasses mentioned in this article, light flint and borosilicate crown, have been included in the test for the effects produced by chilling. These glasses were heated to temperatures well above " B " and chilled by dropping them into water. In both cases the curves obtained on heating these chilled glasses indicate an exothermic transformation, occurring below the region of the endothermic effect usually observed in them. Although indications of such an exothermic effect might be expected from the relaxation of stresses in the chilled glass, it seems highly improbable that the effect here observed could be due to this cause, since the glass was powdered. It seems more probable, however, that this exothermic effect has its origin in the same transformation as the exothermic effect observed on cooling, which was shown in the Scientific Paper No. 358. Chilling also reduces the endothermic effect on heating, and renders the point of maximum cooling (B) less well defined, as shown in Figs. 6 and 7. Chilling a glass, therefore, produces an exothermic effect on heating, as shown by these curves, and reduces the normal endothermic effect.. It would appear, therefore, that these effects are the result of the formation and dissolution of certain structures whose formation may be suppressed by the rapid cooling.

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[11] A discussion of a very similar condition in the devitrification of glass was recently given by N. L. Bowen, *J. Am. Cer. Soc.*, **2**, p. 261, 1919.

In the investigation of the effect produced by annealing, four glasses, medium flint, light flint, light crown, and borosilicate crown, were annealed ten days at an average temperature of 415° C. For all these glasses this temperature is well below even the first indication of an endothermic effect. However, it probably falls in a region where the power of formation has not yet become practically zero. It was nearest to point "A" in the case of medium flint, being about 35° below, and farthest from "A," about 115° C. below, for borosilicate crown. In every case there was distinct evidence that this treatment had increased the magnitude of the endothermic effect, although, in practically all instances, the glass had previously received an annealing treatment in the glass factory. For medium flint and borosilicate crown (see Fig. 7), the effect was by no means so marked as for the light crown and the light flint (see Fig. 6). In the latter case the increase in the magnitude of the cooling effect was almost 60 per cent. As may be noted from the table, there was also a tendency to shift the points "B" and "A" to higher temperatures. From these tests it is evident that a prolonged annealing increases the endothermic effect. From this it may be assumed that annealing glass at these temperatures builds up some formation which is peculiar to the glass at these temperatures, and which disappears completely at approximately the temperature "B."

Indications of an increase in the relaxation time [12] after the glass has been held at certain temperatures for a considerable time, have been noted from time to time in this laboratory while viscosity tests were being made. Such increases may also have a bearing on this point. For, if the glass is held for a considerable period of time below the temperature " B," it is possible that the formation of structures, or aggregates, in the glass increases its viscosity.

EFFECT OF POSSIBLE INHOMOGENEITY IN GLASS.

If such a structure can be revealed, however, it becomes extremely important to determine whether it is detrimental to the purposes for which glass is intended. If it is, the problem of annealing then becomes extremely difficult. But it may also be possible that this structure indicates that the glass is in a more stable condition, and may not at all be injurious to it. If this be the case, it would be well in annealing to produce the maximum possible crystalline character of this type.

At any rate it would seem necessary to strive to attain the greatest structural homogeneity in the glass by proper annealing. For instance, it would appear that the heating of the glass to a temperature where it softens considerably, a procedure which is often found necessary in annealing, is required, not so much to reduce large stresses, which would in any case disappear quite rapidly at much lower annealing temperatures, as to bring about greater structural homogeneity in the glass.

Thus there may be a marked difference from point to point in the structure of glass, when chilled or incompletely annealed. Such a condition is evidenced in the Rupert's drop by the increase in the fluorescing power from the surface to its centre. It would appear that much of the peculiar behavior observed in glass not thoroughly annealed may be ascribed to this state of inhomogeneity. For example, when glass is moulded, or worked in any way, it usually happens that certain parts of the mass are cooled very rapidly to temperatures below even the region of the heat absorption discussed in this paper, while other parts reach this temperature region much more slowly. As a result there would be, on the basis of the above supposition, a certain inhomogeneity of structure in the glass. Furthermore, since glass is usually annealed at a temperature well below "A," it is possible that some of this inhomogeneity persists, even after the glass has been subjected to the usual annealing procedure.

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[12] Relaxation time may be defined as the time required for the viscous or plastic deformation to reach a value equal to the initial elastic deformation, when the viscous or plastic body is being subjected to a constant stress. This ignores the possibility that the rate of deformation immediately after the force has been applied is greater for plastic bodies than at any later time.

From the results on the expansivity obtained by C. G. Peters, it would seem reasonable to assume that those portions of the glass which are most vitreous—that is, most free from the aggregates, or structural configurations, that form in the region of the change in the expansivity which he observed—contract more than the more slowly-cooled and less-vitreous parts. This would probably have the effect of causing the glass to contain, on the whole, greater stresses after it had been subjected to any usual annealing and cooling procedure than should normally be expected—this may be the case even though the glass when examined polariscopically, while being held at the annealing temperature, appears to be almost wholly stress-free. Thus, such an effect would probably cause such articles as have received a very sharp chilling on the surface to fail to respond to a treatment that would be sufficient to anneal satisfactorily a more homogeneous sample of the same glass. A more troublesome possibility, however, is the likelihood of a lack of stability, since if the more vitreous parts tend to go over into a more stable form, they will, on the above assumption of their having a greater expansivity, thereby increase in volume. Consequently, the stresses in the glass may continue to grow, withal very slowly, even at room temperature. This possibility would explain on another basis some of the effects discussed by R. Reiger,[13] who, in seeking to explain the failure of Maxwell's relaxation hypothesis in its simple form to apply fully to the phenomena observed in plastic bodies, concludes that one must assume that there are several relaxation phenomena.

This possible change in volume, which must in most cases be very slight, may also explain why some very carefully annealed glasses, considered only from the standpoint of the stresses present, warp, or undergo other changes sufficient to render them unfit for certain delicate optical instruments. To be sure, if the structural inhomogeneity disappears at the same rate as the stresses, for the annealing temperature employed, such a possibility could scarcely be considered. However, it is highly improbable that the relaxation of the stresses and the formation of the structural homogeneity, will, at any annealing temperature, occur in even approximately the same time, or that the relaxation time for the stresses and the power of formation of these possible structures in the glass bear identically the same relation to the temperature.

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[13] R. Reiger, *Verh. d. D. Physik. Ges.;* 21, p. 421, 1919.

RELATION TO SPECIAL HEAT TREATMENTS.

From the preceding discussion it would seem that the problem of annealing glass for optical instruments, or other delicate apparatus, may involve much more than the mere removal of stresses from glass, and that considerable investigation will be required to show that this removal of the stresses is all that is necessary. On the other hand, for common glassware, or for glass for ordinary optical instruments, it is without doubt sufficient to produce a reasonably stress-free glass. In most cases this can be accomplished very easily by annealing the glass at any temperature within a range of 50° C., or so, below "A" for a reasonable time, depending on the temperature chosen, the viscosity and plasticity of the glass, and the degree of annealing desired, and by cooling it at a rate which is adapted to the rigidity of the glass, and the variation of this rigidity with the temperature.

The lack of stability alone is probably of little importance, if the glass is homogeneous in structure—at least, so far as optical instruments are concerned, since any possible changes in refractive index, etc., due to a slow transformation, are probably small. In thermometers, however, even if the glass is homogeneous, the volume changes due to any instability may be of considerable importance. Thus, it may be that some of the changes observed in the ice-point, and similar phenomena, are due in part to this sort of volume changes, rather than wholly to the stresses. If so, it well may be that a treatment such as that discussed by L. Marchis[14] is valuable in bringing about a condition of equilibrium. His method consisted of subjecting the thermometer to continued small cyclic changes of temperature at low values—a procedure which he found produced a stabilization much more quickly than holding the thermometer at approximately an equivalent physically constant temperature. He explained the phenomena observed on the basis of the existence in the glass of at least one allotropic or chemical modification. In any case, it would appear that some method of annealing should be followed which would both relieve all stresses and bring about a condition of the greatest possible structural stability.

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[14] L. Marchis, "Les Modifications Permanentes du Verre et le Deplacement du Zero des Thermometres," 1898.

Another interesting phase of the problem is met with in the so-called "tempered" glass. It is possible that the resisting power of such glasses is not due wholly to stresses set up in them—although it is generally considered that their increased resisting power can be sufficiently explained[15] by these stresses—but to the possible fact that glass in a more vitreous condition is harder and offers more resistance to sudden blows. Such glasses have been studied by various investigators, especially A. de la Bastie[16] and F. Siemens.[17] The results obtained by the latter would seem to support the above view, since he reported that his hardened glass was practically stress-free.

SUMMARY.

Experimental results are given which show that glass on being heated exhibits within a definite and narrow temperature range a marked increase in its heat absorption, which in many ways corresponds to the cooling effect occurring on a change of state.

From a discussion of the possible causes of this effect; it would seem that it may be caused by one or more types of molecular aggregates, which may have a crystalline structure.

It is also shown that chilling or annealing glass produces the effect that should be expected, if the absorption of heat on heating and its evolution on cooling are due to the formation and dissolution of some such crystalline structure or other molecular aggregate.

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[15] Lord Rayleigh, *Phil. Mag.* (6), **1**, p. 169, 1901.

[16] A. de la Bastie: *Ann. d. Chim. et Phys.* (5), **23**, p. 286, 1881; *Brux. Bll. Pht.*, **14**, pp. 118 and 139, 1875.

[17] F. Siemens: *Dtsche Ind. Ztg.*, **26**, p. 236; *Chem. C. B.* (3), **16**, p. 670; *Nature*, **31**, p. 413, 1885.

The possibility that certain methods of cooling or treatment may cause structural inhomogeneity in glass through the non-uniform formation of these aggregates is also discussed, and attention has been called to the importance of this possibility in the annealing and other special heat treatments of glass.

This would mean that annealing is not alone a question of the removing of stresses, but also of producing homogeneity.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

RESONANCE POTENTIALS AND LOW-VOLTAGE ARCS FOR METALS OF THE SECOND GROUP OF THE PERIODIC TABLE.*

BY

F. L. MOHLER, PAUL D. FOOTE and W. F. MEGGERS.

PREVIOUS publications have discussed resonance and ionization potential measurements for five metals of the second group of the periodic table.[1]

The results showed in every case a resonance potential corresponding by the quantum relation $Ve = h\ \nu$ to the combination series line $1.5\ S - 2p_2$ (Paschen notation) and an ionization potential corresponding to the limiting frequency of this series, $1.5\ S$. Calcium showed in addition a second resonance potential corresponding to the frequency $1.5\ S - 2\ P$, the first line of the principal series of single lines converging at the same frequency $1.5\ S$.

Spectroscopic studies of low voltage arcs in these metals[2] are not entirely in accord with the work on current-potential measurements.

The glow discharge at potentials below the ionization point shows the line $1.5\ S - 2p_2$ in mercury, cadmium and zinc. In cadmium and zinc the line $1.5\ S - 2\ P$ also appeared at the potential corresponding to this frequency, but in magnesium the line $1.5\ S - 2p_2$ was not observed below the ionization potential.

Davis and Goucher[3] have found evidence of the line $1.5\ S - 2P$ in mercury by photo-electric methods.

The result of the spectroscopic study of magnesium and the failure to observe two resonance potentials in metals other than calcium were points demanding further investigation.

The search for a second resonance potential required a more sensitive means of detecting inelastic impact than is possible with the type of three-electrode vacuum tube previously used. In the present work a four-electrode tube of the type developed for study of non-metallic gases and vapors[4] was employed. The hot wire cathode is surrounded by two cylindrical grids and a plate. The grids, of which one is close to the cathode and the other close to the plate, are maintained at the same potential by metallic contact; a variable accelerating field for electrons is used between the cathode and inner grid, and the current reaching the plate against a small retarding field between the outer grid and plate is measured. The electrodes were contained in pyrex glass tubes in which the metals were boiled.

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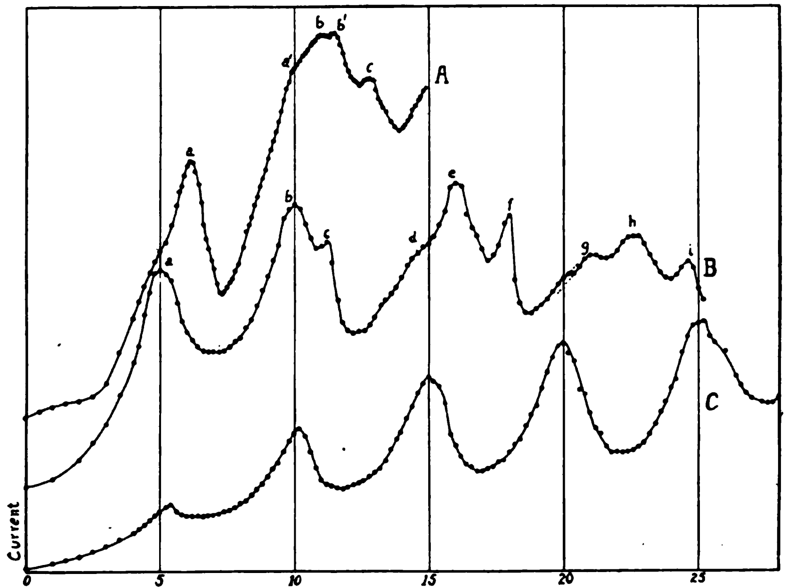
* Published by permission of the Director, Bureau of Standards.

[1] Mercury—Franck and Hertz, *Verh. D. Phys. Ges.* **16**, p. 457, 1914. Also Tate, *Phys. Rev.*, **7**. p. 686, 1916. Cadmium—Tate and Foote, *Bur. Stds. S. P.* 317. Cadmium and Zinc—Tate and Foote, *Phil. Mag.*, 36, p. 64, 1918. Magnesium—Foote and Mohler, *Phil. Mag.*, **37**, p. 33, 1919. Calcium—Mohler, Foote and Stimson, *Bur. Stds. S. P.* 368.

[2] Mercury—Franck and Hertz, *Verh. D. Phys. Ges.*, **16**, p. 512, 1914. Mercury, Zinc and Cadmium—McLennan and Henderson, *Proc. Roy. Soc. A.*, **91**, p. 485, 1915. Magnesium—McLennan, *Proc. Roy. Soc. A*, **92**, p. 574, 1916. Zinc and Cadmium—McLennan and Ireton, *Phil. Mag.*, **36**, p. 461, 1918.

[3] *Phys. Rev.*, **10**, p. 101, 1917.

FIG. 1.



Current-voltage curves in mercury vapor.

Fig. 1 shows some typical curves in mercury vapor. Curve A was obtained at low temperature and vapor pressure; curve B at higher pressure; and curve C at still higher pressure and by a method similar to that used in a three-electrode tube. Curve C shows inflections only at 4.9-volt intervals due to successive resonance collisions of but one type (the lowest energy exchange on account of a small mean free path). All the inflections in curve B may be explained by successive collisions involving two resonance potentials of 4.9 and 6.7 volts corresponding to the two lines $1.5\ S-2p_2$ and $1.5\ S-2\ P$. Inflections at points *a*, *b* and *d* are due to successive collisions of the first type; *c* one of the first and one of the second, *e* two of the first and one of the second, *f* one of the first and two of the second, *g* three of the first and one of the second, *h* two of the first and two of the second, *i* one of the first and three of the second. The curve A shows two additional points. The point *b′* is due to ionization while *a′* gives evidence of a third resonance potential at 8.7 volts. These two points were shown only at low vapor pressure. The failure to observe an inflection arising from one collision of the second type is doubtless due to the overlapping of the effect of the first resonance point. This overlapping of effects limits the precision

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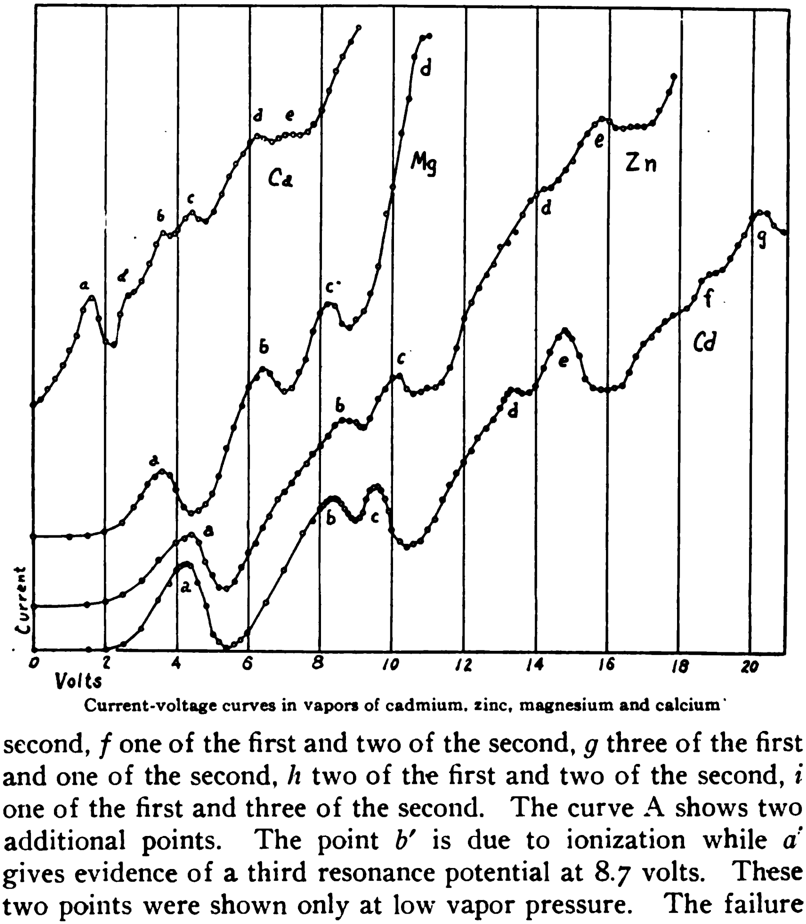
[4] Mohler and Foote, *Bur. Stds.*, *S. P.*, in press. Abstract in JOUR. OPT. SOC., 4, p. 49, 1920.

FIG. 2.



Current-voltage curves in vapors of cadmium, zinc, magnesium and calcium possible by this method. Curves of type C give much higher accuracy. Thus the interval between five inflections is 19.5 ± .2 volts, giving for the observed first resonance potential of mercury 4.88 ± .05 volts while the theoretical value is 4.865 volts.

Fig. 2 illustrates one curve for each of the metals cadmium, zinc, magnesium and calcium. The latter, taken from the work by Mohler, Foote and Stimson,[5] was obtained by use of a three-electrode tube. The curve for cadmium is very similar to the curve B for mercury; the curves for zinc and magnesium less so as the vapor pressure obtainable was limited to lower values on account of the pyrex glass containing tubes. All the inflections are explained by the effect of two resonance potentials. The inflection points correspond to the points on the mercury curve B. Only with calcium does the *single* resonance collision of the second type appear, indicated at a'.

A number of curves were obtained for each element and the mean values are given in Table I as well as the values to be expected in barium and strontium.

As to the possible third resonance potential of mercury, the second lines of either of the absorption series give potentials very close to the observed point.

$1.5\,S-3\,p_2$ $\lambda=1435.6$ $V=8.60$ volts
$1.5\,S-3\,P$ $\lambda=1402.7$ $V=8.80$ volts
Observed mean value $V=8.7$ volts

Although the observed value of 8.7 volts corresponds closely to the second line of either of these series, and although collisions of such types might be expected on the basis of Bohr's theory, further work must be done from the spectroscopic standpoint to substantiate such a conclusion. Foote and Meggers [6] found that collisions corresponding to the second member of a series did not exist in vapors of the alkali metals, but the alkali earth metals may, of course, behave differently.

The similar characteristics of electronic impact in vapors of all the metals of group II of the periodic table require that their spectra in low-voltage arcs should be analogous. Hence the single-line spectrum of each of these metals should be $1.5\,S-2p_2$. As McLennan [7] observed $1.5\,S-2\,P$ for the single-line spectrum

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[5] *Loc. cit.*

[6] Foote and Meggers, *Phil. Mag.*, 40, p. 80, 1920.

[7] *Loc. cit.*

TABLE I.

Ionization and Resonance Potentials.

Metal	Series Paschen notation	Wave length A in vac.	Computed potential	Observed potential
Zinc	$1.5\ S$ (a)	1319.98	9.3524	9.3
	$1.5\ S-2p_2$	3076.88	4.0122	4.18
	$1.5\ S-2P$	2139.33	5.7705	5.65
Cadmium	$1.5\ S$ (a)	1378.69	8.9542	9.0
	$1.5\ S-2p_2$	3262.09	3.7844	3.95
	$1.5\ S-2P$	2288.79	5.3937	5.35
Mercury	$1.5\ S$ (a)	1187.96	10.3918	10.2
	$1.5\ S-2p_2$	2537.48	4.8651	4.76
	$1.5\ S-2P$	1849.60	6.6744	6.45
Magnesium	$1.5\ S$ (a)	1621.72	7.6123	8.0
	$1.5\ S-2p_2$	4572.65	2.6997	2.65
	$1.5\ S-2P$	2853.06	4.3269	4.42
Calcium	$1.5\ S$ (b)	2028.20	6.0867	6.01 (c)
	$1.5\ S-2p_2$	6574.59	1.8777	1.90
	$1.5\ S-2P$	4227.91	2.9198	2.85
Strontium	$1.5\ S$ (b)	2176.94	5.6708	
	$1.5\ S-2p_2$	6894.45	1.7906	
	$1.5\ S-2P$	4608.61	2.6787	
Barium	$1.5\ S$ (b)	2379.28	5.1885	
	$1.5\ S-2p_2$	7913.52	1.5600	
	$1.5\ S-2P$	5537.04	2.2295	

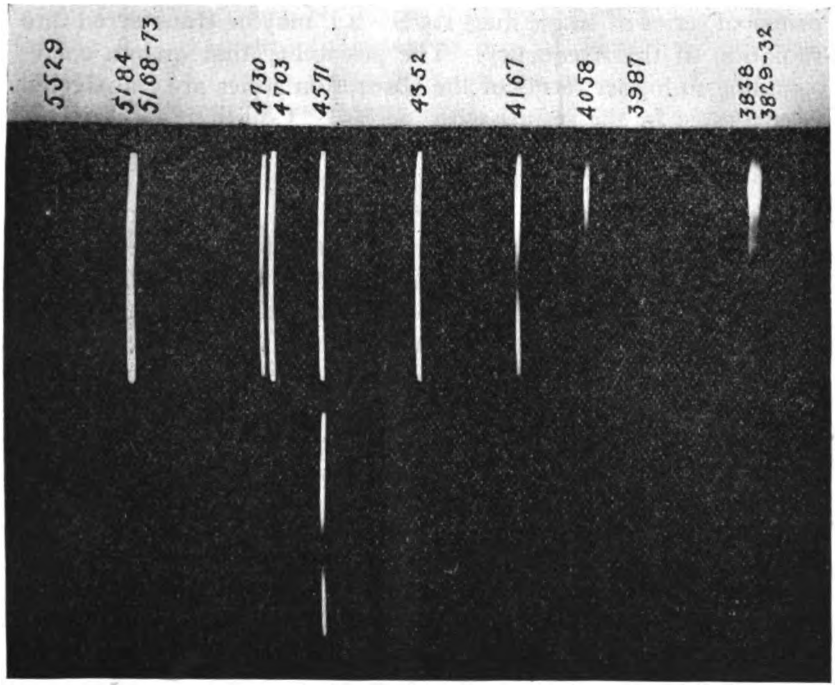
[a] Dunz Tubingen dissertation.
[b] Data furnished by F. A. Saunders.
[c] Mohler, Foote and Stimson.

of magnesium and could not detect the presence of $1.5\ S-2p_2$ below the ionization potential, it was desirable to repeat his experiment. The evidence that ionization is unnecessary to produce the line $1.5\ S-2\ P$ ($\lambda=2852$) is conclusive, so that the present experiment was concerned with determining whether or not the line $1.5\ S-2p_2$ ($\lambda=4571$) is excited below the ionization potential.

A cylindrical anode and a hot-wire cathode along its axis were placed at one end of a long tube with a glass window sealed on the other end. Only the part of the tube near the electrodes was heated. The light of the incandescent cathode was shielded from the window so that the glow discharge appeared against a dark background. This was photographed by a single glass prism Zeiss spectograph of large aperture. Fig. 3 shows the result. The upper spectrum is the arc at 14 volts, the lower at 3.5 volts. The former required a 5-minute exposure, the latter about 4 hours. The current ratio for the two cases was about 50 to 1.

The reproduction masks the relative intensity of the 14-volt arc lines. The lines on either side of $\lambda = 4571$ are much more intense than this line, so that the transition to the single-line spectrum is most marked. The photograph shows conclusively the existence of $1.5\ S - 2p_2$ ($\lambda = 4571$) as the single-line spectrum of magnesium.

FIG. 3.



Spectra of magnesium arc. Upper 14 volts: Lower 3.5 volts showing the single line spectrum $\lambda = 4571$.

It is interesting to note that the 5-minute exposure at 14 volts shows all the arc lines listed in Kayser and Runge's tables in this region. The lines belong to the triplet series and single series. The first lines of the first and second subordinate series of triplets and six lines of the first subordinate series of single lines are shown in the reproduction.

We are accordingly able to state the following general laws for the phenomena of electron impacts with molecules of metals of the second group. There are at least three types of inelastic impact, two resonance potentials, and an ionization potential. (1) Electrons with kinetic energy exceeding one quantum of the frequency of the combination line $1.5\ S - 2p_2$ may lose this amount of energy in exciting radiation of this single frequency. (2) Likewise a quantum of the frequency of the first line of the principal series of single lines $1.5\ S - 2\ P$ may be transferred into radiation of this frequency. The possibility that quanta corresponding to higher terms of the absorption series are transferred is suggested by the results with mercury. Collisions of this type must be very improbable and we feel that the evidence here given is insufficient for deciding this important point. (3) Electrons with energy greater than that corresponding to the common limit of the two absorption series, $1.5\ S$, may ionize the molecule. While the phenomenon of successive inflections corresponding to successive *resonance* collisions shows that in this case the transfer of energy must take place by quanta, there is no corresponding evidence for ionization collisions. It is entirely in accord with atomic theories that this latter energy transfer should not be by quanta. Thus, when an electron with velocity exceeding the ionization potential ejects an electron from an atom, it may divide its remaining kinetic energy with the ejected electron so that the greater its energy before collision the greater will be its loss of energy. The many-line spectrum emitted above the ionization potential is probably to be ascribed to recombination of ionized atoms with electrons. This spectrum consists of the single-line and triplet series, as shown in the 14-volt arc spectrum of magnesium. In addition to the arc spectrum obtained under the condition of our experiment there are a large number of enhanced spark lines, including a set of doublet series. This fact would indicate that there exists for these elements a second ionization potential corresponding to the formation of doubly-charged ions which on recombination give the latter spectrum. It may be possible to observe this second type of ionization by the methods here employed. If so, considerable information would be available in regard to enhanced spectra.

BUREAU OF STANDARDS,
August 5, 1920.

THE PRECISION OF PHOTOMETRIC MEASUREMENTS.

BY

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THE extent to which the advance of Physical Science is dependent on our ability to make more and more precise measurements was discussed most admirably by Vice-President Zeleny in his retiring address before Section B of the American Association for the Advancement of Science at its Columbus meeting, December, 1915.[1] From this standpoint the question of the precision of photometric measurements is of peculiar importance in that in this field, more than in any other, the precision obtainable is limited by other than physical factors; namely, by the ability of the eye to decide when two adjacent areas appear equally bright. This physiological factor seems to set a definite limit to the precision attainable in photometry. We may improve the constancy of our light sources; make better photometer tracks; provide more comfortable and more favorable conditions under which the eye works, such as a head rest, stationary photometer, exclusion of side lights, a "good" photometric field, absence of noise and vibration, most favorable field brightness, etc., but we cannot go beyond the limits set by the sensibility of the zero instrument used, the eye. What these limits are, and whether they have already been reached, are questions of much importance not only to the practical photometrist, but to the investigator who is using photometric methods in pushing forward the boundaries of science.

During the summer of 1915 the writers, in collaboration with a number of observers at the Bureau of Standards, collected a large amount of photometric data,[2] under conditions which called for the highest available precision. Although not contemplated at the time, it has since seemed desirable to examine these observations to ascertain what precision obtains in photometric work. For the purpose of such an analysis, the material available is somewhat unsatisfactory in that it was intended for an entirely different purpose, and therefore leaves untouched a large number of questions of fundamental importance. The results, however, are regarded as of special significance because of the fact that the observers, in making the settings, were not conscious of being studied for the precision of which each was capable. It is thought that actual working conditions have been studied, and that the conclusions, though far from being exhaustive, are free from any disturbing factors, more or less psychological, which must inevitably be present in any work deliberately intended for studying precision.

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[1] *Science,* Vol. xliii, p. 185, 1916.

[2] See paper by Crittenden and Richtmyer, "An 'Average Eye' for Heterochromatic Photometry, and a Comparison of a Flicker with an Equality of Brightness Photometer." *Trans. Ill. Eng. Soc.,* Vol. xi, p. 331, 1916, and *Bureau of Standards Scientific Paper* No. 299.

Precision photometry, as we understand the term to-day, was made possible by the development of the incandescent lamp and the storage battery, and began with the invention of the now well-known Lummer-Brodhun photometer in 1889. Previous to this time a great deal of attention had been given to studying the sensitiveness and theory of operation of the Bunsen screen, notably by Weber,[3] who found the "mean error" of a single observation with the Bunsen photometer to vary from 1.8 to 4.7 per cent. It was later pointed out by Lummer and Brodhun that a part of this variation was undoubtedly due to the fact that Weber used open gas flames. Later observers, of course, used incandescent lamps.

In the paper describing their new photometer,[4] Lummer and Brodhun discuss the theory of the Bunsen screen, and, applying the same theory to their device, show that, for the same sharpness of division between the adjacent fields, their photometer should be about 2.5 times as sensitive as the Bunsen. Actually, they state, the ratio is somewhat larger because of the fact that the dividing line in the Lummer-Brodhun field is somewhat sharper than in the Bunsen field. Lummer and Brodhun state that for their photometer they found the "mean error" of a single setting to be under 0.5 per cent., and that a difference in brightness of 1.5 per cent. between the two fields was easily observable. (These measurements refer to the so-called "equality" field; not to the contrast pattern.) The field size and field brightness used are not given except to state that they were, so far as brightness is concerned, working in the region of maximum sensibility.

Later, in the description of their contrast photometer,[5] which they show ought to be approximately twice as sensitive as the "equality" photometer, they find for the ordinary contrast cube with stationary absorbing glasses (*i.e.,* 8 per cent. contrast) a "mean error" of one setting in a series of 20 to be 0.40 per cent. Maximum sensibility was found with the 3.5 per cent. contrast. In this case the mean error of a single setting was 0.22 per cent.

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[3] *Ann. d. Ph.*, Vol. xxxi, p. 676 (1887).

[4] *Zeit. fur Inst.*, Vol. ix, p. 41 (Feb., 1889).

[5] *Zeit. fur Inst.*, Vol. ix, 461 (Dec., 1889).

Martens and Bechstein [6] also compared the contrast type with the equality as well as several other forms of photometer, and reported the "mean errors" as follows:

Photometer	Mean error
	Per cent.
Lummer-Brodhun contrast	0.38
Martens	0.46
Lummer-Brodhun equality	0.59
Joly diffusion	1.7
Bunsen-Rudorff grease-spot	2.0

In a paper on "The Sensitiveness of Photometers" [7] L. W. Wild reports an examination of the sensibility of a number of different types of photometers. His method was as follows: "The photometer box was moved slowly out of balance to the right, and was then brought back slowly till the balance reappeared, the reading being taken." A similar reading was then taken to the left. "The difference between these two readings (left and right) is a measure of the sensitiveness of the photometer." Each pair of readings was taken six times and the mean of the six used. The following table gives the results:

Photometer	4.32 wpc. carbon against 4.35 wpc. carbon illumination = 1.2 foot candles	4.3 wpc. carbon against 1.4 wpc. osram illumination = 1.65 foot candles	Pentane lamp against incandescent gas mantle
	Per cent.		
Ritchie wedge	2.4	5.0	8.2
Joly paraffin prism	2.5	4.8	8.3
Lummer-Brodhun (equality?)	0.7	3.5	8.0
Bunsen, ordinary single	1.2	3.6	8.7
Bunsen, ordinary double	1.5	5.0	9.8
Bunsen, special single	0.4	3.3	7.8
Bunsen, special double	1.0	4.5	9.0
Simmance-Abady flicker	1.7	1.8	2.1
Wild flicker	0.8	0.8	1.0

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[6] *Jour. f. Gasbeleuchtung,* Vol. xliii, p. 251 (1900).

[7] *Electrician* (London), Vol. lx, p. 122 (Nov., 1907).

To show the effect of even a small color difference in the field the sensibility of the Special Bunsen (single) was determined for the relativity small color difference: 4.32 wpc carbon against 3.85 wpc carbon. The value reported is 2.6 per cent., showing that the photometer is less than one-fourth as sensitive as with a color match.

Kennelly and Whiting[8] studied four different photometers, determining the probable error of a single observation for a color match (0.24 mean spherical candle per watt carbon) and for the color difference given by 0.19 mean spherical candle per watt carbon against 0.55 mean spherical candle per watt tantalum. The results for a color match were as follows:

Photometer	Probable error of a single observation
	Per cent.
Lummer-Brodhun contrast	0.5
Lummer-Brodhun equality	0.75
Lummer-Brodhun concentric	1.0
Bunsen (grease-spot)	1.5

In the heterochromatic measurements the probable error of a single observation varied from 1.4 to 1.9 per cent. With a color match the Lummer-Brodhun contrast photometer was three times as sensitive as the Bunsen. But with a color difference the two photometers had more nearly the same sensibility. It is further stated that the constant errors (errors in mean results) obtained by different individuals averaged only one-third the probable error of a single observation in the case of the homochromatic settings, but *twice* the probable error in the heterochromatic.

In the discussion of the paper Mr. J. B. Klumpp[9] thought that the precision stated for the Bunsen was too low in comparison with the Lummer-Brodhun; that the precision with experienced operators should be from one-fourth to one-half per cent. Mr. P. S. Millar reported experiments with the Bunsen (Leeson disk) and with the Lummer-Brodhun contrast photometer. In a series of 25 settings the former gave a probable error of 0.4 per cent., and the latter of 0.24 per cent.

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[8] Proc. Natl. Elect. Light Assn., 1908. Abstract in *Elect. World,* Vol. li, p. 1104 (1908).

[9] *Elect. World,* Vol. li, p. 1152 (1908).

Later, Edwards and Harrison[10] reported an analysis of some 5000 readings to test the relative accuracy of the Bunsen and the Lummer-Brodhun low-contrast type. They find the average deviation of a single setting to be 0.4 per cent. for the Lummer-Brodhun, and 1.5 per cent. for the Bunsen. They further state that *no marked loss of precision results from small color differences such as with a carbon against a tungsten* incandescent lamp. This conclusion is quite contrary to that given by Wild and by Kennelly and Whiting.

Separating out from the above the results on the Lummer-Brodhun we find the following, for color match:

TABLE I.

Observer	Type	Precision of single observation
Lummer and Brodhun..	Equality..............	" Mean error " (less than) 0.5 per cent.
Lummer and Brodhun..	Contrast (8 per cent.) ..	" Mean error " 0.4 per cent.
Lummer and Brodhun..	Contrast (3.5 per cent.).	" Mean error " 0.22 per cent.
Martens and Bechstein.	Equality..............	" Mean error " 0.59 per cent.
Martens and Bechstein.	Contrast (high?).......	" Mean error " 0.38 per cent.
Wild................	Not given.............	"Sensitivity " 0.7 per cent.
Kennelly and Whiting..	Contrast (high?).......	Probable error 0.5 per cent.
Kennelly and Whiting..	Equality..............	Probable error 0.75 per cent.
Kennelly and Whiting..	Concentric............	Probable error 1.0 per cent.
Millar................	Contrast (high?).......	Probable error 0.24 per cent.
Edwards and Harrison.	Contrast (low)..........	Average deviation 0.4 per ct.

It is rather difficult to reduce these errors to a common denominator. " Mean error," as used by Lummer and Brodhun and by Martens and Bechstein, is not clearly defined. It probably means " average deviation," *i.e.*, the average of the deviations of the several observations from their mean, rather than " mean error " which is seldom used and which is the square root of the mean square deviation. " Sensitivity " as measured, probably gives the average *maximum* departure from the mean. Probable error, E, and average deviation, a.d., are connected by the approximate equation,

$$a.d = 1.2\ E$$

Assigning equal weights to the four values for the 8 per cent. (high) contrast field given above, one obtains 0.4 per cent. as the *average deviation* of a single observation from the mean. The validity of this average is doubtful, for the four values were probably obtained under different conditions as regards field brightness, field size, etc.

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[10] *Trans. Ill. Eng. Soc.*, Vol viii, p. 633 (1913).

With regard to the effect of brightness and size of field on the precision of observations very few definite measurements have been published. König and Brodhun's[11] fundamental data on the least perceptible brightness difference apparently show that no important change in this quantity occurs in the range of brightness which corresponds to illuminations of 1 to 100 foot-candles on the photometer. Uppenborn,[12] however, made experiments with two observers which showed a maximum precision of setting with an illumination of about 30 meter-candles on the photometer. With this illumination the "mean error" of an observation was found to be about 0.5 per cent., while at 5 and at 50 meter-candles it was about 1.0 per cent. The type of photometer is not mentioned.

Uppenborn's observations were made with an exact color match. When color differences are involved in the measurements, an increase in the illumination in general increases the difficulty of equating the brightness of the two parts of the photometric field and tends to reduce the precision. Such measurements can therefore be made more easily and precisely at relatively low illuminations, but reliable quantitative data are not at hand to show how much is gained in precision in any particular case by reduction in brightness of the field.

The present paper is the result of an analysis of nearly 1500 sets of observations of from 10 to 20 settings each (approximately 20,000 settings in all) by 15 different observers. Some of these were experienced photometrists, and all were accustomed to making careful observations in various kinds of laboratory work. They were selected from the much larger group which took part in the work, not because of any peculiarities in their observations, but because they happened to have made the various settings which it was desired to analyze. For simplicity we shall designate the several observers by the letters B, C, D, F, G, H, J, K, M, N, R, S, T, V, and W, respectively.

The apparatus and the method of taking and recording the observations have already been described by the writers in the paper referred to. In all the observations, the left lamp and the photometer were kept stationary. The lamp at the right was moved by turning a wheel beneath the photometer. The settings of this lamp were printed on a sheet of paper fastened to a drum which was placed immediately beneath the carriage, and which, after each observation, was rotated automatically a slight amount so as to prevent one "record" falling on top of a preceding one. The distance of the lamp from the photometer is easily determined by measuring from reference lines on the paper. The photometer was provided with an eye-shield, which also served as a head-rest, and in some cases outside light was further excluded by a cloth thrown over the head. The observer was seated on a stool, the height of which could be adjusted so as to allow a comfortable position while observing.

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[11] Sitzungsber, Akad. Wiss. Berlin, 1888, p. 917.

[12] Uppenborn-Monasch Lehrbuch der Photometrie, R. Oldenbourg, Munich and Berlin, 1912, p. 212.

FIG. 1.

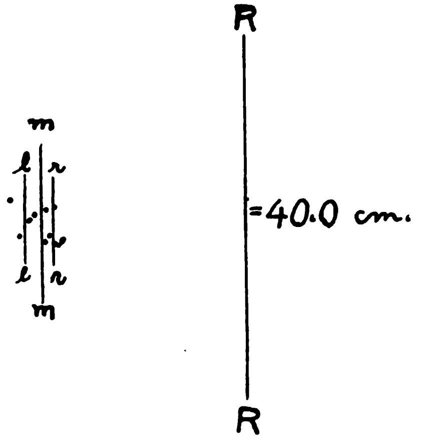

Fig. 1 represents a set of 10 readings. RR is the reference line, which in this case was 40.0 cm. from the photometer. The line mm was located "by eye" to represent the average of the 10 readings. Its distance from the photometer can at once be found. In this set it was 36.81 cm. (Actual tests showed that in this graphical method of locating the mean of a number of observations, unless the observations were very badly scattered, an error of more than one- or two-tenths of a millimetre was seldom made.) It is seen that four of the observations fall on the left of the mean, and six on the right. The line ll is the mean position of the points on the left, and rr the mean of the points on the right. The distance ml gives the average deviation to the left, and likewise mr to the right. In this set ml = 0.23 cm.

and mr = 0.15 cm. The average distance in cm. of the several observations from mm is

$$\text{Aver. dist.} = \frac{4 \times 0.23 + 6 \times 0.15}{10} = 0.18 \text{ cm.}$$

The average deviation of a single observation from the mean, expressed in per cent., is

$$\text{a.d.} = 100 \times \frac{2 \times 0.18}{36.81} = 0.98\%$$

This graphical method of determining average deviation has been used throughout. In like manner the maximum deviation of any single observation from the mean has been determined. This latter value should correspond approximately to the "sensitivity" as determined by Wild.

PRECISION IN HOMOCHROMATIC PHOTOMETRY.

These measurements with a color match were made with the Lummer-Brodhun 8 per cent. contrast photometer, with three different field colors: (1) 4 wpc carbon; (2) a blue resulting from filtering 4 wpc. carbon light through 1 cm. of a copper sulphate solution containing 53 grams per litre; (3) a yellow resulting from filtering 4 wpc carbon light through 1 cm. of a potassium bichromate solution containing 72 grams per litre. (These are the so-called blue and yellow solutions in the paper above referred to.) Although the contrast field was used, the observers in making the settings directed their attention not primarily to the contrast trapezoids, but to the narrow strips between the trapezoids, practically using therefore the equality method of setting, rather than the contrast. The following table gives the results:

Table II.

Color	Observer	Number of sets examined (10 to 20 settings each)	Average a.d. of a single observation	Average maximum deviation
Carbon	C	25	0.301	0.780
Carbon	R	44	.310	.828
Carbon	T	97	.314	.825
		Mean	0.308 per cent.	0.811 per cent.
Blue	C	67	.327	.830
Blue	R	138	.318	.780
Blue	T	74	.299	.723
		Mean	.315 per cent.	.778 per cent.
Yellow	C	40	.306	.824
Yellow	R	33	.342	.912
Yellow	T	71	.309	.813
		Mean	.319 per cent.	.850 per cent.
		Mean of all	.314 per cent.	.813 per cent.

The maximum deviation from the mean is about 2.5 times the average deviation from the mean. These values for average deviation seem to be sufficiently consistent to warrant the statement, that, with the photometer used, a careful observer, accustomed to the apparatus and taking data under favorable conditions, should maintain an average deviation of a single observation from the mean of its set of about 0.3 per cent. In a set of 25 observations the average deviation of the mean (A.D.), *so far as errors of setting are concerned,* should be about 0.06 per cent. It is interesting to note that this is the order of magnitude of the variation to be expected between the mean of sets if the relative voltage of the lamps is maintained constant to .01 per cent. In other words, if errors in photometric measurements of this kind, due to variation in voltage, are to be made small in comparison with the errors due to the photometric settings themselves, the voltage must be maintained accurate to the order of .001 per cent.

PRECISION WITH A SLIGHT COLOR DIFFERENCE.

In the course of measurements on the temperature coefficient of transmission of the two solutions, it was noticed that the potassium bichromate solution became distinctly redder with rising temperature. The measurements were made by placing one cell at 20° C. on one side of the photometer, and the other at temperatures of 25°, 30°, and 35°, successively, on the other side. For the temperature difference 20°–25°, the color difference in the two photometer fields was just barely perceptible. In the case of the temperatures 20° – 30° the difference in color was easily observed, while for the temperatures 20° – 35° it was very marked, although not nearly so great as the difference between a carbon and a tungsten lamp. The measurements were made by eight different observers, some of whom were not accustomed either to the apparatus or to the method of setting, and consequently the precision was somewhat lower. The results are given in Table III.

There appears to be no marked loss of precision arising from a *slight* color difference, since the first of these three values of average deviation, where there is practically no color difference, is almost exactly the average of the other two.

TABLE III.

Photometer: Lummer-Brodhun Contrast.

Color difference (in terms of temp. difference)	Observers	Total number of sets examined	Average a.d. of one setting	Average maximum deviation
			Per cent.	*Per cent.*
20°–25°	C, F, G, H, K, R, S, T.	23	0.393	1.25
20°–30°	C, F, G, H, K, R, S, T.	21	.386	1.04
20°–35°	C, F, G, H, K, R, S, T.	24	.404	1.11

PRECISION IN HETEROCHROMATIC PHOTOMETRY WITH LUMMER-BRODHUN CONTRAST PHOTOMETER.

Two color differences, most commonly met in practice, were studied: (1) 4 wpc carbon against 1.2 wpc tungsten, and (2) 4 wpc carbon against 0.65 wpc gas-filled tungsten. Ten observers took part in the tests. These will be divided into two groups: Group A, containing three observers, C, R, and T, whose work was analyzed in Table II, and who had had a comparatively large amount of experience in making settings with a color difference; and Group P, containing seven observers, only three of whom had had experience even in *homochromatic* measurements. The results are shown in Table IV. For the sake of comparison the average deviation obtained by these groups with a color match (4 wpc carbon) is shown. In all these measurements the effective field brightness was approximately 2.5 millilamberts. (Effective illumination, 25 meter-candles.)

TABLE IV.

Photometer: Lummer-Brodhun Contrast.

Color difference	Observers	Total number of sets examined	Average a.d. of one setting	Average maximum deviation
			Per cent.	*Per cent.*
None:	Group A:			
Carbon-carbon......	C, R, T.....	18	0.345	0.78
Carbon-carbon......	Group P: B, F, G, J, K, S, W.....	42	.58	1.60
Carbon vs. tungsten.	Group A......	6	.60	1.95
Carbon vs. tungsten.	Group P......	14	1.36	3.40
Carbon vs. gas-filled.	Group A......	18	1.00	3.45
Carbon vs. gas-filled.	Group P......	42	1.95	5.20

The progressive decrease in precision, with increasing color difference, is obvious. The proportionate change from color match to the color difference presented by a carbon and a gas-filled tungsten lamp is about the same for Group A as for Group P. Approximately it may be said that with the carbon-tungsten color difference the precision is one-half as great, and with the carbon-gas-filled one-third as great as with a color match.

An attempt was made to measure the transmission of the yellow solution and of the blue solution, mentioned above, by the Lummer-Brodhun photometer. Even with experienced observers the precision was so low that the experiments were abandoned.

PRECISION IN HETEROCHROMATIC PHOTOMETRY WITH THE FLICKER PHOTOMETER (KINGSBURY TYPE).

The arrangement of apparatus was similar in every respect to that with the Lummer-Brodhun. So far as possible, all noise and jar due to the rotating parts of the flicker attachment were eliminated, a precaution much more important in the case of the flicker photometer than in the case of the Lummer-Brodhun.

Within certain limits, the speed of the flicker photometer (light cycles per second) had little effect on the precision, as is shown by the following table:

TABLE V.

Showing Precision as a Function of Flicker Speed.

Effective illumination on photometer field approximately 25 meter-candles.
Observers: C, R, and T.
Color difference[12]: 4 wpc. carbon against 4 wpc. carbon filtered through the yellow (potassium bichromate) solution.

Speed (light cycles per second)	Total number of sets examined	Average a.d. for one setting	Average maximum deviation
		Per cent.	*Per cent.*
18	5	0.99	2.6
15	5	.95	2.1
12	5	.91	2.25
9	5	.94	2.5
6	5	1.32	3.75

Doubling the speed (*i.e.*, from 9 to 18 cycles per second) seemed to have little effect on the precision. In the data hereafter given a constant speed of 12 cycles per second was used.

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[12] For brevity of expression we will hereafter designate this color difference as "carbon-yellow," and similarly the expressions "carbon-blue," "carbon-tungsten," "carbon-gas-filled," etc., will have an obvious meaning.

In the course of the experiments, a large number of preliminary measurements were made on the transmissions of the yellow and the blue solutions by several different observers, only one or two of whom had had even a brief experience with the flicker photometer. The following table therefore gives the precision attained by observers reading the flicker photometer for the first time:

TABLE VI.

Flicker Photometer.

Color difference	Observer	Total number of sets examined	Average a.d. for one setting	Average maximum deviation
			Per cent.	*Per cent.*
Carbon-yellow	B	22	1.12	2.88
Carbon-yellow	C	38	1.19	2.78
Carbon-yellow	F	8	1.22	2.87
Carbon-yellow	K	8	1.45	4.13
Carbon-yellow	M	7	.70	1.89
Carbon-yellow	N	9	.72	1.64
Carbon-yellow	R	36	.54	1.54
Carbon-yellow	S	8	1.48	3.84
Carbon-yellow	T	4	.96	2.21
Carbon-yellow	V	9	1.32	3.40
		Mean	1.07	2.71
Carbon-blue	B	22	0.88	2.27
Carbon-blue	C	28	.73	1.74
Carbon-blue	F	8	.88	2.27
Carbon-blue	K	8	1.06	2.86
Carbon-blue	M	7	.74	1.66
Carbon-blue	N	9	.82	2.16
Carbon-blue	R	31	.47	1.09
Carbon-blue	S	8	1.10	3.04
Carbon-blue	T	4	.94	2.51
Carbon-blue	V	9	1.01	2.23
		Mean	.86	2.18

It is thus seen that even with these color differences, on which measurements with the Lummer-Brodhun photometer were practically impossible, a group of 10 inexperienced observers maintained, in over 3000 individual settings, an average deviation of a single observation from the mean of its set of less than one per cent. At this rate, the precision of the mean of a set of 25 readings, expressed in terms of average deviation, would be about 0.2 per cent.—a figure which amply justifies the statement that the flicker photometer may be called a precision instrument.

It must not be inferred, however, from the above that a group of experienced observers would attain proportionately higher precision. The ability to make precise measurements with the flicker photometer seems to depend on the physiological peculiarities of the observer, and perhaps on his physical and mental condition, even more than in the case of the Lummer-Brodhun. One observer, C, improved markedly in precision as a result of several weeks' experience, while the opposite is true with another observer, R. It is difficult to say to what extent the precision attained depends upon practice.

COMPARISON OF FLICKER WITH EQUALITY-OF-BRIGHTNESS MEASUREMENTS.

With the eye-piece focused on the four-part field of the comparison prism, the flicker attachment, at rest, could be so turned as to bring both fields into view simultaneously. In this position equality-of-brightness measurements by the ordinary method were possible. The photometric field in this case was a circle of about 2° aperture, whose vertical diameter formed the dividing line between the two parts of the field. Using this device a number of settings were made with a color match, and with a carbon-tungsten color difference. These same measurements were repeated with the flicker method. Table VII gives the comparative precision.

TABLE VII.

Comparison of Flicker with Equality-of-Brightness Measurements in 2° Field.

Observers: B, C, F, K, M, N, S, R, V. Two sets each.

Photometer	Color difference	Number of sets examined	Average a.d. for one setting	Average maximum deviation
			Per cent.	*Per cent.*
Equality of brightness.....	None: Carbon-carbon	18	0.82	1.91
Flicker..................	None: Carbon-carbon	18	.87	1.86
Equality of brightness.....	Carbon-tungsten.....	18	1.74	4.07
Flicker..................	Carbon-tungsten.....	18	.99	2.48

Of course, one would not expect this type of equality-of-brightness photometer to give as precise results as the usual Lummer-Brodhun. It is, however, worth noting that while it and the flicker give, for the same group of observers, approximately the same precision with a color match, the loss of precision with the carbon-tungsten color difference is small with the flicker, and much larger with the equality of brightness.

PRECISION OF THE FLICKER PHOTOMETER FOR COLOR DIFFERENCES COMMONLY MET IN PRACTICE.

To study this question advantage was taken of the flicker measurements with carbon-tungsten and with carbon-gas-filled color differences made simultaneously by the same group of observers which made the corresponding measurements with the Lummer-Brodhun contrast. (See Table IV.) In addition, measurements on the yellow and the blue solution, made at the same time, are appended for the sake of comparison. As before, the observers are divided rather arbitrarily into a more experienced group, Group A, and a less experienced group, Group P. The data is summarized in Table VIII.

TABLE VIII.

Photometer: Flicker.

Group A: Observers C, R, and T (more experienced).
Group P: Observers B, F, G, J, K, W (less experienced).

Color difference	Observers	Total number of sets examined	Average a.d. for one setting	Average maximum deviation
			Per cent.	*Per cent.*
None: carbon-carbon	Group A	18	0.68	1.55
	Group P	42	.87	2.09
Carbon-tungsten	Group A	6	.87	1.85
	Group P	14	.90	2.30
Carbon-gas-filled	Group A	12	.68	1.85
	Group P	28	1.13	2.75
Carbon-yellow	Group A	18	.85	2.05
	Group P	42	1.34	3.15
Carbon-blue	Group A	18	.75	1.80
	Group P	42	1.03	2.60

A comparison between Table VIII and Table IV shows that while with the carbon-tungsten color difference, the results obtained by the more experienced observers have a higher precision in the case of the Lummer-Brodhun than in the case of the flicker photometer, with the carbon-gas-filled color difference the advantage is distinctly in favor of the flicker. Further, the order of precision with the flicker photometer is practically the same for all color differences, and averages about 0.75 per cent. (average deviation of a single observation from the mean of its set). Expressed in terms of average deviation, the precision of the mean of a set of 10 readings, taken by the better observers, would be about 0.25 per cent. In the observations with the regular Lummer-Brodhun photometer on the carbon-gas-filled color difference, the average deviation of a single observation from the mean (Table IV), by experienced observers, is 1.00 per cent. and the precision of the mean of a set of 10 readings should be about 0.30 per cent.

The agreement between the means of two or more sets is, however, a better indication of the precision of the measurement than can be obtained from a single set. As indicated in the preceding paragraph, the order of magnitude of the precision of the mean, for carbon-gas-filled color difference is 0.25 per cent. for the flicker photometer, and 0.30 per cent. for the Lummer-Brodhun, as computed from the observations of a single set. It is of interest to ascertain to what extent these values agree with actual measurements.

The data from which Table IV and Table VIII were computed were used for this purpose. In taking these data, each observer made two independent sets of observations for the several color differences, with each photometer. A short time-interval, during which other observations were made, always elapsed between the two sets. The deviation of the two means from their average value was computed, and taken as representing the order of magnitude of the uncertainty of the measurement. Values so obtained were averaged for the ten observers, as shown in Table IX.

TABLE IX.

Agreement Between the Means of Successive Sets of Readings.

Observers: Groups A and P combined (ten in all).

Photometer	Color difference	Average deviation of the mean	
		Observed	Computed*
		Per cent.	*Per cent.*
L–B contrast	None: Carbon-carbon	0.16	0.13
Flicker	None: Carbon-carbon	.19	.22
L–B contrast	Carbon-gas-filled	.90	.43
Flicker	Carbon-gas-filled	.27	.26

* This "computed" value was determined as follows: The average a.d. was computed for each case from the data given in Tables IV and VIII. This average a.d. was then divided by the square root of the average number of readings per set, which, while varying somewhat, was very nearly 15.

The approximate agreement between the observed and the computed values in both flicker measurements and in the contrast photometer with no color difference is too consistent to be accidental. It shows that these measurements are reproducible to a reasonable degree. With the contrast photometer measurements on the carbon-gas-filled color difference the discrepancy between the observed and the computed values is very marked, and demonstrates conclusively what is a matter of common observation, that the individual observations in a single set of heterochromatic measurements taken with the ordinary equality-of-brightness or contrast photometer may agree fairly well among themselves, and the observations of a subsequent set may be in similar agreement, while the two sets may be so far apart that no two observations overlap. In other words, the measurements are not reproducible within the limits to be expected from examining the observations of a single set. It is as if the observer on making his first observation of a set, in his inability to decide when the differently colored fields are of the same brightness, makes a more or less arbitrary judgment, which for his eye, is probably correct within certain rather wide limits. His memory then helps him to make the next and succeeding observations in the same set with a reasonable precision, if the color difference is not too great. If some time elapses between the first and the second set of observations, his memory fails and he has to make a new, and probably a different, decision regarding the equality of brightness, which decision is followed rather consistently throughout that set. If he has practice enough his memory may hold over from one set to the next, and with more practice he may be able to settle upon a criterion which he follows thereafter, and which will differ in general from a similar criterion established by another observer, not so much because of any difference in the color vision of the two, as because of the accidental difference in starting point. If an observer, who had not already established his own criterion, were to make a large number of sets of observations on a given color difference with sufficient time between sets to wipe out the memory of the previous set, it is probable that this difference in accidental starting point could be averaged out, so that the result would give a true index of the observer's color sense, an index which, as shown in the paper by the writers to which reference has already been made, is given by the flicker photometer in a *single* set of observations.

CONCLUSIONS.

It has been shown that when using the Lummer-Brodhun contrast photometer for homochromatic measurements, experienced observers attained a precision, expressed in terms of average deviation of a single observation from the mean of its set, of approximately 0.3 per cent.; but that for heterochromatic measurements the precision decreased rapidly with increasing color difference. Further, except in the case of some observers of long practice, the heterochromatic measurements were not reproducible, the deviations of the means of successive sets being much greater than would be expected from the average deviation of a single observation from the mean of its set.

The precision attainable by the more experienced observers with the flicker photometer was in the neighborhood of 0.75 per cent. for the average deviation of a single observation. This value was not dependent on the color difference used, but was approximately the same for all color differences. Approximately, the ratio of the precision of the flicker to the Lummer-Brodhun photometer expressed in terms of deviation of a single observation, was for a color match 0.5; for the carbon-tungsten color difference, 1.0; and for the carbon-gas-filled, 1.5. Taking into account the greater reproducibility of the mean, it was found that even for the carbon-tungsten color difference the flicker photometer is the more precise instrument. The rather high field brightness used tended to favor the flicker photometer in these comparisons.

As stated at the beginning, the observations analyzed in this paper were not planned as a study of precision of measurement, and the data are by no means complete enough to indicate how the precision can best be increased. It is also realized that precision is only one of many factors which must be considered in choosing a photometer for any particular purpose, but it is a factor to which more study might profitably be given. In particular it should not be assumed that the sensibility or the precision is a constant characteristic of a type of instrument, but due recognition should be given to the physiological character of the ultimate instrument. Further increases in precision appear most likely to result from study of the conditions under which the eye operates best rather than from improvements in instruments.

PRELIMINARY NOTE ON THE RELATIONS BETWEEN THE QUALITY OF COLOR AND THE SPECTRAL DISTRIBUTION OF LIGHT IN THE STIMULUS.*

BY

IRWIN G. PRIEST.

Bureau of Standards.

I. INTRODUCTION.

It has long been established practice and is now recognized as a procedure of unquestioned validity to indicate the relative *brilliance* (brightness) of colors due to heterogeneous stimuli by the ratio of the integrals of light (" luminosity " or " luminous flux ") with respect to wave-length. It is also recognized that the *quality* of a color is determined by the spectral distribution of light, while, on the other hand, many different spectral distributions may excite colors of the same quality.[1]

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[1] Color is the *sensation* due to stimulation of the optic nerve.

The " adequate stimulus " of color is *light.*

Light (luminosity, luminous flux) is radiant power multiplied by the *visibility* of the radiant power in question. (*Radiant power* is the time rate of transfer of radiant energy.)

Relative visibility (function of wave-length) ≡

$$V_\lambda \equiv \frac{P_{V_{\max}}}{P_\lambda}$$

where $P_{V_{\max}}$ is radiant power of the wave-length which requires a minimum of power to excite a given brilliance (brightness) ; P_λ is radiant power of wave-length, λ; and $P_{V_{\max}}$ and P_λ excite colors of equal brilliance.

By introspective analysis, color is found to possess three characteristics, the specification of which completely describes the color. These are: *hue, saturation, brilliance* (brightness). Hue and saturation are determined by the spectral distribution of the stimulus. It is convenient to designate as *quality,* the property of color determined by hue and saturation together. *Brilliance* is determined by the amount of the stimulus. In so far as it is permissible to ascribe quantitative significance to it, brilliance may be regarded as proportional to the logarithm of the stimulus. (Fechner's Law.) (For further discussion of definitions, and bibliography of same, see Priest, Op. Soc. of Am., Com. on Standards and Nomenclature, Sub-com. on Colorimetry, Report, 1919 (Preliminary Draft), Bureau of Standards Library, Washington.)

The discussion in this present paper assumes brightness levels above the Purkinje effect.

But it does not appear that the general relations between colors due to various heterogeneous stimuli and the spectral distributions of these stimuli have ever been definitely formulated.[2] Indeed, it is not probable that any one has ever had available a collection of data sufficient to deal with this question as it is proposed to treat it in this paper. The author has now accumulated a considerable amount of data of diverse kinds which makes it possible to formulate one general condition which is always satisfied by two different spectral distributions which excite the same color. The purpose of this paper is to state this condition, together with some of the evidence supporting it.

II. DESCRIPTION OF ORIGINAL DATA USED AND THE GENERAL RELATION FOUND.

The primary data consist of pairs or sets of spectral distributions of radiant power which have been found by direct observation to excite colors of the same or nearly the same quality, although the distributions themselves differ markedly. These data may be classified as follows:

1. Data on Bittinger's Camouflage Paints.

There are six pairs of colored paints. The colors excited by the sunlight, reflected by the two members of each pair, are perfectly matched, or very nearly so, notwithstanding the fact that the spectral reflections of the two matched members are very different[3] (Figs. 1 and 2). The colors include two greens, two blues, a reddish purple and a brown.

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[2] The manner of treatment followed in this paper is perhaps vaguely foreshadowed by the work of Sir Isaac Newton (*Optics*, London, 1730, 4th Ed., pp. 134 to 141), and H. Grassman (*Zur Theorie der Farbenmischung*, Pogg. Ann. 89, pp. 69–84; 1853: *Ges. Werke*, Leipzig, 1904, Vol. 2, part 2, pp. 161–173). It is obvious, however, on consideration of the nature of the data dealt with in the present paper, that the similarity of treatment can be only of the most general nature, for such quantitative data were necessarily entirely lacking to these early investigators. In modern works the closest approaches which the author has found to the problem are given by Parsons, "Introduction to the Study of Color Vision," Cambridge, 1915, Part i, Sec. ii, Chap. iii; H. E. Ives, "The Transformation of Color Mixture Equations," *Jour. Frank. Inst.*, Dec. 1915, pp. 673–701; and Luckiesh, "The Physical Basis of Color Technology," *Jour. Frank. Inst.*, July and August, 1917.

[3] Report to Chas. Bittinger on B. S. Test No. 28,479, June, 1920.

FIG. 1.

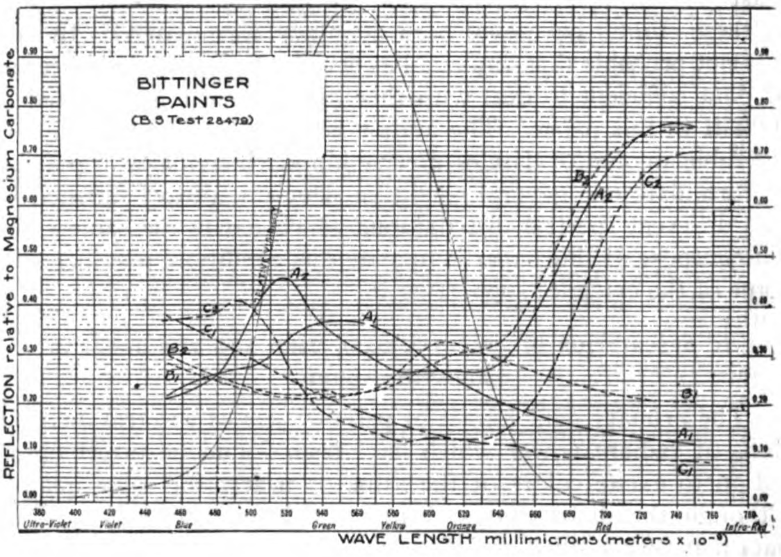
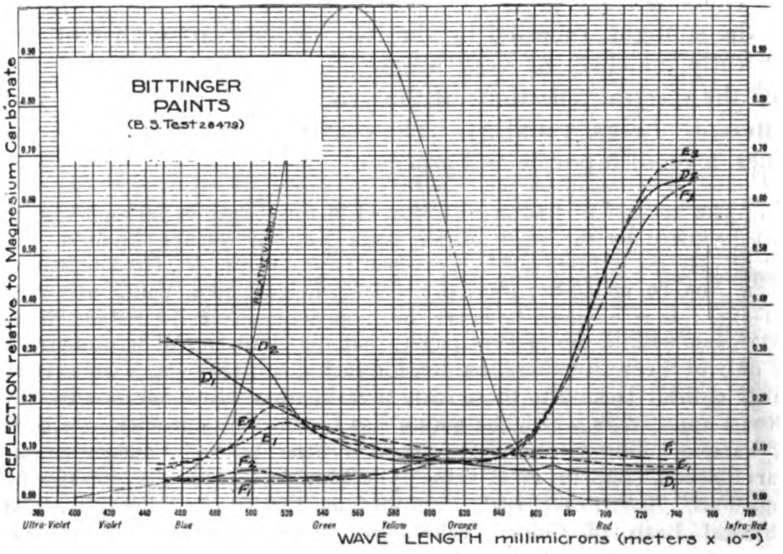

BITTINGER PAINTS
(B. S. Test 28479)
REFLECTION relative to Magnesium Carbonate
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
RELATIVE VISIBILITY
A₁
A₂
B₁
B₂
C₁
C₂
380
400
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
740
760
780
Ultra-Violet
Violet
Blue
Green
Yellow
Orange
Red
Infra-Red
WAVE LENGTH millimicrons (meters x 10^{-9})

FIG. 2.


BITTINGER PAINTS
(B. S. Test 28479)
REFLECTION relative to Magnesium Carbonate
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
RELATIVE VISIBILITY
D₁
D₂
E₁
E₂
F₁
F₂
380
400
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
740
760
780
Ultra-Violet
Violet
Blue
Green
Yellow
Orange
Red
Infra-Red
WAVE LENGTH millimicrons (meters x 10^{-9})

2. Data on Leucoscope[4] *Color Matches.*

There are two sets of data, each set comprising three spectral distributions. In each set the color due to the radiant power of a black body at a given temperature is approximately matched by the colors due, respectively, to two radiant powers of a markedly banded distribution, there being several bands with sharp maxima and minima distributed through the spectrum. The minima are actually zero; and the minima in one distribution coincide in wave-length with the maxima in the other distribution which excites the same or nearly the same color. The temperatures mentioned above are 2000° and 3000° K. (Figs. 9 and 10).

3. Data on Yellow Glasses, Yellow Oils and Color Matches of the Same Obtained by Rotatory Dispersion on the Arons Chromoscope.[5]

All of these stimuli are characterized by more or less absence of energy of short wave-lengths (blue and green), while the oils generally have marked absorption bands at about wave-lengths 610 and 670 millimicrons which do not occur in the glasses nor in the chromoscope[6] (Figs. 11 and 12). The colors range from about 35 yellow + 1 red to 35 yellow + 20 red on the Lovibond color scale. (Only part of the data available are published in this paper.)

These data have been treated as follows:

1. In the case of the data under 1 and 3, the observed or computed spectral reflection or transmission has been multiplied, by relative visibility and the relative energy of the sun, wave-

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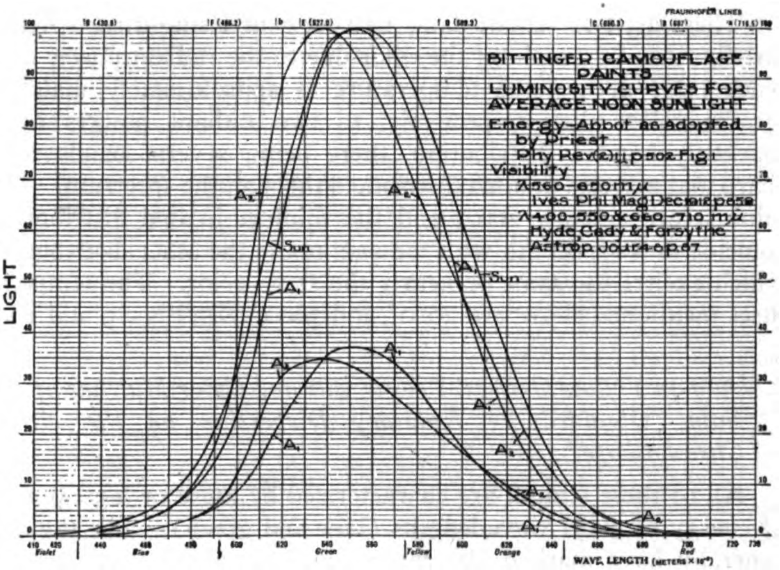
[4] Diro Kitao, Zur Farbenlehre, Inaug. Dis., Goettingen, 1878, and Abh. Tokio University, No. 12, 1885; Koenig, *Ann. der Phy.*, 17, pp. 990–1008, 1882; Brodhun, *Ann. der Phy.*, 34, pp. 897–918, 1888; Priest, "A New Study of the Leucoscope," forthcoming paper in JOUR. OF OPTICAL SOC. OF AMERICA.

[5] Leo Arons, *Ann. der Phy.* (4), 39, pp. 545–568, 1912; Priest, *Pro. Soc. Cot. Prod. Analysts*, May, 1914. (There are several serious misprints.)

Priest, "The Application of Rotatory Dispersion to Colorimetry," *Phy. Rev.* (2), 15, pp. 538, 539, 1920; and forthcoming paper, JOUR. OPTICAL SOC. OF AMERICA.

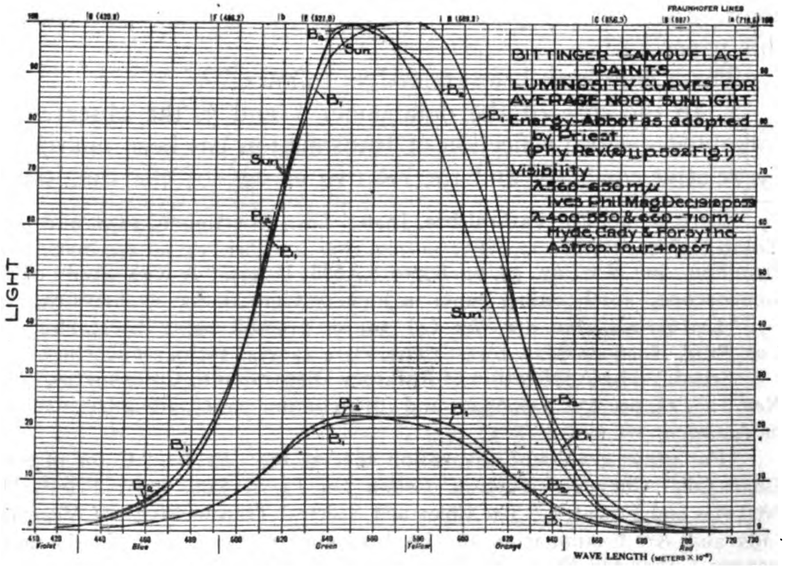
[6] For other graphs of absorption spectra see: Priest, "Color of Soya Bean Oil," Chemists' Section, *Cotton Oil Press*, Jan., 1920; Priest, McNicholas and Frehafer, "The Color and Spectral Transmissivity of Vegetable Oils and An Examination of the Lovibond Color Scale," forthcoming papers, *Cotton Oil Press* and *B. S. Tech. Pap.*

FIG. 3.



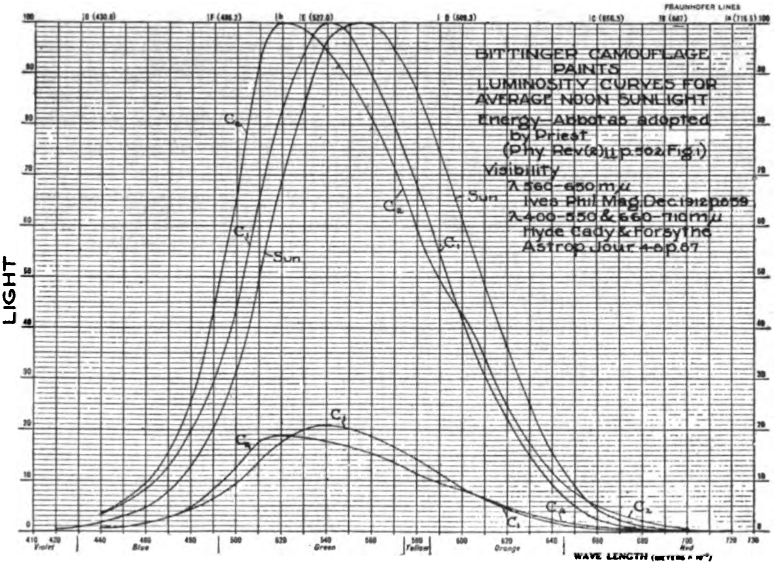
NOTE.—The lower curves represent the product:—*reflection* × *sunlight* × *visibility*. The higher curves are this same product *reduced to 100 at the maximum*.

FIG. 4.



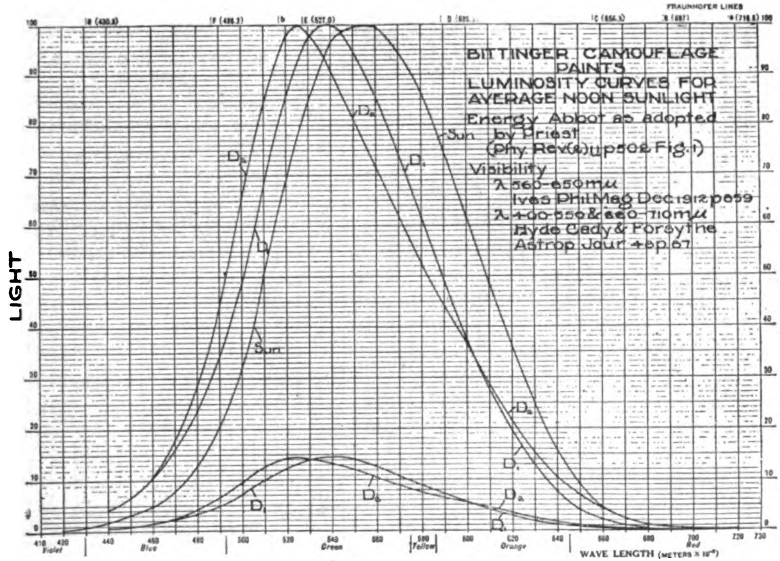
NOTE.—The lower curves represent the product:—*reflection* × *sunlight* × *visibility*,. The higher curves are this same product *reduced to 100 at the maximum*,

FIG. 5.



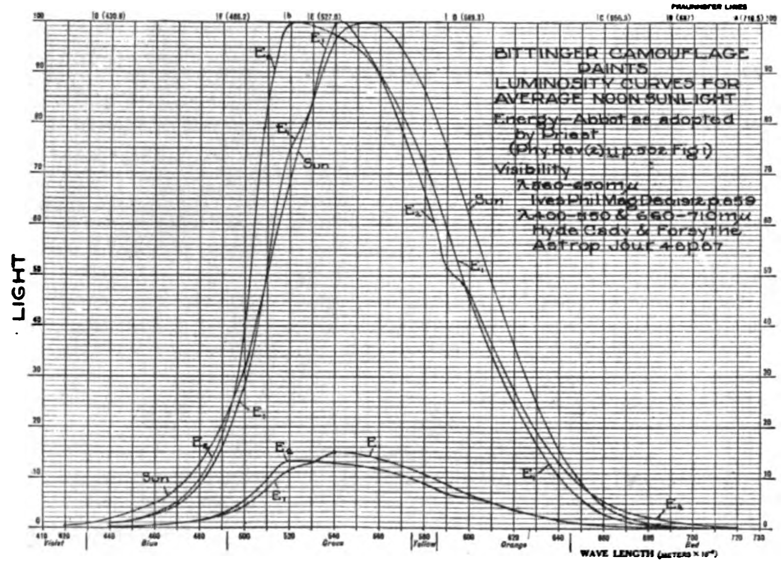
NOTE.—The lower curves represent the product:—*reflection* × *sunlight* × *visibility*. The higher curves are this same product *reduced to 100 at the maximum.*

FIG. 6.



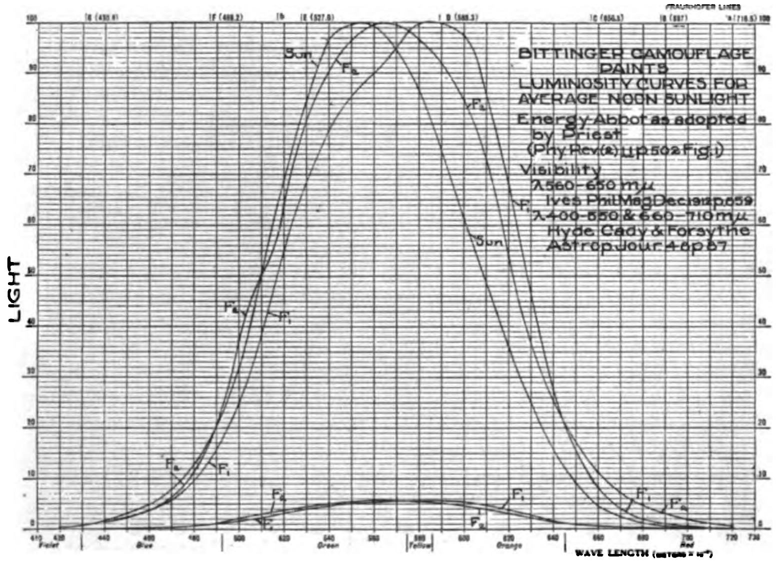
NOTE.—The lower curves represent the product:—*reflection* × *sunlight* × *visibility* The higher curves are this same product *reduced to 100 at the maximum.*

FIG. 7.



NOTE.—The lower curves represent the product:—*reflection* × *sunlight* × *visibility*. The higher curves are this same product *reduced to 100 at the maximum.*

FIG. 8.



NOTE—The lower curves represent the product:—*reflection* × *sunlight* × *visibility*. The higher curves are this same product *reduced to 100 at the maximum.*

FIG. 9.

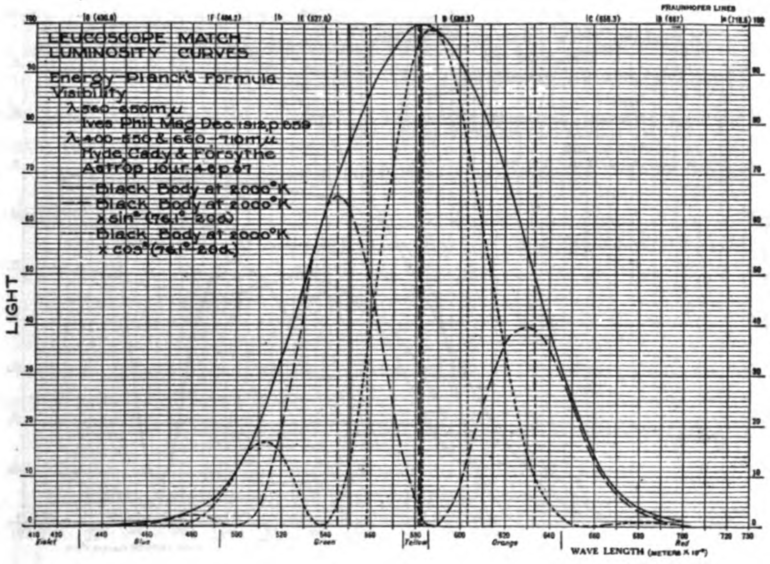

LEUCOSCOPE MATCH
LUMINOSITY CURVES
Energy–Planck's Formula
Visibility
λ 560–650 mμ
Ives Phil Mag Dec. 1912, p 659
λ 400–550 & 660–710 mμ
Hyde, Cady & Forsythe
Astrop Jour. 46 p 67
Black Body at 2000°K
Black Body at 2000°K × sin² (76.1°–20α)
Black Body at 2000°K × cos² (76.1°–20α)
FRAUNHOFER LINES
G (430.8)
F (486.2)
b
E (527.0)
D (589.3)
C (656.3)
B (687)
A (718.5)
LIGHT
0
10
20
30
40
50
60
70
80
90
100
410
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
730
Violet
Blue
Green
Yellow
Orange
Red
WAVE LENGTH (METERS × 10⁻⁹)

FIG. 10.

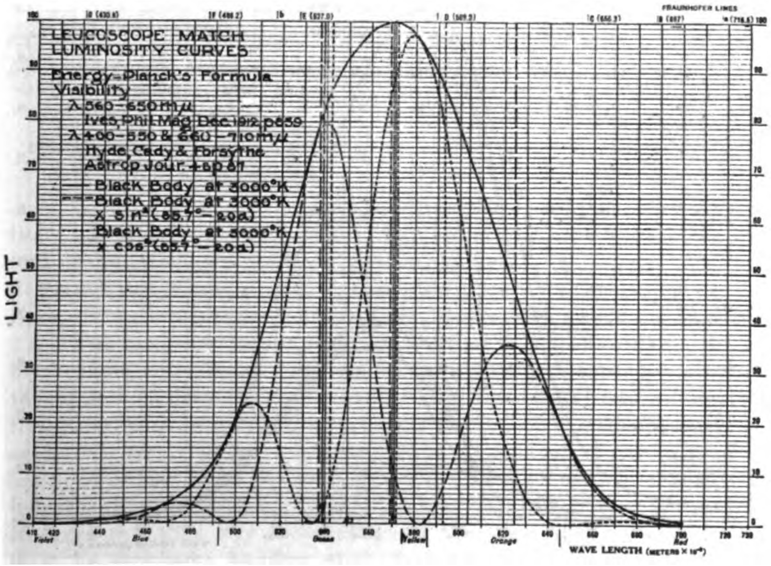

LEUCOSCOPE MATCH
LUMINOSITY CURVES
Energy–Planck's Formula
Visibility
λ 560–650 mμ
Ives, Phil Mag Dec 1912 p 659
λ 400–550 & 660–710 mμ
Hyde, Cady & Forsythe
Astrop Jour. 46 p 61
Black Body at 3000°K
Black Body at 3000°K × sin² (65.7°–20α)
Black Body at 3000°K × cos² (65.7°–20α)
FRAUNHOFER LINES
G (430.8)
F (486.2)
b
E (527.0)
D (589.3)
C (656.3)
B (687)
A (718.5)
LIGHT
0
10
20
30
40
50
60
70
80
90
100
410
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
730
Violet
Blue
Green
Yellow
Orange
Red
WAVE LENGTH (METERS × 10⁻⁹)

FIG. 11.

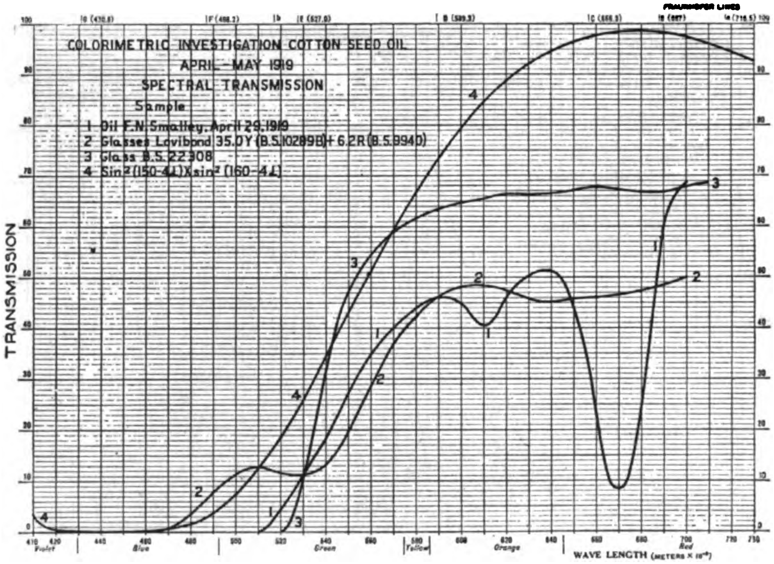
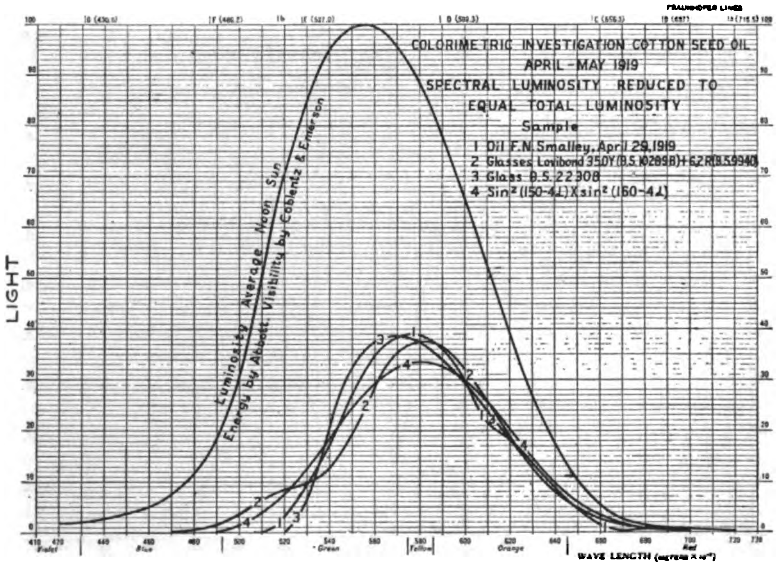

COLORIMETRIC INVESTIGATION COTTON SEED OIL
APRIL - MAY 1919
SPECTRAL TRANSMISSION
Sample
1 Oil F.N. Smalley, April 29, 1919
2 Glasses Lovibond 35.0Y (B.S.10289B) + 6.2R (B.S.9940)
3 Glass B.S. 22308
4 Sin² (150-4⊥) X sin² (160-4⊥)
FRAUNHOFER LINES
G (430.8)
F (486.2)
b
E (527.0)
D (589.3)
C (656.3)
B (687)
a (718.5)
TRANSMISSION
0
10
20
30
40
50
60
70
80
90
100
410
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
730
Violet
Blue
Green
Yellow
Orange
Red
WAVE LENGTH (METERS × 10⁻⁹)
1
2
3
4

FIG. 12.


COLORIMETRIC INVESTIGATION COTTON SEED OIL
APRIL - MAY 1919
SPECTRAL LUMINOSITY REDUCED TO
EQUAL TOTAL LUMINOSITY
Sample
1 Oil F.N. Smalley, April 29, 1919
2 Glasses Lovibond 35.0Y (B.S.10289B) + 6.2R (B.S.9940)
3 Glass B.S. 22308
4 Sin² (150-4⊥) X sin² (160-4⊥)
Luminosity Average Noon Sun
Energy by Abbott. Visibility by Coblentz & Emerson
FRAUNHOFER LINES
G (430.8)
F (486.2)
b
E (527.0)
D (589.3)
C (656.3)
B (687)
a (718.5)
LIGHT
0
10
20
30
40
50
60
70
80
90
100
410
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
720
730
Violet
Blue
Green
Yellow
Orange
Red
WAVE LENGTH (METERS × 10⁻⁹)
1
2
3
4

length by wave-length. In the case of the leucoscope data (2), relative energy by Planck's formula has been multiplied by visibility and the rotatory dispersion transmission. In all cases the resulting function of wave-length is *light* or "luminosity"; and the graph with this function as ordinate and wave-length as abscissa is a so-called "luminosity curve." For this purpose I have used data as follows:

(*a*) Spectral distribution of the sun's energy from Abbot as adopted by Priest.[7]

(*b*) Visibility: For wave-lengths 560 to 650 millimicrons, by Ives[8]; for wave-lengths 400 to 550 and 660 to 700, by Hyde, Forsythe and Cady.[9] (The author's own visibility, as determined by Coblentz and Emerson, is in close agreement with this average data.) In Table III and Fig. 12, visibility by Coblentz and Emerson[10] has been used.

2. The "luminosity curves" mentioned above have been plotted.

3. The wave-length of the centre of gravity of each of these curves has been determined.[11] This wave-length is designated as λ_c.

Considering all of these data (Tables I, II, III), the following rule has been found to hold without exception:

If any two lights, however different in spectral distribution, excite colors of the same quality, the wave-lengths of the centres of gravity of their spectral distributions are coincident.[12] The

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[7] *Phy. Rev.* (2), 11, p. 502, 1918, Fig. 1, open circles.

[8] *Phil. Mag.*, Dec., 1912, p. 859.

[9] *Astrop. Jour.*, **48**, p. 87, 1918.

[10] *B. S. Sci. Pap.* 303, Table 5.

[11] By means of an Amsler planimeter, making several check determinations. In nearly all cases, check determinations have been made by different operators in order to avoid blunders. In a few cases, check determinations have also been made by mechanical balancing on a knife edge. The final results obtained are accurate to about 0.5 millimicron.

[12] The generality of this conclusion is, of course, limited by the extent of the data considered. The reader will note, however, the diversity of the data, including the confirmatory data on monochromatic analysis in the latter part of this paper. The converse of this proposition is not necessarily true; two spectral distributions of light may have the same wave-length of centre of gravity and *not* excite colors of the same quality if the lights in the two cases are distributed over different ranges of wave-length. In order that they may excite colors of the same quality another condition must be satisfied. Further study is being given to the formulation of this condition.

TABLE I.

Bittinger's Paints.

Paint ↓	λ → Color ↓	Reflection relative to $MgCO_3$													
		460	500	520	540	560	580	600	620	640	660	700	740	Total sun-light	λ_c
A_1	Green*	0.23	0.28	0.33	0.37	0.36	0.34	0.28	0.24	0.21	0.18	0.15	0.13	0.320	558.2
A_2		0.22	0.38	0.45	0.36	0.31	0.27	0.27	0.27	0.28	0.39	0.67	0.77	0.324.	555.8
B_1	Reddish purple	0.27	0.22	0.21	0.21	0.22	0.26	0.32	0.32	0.29	0.27	0.23	0.21	0.248	565.4
B_2		0.28	0.23	0.22	0.23	0.22	0.24	0.28	0.31	0.33	0.43	0.70	0.76	0.246	564.4
C_1	Blue	0.36	0.29	0.25	0.22	0.19	0.16	0.14	0.13	0.12	0.10	0.09	0.09	0.196	548.8
C_2		0.37	0.39	0.28	0.18	0.15	0.13	0.13	0.13	0.14	0.22	0.55	0.71	0.191	547.1
D_1	Blue	0.32	0.22	0.18	0.15	0.12	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.135	546.1
D_2		0.32	0.30	0.21	0.13	0.10	0.09	0.09	0.09	0.09	0.15	0.47	0.65	0.139	544.4
E_1	Green	0.08	0.13	0.16	0.16	0.14	0.12	0.11	0.10	0.10	0.09	0.08	0.07	0.132	555.9
E_2		0.08	0.17	0.19	0.13	0.12	0.10	0.10	0.10	0.11	0.15	0.47	0.69	0.125	554.2
F_1	Reddish brown	0.04	0.04	0.05	0.05	0.05	0.07	0.09	0.11	0.10	0.11	0.10	0.09	0.0633	571.8
F_2		0.05	0.07	0,05	0.05	0.06	0.06	0.08	0.08	0.09	0.14	0.42	0.63	0.0646	568.2

* A_1 is a slightly more yellowish green than A_2, which corresponds with their respective values of λ_c. The other pairs (paints designated by the same letter) are very closely color-matched. Compare with Figs. 1 to 8, inclusive.

TABLE II.

Leucoscope.

Fig. No.	Curve	λ_c	Relative color by direct observation
9	Dashed	581.5	Pale bluish.
	Solid	582.6	
	Dotted	583.2	Yellowish, more saturated.
10	Dashed	568.9	Bluish.
	Solid	570.8	
	Dotted	571.9	Yellowish.

NOTE: Compare with spectral light distributions in Figs. 9 and 10. The three distributions shown in each figure excite colors of the same or nearly the same quality. The slight relative differences in quality are indicated in the last column of this table, and it will be observed that they correspond with the small differences in λ_c.

TABLE III.

Oil, Glasses and Rotatory Dispersion (Arons Chromoscope).

Numbers of curves, Figs. 11 and 12	λ_c
1	583.2
2	581.8
3	583.3
4	583.4

NOTE: Compare with spectral transmission curves in Fig. 11 and luminosity curves in Fig. 12. These four samples excite colors of the same or nearly the same quality in daylight.

small discrepancies are of the same order of magnitude as the wave-length hue limen, that is, about 1 millimicron (metre × 10^{-9}).[13] In the case of colors, nearly, but not perfectly matched in quality, the small differences observed in λ_c are in the sense to be expected from the apparent slight difference in hue. (A_1 and A_2, Table I; and Table II.)

III. RELATION TO MONOCHROMATIC ANALYSIS AND CONFIRMATION BY PREVIOUS DATA.

"Monochromatic analysis"[14] may be regarded as a special case of finding, by trial, a particular type of spectral distribution which will excite color of the same quality as another distribution.[15] In the mixture of homogeneous light and sunlight determined by this method, the wave length of the centre of gravity will be given by:

$$\lambda_c = \frac{\Lambda l_\Lambda + \lambda_{cs} l_s}{l_\Lambda + l_s} \tag{1}$$

where

Λ = wave length of dominant hue,
λ_{cs} = wave length of centre of gravity for sunlight,
= 560.0 millimicrons,
l_Λ and l_s are respectively the amounts of homogeneous light and sunlight mixed.

If $\Pi = \frac{l_\Lambda}{l_\Lambda + l_s}$, this reduces to:

$$\lambda_c = \Pi\Lambda + \lambda_{cs}(1 - \Pi). \tag{2}$$

Λ and Π being the data given by monochromatic analysis, it is obvious that λ_c for the mixture of homogeneous light and sunlight can be readily computed by this equation. If the rule stated above[16] is universally valid, this value of λ_c should equal the value of λ_c for any spectral distribution which excites color of the same quality as the color excited by the mixture of homogeneous light and sunlight specified by Λ and Π. I have tested this in a number of cases and have confirmed the rule in every instance.

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[13] Steindler, *Wien. Sitz.* 115, 2 A., p. 39, 1906; Nutting, *Bull. B. S.* 6, pp. 89–93, 1909. (The decimal point in the tabulated values column 2, Table II, is one place too far to right.) Jones, JOUR. OPTICAL SOC. OF AMERICA, 1, pp. 63–77, 1917.

[14] Nutting, *Bull. B. S.* 9, p. 1, 1913.

[15] Op. Soc. of Am., Com. on Standards and Nomenclature, Sub-com. on Colorimetry, Report, 1919 (Preliminary Draft), pp. 28 and 38. Copy in Bur. Stands. Library, Washington.

[16] Page 397.

The data used for this purpose may be classified as follows:

1. Jones' data on Λ *and* Π *for a number of fairly reproducible light sources,*[17] *together with Hyde and Forsythe's data on the color temperature of the same sources.*[18] We have computed λ_c from "color temperature" on the basis of Planck's formula and the same visibility as described above (Ives and Hyde, Forsythe and Cady); and also λ_c from Jones' values of Λ and Π. The results are compared in Table IV.

TABLE IV.

Source	From color temp.		From Monoc. analysis		
	Temp.	λ_c	Λ	Π	λ_c
Hefner	1875° K.	585.0	593.0	0.86	588.3
Pentane	1914	584.0	592.0	0.85	587.1
Tung, 1.25 w. p. c.	2385	577.2	588.0	0.65	578.0
Acetylene	2360	577.5	585.5	0.64	576.4

2. Jones' data on Λ *and* Π *for Soya bean oil of color 35 yellow +7.2 red, Lovibond scale,*[19] *together with our own average value of* λ_c *derived from the actual spectral distribution of sunlight transmitted by oil of this same color on the Lovibond scale.* The results are:

	λ_c
From Jones' $\Lambda = 587.7$ and $\Pi = 0.84$	583.3
From our distribution data	583.4

It appears on consideration of these results, and those in Table IV, that the rule in regard to equality of the values of λ_c for different spectral distributions which excite colors of the same quality is confirmed to within the reliability of the data. It is to be remembered that Λ and Π on one hand and "color temperature" on the other were not determined for the same sources, but for sources more or less accurately reproduced from specifications; and it will be noticed that the agreement of the values of λ_c is closest for the source which is the most accurately reproducible, the vacuum tungsten lamp at 1.25 w.p.c.

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[17] L. A. Jones, *I. E. S.* 9, p. 691, 1914.

[18] Hyde and Forsythe, *Jour. Frank. Inst.*, 183, pp. 353, 354, 1917.

[19] Wesson, *Cotton Oil Press*, Chemists' Section, Table I, p. 66, July, 1920.

The following corollaries from equation (2) present points of interest:

1. If $\Pi = 1$ or if $\Lambda = \lambda_{cs}, \lambda_c = \Lambda$.

2. If $\Pi \neq 1$ and $\Lambda \neq \lambda_{cs}$, Λ lies on the side of λ_c remote from λ_{cs}.

3. If Π = zero, Λ is indeterminate as would be expected (since, if Π= zero, λ_c must equal λ_{cs}).

The author is indebted to Paul D. Foote for advice on the method of determining the centre of gravity and to K. S. Gibson, H. J. McNicholas, Mabel K. Frehafer, C. M. Blackburn, M. C. Malamphy, C. L. Snow, J. H. Turner, Jr., and Gertrude Clemens for assistance in observations and in carrying through the intricate computations and plotting of curves necessary to obtain the above results.

NATIONAL BUREAU OF STANDARDS,
August 9, 1920.

NOTE ON THE RELATION BETWEEN THE FREQUENCIES OF COMPLEMENTARY HUES.*

BY

IRWIN G. PRIEST.

THERE are available considerable data on the wave-lengths of complementary hues.[1] It appears that such data have always been presented and discussed in terms of wave-length, and not in terms of frequency.[2] It has been suggested that the curves relating complementary wave-lengths "nearly resemble" (rectangular) "hyperbolæ."[3] Inspection of Fig. 1 will show that the approximation to a rectangular hyperbola is not at all close; the alleged likeness is rather far-fetched.

It might reasonably be expected that the relation between frequencies would be simpler than that between wave-lengths. From the point of view of physiologic optics, wave-length can be nothing more than a purely arbitrary reference scale; while it is at least a reasonable hypothesis that some comparatively simple relations may exist between the retinal response and the *frequency* of the stimulus. The author has previously shown that retinal visibility as a function of frequency is simpler than as a function of wave-length.[4] In this paper it is proposed to show that the relation between the frequencies of complementary hues can be represented in a simple way which is at least suggestive of simple relations between the retinal response and frequency.

The data mentioned above have been replotted, as shown in Fig. 1. A curve has been drawn to represent, as well as may be, these data. From this curve pairs of complementary wave-

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* Published by permission of the Director, Bureau of Standards.

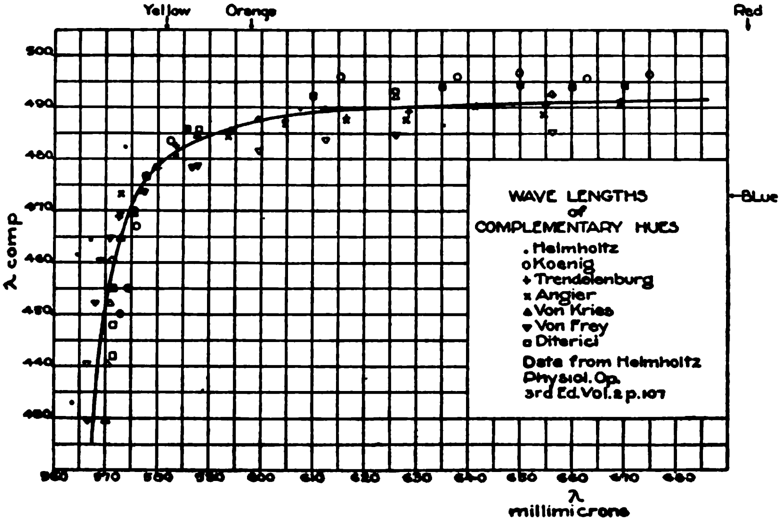
[1] Helmholtz, v. Kries, v. Frey, Koenig, Angier, Trendelenburg and Diterici as quoted in Helmholtz: "Physiol. Optics," 3rd Ed., Vol. 2, p. 107.

[2] Helmholtz: "Physiol. Optics," 2nd Ed. (1896), pp. 316–319. Helmholtz: "Physiol. Optics," 3rd Ed. Vol. 2, pp. 105–107. Parsons: "Introduction to Study of Color Vision" (Cambridge, 1915) Part I, Sec. II, Chap. III. Rood: "Modern Chromatics" (New York, 1879), p. 175. Köllner: "Störungen des Farbensinnes" (Berlin, 1912), p. 14. Luckiesh: "Color and Its Applications" (New York, 1915), pp. 59 and 75. Grünberg: Sitzb. Ak. Wein, II A, 113, pp. 627–636; 1904.

[3] Parsons: "Introduction to Study of Color Vision," p. 35. Grünberg: Sitzb. Ak. Wien, II A, 113, pp. 627–636; 1904.

[4] *Phy. Rev.* (2) 11, 498, 1918.

FIG. 1.


Yellow
Orange
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λ comp
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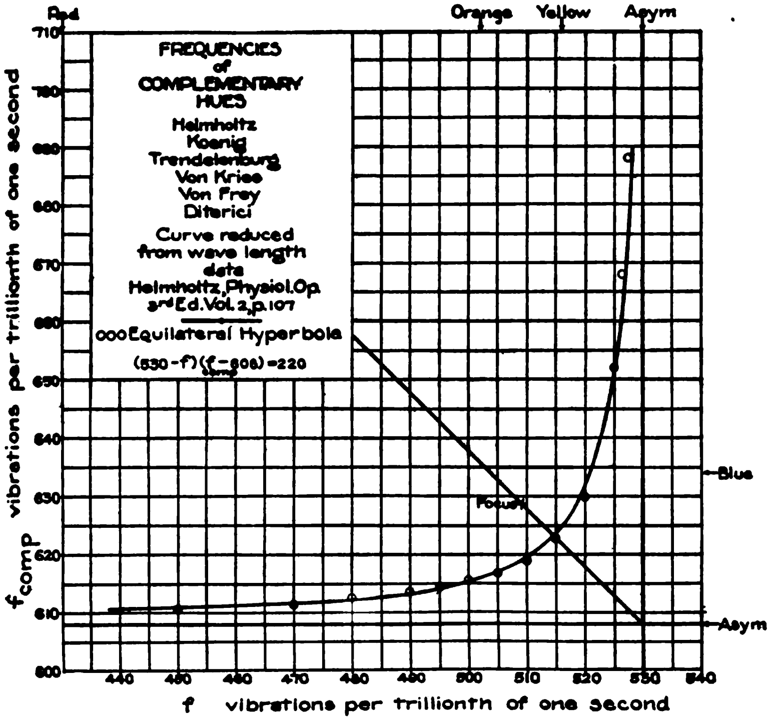
680
λ
millimicrons
WAVE LENGTHS
of
COMPLEMENTARY HUES
. Helmholtz
o Koenig
+ Trendelenburg
x Angier
▲ Von Kries
▼ Von Frey
▫ Diterici
Data from Helmholtz
Physiol. Op.
3rd Ed. Vol. 2 p. 107

FIG. 2.


Red
Orange
Yellow
Asym
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700
690
680
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660
650
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630
620
610
600
f_{comp} vibrations per trillionth of one second
FREQUENCIES
of
COMPLEMENTARY
HUES
Helmholtz
Koenig
Trendelenburg
Von Kries
Von Frey
Diterici
Curve reduced
from wave length
data
Helmholtz, Physiol. Op.
3rd Ed. Vol. 2, p. 107
ooo Equilateral Hyperbola
$(530-f)(f_{comp}-608)=220$
Focus
Blue
Asym
440
450
460
470
480
490
500
510
520
530
540
f vibrations per trillionth of one second

lengths have been read. These wave-lengths have been converted into frequencies;[5] and the relation of frequencies has been plotted (solid curve, Fig. 2). The circles in Fig. 2 have been plotted from the equation of a rectangular hyperbola:

$$(530-f)\ (f_{comp}-608)=220$$

That is: the relation between the frequencies of complementary hues can be represented closely by a rectangular hyperbola of which the asymptotes are:

530 and 608

and the focus is:

509.2, 628.8

There is at present considerable interest in another redetermination of the visibility function. In order that such a determination might yield the largest results, the author would suggest that visibility should be represented as a function of frequency and the visibility of each subject should be correlated with:

1. The frequencies of his spectral complementaries.
2. His hue sensibility to frequency differences as a function of frequency.
3. The "black-body" spectral energy distribution which he recognizes as gray.[6]
4. The relative intensities of homogeneous complementaries required to be mixed to color match gray specified by the findings under (3) above.

By the correlation of these functions we might reasonably expect to gain a new insight into the various phenomena of color and perhaps elucidate somewhat the nature of the retinal response and its relation to the stimulus.

The author is indebted to Mr. C. M. Blackburn and Mr. J. A. Turner, Jr., for assistance in reducing the data, and to Miss Gertrude Clemens for the preparation of the figures.

NATIONAL BUREAU OF STANDARDS.
July 26, 1920.

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[5] f (in vibrations per trillionth of one second) $= \dfrac{300{,}000}{\lambda \text{ (in millimicrons)}}$

[6] Troland: Trans. I. E. S., 13, pp. 26–27; 1918. Priest: Trans. I. E. S., 13, pp. 75–77; 1918.

REVIEWS.

THE ORGANIZATION OF INDUSTRIAL SCIENTIFIC RESEARCH, C. E. Kenneth Mees, D.Sc., Director of the Research Laboratory, Eastman Kodak Company. 170 pages, 5½ × 8, illustrated with 10 charts; $2.00 net, postpaid; McGraw Hill Book Company.

The experience of Dr. Mees both in England and in the United States abundantly qualifies him to write a most timely and instructive book upon the organization of industrial, scientific research. His book is stated to be intended for the manufacturer who is ready to organize a research laboratory, but who wants to know what such a laboratory will cost to start and maintain, what should be expected of it, what position in the organization it should occupy, how it should be directed, and where competent men may be secured for it.

Following a most comprehensive introduction, there are chapters on research and coöperative laboratories, on their relation to the plant, on the organization, staff, direction, equipment, and building design; there is also a very complete bibliography and an index of laboratories. The manufacturer will find here concise and ready answers to perhaps all but the last question—How may competent research men be obtained?

The book, if apparently written for the business man, will be nevertheless of the greatest interest to scientific workers and all who are responsible for, or having to do with, scientific production. The pages teem with words of sound advice regarding the relation of the scientific man to his problem, to the organization of which he is a part, to his fellows, to publication, to the community. Much of what is so ably set forth is applicable not only to the laboratories of industrial establishments but to centres of research generally, and one cannot be but impressed by the orderly and systematic method of conducting research here set forth.

Dr. Mees considers very promising the future of coöperative laboratories, for types of industry with many relatively small or scattered units, especially in those industries in which little scientific work has been done and where no strong feeling of exclusiveness and secrecy prevails. He well states the limitations of the university in its relation to industrial research, but could have put greater emphasis upon the possibilities of the coöperative regional aspects; for example, the inter-relation of certain of the technical schools with nearby industries, as exemplified in the Sheffield Scientific School of Yale University and the brass industry; the Carnegie Technical School and the steel industry, and others. The "Technology Plan" of the Massachusetts Institute, of very far reaching possibilities, has been inaugurated since this book was written.

Dr. Mees seems to stand on less sure ground when discussing the relations of government to research, and appears, for example, to consider with some hesitancy the advantages to be gained from the recent British developments as illustrated by the formation of the Department of Scientific and

Industrial Research. Again, the rôle of the Government of the United States in the fostering of industrial research is hardly given in its proper perspective; thus the great, industrial, scientific, research organization which is the Department of Agriculture is passed over all. too briefly. The part played both directly and indirectly by such government institutions as the Bureau of Standards in organizing, executing, and stimulating industrial research in its own and other laboratories, and its service to industry in furnishing both ideas and men accomplished in research, could have well received more attention.

Industrial laboratories are classified in three general divisions: (1) works laboratories exerting analytical control over materials, processes, and product; (2) industrial laboratories working on improvement in product and in process, tending to lessen cost of production and to introduce new products on the market; (3) laboratories working on pure theory and on the fundamental sciences associated with the industries; and their respective scope and relation one to the other are well set forth. In this connection an interesting phenomenon is to be noted in the evolution of industrial research laboratories that in certain striking instances they have been practically compelled not only to act as development laboratories and semi-manufacturing plants, but actually to take over the permanent manufacturing of special products originating in the laboratory; in other words, industry developing from the laboratory rather than an industry establishing a laboratory.

Laboratories may also be grouped as "divergent" or "convergent" depending upon whether the problems investigated are all connected with one subject or cover a disjointed or scattered range of subjects. For either type there are two forms of organization possible, designated as "departmental" and "cell" systems which are discussed in detail.

The chapters on the staff and direction of a research laboratory are perhaps of greatest interest to the scientific man; they are based largely on the highly satisfactory experience and practice of the author at the Eastman Kodak Plant, although he has drawn generously from the writings and experience of others.

It is encouraging to note that Dr. Mees frowns on secrecy, although maintaining this is necessary in very exceptional cases, and secret processes should be confined to very few people. The spirit of the laboratory should be always to encourage publication.

G. K. BURGESS,
August, 1920.

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THE TESTING OF PLANE SURFACES BY INTERFERENCE METHODS.*

BY
C. G. PETERS and H. S. BOYD.

I. INTRODUCTION.

THE planeness of an optical surface is most readily and accurately determined by means of the interference fringes seen in monochromatic light. In making this test it is usually assumed that straight fringes equally spaced indicate that the surface is plane and that curved fringes indicate how much the surface deviates from true plane. These assumptions are correct for normal incidence; but when the surface is viewed obliquely, as is usually the case, they may introduce considerable errors in the determination of planeness. The purpose of the present paper is to point out the best conditions for making the test and the errors introduced by treating oblique incidence as normal.

II. INTERFERENCE OF LIGHT.

If two trains of waves from one point in a source having traversed different paths fall upon a point on the retina of the eye the resultant amplitude of vibration determines the brightness. If they are " in step " maximum brightness results; if, however, the troughs of the one arrive with the crests of the other, destructive interference takes place resulting in relative darkness. If the two trains travel different distances so that the difference in path is some whole number of wave-lengths, then the waves will reach the eye in phase. If the difference in path is equal to some whole number of wave-lengths plus one-half wave-length, the waves in the two trains will be in opposite phase so that destructive interference takes place. The conditions for interference are realized when light from an extended source S (Fig. 1) falls on a thin transparent film. Part of the light is reflected from the first surface, ABCD, and the remainder is transmitted to the second surface, ABGF, where partial reflection again takes place. Since the wave trains reaching the eye E from these two reflections have travelled over different distances, reinforcement or destructive interference therefore can occur. When white light is used and the film is sufficiently thin, a few brightly colored bands are seen across the surface. If monochromatic light, that is, light of one color or of very limited spectral extent, be employed, alternate light and dark bands or interference fringes extend across the film.

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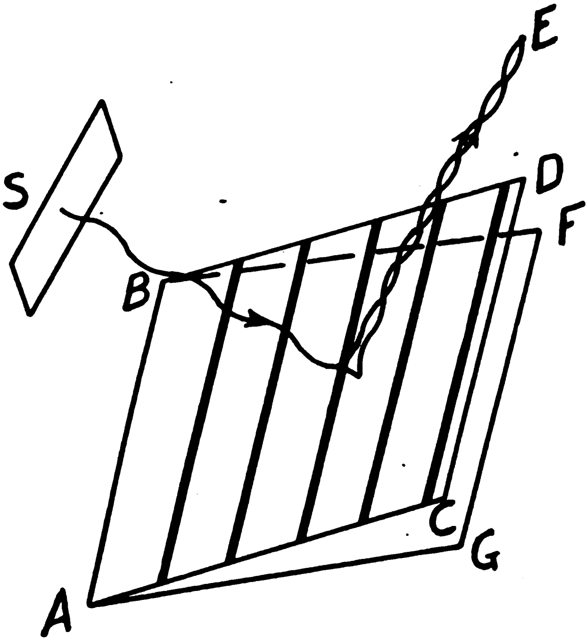
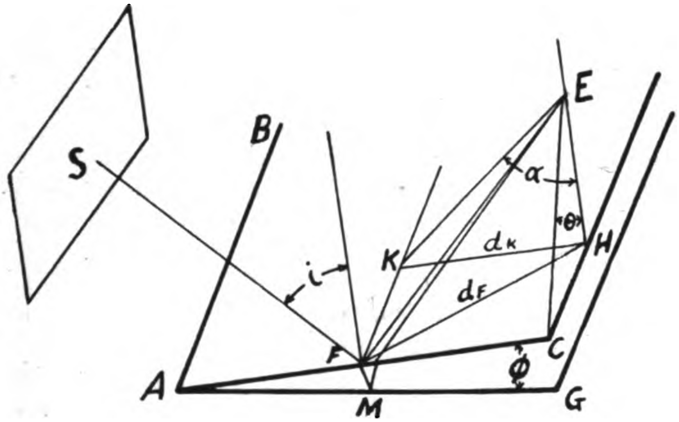
FIG. 1.



Interference in a thin film.

Suppose we have two plane surfaces ABC and ABG (Fig. 2) which are in contact along the line AB and inclined at a slight angle ϕ. A ray of light from a point in the source S reaches the point F, making an angle of incidence i with the normal to the surface ABC. At F this ray is split into two parts, one of which reaches the eye E along the path FE, while the other, reflected at M, travels along the path FME. The effect produced at the eye by these two parts will depend upon the difference in length Nλ of the two paths, where λ is the wave-length of the light and N the number of waves, or order of interference as it is usually called. If N is any integer the waves will be out of phase, due to half a wave-length phase change[1] by reflection at the denser medium, and the point F will in that case appear dark. The difference in path of these two rays coming from F is given by the equation.[2]

FIG. 2.



Interference in a thin film.

$$N\lambda = 2\frac{T + P\tan\phi\tan\alpha}{\sqrt{1 + \tan^2\alpha + \tan^2\theta}} \qquad (1)$$

where P is the perpendicular distance (EH) from the eye to the surface; T is the distance between the surfaces at H; and α and θ are equal to the respective projections of the reflection angle FEH on the two planes through EH perpendicular and parallel to AB.

The numerator of the fraction in equation (1) represents the thickness t of the film at any point say F and $\tan^2\alpha + \tan^2\theta$ can be replaced by $\frac{d^2}{P^2}$ where d is the distance H to F. Equation (1) then becomes

---

[1] Only under special cases is this phase change exactly a half wave length; but it is seldom necessary, as in the present case, to determine its exact value.

[2] Michelson: *Phil. Mag.*, Vol. 13, p. 239; 1882.

$$N\lambda = \frac{2t}{\sqrt{1 + \frac{d^2}{P^2}}} \tag{2}$$

from which

$$t = \frac{N\lambda}{2}\sqrt{1 + \frac{d^2}{P^2}} \tag{3}$$

Therefore from a knowledge of the order N and the distances d and P the exact thickness of the film at any point can be determined. If one boundary surface of the film is plane it is possible to determine the nature of the other surface from measurements of the film thickness at various positions. This is the method commonly used for testing the planeness of optical surfaces.

III. TEST FOR PLANENESS OF SURFACES.

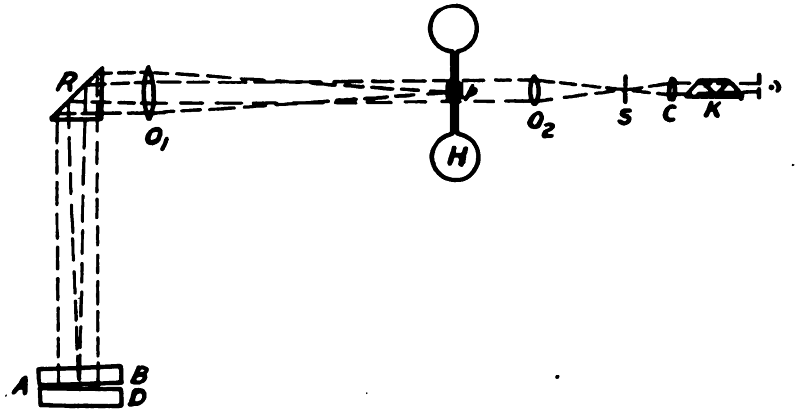
1. *Perpendicular Incidence.*—In testing the planeness of polished surfaces, such as are produced on prisms, surface plates, precision gages or micrometer anvils, a test plate is placed in quite close contact with, and slightly inclined to, the surface to be tested, thus forming a thin wedge-shaped film discussed in the previous section. This test plate is of glass, one surface of which has been polished accurately true plane and tested against a master true plane or liquid surface of large extent. The accuracy of the test is of course limited by the irregularities of the test plate surface. It is very difficult to make glass surfaces fifty to seventy-five millimeters in diameter plane within .25 micron, and to reduce this error requires exceptional skill. For ordinary work, however, test plates of the accuracy just mentioned are sufficient, but when it is necessary to determine irregularities of a few hundredths of a micron in the surface under test, great care must be exercised in selecting and testing the test plate.

In order to give a definite value to the wave-length λ the thin wedge-shaped film of air, formed between the plane surface of the test plate and the surface under test, is illuminated with monochromatic light. A convenient source of monochromatic light is a bunsen flame in which is inserted a piece of asbestos soaked in a salt solution, or a ground glass plate illuminated either by a helium lamp operated on a 5000-volt A. C. circuit, or by a mercury vapor lamp covered with a green absorption screen. The wave-lengths of the most effective visible radiation from these sources are approximately:

Sodium (yellow) = .589 micron
Helium (yellow) = .588 micron
Mercury (green) = .546 micron

A colored glass screen illuminated by an incandescent lamp or ordinary daylight may be used as a source if high precision is not desired, but the light will not be sufficiently monochromatic to allow assignment of a definite value to the most effective wave-length.

FIG. 3.



Pulfrich interferometer.

A very convenient instrument for illuminating the film and at the same time viewing the interference fringes on the film is one designed by Pulfrich[3] and shown in Fig. 3. The light from a helium lamp H is focused upon a small total reflection prism p. After being collimated by the lens O_1 it is reflected by the prism R down to the interferometer ABD which is in the focal plane of the lens O_1. The rays reflected by the film surfaces are brought to a focus by the lenses O_1 and O_2 upon the slit S and images of the interference pattern and the reference marks on the test plate are viewed with the eyepiece C. The direct vision prism K separates the fringe patterns due to the helium light of different wave-lengths. A pair of crosswires located at S and operated by a micrometer head make it possible to measure small displacements of the fringes from the straight reference lines ruled on the test plate AB. With this instrument and an exceedingly true test plate, the planeness of an unknown surface can be measured with an accuracy of .025 micron. The only objection to the instrument is that the field is limited to about 2.5 cm., so in testing a large surface only a small portion can be seen at one time. Instruments of similar optical design described by Schultz,[4] Shönrock,[5] and Laurent [6] eliminate this objection by having fields of view as large as 25 cm. With these four instruments the rays of light coming from the source to the film surfaces are made parallel by the lens systems and when reflected back to the eye pass along the perpendicular to the first surface; and this condition holds over the whole field of view, that is, all points in the film are at the foot of the perpendicular from the eye. Hence in equation (3) d is zero, and

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[3] Pulfrich: *Zeits. f. Inst.*, Vol. 18, p. 261; 1898.

$$2t = N\lambda \tag{4}$$

which states that the difference in path of the two interfering trains is simply equal to the double thickness of the film (the double distance signifies that light travels down and back through the film). From this equation it is evident that where N is constant, that is along any one fringe, t is also constant; hence, the fringes trace lines of equal separation of the two surfaces.[7] Starting from the line of contact AB of the two plane surfaces (Fig. 4) and moving to a wider part of the film, when $2t=\lambda$ interference takes place and the first dark fringe f_1 will be a straight line parallel to AB. When $2t = {}^3/_2\,\lambda$ the wave trains reinforce each other and a light fringe is produced. Moving to a still thicker part of the wedge where $2t=2\lambda$, a second dark fringe f_2 will occur, etc. From this, it is evident that if the surfaces are plane the fringes will be straight lines, equally spaced and parallel to the line of intersection of the surfaces. The next dark fringe always occurs on passing to where the double separation increases by λ, hence, the distances between fringes depends on the inclination of the surfaces.

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[4] Schultz: *Zeits. f. Inst.*, Vol. 36, p. 252; 1914.

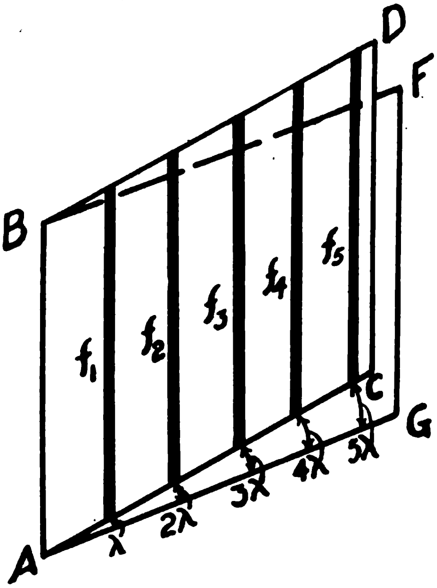
[5] Brodhun and Schönrock: *Zeits. f. Inst.*, Vol. 22, p. 355; 1902.

[6] Laurent: *Comptes Rendus*, Vol. 96, p. 1035; 1883.

[7] The fact should be stressed here, however, that the fringe marks the line of constant thickness of the film only when the direction of view is perpendicular to the film over the whole film. Observed obliquely, straight fringes do not indicate that the tested surface is plane, as shown below.

If a plane surface be brought in contact with a convex spherical surface (Fig. 5) then at C the point of contact, 2t, is equal to zero. Radially from this point the separation of the surfaces increases uniformly in all directions, so the fringes, and hence the lines on which $2t = N\lambda$, are concentric circles around the point of contact as a centre. On any ring the distance of the spherical surface from the plane is equal to the number of the ring counting from the point of contact times $\frac{\lambda}{2}$. By pressing down at A, the plane surface can be made to roll on the spherical surface shifting the point of contact and with it the centre of the ring system in the direction of A, or, toward the point of application of the pressure. With a convex surface therefore the centre of the ring system lies at the point of minimum separation.

FIG. 4.

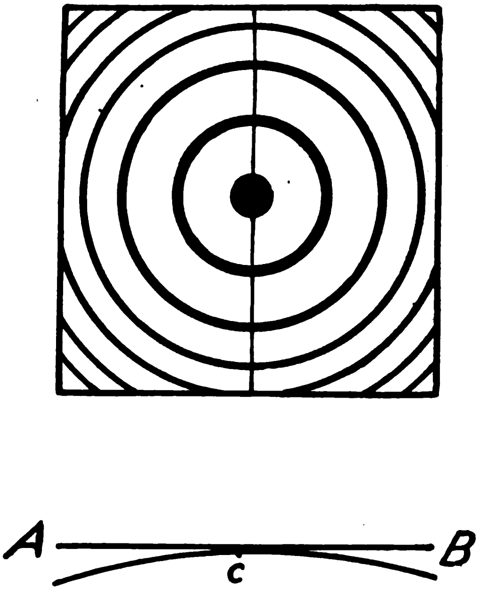


Fringes observed at normal incidence from plane surfaces.

If one of the surfaces be concave and spherical (Fig. 6), a similar system of concentric circular fringes is produced, but in this case the centre of the system lies at the point of maximum separation. Pressing down on A causes the centre of the ring system to shift toward B, the direction of increasing separation, away from the point of application of the pressure. Thus a slight pressure on one edge of the plane surface AB serves to indicate whether the curved surface is convex or concave.

With one surface plane and the other irregular the fringes are irregular curves each of which follows the line of equal separation of the surfaces. Whether the irregularity is a projection or depression can be determined by applying a slight pressure to one edge of the upper surface and noting the direction of shift of the fringes.

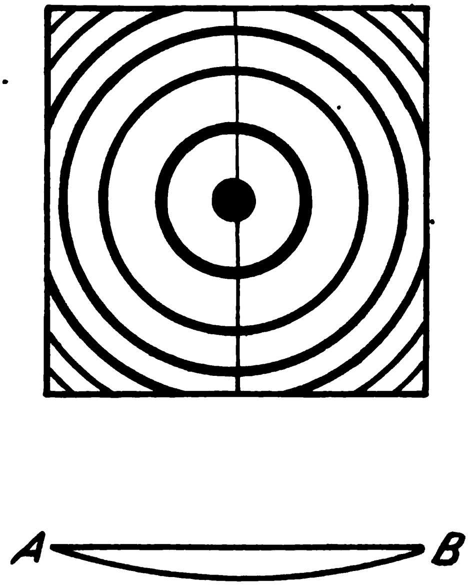
FIG. 5.



Fringes observed at normal incidence from convex surface.

The amount a curved surface deviates from a true plane can be readily estimated as follows: Draw a straight line FK (Fig. 7) across the centre of the true plane surface, parallel to the line of contact AB of the surfaces. Bring this line tangent to one of the fringes at say the point H. It is evident that this line represents the direction a fringe through H would take if the irregular surface could be converted into a plane tangent to the irregular surface at H. The fractional part of the distance between two fringes by which the fringe EL deviates from the straight line FK gives the fractional part of a half wave length by which the irregular surface deviates along FK from true plane. With AB as the line of contact, the point F is estimated to be one-fourth wave-length above and K one-half wave-length below the surface tangent at H.

FIG. 6.



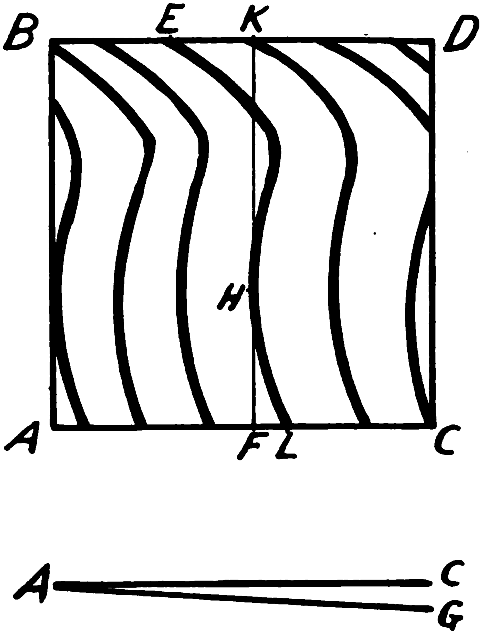
Fringes observed at normal incidence from concave surface.

2. *Errors of Interpretation.*—Although the theory is quite explicit, it is a very common mistake to assume that straight fringes always indicate planeness, when a surface under test is placed on a true plane; and conversely, curved fringes indicate that the surface is not a true plane. Both of these interpretations may be wrong. For example, the operator will view the light reflected by the film under oblique incidence; and to make sure that the fringes are straight, he may lower his eye to sight along them. Or, the fringes from one position appearing curved, he may shift

to another and find them quite straight, then draw his conclusions from what he observes under these, to him, best conditions.

It is important to know something about the magnitude of the errors which may be introduced by the assumption that

FIG. 7.



Fringes observed at normal incidence from irregular surface.

straight fringes always indicate planeness and curved ones indicate curvature of the surface tested. The treatment of this falls under the general case shown in Fig. 2 where the thickness of the film at any point is given by equation (3),

$$t = \frac{N\lambda}{2}\sqrt{1 + \frac{d^2}{p^2}}$$

which goes over, for normal incidence, into equation (4),

$$t = \frac{N\lambda}{2}$$

Suppose, with the eye at E (Fig. 2) a straight fringe is observed along FK. With one of the film surfaces true plane, how much does the other diverge from a plane between F and K? The actual error in determining the thickness t at any point by (4) when (3) should be used is obviously

$$E = \frac{N\lambda}{2}\left(\sqrt{1 + \frac{d^2}{P^2}} - 1\right) \tag{5}$$

and the relative error,

$$\begin{aligned} \varepsilon &= \frac{E}{t} \\ &= 1 - \frac{P}{\sqrt{P^2 + d^2}} \end{aligned} \tag{6}$$

The actual deviation from planeness between F and K is

$$X = t_F - t_K$$

The error introduced by using (4) instead of (3) is

$$\begin{aligned} S &= \epsilon_F\, t_F - \epsilon_K\, t_K \\ &= X\epsilon_F + (\epsilon_F - \epsilon_K)t_K \\ &= X\left(1 - \frac{P}{\sqrt{P^2 + d_F^2}}\right) + \left(\frac{P}{\sqrt{P^2 + d_K^2}} - \frac{P}{\sqrt{P^2 + d_F^2}}\right)t_K \end{aligned} \tag{7}$$

This equation is seen to be consistent in that S becomes zero for normal incidence at both F and K, in which both d_F and d_K are zero.

Sighting along a line from a normal position over K, $d_F = 0$, $\therefore \epsilon_K = 0$.

$$\begin{aligned} S &= \epsilon_F\,(X + t_K) \\ &= \epsilon_F\, t_F \end{aligned} \tag{8}$$

which states that the error in determining how much the tested surface deviates from the plane is simply equal to that in determining the thickness of the film at F by using equation (4) instead of (3).

If one observes the fringe obliquely at both F and K (d_F and d_K increasing and at the same time approaching equality), we have, for a given length FK, the error arising from the true deviation X increasing with the angle of incidence, while that arising from the thickness t_K of the film is decreasing. Consider some numerical examples:

(a) With the eye 25 cm. above the centre of a 15-cm. surface being tested on a true plane, and viewing a diametral fringe

$$P = 25,\ d_F = 7.5,\ d_K = 0$$

Applying (7), $S = .042\ X + .042\ t_K$

showing that the surface under test judged to be plane is really convex by more than 4 per cent. of the film thickness t_K at the centre of the plate. With such a method, since S decreases with t_K the fringes should change in curvature as the plate settles.

(b) With the eye moved back 25 cm. in a direction at right angles to the fringe—P remaining constant—and viewing the centre K and farthest edge F, $d_F = \sqrt{25^2 + 7.5^2}$ $d_K = 25$, we have from (7)

$$S = .308\ X + .015\ t_K$$

(c) With the eye moved back 50 cm. instead of 25,

$$d_F = \sqrt{50^2 + 7.5^2} \quad d_K = 50.$$

$$S = .577\ X + .004\ t_K$$

A comparison of (a), (b) and (c) shows that the error arising from the actual deviation X increases rapidly with slanting inspection, while the error arising from the thickness t_K of the film decreases rapidly.

If in (b) and (c) one sights respectively along instead of across the fringes

(d) $d_F = 32.5 \quad d_K = 25$

$\therefore S = .39\ X + .10\ t_K$

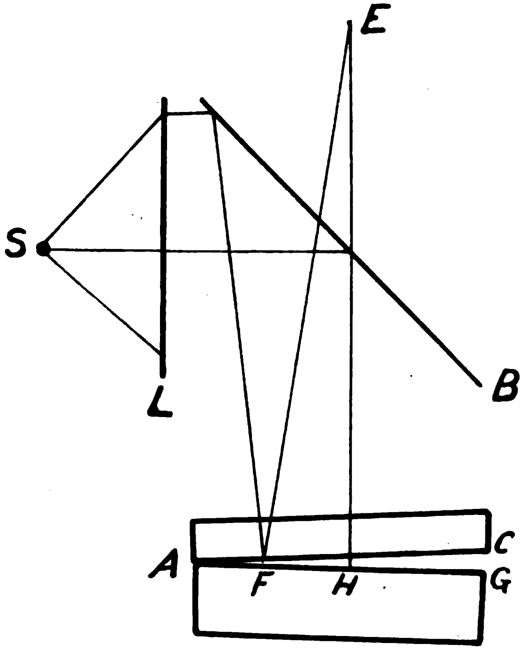
(e) $d_F = 57.5 \quad d_K = 50$

$\therefore S = .60\ X + .045\ t_K$

Here a comparison of (d) and (e) with (b) and (c) shows that sighting along the fringes obliquely increases the error arising from straight fringes at about the same rate as when sighting across them, insofar as the part arising from the actual deviation X from planeness is concerned; while the error arising from the thickness t_K of the film is six times as large in (d) as in (b) and ten times as large in (e) as in (c). For surfaces anywhere near true, X is likely to be small as compared with the thickness of the film. In which case these examples show that sighting along the fringes may go a long way toward straightening them and should therefore be especially avoided.

An improvement over the customary apparatus for testing planeness of a surface would be the addition of a thin plate of glass B set at an angle of 45° to the perpendicular EH from the eye to the surface (Fig. 8), but even then care should be taken to use only the small portion of the film near the foot of the perpendicular EH if a high degree of accuracy is desired, as this apparatus eliminates only the larger errors illustrated by examples (b), (c), and (d) and (e) above. In fact, if ϵ is to be kept less than one per cent., the ratio d/P should not exceed $^1/_7$, as may be determined from equation (7).

FIG. 8.



Satisfactory arrangement for illuminating test surface.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A COLORIMETER OPERATING ON THE SUBTRACTIVE PRINCIPLE.

BY

L. A. JONES.

DURING the recent war, the writer was engaged in the conduct of research work dealing with the question of marine camouflage. In connection with this, it became necessary to make measurements of the color of the sky, just above the horizon, under various conditions of weather through the regions in which vessels were subject to attack by submarines. At that time no instrument suitable for this work was available on the market, and it was necessary, therefore, to design and construct special apparatus suitable for the purpose. After carefully considering the requirements of such apparatus, and the materials readily available for use in their construction, it was decided that a colorimeter utilizing the subtractive principle of color mixture offered the greatest promise as to quickness of construction, portability, and simplicity of operation.

Since such an instrument is well adapted to many commercial uses, such as the measurement of the color of fabrics, wall coverings, papers, paint, glass, liquids, etc., it seems desirable to present at this time a description of the instrument and the principle upon which it operates. In considering the various means of measuring and specifying color, the conclusion that the monochromatic method[1] of color analysis offers the most satisfactory solution seems unavoidable. This follows from a consideration of the fundamental nature of color and the factors necessary for its specification. When radiant energy is incident upon the retina of the normal eye, the sensation of light is produced. This sensation may be said to consist of two factors, brightness and color, the former being dependent upon the intensity and the latter upon the quality of the incident radiation. The quality of the radiation may be specified by stating its wave-length, in case radiation of but one frequency is present; or, in case it is composed of a mixture of frequencies, by stating the wave-length and intensity of each of the component elements. This constitutes complete objective specification of the quality of the radiation and hence of the color. The retina, however, being a synthetic rather than analytic receiving organ, does not recognize the individual component part of the radiation as such, but receives the mixed radiation at a single stimulus producing a single sensation. This sensation, as stated previously, may be specified by two factors, brightness and color, and the latter factor may also be split into two components, hue and saturation. Hue refers to the position in the spectrum of the dominant quality, while purity expresses the proximity of the color to the condition of monochromatism. The factors necessary for the complete specification of the sensation resulting from the action of radiant energy upon the retina are brightness, hue, and saturation.

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[1] Nutting, P. G.: *Bull. Bur. Stand.*, Vol. 9, 1913, p. 1.

By the monochromatic method of color analysis these factors are measured, and a specification of color in terms of the various subjective factors is obtained.

Monochromatic analysers, while they must be regarded as most suitable for the measurement of color in absolute terms, unfortunately are somewhat complicated in design, and up to the present time no satisfactory portable model suitable for field use has been developed. The three-color additive method of color mixture has been applied to the measurement of color and several different types of instruments employing this fundamental principle have been produced. Such instruments involve the use of either of three arbitrary color screens (red, green, and blue) so arranged that the intensity of the light transmitted by each can be varied at the will of the operator, this transmitted light being subsequently mixed to match the unknown, or of three fairly narrow bands suitably chosen from the spectrum which are mixed in the proportions to match the unknown. Three-color additive instruments of the spectral type are particularly complicated and objectionable as field instruments, while other instruments of this class due to structural peculiarities lack portability and simplicity of operation. While the subtractive instrument is subject to some of these objections, it is free from many of them, especially from the necessity for motor-driven parts and adjustable slits, which in order to function properly are expensive to construct.

The subtractive instrument described in the paper has been found very satisfactory in field work, and in view of its portability, simplicity of operation, and freedom from the necessity of delicate adjustment of parts should be found very useful in many branches of commercial industry. The color wedges which constitute the essential part of the instrument are reproducible with considerable precision and under the conditions existing in the instrument they show practically no fading over long periods of time. For many classes of work the specification of color in terms of the constants of the instrument is quite sufficient, but if it be desired to express color in terms of the fundamental factors (hue and saturation) it is only necessary to calibrate the instrument by comparing it with the absolute instrument, the monochromatic analyzer.

It may be well at this point to explain more fully the meaning of the terms "additive mixture" and "subtractive mixture." While these terms have been used to a considerable extent, especially in connection with various processes of color photography, they may not be commonly understood. Perhaps the best way of explaining these terms is by the citation of concrete examples. Suppose, for instance, two projecting machines are mounted side by side and each illuminate the same screen surface. Now in front of one of the projecting lenses let a green filter be placed, while before the other is placed a red filter. One projector acting alone would illuminate the screen with green light while the other acting alone would illuminate the screen with red light. The two acting simultaneously result in a screen illumination which is yellow, and this may be taken as a typical example of the additive mixtures of red and green.

Now assume that only one projecting machine is used and that both the red and green filters are placed in front of the projecting lens. The light which reaches the screen must pass through both filters, but since the green filter transmits no red light and the red filter transmits none of the light passed by the green filter, the screen is not illuminated and we may say that this subtractive mixture of red and green produces darkness or black.

In order to obtain a yellow screen illuminated by the subtractive method, it is only necessary to place in front of the projecting lens a filter which absorbs the blue component of the sensibly white light emitted by the light source, the yellow in this case being obtained by the subtraction of the blue from the white light of the projecting beam. Suppose now that in front of the projecting lens of one of the lanterns is placed a blue-green filter and also a yellow filter. The screen will now be illuminated by green light, the red component originally present in the white light of the source being subtracted by the blue-green (that is, minus red) filter, while the blue component is subtracted by the yellow filter(that is, the minus blue), the resultant screen illumination being due to the light transmitted by both filters, this being the green component.

Thus in the case of additive mixture the resulting color is built up by mixing lights of various colors, while in the case of the subtractive method the desired color is arrived at by subtracting certain parts from the light used as a source of illumination. For a color mixing device which is designed to match all possible colors by the additive method, a properly selected set of filters transmitting the primary colors—that is, red, green, and blue-violet—are used, but if the subtractive method is to be employed, the three primary filters cannot be used since they are practically mutually exclusive and it is necessary to adopt the three colors complementary to the primaries, namely minus red, that is blue-green; minus blue, that is, yellow; and minus green, which is magenta or purple.

FIG. 1.

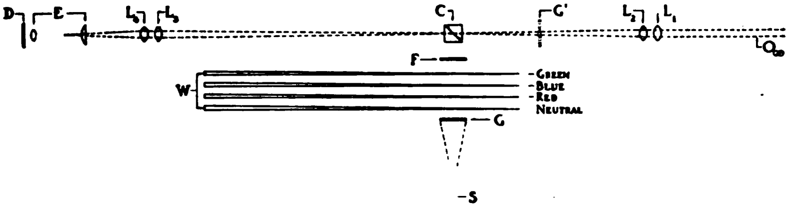
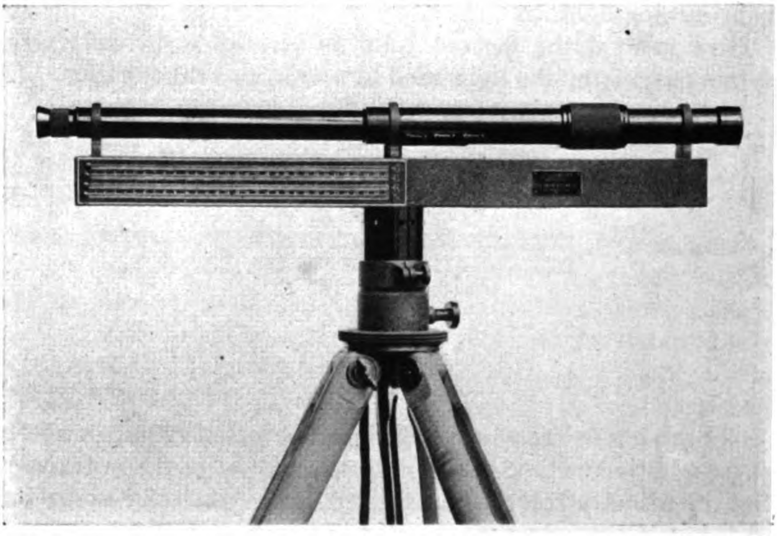


Diagram of optical parts.

In Fig. 1 is presented a diagram showing the arrangement of the essential parts of the subtractive colorimeter. The lens L_1L_2 forms an image of the surface whose color is to be measured in the plane of the photometer tube C. This image is examined by means of the lens L_3L_4, and the eye-piece system E. The lens system L_1L_2 is adjustable so that any surface at a distance greater than 3 feet from the photometer cube may be focused in the cube plane. On the axis perpendicular to and intersecting the line of sight at the dividing line of the photometer cube is placed the light source S with a diffusing screen at G as indicated. A filter, F, reduces the color of the light illuminating the photometer cube from the source S to a subjective match with noon sunlight. Four gelatine wedges are placed as indicated by W, each wedge being adjustable in a horizontal direction so that it may be inserted to any extent or withdrawn completely from the path of the beam of light from the lamp S. A set of neutral gray filters is provided at G′ in order that the intensity of the light from the object may be controlled so that it shall fall within the range of the instrument. The colored wedges used are as indicated in the figure, one being minus green, one minus blue, and one minus red, while the fourth is neutral tint and is provided in order that an intensity balance may be obtained. In making a measurement of color, the wedges are so adjusted that the light transmitted by them matches that from the object under consideration. By using three wedges of the complementary colors as indicated, that is, minus green, which transmits the red and blue regions; minus blue, which transmits the red and green; and minus red which

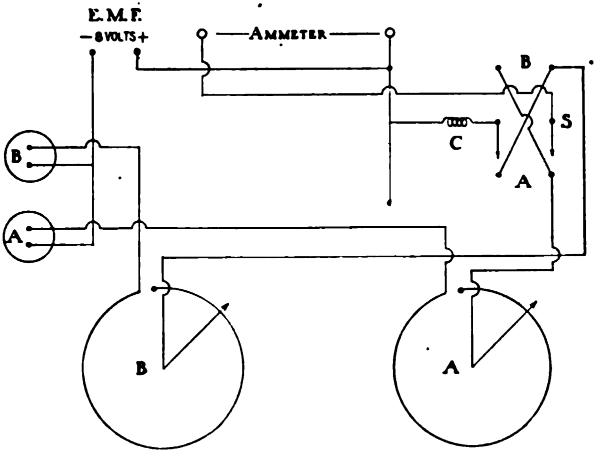
FIG. 2.



Instrument mounted for field work.

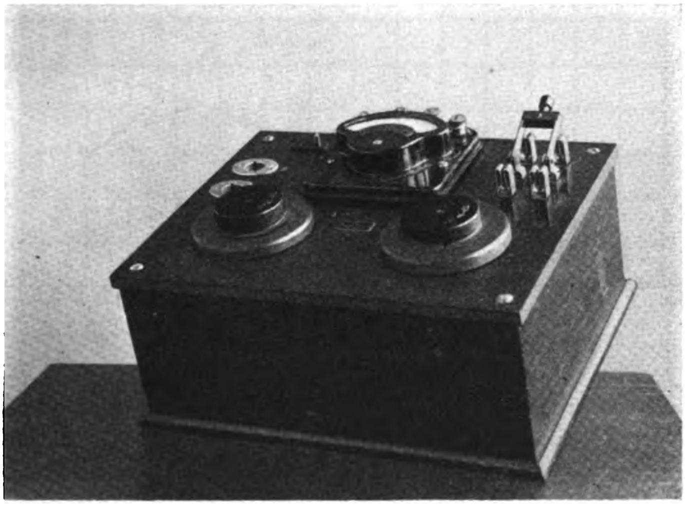
transmits the green and blue, any color whatever can be matched by proper adjustment. The position of each wedge is indicated by a pointer attached to the wedge frame and reading upon a graduated scale. A photograph of the finished instrument is shown in Fig. 2.

FIG. 3.



Wiring diagram of control.

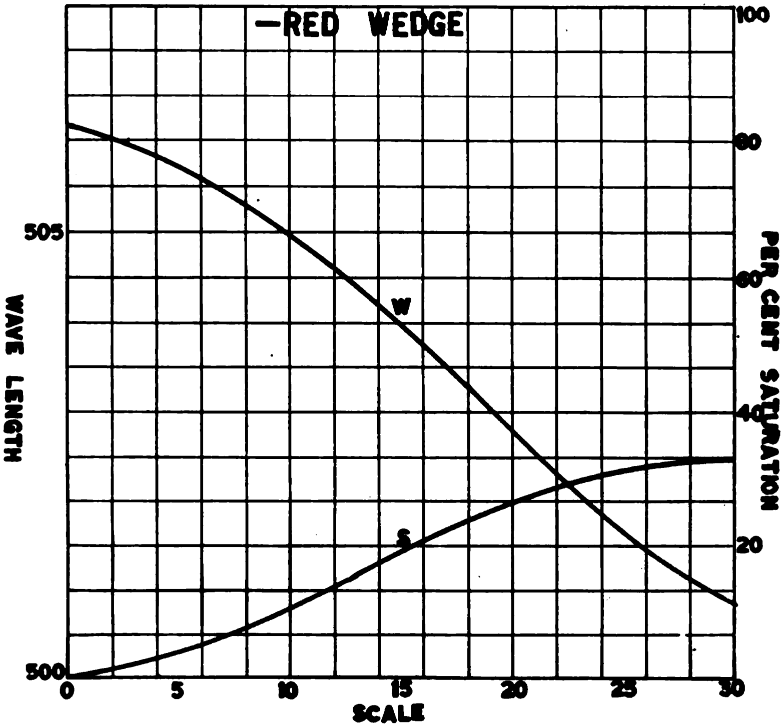
FIG. 4.



Control box.

With this instrument color measurement is made by matching light from the object with light from the source S and transmitted by the wedges. It is necessary therefore that the lamp S be operated under fixed conditions in order that the emitted light shall be of constant color. A control box is therefore provided containing the necessary variable resistance and an indicating instrument. Since the quality and intensity of the light emitted by a lamp run at a fixed current may change with the age of the lamp, it is necessary to provide a means of checking this lamp from time to time and of finding the current at which a renewal lamp must be operated. For this purpose a standard lamp is provided which may be attached to the instrument in front of the lens system L_1L_2. This standard lamp is seasoned and the current determined at which it must be operated in order that the light

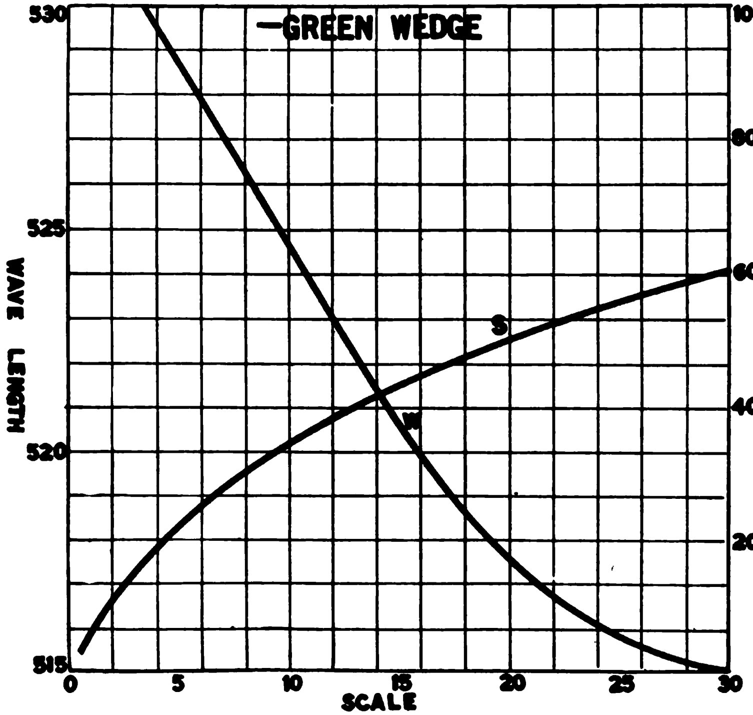
FIG. 5.



Hue and saturation curves for minus red wedge.

emitted by it when screened with the proper filter, which is mounted as an integral part of the standard lamp house, shall match noon sunlight in color. In checking the working lamp against the standard, it is necessary to operate both lamps under fixed conditions. The control box is, therefore, provided with two connections so that both lamps may be operated simultaneously and the current in either read on the ammeter, the necessary change in the connection being made by a double pole double throw switch. A diagram of the electrical connection of the control box is shown in Fig. 3, and a photograph of the control box in Fig. 4.

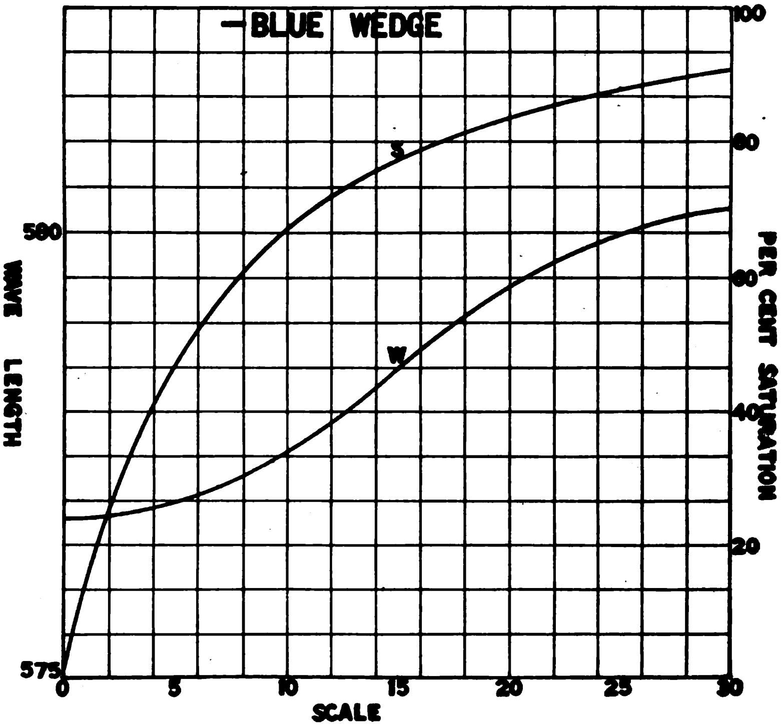
FIG. 6.



Hue and saturation curves for minus green wedge.

The three colored wedges used in the instrument have been analyzed on the monochromatic analyzer, and the results are given in Figs. 5, 6, and 7. The abscissæ values are the scale reading numbers, while the ordinates at the left of each figure (marked wave-length) are the wave-lengths of the dominant hue. The ordinate scale at the right of these figures gives the values of saturation in per cent. The curve marked W in each case refers to the wave-length of the dominant hue while that marked S is the curve of saturation. The dye concentration used in the manufacture of these wedges is so adjusted as to give wedges of such gradient that the scale of the instrument will be fairly open for the reading of colors of low saturation, and at the same time permit the reading of high saturation colors. With the wedges actually used, no colors other than those existing in the spectrum have been found of such saturation as to be beyond the range of the instrument.

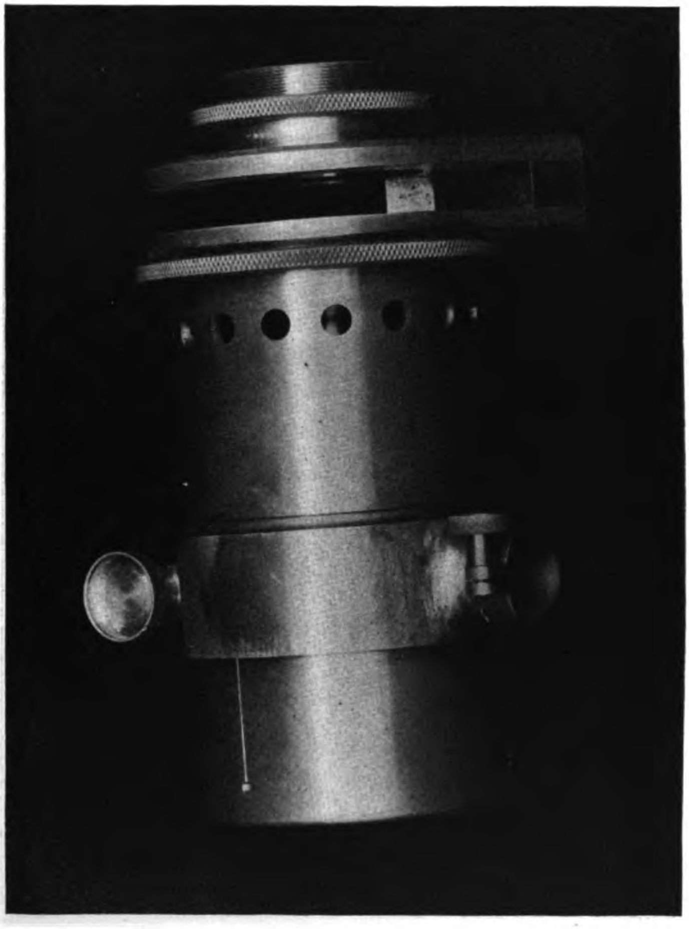
Fig. 7.



Hue and saturation curves for minus blue wedge.

While some difficulty was experienced with the first wedges manufactured for this purpose, in that they were not as stable as could be desired, this has been very largely overcome, methods having been worked out for properly stabilizing the dyes used. As an illustration of the stability obtained, it might be mentioned that one of these wedges was exposed to daylight continuously for over a month at the end of which time no perceptible fading had occurred. When it is considered that fading is due largely to the extreme blue and ultra-violet light, and that this component

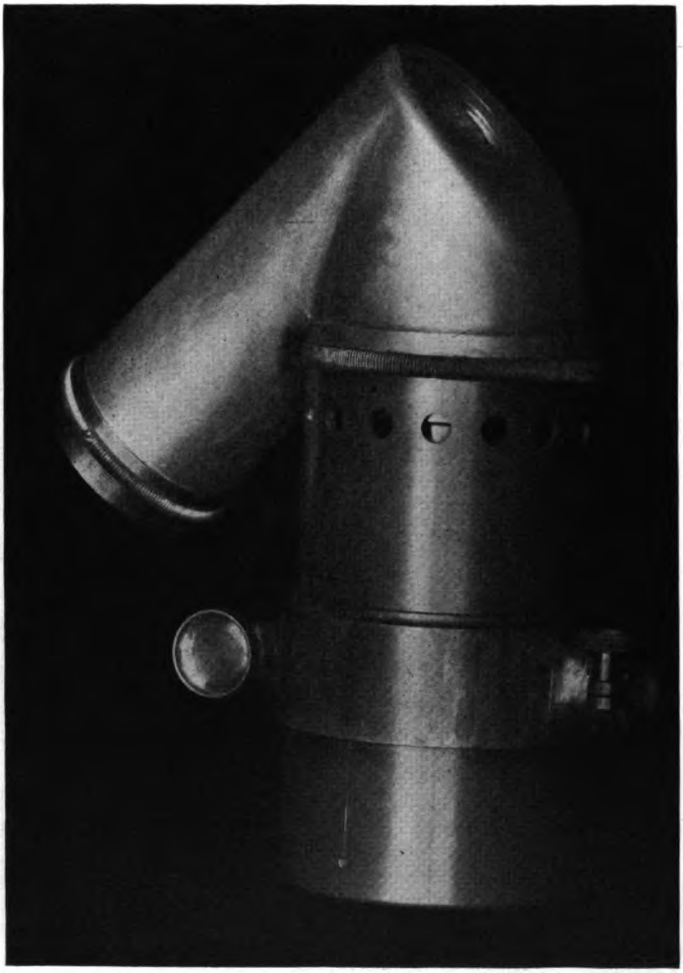
FIG. 8.



Attachment for holding transparent samples.

is much stronger relatively in daylight than in the light from the tungsten lamp used in this instrument, it will be seen that there is little danger of fading. The wedges are enclosed in a dust-proof housing so as to be completely protected from injury. While the instrument was designed primarily for the measurement of the color of objects at some distance, it can be easily adapted to other

Fig. 9.



Attachment for holding reflecting samples.

purposes. Special attachments have, therefore, been built for holding samples of materials to be measured. One of these shown in Fig. 8 is designed for holding transparent solids such as colored glass or filters of celluloid or gelatine. The sample being placed in the slot, the attachment is fastened to the objective end of the instrument by means of the threaded ring above. A lamp for providing the necessary illumination of the sample is mounted in the housing below, in front of which is carried an opal glass diffusing screen and filter of proper quality to reduce the light to sunlight quality.

A second attachment for use in measuring the color of reflecting surfaces such as fabrics, paper, etc., is shown in Fig. 9. This also is attached to the objective end of the instrument by a threaded ring. The lamp providing the necessary illumination is mounted in the housing. The sample to be measured is held in contact with the upper right-hand surface by means of a spring clip not shown in the figure. The illumination on the sample is from a disk of diffusing glass which subtends a relatively large angle at the point of observation on the sample. The mean angle of incidence of this illumination is 45° while the sample is preserved normally. A third attachment for the measurement of liquids is provided, the liquid in question being placed in a flat-bottomed glass tube which can be easily inserted and removed from its holder. Only one lamp housing for the illumination of the samples being measured is provided with each instrument, this being so constructed that it can be attached to any one of the three different fixtures provided for samples of different character.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.,
October 29, 1920.

TRANSMISSION AND REFRACTION DATA ON STANDARD LENS AND PRISM MATERIAL WITH SPECIAL REFERENCE TO INFRA-RED SPECTRORADIOMETRY.*

BY

W. W. COBLENTZ.

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I. INTRODUCTION.†

THIS paper gives exact data on the spectral transparency and in particular, the refractivity of materials which are useful for prisms and lenses for spectroradiometers.

The data on refractive indices were taken from smooth curves, drawn through values which were collected from various sources and reduced to a common temperature.

It is important, especially in work of the highest precision (such as, for example, the determination of the constant of spectral radiation), to use the most precise instruments and optical data available. It is, therefore, relevant to discuss very briefly some recent designs of optical instruments suitable for spectroradiometry.

The pioneering investigation of the infra-red refractive indices of a substance dates back to 1886 when Langley determined the dispersion of rock salt to about 5μ. In these determinations a spectrometer having an image forming mirror of long focal length was used. In subsequent determinations of the refractive indices of rock salt and of fluorite, the image forming mirror of the spectrometer used by Langley had a focal length of 4 to 4.7 metres. The apparatus was in a large enclosure which could be maintained at a constant temperature. Langley[19] was therefore justified in calling attention to the very high precision attainable "owing, if to no other reason, to the far greater size of the apparatus employed, where size is a most important element of accuracy." Other experimenters,[20, 23, 24] using his methods, but having spectrometer mirrors of only about one-twelfth the focal length, have attempted to produce similar data, which, unfortunately, have been given the widest recognition in tables of physical constants. These published results, especially the older ones, have been very confusing to the writer who, for some years, has been confronted with the task of obtaining reliable refractive indices.

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* Published by permission of the Director, Bureau of Standards.

† References are given in a classified bibliography at the end of this paper.

The recent measurements on rock salt [26] and on fluorite [28] by Paschen, when corrected for temperature,[33, 34] are in agreement with Langley's [19] measurements. The numerical values, given in the present paper, have been adopted after a careful study of all the data available.

The Spectroradiometer.—For measuring thermal radiation intensities, in the ultra-violet part of the spectrum, one may use a spectrometer having achromatic lenses of quartz-fluorite. However, the scarcity of clear fluorite, for large sized lenses, makes such apparatus very expensive.

Pflüger [1] used simple lenses of fluorite, 4 cm. in diameter (32 cm. focal length) and a fluorite prism. An inexpensive spectroradiometer of high light-gathering power was made by Coblentz [2] by using simple plano-convex lenses (6 cm. in diameter and 20 cm. focal length) and a prism of quartz. Pfund [5] has described similar apparatus, in which the radiometer is kept in focus automatically, in different parts of the spectrum.

The apparatus may be designed also as an illuminator for separating the visible from the ultra-violet of a source like the sun or a quartz mercury vapor lamp.[4]

For spectroradiometric measurements in the visible spectrum and the infra-red to about 0.8μ we can use a spectrometer with visually achromatized lenses of glass. Here, also, it is desirable to use apparatus having a high light-gathering power, such as one obtains with lenses 6 cm. in diameter and 20 cm. focal length.[2, 3, 4]

A common property of all metals is a low reflectivity in the ultra-violet and in the violet-blue part of the visible spectrum.[15, 16, 17] Furthermore, the spectral reflectivity in the short wave-length is greatly reduced on tarnishing of the metal. Hence concave mirrors of metals have never been used extensively for spectroradiometric measurements in the visible and in the ultra-violet spectrum.

Because of the lack of achromatism (and the opacity of the material) lenses of glass, quartz, fluorite, etc., achromatized for the visible spectrum, have not been used extensively in infra-red spectroradiometric work. Lehmann[46] has described an infra-red spectrograph achromatized for $\lambda = 0.589\mu$ and $\lambda = 1.529\mu$.

A concave mirror is achromatic (also astigmatic) and hence spectrometers with collimating and image-forming mirrors, instead of lenses, have been used almost exclusively for infra-red spectral radiation intensity measurements. In the infra-red spectrum, beyond 2μ most of the metals have a very high reflecting power (90 to 98 per cent.) and concave metal mirrors, or metal-on-glass mirrors, are, therefore, especially useful for infra-red investigations.

Recent designs of spectrometers having collimating and image-forming mirrors are described in papers by Coblentz[6] and by Gorton.[9] Vacuum spectrometers have been described and used by Trowbridge[7] and by McCauley.[8]

In order to obtain the spectral energy distribution of an incandescent substance, it is necessary to correct the observations for absorption of radiation by the mirrors and by the prism. The proper formula for eliminating the absorption in a wedge is given by Paschen,[29] and by Coblentz,[39] who gives also the numerical factors for eliminating the absorption in a wedge of quartz.

It is beyond the scope of this paper to discuss the construction and operation of the instruments (bolometers, thermopiles, etc.) used for measuring the thermal radiation intensities. References are given in the appended Bibliography[10] on "Radiometers."

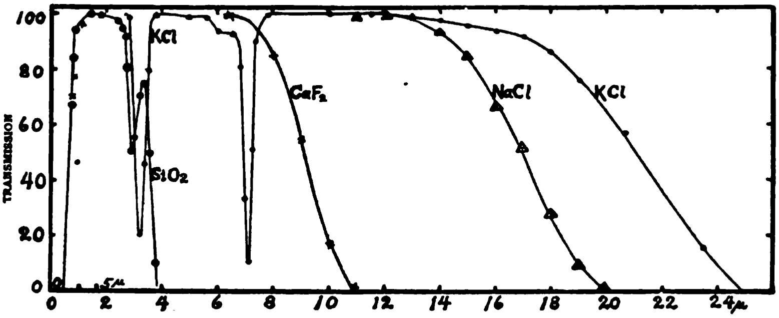
Spectrometer Calibration.—In most spectroradiometric work it is necessary to know the wave-lengths at which the thermal radiation intensities are measured. In the visible spectrum it is an easy matter to note the spectrometer settings for the emission lines of some source (*e.g.*, the mercury arc or helium gas in a

Plücker tube), the wave-lengths of whose emission lines are known. Similarly in the ultra-violet the emission lines of mercury, cadmium, zinc, etc., may be noted with a fluorescent canary glass screen, or radiometrically with a thermophile [1] or bolometer.

The spectrometer circle may be calibrated for wave-lengths in the infra-red spectrum to 1μ, by noting the emission lines [12] of sodium and potassium in a carbon arc; also the emission lines of a quartz mercury vapor lamp, and of helium in a vacuum tube.

Beyond 2μ, where the emission lines are usually weak (except the strong emission band of carbon dioxide at 4.4μ in the bunsen flame) one can calibrate the prism by noting sharp absorption bands,[13, 14] such as, for example, the bands of sylvite, KCl, illustrated in Fig. 1.

FIG. 1.



Transmission of quartz (SiO_2), fluorite (CaF_2), rock salt (NaCl), and sylvite (KCl).

For work requiring great accuracy, the proper method of calibration is by calculating the minimum deviation settings for different wave-lengths, using the refractive indices and the angle of the prism. For this purpose. the yellow sodium lines, or, better, the yellow helium line, $\lambda = 0.5875\mu$, is used as a reference point on the spectrometer circle. The minimum deviation settings for the various infra-red wave-lengths are computed from the corresponding refractive indices, and referred to the yellow helium line as a basis. After this the bolometer or thermopile is adjusted upon the yellow helium line, then on rotating the spectrometer through a certain angle, say 2°, the corresponding wave-length is, say, 6μ, while a rotation of 4° places the bolometer at about 8.7μ in the spectrum of a 60° fluorite prism (*loc. cit.*,[6] page 49).

Elimination of Scattered Radiation in Spectral Energy Measurements.—In the design of optical instruments, there are opportunities for great improvement in this respect., Take, for example, the image-forming telescope of a spectrometer. The telescope tube should be large and suitably diaphragmed so that, when the violet end of the spectrum is incident upon the radiometer receiver, the infra-red end of the spectrum cannot be reflected from the side of the tube and impinge upon the receiver. Furthermore, the bevelled edges of the exit slits of the spectrometer should face outwards [18] instead of facing the image-forming lens, as obtains in commercial instruments.

By using suitably constructed optical instruments, the scattered radiation is practically eliminated. What little remains may be obviated by using, before the entrance slit of the spectrometer, a shutter,[22, 5, 18] which is opaque to the region of the spectrum under investigation but which transmits the scattered radiations. In this manner, the scattered radiations are incident upon the radiometer all the time and, hence, do not affect the energy measurements. Using a spectrometer which is provided with slits and diaphragms, as just mentioned, it has been found [4] that the scattered radiation was immeasurable, and hence negligible in comparison with the intensities under investigation.

II. OPTICAL CONSTANTS OF GLASS.

In the ultra-violet end of the spectrum, ordinary crown glass is transparent to about 0.3μ, while the flint-silicate glasses absorb strongly throughout the blue and violet end of the spectrum.

In the infra-red spectrum, all glasses [38, 40] begin to absorb at about 2μ, and, for a thickness of 1 cm., they are practically opaque to radiations of wave-length greater than 3μ. Glasses containing traces of iron impurities have an absorption band at 1μ.

The refractive indices of various glasses have been determined by Rubens.[20] He determined the refractive indices also of water, xylol, benzol, etc. However, in view of the fact that the refractive indices depend upon the composition of the glass, no refraction data are given in this paper. Practically no infra-red work is being done with glass prisms.

III. OPTICAL CONSTANTS OF CARBON DISULFID.

Carbon disulfid is quite transparent in the infra-red. In the region to 3μ, Rubens [20] found an absorption of only 5 to 7 per cent. for a 1 cm. thickness. Beyond 4μ there are a number of very large absorption bands.[36]

As illustrated in Fig. 3, carbon disulfid has a very much larger dispersion than quartz, etc., in the region of 0.5 to 2μ, and hence is especially adapted for certain fields of spectroradiometry.

The infra-red refractive indices of carbon disulfid were determined by Rubens.[20] Those in the visible and in the ultra-violet were determined by Flatow.[32] Rubens's values of the infra-red refractive indices are given in Table I.

TABLE I.

Indices of Refraction of Carbon Disulfid in Air at 15° C. (Rubens).

Wave-lengths $\mu = 0.001$ mm.	Refractive Index, n	Log n
0.434μ	1.6784	0.2248955
0.485	1.6550	.2187980
0.590	1.6307	.2123741
0.656	1.6217	.2099705
0.777	1.6104	.2069338
0.823	1.6077	.2062050
0.873	1.6049	.2054480
0.931	1.6025	.2047980
0.999	1.6000	.2041200
1.073	1.5978	.2035224
1.164	1.5960	.2030329
1.270	1.5940	.2024883
1.396	1.5923	.2020249
1.552	1.5905	.2015337
1.745	1.5888	.2010692
1.998	1.5872	.2006317

IV. OPTICAL CONSTANTS OF QUARTZ.

Quartz is one of the most useful materials for prisms. It is extremely transparent to ultra-violet radiations. Pflüger [41] found a transmission of 94 per cent. at 0.222μ and 67 per cent. at 0.186μ, for a sample of crystalline quartz, 1 cm. in thickness. Some samples of amorphous quartz have been found to be more opaque than crystalline quartz; but this may be the result of contamination in melting.

The infra-red transmission of quartz has been determined by various observers. A characteristic absorption band occurs at about 2.95μ. A sample 1 cm. in thickness is practically opaque [38] to radiations of wave-length greater than 4μ (see Fig. 1).

The absorption, reflection and dispersion constants of quartz are given in a paper by Coblentz,[39] who determined the transmission of samples 3 cm. in thickness. The paper gives also factors for eliminating the absorption in a quartz prism.

In the short wave-lengths the refractive indices of quartz have been determined by Martens.[31] In the infra-red there are important determinations by Rubens,[21, 23] Carvallo,[35] and Paschen.[27] Carvallo's data extend to 2.2μ and in the region of 1.45 to 1.8μ they are slightly lower (by several units in the fifth decimal place) than three determinations made by Paschen. In this region of the spectrum, the data must therefore be considered uncertain with the doubt in favor of Carvallo's data. This uncertainty affects Warburg's [43] determination of the spectral radiation constant by perhaps 0.2 to 0.4 per cent.

The refractive indices (ordinary ray) of quartz at 18° C. are given in Table II. They are taken from a graph of sufficient size to permit reading the data to 1 or 2 units in the fifth decimal place. In many cases the values agree exactly with Carvallo's measurements. Paschen's data may be recognized by the fact that his wave-lengths are given to the fifth decimal place.

V. OPTICAL CONSTANTS OF FLUORITE.

Fluorite is very transparent to radiations of wave-lengths extending from 0.2μ to 10μ (see Fig. 1). Pflüger [41] found a transmission of 86 per cent. at 0.23μ and 70 per cent. at 0.186μ, for a sample 1 cm. in thickness. Lyman [42] examined fluorites from various sources, and of various colors, and found that they are opaque to radiation of wave-lengths less than about 0.12μ. Coblentz [38] examined green fluorites with a view of determining their suitability for prisms. He found numerous sharp absorption bands, in the infra-red, which would render such material unsuitable for prisms.

The refractive indices of fluorite have been determined by various observers,[21, 28] and repeatedly by Paschen.[23, 24, 25, 28] Applying temperature coefficients of refraction,[33, 34] it is found that Paschen's determinations, especially the latest ones [28] which were obtained with an improved spectrometer, coincide with the dispersion curve of fluorite determined, to 3.5μ, by Langley.[19] Beyond 4μ the dispersion of fluorite is much larger than at 1.5 to 2μ and there is better agreement among the various determinations of the refractive indices. Furthermore, slight deviations have less effect upon spectral radiation measurements.

TABLE II.

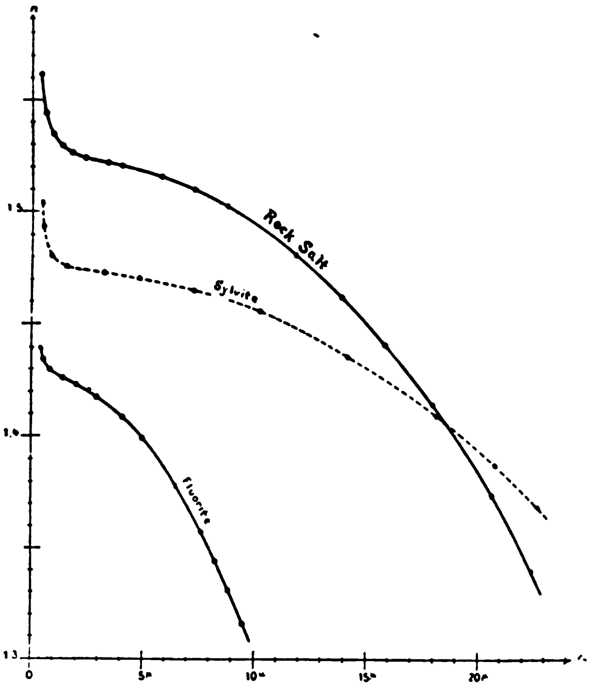
Indices of Refraction of Quartz in Air at 18° C. (*Carvallo, Paschen*).

Wave-lengths μ = 0.001 mm.	Refractive Index, n	Log n
0.54609μ	1.54617	0.1892573
.58758	.54430	.1887317
.58932	.54424	.1887148
.61577	.54323	.1884306
.66784	.54154	.1879548
.6731	.54139	.1879126
.6950	.54078	.1877404
.70654	.54048	.1876561
.72817	.53995	.1875066
.7711	.53895	.1872245
.8007	.53834	.1870523
.8325	.53773	.1868801
.84467	.53752	.1868208
.8671	.53712	.1867078
.9047	.53649	.1865298
.9460	.53583	.1863432
.9914	.53514	.1861480
1.01406	.53486	.1860405
1.0417	.53442	.1859443
1.08304	.53390	.1857970
1.0973	.53366	.1857291
1.12882	.53328	.1856214
1.1592	.53283	.1854940
1.17864	.53263	.1854373
1.2288	.53192	.1852361
1.3070	.53090	.1849468
1.3195	.53076	.1849071
1.3685	.53011	.1847226
1.3958	.52977	.1846162
1.4219	.52942	.1845268
1.47330	.52879	.1843478
1.4792	.52865	.1843081
1.4972	.52843	.1842427
1.52961	.52800	.1841234
1.5414	.52782	.1840722
1.6087	.52687	.1837921
1.6146	.52680	.1837822
1.6815	.52585	.1835118
1.7487	.52486	.1832300
1.76796	.52462	.1831616
1.8487	.52335	.1827997
1.9457	.52184	.1823690
2.0531	.52005	.1818579
2.06262	.51991	.1818178
2.1719	.51799	.1812689
2.35728	.51449	.1802664
2.3840	.51400	.1801259
2.4810	.51200	.1795518
2.575	.51100	.1792645
2.65194	.50824	.1784704
2.79927	.50474	.1774614
3.09393	.49703	.1752305

In Table III, the refractive indices of fluorite to 3.5μ are taken from the smooth curve published by Langley.[19] In many cases Paschen's original wave-lengths are retained. As already mentioned, the corresponding refractive indices, when corrected for temperature of the prism, fall exactly upon Langley's curve of refractive indices.

Beyond 3.5μ the refractive indices are taken from the smooth curve (extended from 3μ) through the various determinations of Paschen[25] and Rubens.[20] All these data are reduced to 20° C.

FIG. 2.



Dispersion curves of rock salt, sylvite and fluorite (Rubens).

Langley's data are referred to the "A line," λ = 0.7604μ. The most convenient reference point for adjusting a spectroradiometer in the spectrum is the yellow helium line, λ = 0.58758μ. Until recently, when Paschen[28] determined the refractive index of this line, there has been some uncertainty in infra-red spectral radiation measurements[44] requiring the highest accuracy. An error of 5″ (or $n = 3 \times 10^{-5}$) in the determination of this refrac-

TABLE III.

Indices of Refraction of Fluorite in Air at 20° C. (Langley, Paschen, Rubens).

Wave-lengths $\mu = 0.001$ mm.	Refractive Index, n	Log n
0.48615μHβ	1.43704	0.1574695
.58758 He	.43388	.1565120
.58932 Na	.43384	.1564995
.65630 Hα	.43249	.1560916
.68671	.43200	.1559430
.72818 He	.43143	.1557601
.76653 K	.43093	.1556184
.88400	.52980	.1552753
1.0140 Hg	.42884	.1549835
1.08304 He	.42843	.1548589
1.1000	.42834	.1548316
1.1786	.42789	.1546948
1.250	.42752	.1545822
1.3756	.42689	.1543905
1.4733	.42642	.1542474
1.5715	.42596	.1541073
1.650	.42558	.1539916
1.7680	.42502	.1538210
1.8400	.42468	.1537173
1.8688 He	.42454	.1536747
1.900	.42439	.1536274
1.9153	.42431	.1536046
1.9644	.42407	.1535313
2.0582 He	.42360	.1533880
2.0626	.42357	.1533789
2.1608	.42306	.1532232
2.250	.42258	.1530766
2.3573	.42198	.1528936
2.450	.42143	.1527255
2.5537	.42080	.1525329
2.6519	.42018	.1523460
2.700	.41988	.1522517
2.750	.41956	.1521538
2.800	.41923	.1520528
2.850	.41890	.1519518
2.9466	.41823	.1517467
3.0500	.41750	.1515231
3.0980	.41714	.1514128
3.2413	.41610	.1510939
3.4000	.41487	.1507134
3.5359	.41376	.1503788
3.8306	.41119	.1495855
4.000	.40963	.1491051
4.1252	.40847	.1487476
4.2500	.40722	.1483620
4.4000	.40568	.1478864
4.6000	.40357	.1472341
4.7146	.40233	.1468502
4.8000	.40130	.1465311
5.000	.39908	.1458426
5.3036	.39522	.1446427
5.8932	.38712	.1421141
6.4825	.37824	.1393312
7.0718	.36805	.1361020
7.6612	.35675	.1324998
8.2505	.34440	.1285290
8.8398	.33075	.1240965
9.4291	.31605	.1192724

tive index affects the constant of spectral radiation by 0.3 per cent.

The dispersion curve of fluorite is illustrated in Figs. 2 and 3 (from Rubens').

In the infra-red, the variation of the refractive index of fluorite[34] with temperature decreases slowly with wave-lengths. At 1μ the coefficient of variation amounts to about $\Delta n = 0.000\,012$ and at 6.5μ it amounts to about $\Delta n = 0.000\,009$, for 1° rise in temperature.

VI. OPTICAL CONSTANTS OF ROCK SALT.

Rock salt is uniformly transparent from 0.2μ in the extreme ultra-violet[41] to 12μ in the infra-red[23] (see Fig. 1). In the region of 15μ the absorption increases rapidly. A plate of rock salt 1 cm. in thickness is completely opaque[22] to radiation of wave-lengths greater than 20μ. The refractive indices of rock salt have been determined in the short wave-lengths by Martens,[30, 31] and in the infra-red by Langley,[16] by Rubens,[21, 22] and by Paschen.[26] In the region of 1 to 3μ there is considerable disagreement among the older determinations. However, the recent work of Paschen[26] is in excellent agreement with Langley's measurements which are, without doubt, very accurately determined.

The infra-red refractive indices of rock salt, at 20°, are given in Table IV. The first part of the table, to 5μ, consists principally of Langley's (and Paschen's corrected for temperature) measurements as read from the smooth curve (*loc. cit.*,[19] p. 235, plate XXIX). Beyond 5μ, to 16μ, the refractive indices are principally Paschen's measurements, corrected for temperature;[34] also some of Rubens' measurements and several interpolated values.

The temperature coefficient of refraction of rock salt[19, 33, 34] decreases slowly with wave-length; amounting to about $\Delta n = 0.000\,038$ at 1μ and $\Delta n = 0.000\,025$ at 9μ, for 1° rise in temperature.

The general outline of the dispersion curve of rock salt is illustrated in Fig. 2 (from Rubens).

VII. OPTICAL CONSTANTS OF SYLVITE.

Of all the substances which are otherwise suitable for prisms, sylvite, KCl, is transparent throughout the greatest part of the infra-red spectrum. A plate 1 cm in thickness transmits[22] radia-

TABLE IV.

Indices of Refraction of Rock Salt in Air at 20° C. (Langley, Paschen, Rubens).

Wave-lengths $\mu = 0.001$ mm.	Refractive Index, n	Log n
0.5893μ	1.54427	0.1887232
.6400	.54141	.1879182
.6874	.53930	.1873233
.7604	.53682	.1866230
.7858	.53607	.1864110
.8835	.53395	.1858112
.9033	.53361	.1857149
.9724	.53253	.1854090
1.0084	.53206	.1852758
1.0540	.53153	.1851255
1.0810	.53123	.1850404
1.1058	.53098	.1849695
1.1420	.53063	.1848702
1.1786	.53031	.1847794
1.2016	.53014	.1847312
1.2604	.52971	.1846091
1.3126	.52937	.1845126
1.4874	.52845	.1842512
1.5552	.52815	.1841660
1.6368	.52781	.1840693
1.6848	.52764	.1840238
1.7670	.52736	.1839414
2.0736	.52649	.1836960
2.1824	.52621	.1836142
2.2464	.52606	.1835716
2.3560	.52579	.1834947
2.6505	.52512	.1833040
2.9466	.52466	.1831730
3.2736	.52371	.1829024
3.5359	.52312	.1827341
3.6288	.52286	.1826600
3.8192	.52238	.1825231
4.1230	.52156	.1822891
4.7120	.51979	.1817836
5.0092	.51883	.1825092
5.3009	.51790	.1812432
5.8932	.51593	.1806792
6.4825	.51347	.1799738
6.80	.51200	.1795518
7.0718	.51093	.1792443
7.22	.51020	.1790345
7.59	.50850	.1785453
7.6611	.50822	.1784647
7.9558	.50665	.1780124
8.04	.6064	.1779403
8.8398	.50192	.1766468
9.00	.50100	.1763807
9.50	.49980	.1760333
10.0184	.49462	.1745308
11.7864	.48171	.1707632
12.50	.47568	.1689922
12.9650	.47160	.1677898
13.50	.4666	.1663117
14.1436	.46044	.1644837
14.7330	.45427	.1626450
15.3223	.44743	.1605976
15.9116	.44090	.1586338
14.93	.4149	.1507257
20.57	.3735	.1378287
22.3	.3403	.1272020

tions to 24μ (see Fig. 1). In the region of 5μ to 10μ the dispersion is small. Furthermore, there are sharp absorption[11, 13] bands at 3.18μ and 7.08μ. Hence, sylvite is the most useful for investigations in the region of the spectrum extending from 10 to 20μ.

In the short wave-lengths the refractive indices of sylvite have

TABLE V.

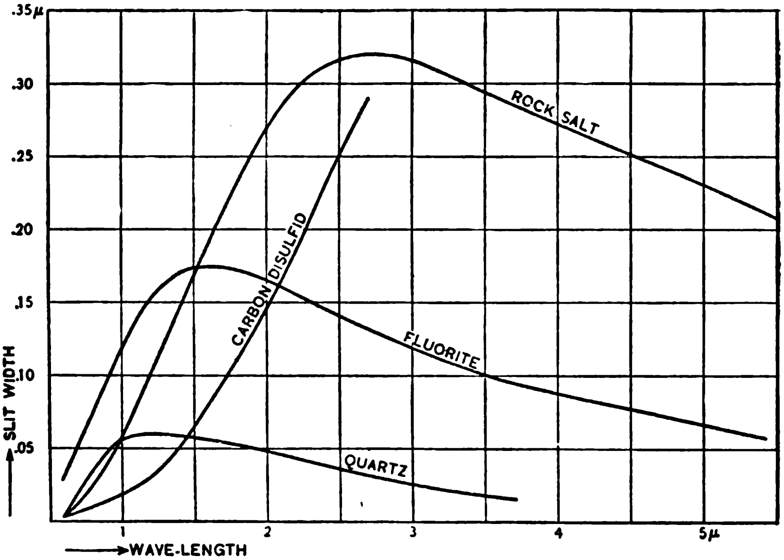
Indices of Refraction of Sylvite in Air at 15° C.

Wave-lengths μ = 0.001 mm.	Refractive Index. n	Log n
0.5893μ	1.49044	0.1733145
.656	.48721	.1723723
.7858	.48328	.1712232
.845	.48230	.1709361
.884	.48142	.1706782
.9822	.48008	.1702862
1.003	.47985	.1702177
1.1786	.47831	.1697655
1.584	.4765	.1692335
1.7680	.47595	.1690717
2.3573	.47475	.1687184
2.9466	.47388	.1684621
3.5359	.47305	.1682164
4.125	.47215	.1679521
4.7146	.47112	.1676481
5.3039	.47001	.1673202
5.50	.46962	.1672050
5.8932	.46880	.1669627
6.50	.46750	.1665781
7.00	.46625	.1662080
7.661	.46450	.1656894
8.00	.46350	.1653927
8.2505	.46272	.1651642
8.8398	.46086	.1646086
9.500	.45857	.1639273
10.0184	.45672	.1633760
10.500	.45475	.1627883
11.00	.45263	.1621550
11.786	.44919	.1611253
12.50	.44570	.1600782
12.965	.44346	.1594048
14.144	.43722	.1575232
15.00	.4320	.1559430
15.912	.42617	.1541713
16.50	.42230	.1529912
17.00	.41885	.1519365
17.680	.41403	.1504586
18.10	.4108	.1494655
19.00	.4031	.1470886
20.00	.3939	.1442316
20.60	.3882	.1424520
22.5	.3692	.1364669

been determined by Martens.[21] In the infra-red we have various determinations by Rubens[22] (with Nichols and with Trowbridge) by Trowbridge,[15] and by Paschen.[26]

The infra-red refractive indices of sylvite are given in Table V. They are read from the smooth curve (practically Paschen's curve) drawn through the various determinations, all of which are in close agreement, except at 9 to 11μ where the older determinations do not agree very well with Paschen's data.

FIG. 3.



Comparative dispersion of prisms at different wave-lengths in the spectrum.

The temperature coefficient of refraction of sylvite observed by Liebreich[34] decreases from

$$\Delta n = 0.0000364 \text{ at } \lambda = 0.589\mu \text{ to}$$

$$\Delta n = 0.0000031 \text{ at } \lambda = 8.85\mu.$$

VIII. SUMMARY; COMPARISON OF DISPERSIVE MATERIALS.

In Fig. 3 is given the width that a radiometer receiver of 4′ of arc, subtends in wave-lengths, in different parts of the spectrum produced by prisms of carbon disulfid, quartz, fluorite and rock salt. These data are required for reducing the spectral energy distribution from the prismatic into the normal spectrum.[6]

From these curves it is evident that, in the region of 0.5 to 1.5μ, a carbon disulfid prism is the most useful for producing a large dispersion.

The next best prism material is quartz, which is the most useful in the region of the spectrum extending from the visible to 2.8μ in the infra-red. Beyond this point, a quartz prism is too opaque for practical work.

From the standpoint of dispersion and transparency, a fluorite prism is the most useful in the region of 2μ to 9μ. However, the material is difficult to obtain and the next best substance is rock salt, which permits measurements to 14μ, when using a 60° prism, and to 16μ when using a 30° prism. By enclosing the spectrometer [14] and by keeping the prism covered when not in use, the faces of a rock salt prism are easily protected from moisture.

There are but few sylvite prisms in existence and their usefulness is confined to that part of the spectrum extending from 10 to 20μ.

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[22] RUBENS (with NICHOLS and with TROWBRIDGE): *Ann. der Phys.* (3), **60**, pp. 418 and 724; **61**, p. 225, 1897.
[23] PASCHEN: *Ann. der Phys.* (3), **53**, pp. 301, 337 and 812; 1894.
[24] PASCHEN: *Ann. der Phys.* (3), **56**, p. 762; 1895.
[25] PASCHEN: *Ann. der Phys.* (4), **4**, p. 299; 1901.
[26] PASCHEN: *Ann. der Phys.* (4), **26**, pp. 120 and 1029; 1908.
[27] PASCHEN: *Ann. der Phys.* (4), **35**, p. 1005; 1911.
[28] PASCHEN: *Ann. der Phys.* (4), **41**, p. 670; 1913.
[29] PASCHEN: *Sitzber. Akad. Wiss.,* Berlin, **22**, p. 405; 1899.
[30] MARTENS: *Ann. der Phys.* (4), **6**, p. 603; 1901.
[31] MARTENS: *Ann. der Phys.* (4), **8**, p. 459; 1902.
[32] FLATOW: *Ann. der Phys.* (4), **12**, p. 85; 1903.
[33] MICHELI: *Ann. der Phys.* (4), **7**, p. 772; 1902.
[34] LIEBREICH: *Verh. Phys. Gesell.,* **13**, p. 1; 1911.
[35] CARVALLO: *Compt. Rend.,* **126**, p. 728; 1898.
[36] COBLENTZ: *Publ. No. 35, Carnegie Inst. of Washington,* p. 16, Fig. 50; 1905.
[37] COBLENTZ: *B. S. Bulletin,* **7**, p. 652; 1911.
[38] COBLENTZ: *B. S. Bulletin,* **9**, p. 112; 1912.
[39] COBLENTZ: *B. S. Bulletin,* **11**, p. 471; 1914.
[40] COBLENTZ, EMERSON AND LONG: *B. S. Bulletin,* **14**, p. 653; 1918.
[41] PFLÜGER: *Phys. Zs.,* **5**, 215; 1904.
[42] LYMAN: *Astrophys. Jour.,* **23**, p. 197; 1906. *Ibid.,* **25**, p. 50; 1907.
[43] WARBURG AND MÜLLER: *Ann. der Phys.* (4), **48**, p. 410; *Zeitsch. für Instrk.,* **35**, p .98; 1915.
[44] COBLENTZ: *B. S. Scientific Paper,* No. 406, "The Present Status of the Radiation Constants"; 1920.
[45] TROWBRIDGE: *Ann. der Phys.* (3), **65**, p. 595; 1898.
[46] LEHMANN: *Ann. der Phys.* (4), **39**, p. 62; 1912.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A NEW STUDY OF THE LEUCOSCOPE AND ITS APPLICATION TO PYROMETRY.[*]

BY

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[*] Published by permission of the Director, Bureau of Standards. This paper is an amplification and extension of a paper under similar title by Priest and Foote, presented orally at the New York meeting of the Optical Society of America, February 27, 1920, but not printed. For lack of time, Dr. Foote has been obliged to abandon the original plan of completing this work as a joint investigation. The present author desires to express, however, his hearty appreciation of Dr. Foote's assistance and advice and also of the lively interest he has taken in the work. The author is also indebted to Dr. K. S. Gibson, Messrs. C. M. Blackburn, H. J. McNicholas and Mark Malamphy for assistance. Most of the figures have been prepared by Mr. Chester L. Snow and Miss Gertrude Clemens.

INTRODUCTION.†

1. *History of the Leucoscope.*

"Books are sepulchres of thought;
The dead laurels of the dead
Rustle for a moment only,
Like withered leaves in lonely
Churchyards at some passing tread."—*Longfellow.*

THIS paper deals with a new study of an old optical instrument, practically unknown to contemporary science—the leucoscope. The invention of this instrument is commonly attributed to Helmholtz [1] (about 1878–1882), but it appears that a very similar instrument under another name, "Farbenmesser," was really practically used by Rose in the diagnosis of color-blindness at an earlier date (1863).[2] The name, "leucoscope" (*Greek:* white + view), was apparently coined by Kitao,[3] a pupil of Helmholtz, in his doctor's thesis, *Zur Farbenlehre,* 1878. From 1878 to 1888 a number of papers dealing with this instrument were published by Helmholtz's pupils, Kitao, Koenig and Brodhun.[4]

The potentialities of the instrument as a means of investigation in two distinct fields, vision and the character of the emission of light sources (in modern terms, spectral distribution, color temperature) were clearly pointed out by these early investigators. Koenig, particularly, made a rather extensive series of leucoscopic observations [5] on various light sources, including the just then newly introduced carbon filament lamps[6] of Swan and Edison; and tabulated the latter results as a function of current and candlepower. Notwithstanding these early promises of its utility in two important fields of scientific investigation, the instrument appears to have fallen into almost complete oblivion since the appearance of Brodhun's paper in 1888.[7] So far as can be found, no later papers dealing with it have been published. There is indeed occasional casual reference to it in works dealing with vision;[8] but these merely serve to indicate that no practical use is made of the instrument, and indeed that the authors themselves have but slight acquaintance with it. So far as I can find, recent and contemporary authorities[9] on pyrometry, spectral distribution and color temperature make absolutely no mention of this instrument; and it is apparently quite unknown to present-day pyrometry. It is not indexed and apparently not mentioned in many of the best known standard works of reference on physics, optics, chromatics and pyrometry.[10] In declining to quote price on a leucoscope in reply to a recent request, the original makers[11] state that there never was any demand for the instrument and that they have not made one for about twenty years. The leucoscope has been buried in the uncut pages of scientific papers.

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† Prospective readers of this paper should first familiarize themselves with a paper by Koenig, "Das Leucoscope . . .," *Ann. der Phy. und Chem.,* 17, pp. 990–1008, year 1882. A complete bibliography is given in Appendix II to the present paper; but an acquaintance with this one paper by Koenig will be sufficient to give the reader an understanding of the subject, and prepare him to read the present paper intelligently.

[1] Cf. papers by Koenig and Brodhun (see Appendix II, this paper) and Webster's International Dictionary, 1910 ed.

[2] Rose's instrument could not, however, have been used by trichromatic observers as the leucoscope is, because he prescribes a fixed thickness of quartz of only 5 mm. (*Archiv für path. Anat.* **28**, p. 36), whereas from about two to four times this thickness is necessary to make use of the leucoscope principle. (See Section 1–2 below.)

[3] There appears to be some doubt and controversy as to the relative contributions of Kitao, and his master, Helmholtz, in the original design and naming of the instrument. (Cf. last section of Kitao's second paper. For reference, see Appendix II.)

[4] See bibliography, Appendix II to this paper.

[5] *Ann. der Phy. und Chem.,* 17, pp. 1003–1008; year 1882.

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[6] "The carbon filament lamp was developed about the year 1878 . . ."—Solomon: "Electric Lamps," p. 95; London, 1908.

"In 1879, Swan exhibited a lamp with a filament of carbon in a vacuum bulb, and followed this by various improvements."—New Int. Enc. (1918 Ed.), Vol. 21, p. 715.

"The Edison incandescent lamp was first exhibited in 1879 . . . ."—New Int. Enc. (1918 Ed.), Vol. 7, p. 599.

On the 20th of October, 1880, Sir Joseph Wilson Swan gave at Newcastle the first public exhibition on a large scale of electric lighting by means of glow lamps—Enc. Brit. (11th Ed., 1911), Vol. 26, p. 179, under "Swan, Sir Joseph Wilson."

[7] The author is indebted to his former associate, Mr. P. V. Wells, for having first directed his attention to the leucoscope. Mr. Wells happened to notice Koenig's paper in the *Annalen der Physik* while searching the literature on another subject.

[8] Norris and Oliver: "System of Diseases of the Eye," Vol. II ("Detection of Color Blindness," Wm. Thompson), p. 347. Tscherning: "Physiologic Optics," Eng. trans. by Weiland, p. 270; The Keystone Press, Philadelphia, 1904.

[9] For example, G. K. Burgess, Block, Hyde, H. E. Ives, L. A. Jones, P. D. Foote. Cf. also Appendix II to this paper.

[10] Cf. Appendix II.

[11] Schmidt and Haensch, Cf. *Zeit. für Instk.*, 3, p. 20, Jan., 1883.

2. Design and Essential Principles of the Leucoscope.

The essential principle of the leucoscope design is this: Two juxtaposed images of a rectangular aperture produced by double refraction are viewed through a nicol prism and a variable thickness of crystalline quartz, the nicol being between the eye and the quartz, and the optic axis of the latter being coincident with the line of sight.[12]

To a normal observer, with "white" light illumination, the two images appear, in general, in complementary colors, of unequal brilliance. If another nicol be interposed between the doubly refracting part (calcite rhomb) and the light source, it may be set so as to equalize the brilliance of the two images, leaving them still complementary in color.

Experimental spectroscopic analysis, as well as theoretical consideration based on the rotatory dispersion of the quartz, shows that the spectra of both images are banded [13] and, in particular:

(1) The bands become narrower and more closely spaced as the thickness of quartz is increased.

(2) The system of bands moves through the spectrum as the ocular nicol is rotated.

(3) The bright bands of one image coincide with the dark bands of the other, and conversely; that is, the wavelengths of the maximum and minimum intensity in the two images respectively coincide. The two images are physically complementary in the sense that the light absent from one is in the other.

Trichromatic observers find the following phenomena:

(1) For all thicknesses of quartz greater than about 9 mm., there are found certain positions of the ocular nicol (one in each quadrant), for which both images are very pale (very nearly

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[12] For diagrams of design, consult any one of the following papers: Koenig: *Ann. der Phy. und Chem.*, 17, p. 990; Koenig: *Zeit. für Instk.*, 3, p. 20; *Brodhun*: *Ann. der Phy. und Chem.*, 34, p. 897.

[13] Cf. Appendix I and Figs. 13 and 14, this paper.

white) and, accordingly, very nearly color matched. One may be a very pale blue; the other a very pale yellow; neither may be recognized as more reddish than the other. *This is the criterion for the leucoscope setting* which may be made with considerable accuracy for thicknesses of quartz not much in excess of 20 mm.

(2) For great thicknesses of quartz both images are very pale for *all* positions of the ocular nicol; consequently, the setting by the above criterion cannot be made accurately.

3. *Summary of Conclusions from Early Work.*

The findings of Kitao, Koenig and Brodhun, insofar as they bear on our present interest in the leucoscope, may be summarized as follows:[14]

(1) The setting of the ocular nicol ("leucoscope reading"), by a trichromatic observer, for the condition of closest approach to color match of the two images (neither image reddish with respect to the other) depends upon:

a. The thickness of quartz.

b. The observer.

c. The spectral distribution of energy of the source (color temperature).

(2) Color-blind as well as normal trichromatic observers may make a setting, their criterion being *perfect* rather than approximate color match; and, indeed, they can make the match with a much smaller quartz thickness than the trichromatic observer. They also find the setting to depend upon the quartz thickness and the source of light.[15]

(3) There may or may not be a systematic difference in the settings made by the two eyes of the same observer.[16]

(4) The personal equation in the leucoscope reading cannot be explained in terms of the Young-Helmholtz theory alone.[17]

(5) The personal equation in the leucoscope reading can be explained by assuming individual differences in the relative visi-

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[14] In some cases the obvious modern interpretation of the conclusions of the original papers has been restated in modern terms.

[15] Brodhun: See Appendix II for reference.

[16] Kitao: *Zur Farbenlehre,* pp. 29–31.

[17] Kitao: *Zur Farbenlehre,* pp. 6 and 21.

bility of radiant power. (Modern interpretation of Kitao's conclusions.[18])

(6) The leucoscope is a valuable instrument for the diagnosis of color-blindness.[19]

(7) The leucoscope is a valuable instrument for studying the spectral distribution or color temperature of incandescent solids. (*Modern interpretation of the results of Koenig and Kitao.*[20])

(8) For a constant light source and the same observer, the rotation of the ocular nicol is approximately but not *exactly* a linear function of the thickness of quartz.[21] (Koenig emphasizes the fact that the relation is *not linear;* but on plotting his data,[22] it is found that the departure from a linear relation is *not much* greater than the experimental uncertainty.)

(9) Twenty millimetres of quartz is a suitable standard to use for comparative observations on different light sources.[23]

4. Limitations of Early Work.

It will be noticed, in the section above, that, in order to present them cogently, it has been necessary to restate or interpret in modern terms the original conclusions of Kitao and Koenig. This is because concepts now current were not well formulated; and quantitative data now available did not exist when their papers were published. They could not correlate and formulate their results to the best advantage because of a lack of fundamental data and fundamental concepts of two kinds, viz.:

(1) Visibility of radiant power.[24]

(2) The spectral distribution of energy as a function of temperature—temperature scale for the complete radiator ("black body") and incandescent solids—color temperature.[25] The very

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[18] *Zur Farbenlehre,* p. 21, and *Ab. Tok. Univ.,* **12,** pp. 30–31.

[19] Rose.

Koenig: *Centb. für Prak. Augenheilk.,* **8,** pp. 375–377. Brodhun: *Ann. der Phy. und Chem.,* **34,** p. 918.

[20] Koenig: *Ann. der Phy. und Chem.,* **17,** pp. 1003–1008. Kitao: *Abh. Tokio University No. 12,* pp. 34–58.

[21] Koenig: *Ann. der Phy. und Chem.,* **17,** pp. 1001–1002.

[22] *Ann. der Phy. und Chem.,* **17,** p. 1001.

[23] Koenig: *Ann. der Phy. und Chem.,* **17,** p. 1004.

[24] Koenig's determination of visibility, the earliest that is now given any consideration, was not made until about 1890.

[25] Wien's distribution law was published in 1896; Rayleigh's, in 1900; Planck's, in 1900. The scale of color temperature has been established only in very recent years by Hyde and others.

terms in which the results could have been most simply and instructively stated were lacking. It is interesting to notice how this lack of data and concepts prevented the consummation of this early work, and how when these data and concepts became available, the work had apparently been forgotten.

5. *Purpose and Scope of the Present Paper.*

The purpose of the present paper is to give an introduction to a modern treatment of the leucoscope in terms of knowledge now available. The experimental basis consists of leucoscopic observations on light sources of known temperature by three observers of previously known visibility,[26] and not "color-blind." These observations were made under the following conditions and circumstances [27]:

(1) The criterion for the leucoscope setting was that adopted by Koenig.[28] viz.: An approximate match of color quality (hue and saturation) is made *such that neither field can be recognized as reddish relative to the other.* For the lower temperatures, this results in a nearly or quite perfect match of color quality; for the higher temperatures, one field is very pale blue, the other, very pale yellow. No attempt was made to match brilliance.

(2) One standard thickness of quartz, 20.000 mm.. as recommended by Koenig [29] for the comparison of different sources was used in all observations. (The correctness of this thickness was checked by comparing its measured rotation for homogeneous light with Lowry's tables.[30])

(3) Temperatures are referred to the color temperature scale of Hyde;[31] and are expressed in degrees Kelvin (absolute tem-

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[26] I. G. Priest, P. D. Foote and H. J. McNicholas. Visibility by Coblentz and Emerson, *B. S. Sci. Pap.* 303.

[27] For further details, see also II below.

[28] *Ann. der Phy. und Chem.*, 17, p. 999.

[29] *Ann. der Phy. und Chem.*, 17, p. 1004.

[30] *Phil. Trans. Roy. Soc. A.*, 212, p. 287; year 1912–13.

[31] Only color temperatures between about 1700° and 2400° K are referable *directly* to Hyde's scale as established by the Nela Research Laboratory. Light of spectral distribution corresponding to temperatures above 2400° K is obtained directly from gas-filled lamps and by rotatory dispersion as explained under II–2 below. Temperatures below 1700° K are from a small black body. See also II–2 below.

perature), except in one figure, where degrees Centigrade are used.

(4) Each observer used only his right eye.

(5) One observer made many repeated observations at various times over a period of several months so as to test the constancy of reading. Another repeated his first calibration after a period of several months. The third made calibrations on two successive days.

(6) The observer's eye was protected from extraneous light by a closely fitting eye-shade.

The primary data are presented in the form of calibration curves of leucoscope reading against temperature, arranged so as to show the personal equation and its degree of constancy with time.

These data are discussed with regard to (1) precision, (2) the personal equation, (3) the variation of the personal equation with time, (4) possible use of the leucoscope as a pyrometer.

Two new fundamental laws of the leucoscope are then derived from these data, and stated in terms relating the leucoscope reading to the spectral distribution of light, i.e., *to the spectral distribution of energy of the light source and the observer's relative visibility of radiant power.*

As illustrative of the use of the instrument and without attaching any permanent value or importance to them (because of their preliminary nature), leucoscopic determinations of the color temperature of several sources are given, as well as a series of determinations of the apparent color temperature of sunlight at the earth's surface showing its diurnal variation.

II. APPARATUS AND METHODS USED IN THE PRESENT INVESTIGATION.

1. *The Arons Chromoscope and Its Use as a Leucoscope.*

No leucoscope originally designed as such was available for this investigation. However, the Arons chromoscope[32] is so designed that it may be used to make leucoscopic observations (with some disadvantages noted below). The essential parts of this instrument as used in this investigation are shown in Fig. 1 and explained in the accompanying key.

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[32] Leo Arons. *Ann. der Phy.* (4), 39, pp. 545–568; 1912.

In using this instrument as a leucoscope the Lummer-Brodhun cube (LB) is removed and the analyzing nicol (A) is replaced by the Wollaston prism (W). The principal optical parts, in order from the eye, are then: (1) the observing telescope, (2) the Wollaston prism (W), (3) the quartz plates ($Q_1 - Q_7$), and (4) the nicol, (P). The quartz plates, $Q_{I} - Q_{IV}$, do not function in the leucoscope, and are removed except that one of them is used in obtaining light corresponding to very high temperatures as explained below. Rotation of nicol, P″, relative to nicol P′, serves to adjust the field to a comfortable working brilliance.

One sees, on looking into the instrument, two circular images due to the Wollaston prism. These images are each about 3° in diameter and overlap about one-quarter the diameter of either. It will be noticed that this arrangement is the reverse of the order of parts in the original leucoscope in that the element producing the double image (the Wollaston prism, W) lies between the eye and the quartz instead of between the quartz and the source. This results in the disadvantage that the images rotate as the Wollaston prism is rotated to make the leucoscopic match. A few observations indicate a possible small systematic difference between readings in opposite quadrants, that is, with the positions of the images reversed—the whole field of observation turned through 180°. All readings reported in this paper were made in the same quadrant and are therefore comparable, although their absolute significance may be questioned; that is, we may expect the same observer to obtain slightly different values with a properly designed instrument.

Another disadvantage is the overlapping of the images. This would be a very serious matter if brilliance (intensity) matches were involved, but is not very troublesome in making the matches of color quality, which is all that is done. These disadvantages, although not very serious, were only tolerated because it was not practicable to make promptly any better arrangement for this preliminary work. *It may probably be assumed that the precision and accuracy with a properly designed instrument would be better than that found in this preliminary investigation.*[33]

Leucoscopic readings, by the criterion of Koenig described above, were made by rotating the Wollaston prism with the

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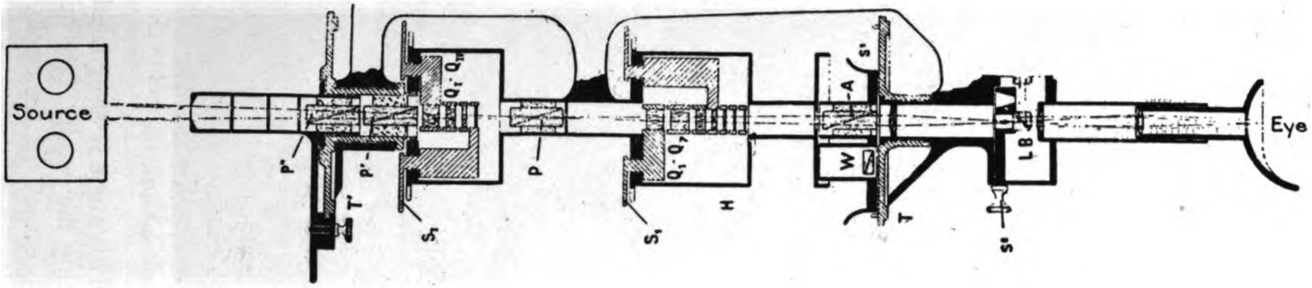
[33] It is now planned to have a new model instrument constructed in the Bureau of Standards instrument shop.

FIG. 1.



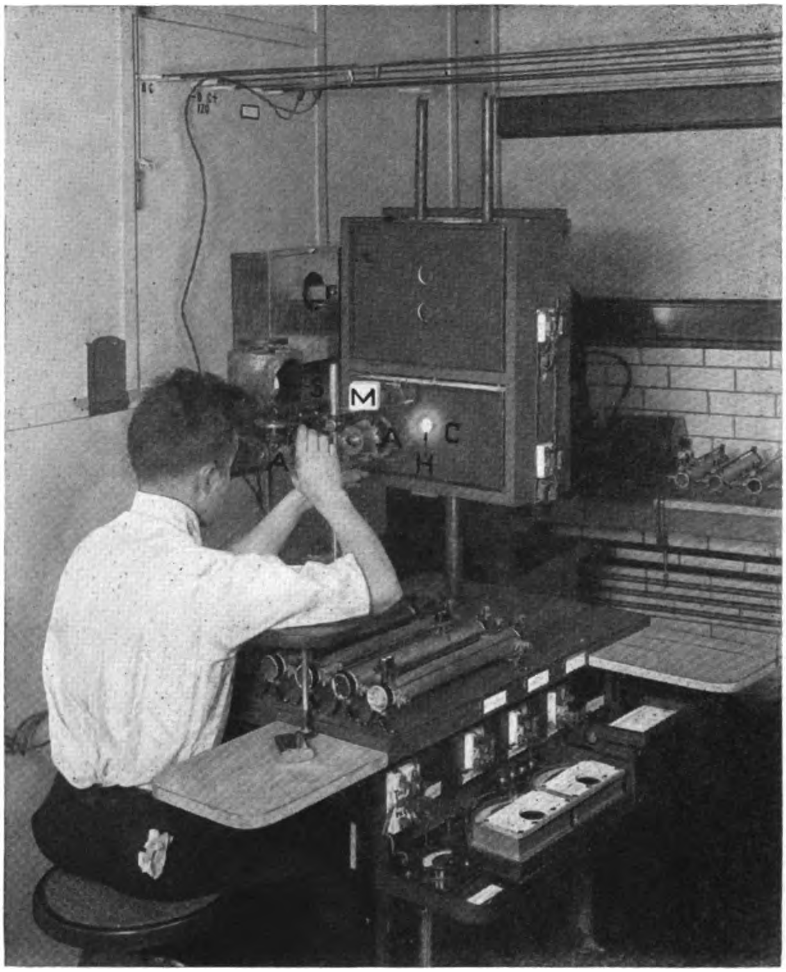
PART OF ARONS CHROMOSCOPE.
(From *Ann. der Phy.* (4) 39, p. 547.)

Key.

LB—Lummer-Brodhun cube, *removed in the present use of the instrument.* A—Nicol prism, *removed and replaced by W in the present use of the instrument.* W—Wollaston prism, *used in place of A* and rotating with the divided circle T. (In this instrument W and A are mounted on a slide so that *either* may be brought into the axis of the instrument at will.) Q_1–Q_7—Quartz plates. *In the present work, all of these plates except three of thicknesses respectively 8, 8 and 4 mm. were removed.* P—Nicol prism in fixed position. Q_I–Q_{IV}—Quartz plates, *not used as part of leucoscope*, but one of them (1 mm.) used in connection with P and P′ to obtain light of spectral distribution corresponding to very high temperatures, as explained in the text. P″–P′—Nicol prisms. Rotation of P″ relative to P′ serves to vary absolute brilliance of both images.

divided circle, T (Fig. 1), in the same direction as the rotation of the plane of polarization by the quartz, angles being measured from zero for the extinction position of the extraordinary image with all quartz removed. This angle is designated as the "leucoscope reading." (In reducing the results, care has been taken to correct for the small "zero error" of the circle.)

FIG. 2.



Assembled apparatus.

2. *Pyrometric Calibration—Temperature Scales.*

The pyrometric calibration has been made in part by pointing the instrument into a black body at measured temperatures and in part by pointing it at a magnesia surface illuminated by lamps at known color temperature. Care was taken to obtain a uniform field in each case. A single determination usually consisted of five observations. (In some cases only three, and in others, ten.)

The earliest quantitative observations of any reliability or importance were made on a small black body of the type used by Fairchild in the Bureau of Standards' pyrometric laboratory,[34] and kindly loaned and operated for this purpose by Dr. Foote. Temperatures were read by Dr. Foote by means of a disappearing filament optical pyrometer immediately before and after each leucoscopic determination; and are estimated by him to be reliable to about 5° C. However, this estimate of accuracy applies only to the measured brightness temperature for wavelength 650 millimicrons. We do not know how closely the small black body in question approximates to an ideal complete radiator, and therefore cannot state how closely these measured brightness temperatures should agree with color temperatures. With this source, a temperature range from about 800° to 1600° C (1173°–1873° K) was obtained (Fig. 5). To obtain higher temperatures standard incandescent filament lamps were used, either directly or by an intermediate comparison source, and the observations made on the light reflected from a magnesium oxide or carbonate surface illuminated by the lamp.

A point at 2360° K was obtained by means of a vacuum tungsten lamp operated at a voltage previously determined to give a color match with a standard acetylene flame assumed to have this color temperature, as indicated by the most recent findings of Coblentz as well as those of Hyde, Forsythe and Cady.[35]

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[34] This radiator was made of a sillimanite crucible within an outer crucible of alundum, the space between being filled with fused alumina. All sides except the aperture were heated electrically by a heater of platinum-rhodium wire. This furnace was cemented with alundum cement to give practically a one-piece furnace. It was supported on four porcelain legs about 15 cm long; and was freely exposed to the air. The dimensions were: inside diameter, 23 mm; inside length on line of sight, 30 mm; and aperture, 12 mm.

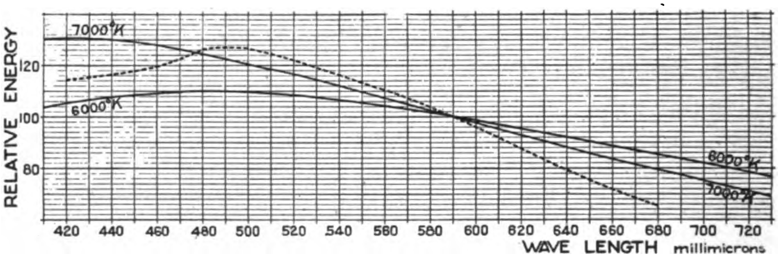
[35] Hyde, Forsythe and Cady: *Phy. Rev.* (2), 13, pp. 157–158, 1919; *Phy. Rev.* (2), 14, p. 379, 1919. Coblentz: *Journ. Frank. Inst.*, pp. 399–401; September, 1919. *B. S. Sci. Pap. 362*, Feb., 1920.

A point at 2830° K was obtained by means of a gas-filled tungsten lamp (B. S. Lamp 1717) of known spectral energy distribution, previously determined radiometrically by Dr. W. W. Coblentz. The color temperature, 2830° K, was derived as follows:

(1) The ratio of energy at wave-length 480 millimicrons to energy at wave-length 680 millimicrons was computed for a complete radiator (Planck's equation $c_2 = 14350$) for various temperatures, and plotted as a function of temperature.

(2) The same ratio for this lamp was found from Coblentz's data, and the corresponding temperature read from the graph just mentioned.

FIG. 3.



Comparison of Spectral Energy Distributions of Complete Radiator at 6000° and 7000° K with Spectral Distributions Obtained by Method of Rotatory Dispersion.
Actual source: B. S. lamp 1716 at 144.1 volts, 22 l.p.w.
Quartz thickness = 1.000 mm. $\phi = 4.5°$ (see Appendix I).
Solid curves: complete radiator by Planck's formula ($c_2 = 14350$) for 6000° and 7000° K.
Dotted curve: B. S. lamp 1716 at 144.1 volts, modified by rotatory dispersion. Relative energy of lamp x $\sin^2 (4.5 - a_\lambda)$ (see Appendix I).

The points at 3000°, 3200°, 4000°, 5000°, and 7000° K were obtained by the artifice of rotatory dispersion, a system consisting of a quartz plate between nicols being inserted between the source and the leucoscope so that the optic axis of the quartz is coincident with the line of sight. The effect is briefly this: Owing to the rotatory dispersion, the nicols and quartz plate constitute, in effect, a selective light filter, the spectral transmission of which is determined by the thickness of the plate and the angle between the principal planes of the nicols.[36] It is possible to choose this thickness and angle so as to make a *blue filter* which will reduce the actual known spectral distribution of a lamp at operable temperature, approximately to the spectral distribution (by Planck's formula) of a theoretical black body at some assigned high temperature. To accomplish this end in the present instance, use has been made of nicols P′ and P and a 1-mm. quartz plate at $Q_{I}-Q_{IV}$ (Fig. 1). The complete specifications are recorded in the legends to Fig. 6. The point 7000° K is uncertain because the energy distribution obtained is not strictly that of a black body (Fig. 3). The quantitative significance of the points at 3000°, 3200°, 4000°, and 5000° K may also be questioned because of the uncertainty of spectral distribution of energy in the standard acetylene flame,[37] on which they depend (Cf. legend, Fig. 6). The distribution given by Coblentz and Emerson[38] was first assumed. The resultant spectral distributions have now been recomputed assuming the spectral distribution of a complete radiator at 2360° K (Planck's $c_2 = 14350$) for the acetylene flame. The resultant distributions on both assumptions are compared with the theoretical (Planck's formula, with $c_2 = 14350$) distributions in Fig. 4. On account of these uncertainties, no very definite significance can be ascribed to the calibration above 2830° K. *It will be noticed, however, that, in spite of these circumstances: (1) a smooth calibration curve has been obtained (Fig. 6); (2) the uncertain part is consistent with a reasonable extrapolation of the more certainly established part (Fig. 6); (3) the high temperature calibration is consistent with a reasonable assumption for the color temperature of noon sunlight at the earth's surface* (Cf. Section VI and Fig. 15, below); *(4) the high temperature calibration is consistent with a reasonable assumption for the temperature of the sun itself* (Cf. Sec. VI, below); *(5) the calibration at about 3700° K is consistent with the probable temperature of the electric arc.* (See Sec. IV–5, below.)

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[36] Cf. Arons: *Ann. der Phy.* (4), 33, pp. 810–819, year 1910. Priest: *Phy. Rev.* (2), 10, pp. 208–212, year 1917; *Phy. Rev.* (2), 11, p. 502, year 1918; *Phy. Rev.* (2), 15, pp. 538–539, year 1920; and forthcoming papers "The Application of Rotatory Dispersion to Colorimetry," . . . See also Appendix I, this paper.

In the present state of uncertainty regarding both the spectral distribution of the acetylene flame and the value of Planck's constant, c_2, it does not appear worth while to discuss this ques-

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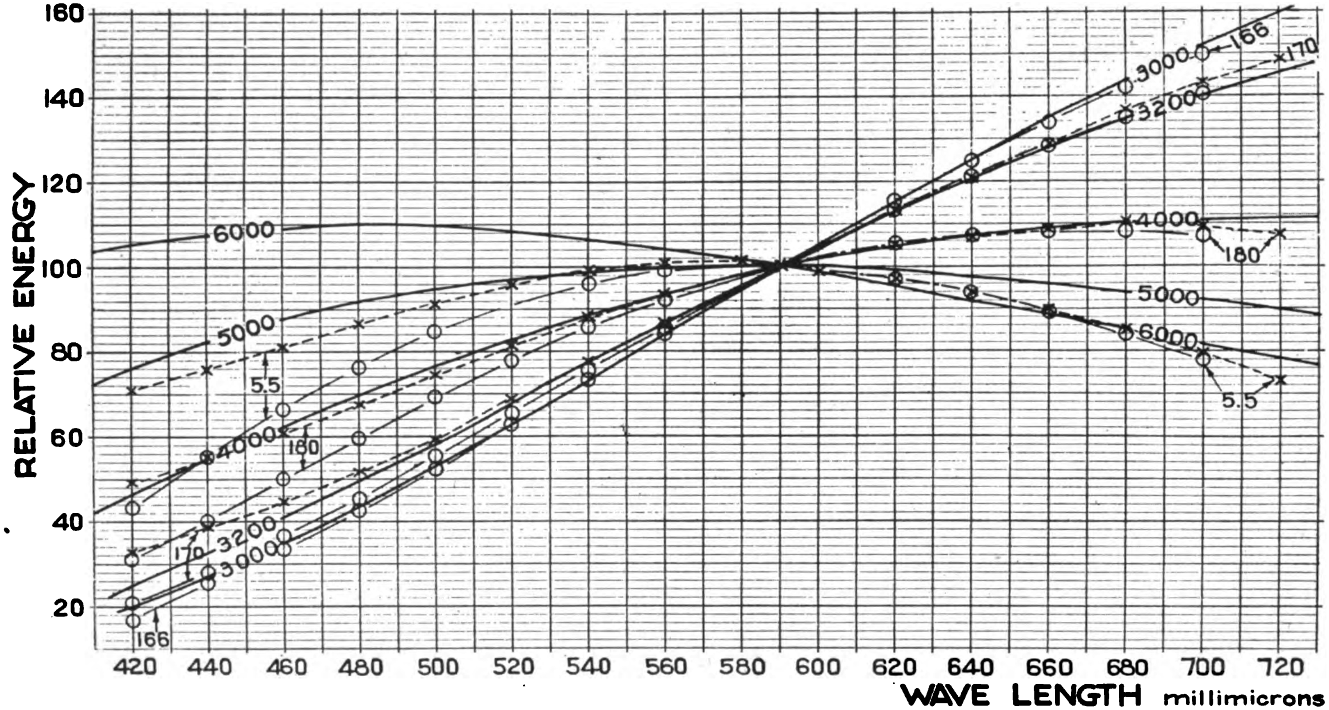
[37] Cf. Hyde, Forsythe and Cady: *Phy. Rev.* (2), 13, pp. 157–158, 1919; *Phy. Rev.* (2), 14, p. 379, 1910. Coblentz: *Jour. Frank. Inst.*, pp. 399–401, Sept., 1919; *B. S. Sci. Pap. No. 362*, Feb., 1920.

[38] *Bull. B. S.*, 13, p. 363; year 1916.

FIG. 4.



Comparison of Spectral Energy Distributions of Complete Radiator with Spectral Distributions Obtained by Method of Rotatory Dispersion. Actual source at color temperature of acetylene flame. Quartz thickness=1.000 mm.
Solid curves: Complete radiator by Planck's formula ($c_2=14350$). Numbers attached to these curves indicate temperature in degrees K.
Dotted curves, crosses: Acetylene flame as given by Coblentz and Emerson (Bull. B. S. 13, p. 363), modified by rotatory dispersion.
Dashed curves, circles: Complete radiator at 2360° K, modified by rotatory dispersion.
Numbers attached to dotted and dashed curves indicate values of ϕ in the formula $\sin^2(\phi - a_\lambda)$. (See Appendix I.)

tion further. The calibration curve given (Fig. 6) is probably as significant as can be determined under the present circumstances; and it is considered worth while to publish it as a preliminary roughing out of the complete leucoscope-temperature function for an approximately normal observer for all temperatures between about 1000° and 7000° K.

Probably the most reliable part of the pyrometric calibration is that based on a standard vacuum tungsten lamp supplied by the Nela Research Laboratory,[39] and certified by that laboratory to operate at given color temperatures, between 1700° and 2400° K, when maintained at prescribed voltages. This lamp was used as a standard both directly and indirectly. It was first determined that the selective reflection of a particular block of magnesium carbonate was negligible for this purpose. In the observations of April 1, 1920 (Fig. 7), this magnesium carbonate block being viewed by the leucoscope was illuminated directly by this standard lamp at various voltages, giving a series of color temperatures at which leucoscopic readings were made. This arrangement did not give sufficient light for accurate observations at low temperatures. To overcome this difficulty and to save the standard lamp, the following indirect method was adopted in later work: The standard lamp (at or near S, Fig. 2) illuminated a surface of the magnesium carbonate block placed so (M, Fig. 2) that the reflected light illuminated part of the Arons chromoscope (A, Fig. 2) field by means of the Lummer-Brodhun cube (LB, Fig. 1).[40] All of the quartz plates (Q_1–Q_7 and Q_I–Q_{IV}, Fig. 1) being removed, the other part of the field was illuminated by light reflected through the hole H (Fig. 2), from the magnesium oxide coated interior wall of the box C (Fig. 2), the interior of this box being illuminated by two seasoned vacuum tungsten lamps in parallel. The room was darkened and the block shaded so that no appreciable light other than that from the standard lamp fell upon the magnesium carbonate surface. With this arrangement, the standard lamp was operated at the voltages prescribed to give certain color temperatures (1719, 1849, 1965, 2070, 2164, 2250, 2331, 2409° K). The current in the lamps in the box, C (Fig. 2), was then adjusted to give a color match. This calibration of the current in the comparison lamps in terms of color temperature of the standard lamp was repeated many times by the author with different arrangements of the photometric field; and an average calibration curve plotted. It was also checked at 2360° K to within 8° by the means of numerous observations on another Nela standard lamp. Subsequent calibration of the leucoscope was made by observations on the magnesia-coated wall of the box, C, with the comparison lamps in this box, operating at measured currents; and corresponding color temperatures were obtained from the current-temperature calibration curve just mentioned. The observations of August 25–26, 1920, by Foote, McNicholas and Priest (Fig. 7), were made in this way. The chief advantage of this procedure over direct use of the standard lamp is that greater brightness in the leucoscope can be obtained, which is necessary at the low temperatures.

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[39] Letter and Report on Calibration, Nela Research Laboratory to Bureau of Standards, March 8, 1920.

[40] Cf. also Arons: *Ann. der Phy.* (4), 39, p. 545.

III. EMPIRIC RESULTS OF PYROMETRIC CALIBRATION.

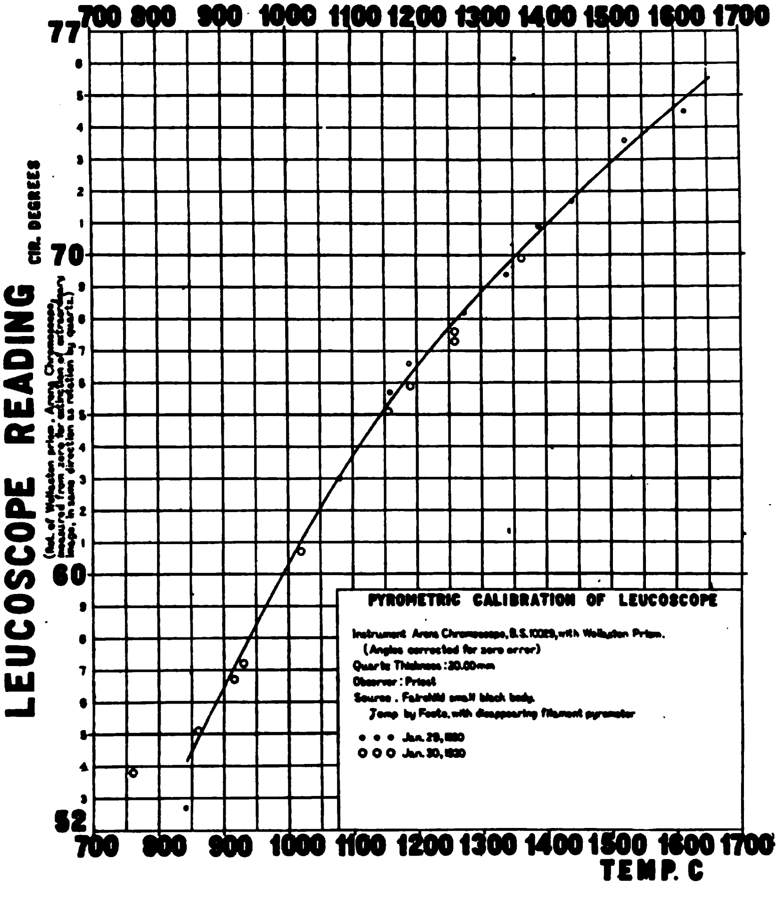
The two calibrations, January 29 and 30, 1920, made with the small black body (about 800–1600° C) and covering the range of industrial furnace temperatures, are shown in Fig. 5. The complete calibration for various temperatures from data obtained between January 29 and February 4, 1920, is shown in Fig. 6. In Fig. 7 are shown:

(1) Calibrations based on the Nela standard lamp as follows:
  a. By Priest, April 1, August 25 and August 26, 1920.
  b. By Foote, April 1, August 25 and August 26, 1920.
  c. By McNicholas, August 25 and August 26, 1920.
  d. Summarized data by Priest at 2000° K. (Mean of all observations, July 8, 9, 12, 16, shown in detail in Table I.)

(2) Summarized previous data by Priest on other sources, obtained between January and August, 1920. (The January-February data have already been shown in Fig. 6, but are reproduced here for the sake of comparison.)

In order to exhibit in detail the precision and the reproducibility of the readings of a practiced observer from time to time, Tables I and II are presented. These data probably represent the most consistent work that can be done by this observer with the present apparatus, but not necessarily the best that could be done with a more favorably designed instrument.[41] These data refer to two temperatures, namely, 2000° and 2830° K. Independent of any question as to whether or not these are true temperatures on any scale, they were accurately repro-

FIG. 5.



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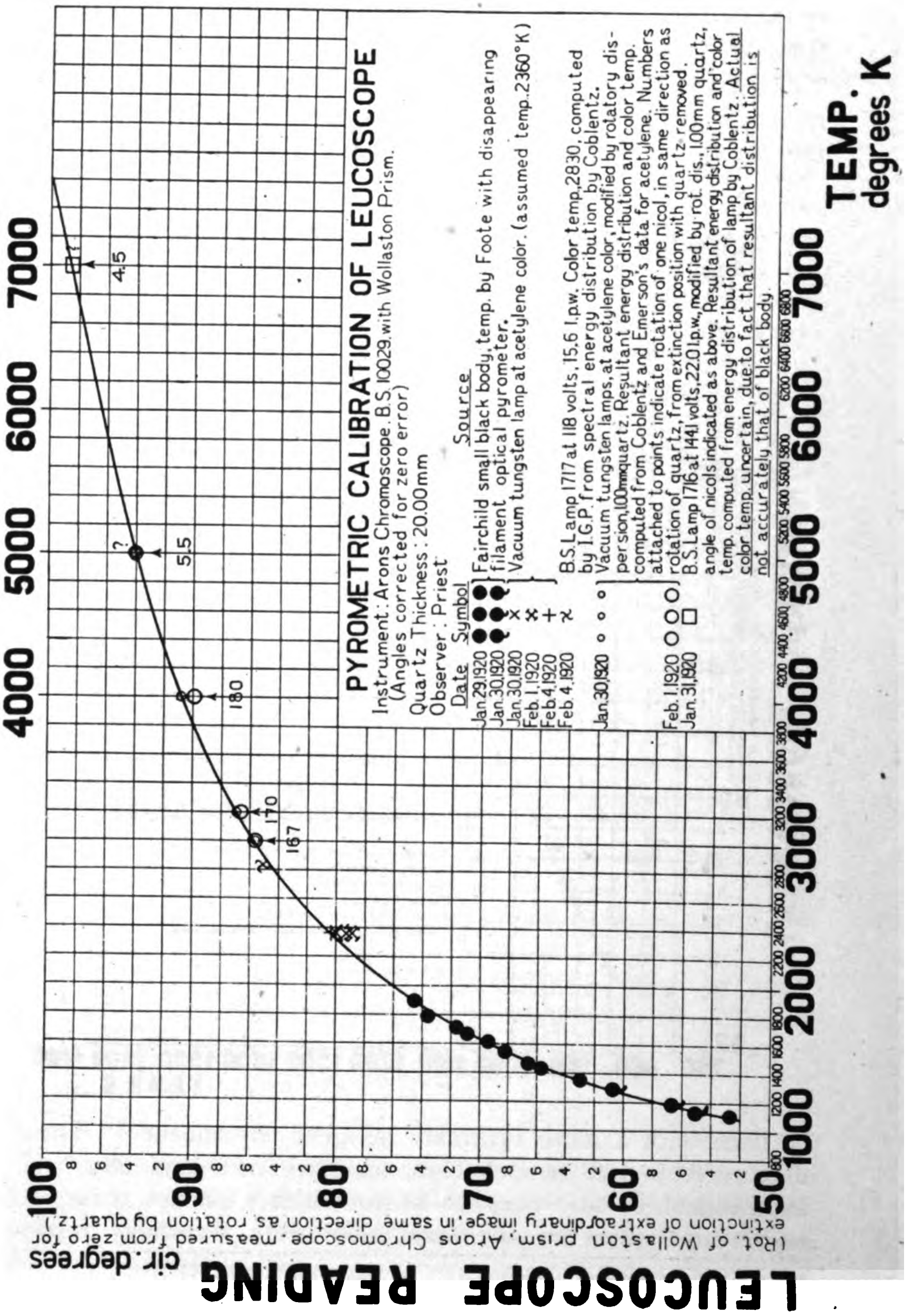
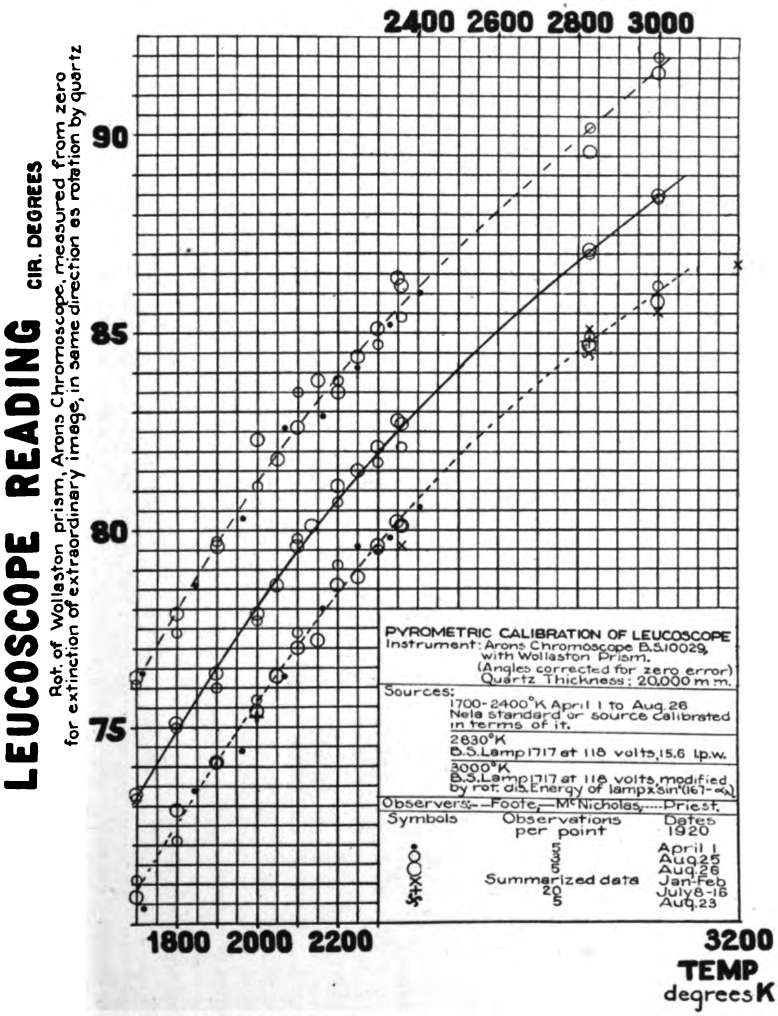
[41] Cf. discussion of disadvantages of instrument under II above.

FIG. 6.


PYROMETRIC CALIBRATION OF LEUCOSCOPE
Instrument: Arons Chromoscope. B.S. 10029. with Wollaston Prism.
(Angles corrected for zero error)
Quartz Thickness: 20.00mm
Observer: Priest
Date
Symbol
Source
Jan.29.1920 ● Fairchild small black body, temp. by Foote with disappearing
Jan.30.1920 ● filament optical pyrometer.
Jan.30.1920 × Vacuum tungsten lamp at acetylene color. (assumed temp. 2360°K)
Feb.1.1920 ×
Feb.4.1920 +
Feb.4.1920 ~
Jan.30.1920 ∘ B.S. Lamp 1717 at 118 volts, 15.6 l.p.w. Color temp. 2830, computed by I.G.P. from spectral energy distribution by Coblentz.
Feb.1.1920 ○ Vacuum tungsten lamps, at acetylene color, modified by rotatory dispersion, 100mm quartz. Resultant energy distribution and color temp. computed from Coblentz and Emerson's data for acetylene. Numbers attached to points indicate rotation of one nicol, in same direction as rotation of quartz, from extinction position with quartz removed.
Jan.31.1920 □ B.S. Lamp 1716 at 144.1 volts, 22.0 l.p.w., modified by rot. dis., 100mm quartz, angle of nicols indicated as above. Resultant energy distribution and color temp. computed from energy distribution of lamp by Coblentz. Actual color temp. uncertain, due to fact that resultant distribution is not accurately that of black body.
4.5
?
5.5
?
180
170
167
LEUCOSCOPE READING
cir. degrees
(Rot. of Wollaston prism, Arons Chromoscope, measured from zero for extinction of extraordinary image, in same direction as rotation by quartz.)
50
60
70
80
90
100
TEMP. degrees K
1000
2000
3000
4000
5000
6000
7000

duced from time to time by reproducing the measured currents in the lamps, the whole arrangement of leucoscope, lamps, and accessory apparatus remaining undisturbed during the period

FIG. 7.



of these observations. The consideration of these data thus affords us a pure test of the reproducibility of one observer's readings at fixed points on the temperature scale. (Constancy of the personal equation.)

TABLE I.

Constancy of Leucoscopic Readings with Time.

Temperature—2000° K (referred to Nela Standard Lamp). Observer—Priest.

	Leucoscope reading (corrected for zero error)				Departures of single observations from daily means (without regard to sign)							
Date, 1920	July 8	July 9	July 12	July 16*	July 8		July 9		July 12		July 16*	
Hour	9 : 50 A.M.	11 : 40 A.M.	2 : 20 P.M.	9 : 00 A.M.	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K
Single observations 1	75.1	75.1	75.9	75.1	0.1	8	0.1	8	0.5	40	0.1	8
2	75.1	75.0	75.1	75.7	0.1	8	0.2	16	0.3	24	0.5	40
3	75.0	75.8	75.6	75.1	0.0	0	0.6	48	0.2	16	0.1	8
4	74.7	75.5	75.2	75.0	0.3	24	0.3	24	0.2	16	0.2	16
5	75.0	74.8	75.2	75.1	0.0	0	0.4	32	0.2	16	0.1	8
Mean	75.0	75.2	75.4	75.2		8		26		22		16
Final mean	75.20				14° K							
Daily mean minus final mean	−0.2	0.0	+0.2	0.0								
Temperature equivalents of above differences	16° K	0	16° K	0								
Mean of above temp. "errors" (without sign)	8° K											

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* NOTE: Observer not feeling very well, July 16.

TABLE II.

Constancy of Leucoscopic Readings with Time.

Temperature—2830° K (referred to B. S. Lamp 1717 at 118 volts, 15.6 l. p. w.). Observer—Priest.

Date, 1920	Leucoscope reading (corrected for zero error)					Departures of single observations from daily means (without regard to sign)									
	July 8	July 9	July 12	July 16*	Aug. 23†	July 8		July 9		July 12		July 16*		Aug. 23†	
Hour	9 : 05 A.M.	11 : 45 A.M.	2 : 10 P.M.	9 : 10 A.M.	2 : 50 P.M.	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K	Degrees circle	Degrees temp. K
Single observations 1	84.5	84.8	85.7	84.3	84.0	0.1	12	0.1	12	0.5	60	0.2	24	0.5	60
2	84.3	85.2	84.7	84.9	84.8	.1	12	.3	36	.5	60	.4	48	.3	36
3	84.2	84.5	84.5	84.2	85.3	.2	24	.4	48	.7	84	.3	36	.8	96
4	84.4	84.6	85.9	84.2	84.2	.0	0	.3	36	.7	84	.3	36	.3	36
5	84.6	85.5	85.2	84.8	84.3	.2	24	.6	72	.0	0	.3	36	.2	24
Mean	84.4	84.9	85.2	84.5	84.5		14		41		58		36		50
Final mean	84.7					40° K									
Daily mean minus final mean	−0.3	+0.2	+0.5	−0.2	−0.2										
Temperature equivalents of above differences	−36° K	+24° K	+60° K	−24° K	−24° K										
Mean of above temp. "errors" (without sign)	34° K														

NOTES: * Observer not feeling very well, July 16. † First observations after over one month without any practice.

IV. DISCUSSION OF THE EMPIRIC RESULTS AS TO PYROMETRIC ACCURACY.

1. *Precision.*

The following estimates of precision of reading (agreement of observations made in rapid succession) are made from the early work done in January and February, 1920. The precision with a properly designed instrument would probably be better.

Temperature degrees K	Average departure of a *single* observation from the mean of a number of observations
1000–2000	10° to 25°
2360	30° to 60°
3000–4000	50° to 100°
4000–5000	100° to 200°

The reader may study the precision at two temperatures (2000° and 2830° K) in detail from Tables I and II, from which the following may be concluded:

Temperature degrees K	Average departure of *single* observation from mean	Maximum departure of *single* observation from mean
2000	8° to 26°	48°
2830	14° to 58°	96°

It will be noticed that the precision falls off notably with increasing temperature. This would appear, however, to characterize measurements of *color temperature* generally, rather than the leucoscope particularly. The reason for it lies in the fact that the quality of color changes much less rapidly with temperature at high temperatures than at low temperatures. The basis of this is apparent in the graphs in Figs. 9 and 10.

2. *The Personal Equation.*

The pyrometric calibration curves of three different observers will be found in Fig. 7. These three observers represent rather extreme types of visibility among non-colorblind individuals. The visibility curves of these observers taken from Coblentz and Emerson's original plots are shown in Fig. 8. The classification of their relative visibilities according to Coblentz and Emerson [42] is as follows: Foote, subnormal blue; McNicholas, normal; Priest, subnormal red.

A marked personal equation is clearly shown, the differences between the different observers' readings at all temperatures (1700°–3000° K), being many times the uncertainty of any one observer's reading.[43]

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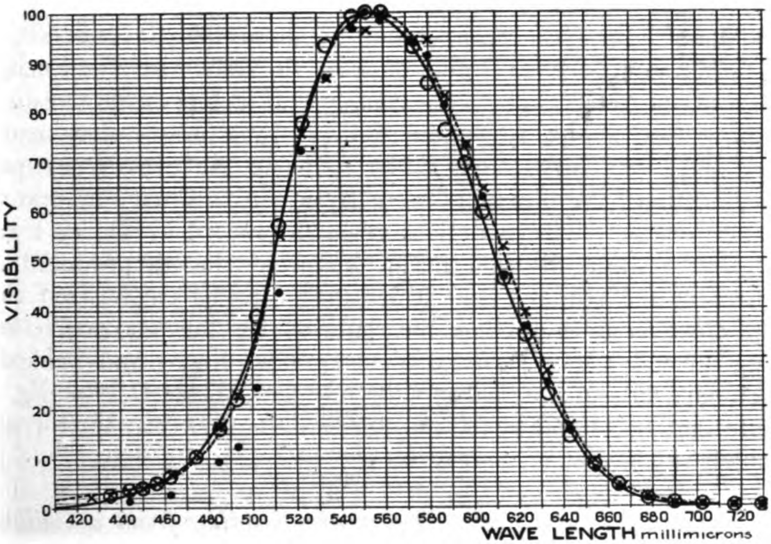
[42] *B. S. Sci. Pap. 303*, pp. 184–185.

3. *The Constancy of the Personal Equation at Different Times.*

From the agreement of observations on different days and at different hours as shown in Figs. 5, 6, and 7 and Tables I and II, we conclude that the personal equation is constant, that is, that the change of reading from hour to hour, day to day, and month to month does not exceed the uncertainty indicated by the

FIG. 8.



Relative Visibility of Radiant Power.
Solid curve: Average data adopted as standard in this paper. 400–550 and 660–710 millimicrons, Hyde, Forsythe & Cady, *Astrop. J.* 48, p. 87. 560–650 millimicrons, Ives, *Phil. Mag.* Dec., 1912, p. 859.
Dashed curve: Coblentz and Emerson's average of 125 observers *B. S. Sci. Pap.* 303, p. 219.

Solid circles: Foote
Crosses : McNicholas
Open circles: Priest
Data by Coblentz and Emerson *B. S. Sci. Pap.* 303, pp. 184–185.

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[43] Similar calibrations for a number of observers of known visibility have just been completed with the same apparatus by C. M. Blackburn under the author's direction. Time has not yet been available to reduce and fully study these data. They will be published later. In the meantime, we refrain from attempting to make any correlation between the personal equations of visibility and leucoscope reading.

differences between closely successive readings. It will be noticed that the observations on which this conclusion is based extend over several months and include work in both morning and afternoon. Some of the observations were made in a dark room, others in daylight.

Kitao found that previous fatigue by:

(a) spectral red, green or violet,
(b) spectral red and violet (purple),
or (c) "white light,"

did not affect the leucoscope setting; while similar previous fatigue by other homogeneous stimuli destroys the color match and makes *any* setting temporarily *impossible*.[44] In the present investigation, no special tests of the effect of previous fatigue were made; nor were any precautions taken to bring the eye to any particular state; the observers were subject to ordinary laboratory conditions. It will be noticed from the record in Tables I and II that a slight disorder in the observer's health produced no effect in the reading, and it is further to be emphasized that the observer does not lose his criterion for the leucoscopic setting through a long period of disuse. (See Fig. 7 and Table II. Compare data of August 23 with July data, Table II. In Fig. 7, note that Dr. Foote made no observations between April 1 and August 25.)

4. *Total Uncertainty of Temperature Reading from a Calibration Curve.*

This discussion is based on the data shown in Fig. 7 (Cf. Sec. III above) as follows:

(1) Small black dots, each representing the mean of five immediately successive observations, April 1, 1920, by Foote and Priest.

(2) Small open circles, each representing the mean of three immediately successive observations, August 25, 1920, by Foote, McNicholas, and Priest.

(3) Large open circles, each representing the mean of five immediately successive observations, August 26, 1920, by Foote, McNicholas, and Priest.

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[44] *Zur Farbenlehre*, pp. 21–29.

Foote is a skilled observer of long experience in optical pyrometry; but has made scarcely any leucoscopic observations other than those shown in Fig. 7.

McNicholas is a skilled photometrician of exceptional precision; but had made absolutely no leucoscopic observations prior to those shown for August 25.

Priest's experience with the leucoscope dates from the latter part of January, 1920, and is fully shown in this paper.

The point x at temperature 2360° K is not strictly comparable with the data in April and August because different standard sources were used.

Dispensing with any mathematical adjustment, curves judged by appearance to represent these data about as well as possible have been drawn as follows:

Dashed curve, Foote's calibrations, April 1, August 25, 26.

Solid curve, McNicholas' calibrations, August 25, 26.

Dotted curve, Priest's calibrations, April 1, Aug. 25, 26.

The individual readings of the three observers are so far apart that there is no chance of confusing their respective data in the plots.

Taking the departure of each point from the curve, we find the following average and maximum departures:

	Average	Maximum
Foote	25° K	75° K
McNicholas	13°	45°
Priest	20°	58°

Omitting consideration of the points determined by only three observations, that is, considering only those determined by five observations, we find the following maximum departures:

Foote	75° K
McNicholas	30°
Priest	52°

We conclude that,

The average error of a leucoscopic determination of temperature at about 2000° K, based solely on color quality (i.e., paying no attention to equating brilliance) *and derived from five observations by a qualified observer from his own calibration curve is about 15° C, and that the actual error rarely attains 50° C. At lower temperatures, throughout the range of indus-*

trial furnace temperatures, the uncertainty will be less than this. There has been as yet no opportunity to study thoroughly the reproducibility of readings at these lower temperatures.

5. Possible Practical Use of the Leucoscope as a Pyrometer.

It is evident from what has preceded that *the leucoscope may be regarded as a type of optical pyrometer.* Indeed, it has the distinction of being a *strictly* and *purely* optical pyrometer, *requiring no electrical or other accessory apparatus.* It shares this distinction with the sensitive tint pyrometer of Mesuré and Nouel[45] of which it may perhaps be considered a special case. The only essential difference in the design of the two instruments is that Mesuré and Nouel substitute a nicol for the double image rhomb.[46] This results in the observer's seeing only one uniform field instead of two juxtaposed fields to be compared and matched according to a stated definite criterion. (Cf. I–2 above.) The criterion for setting is only the so-called " sensitive tint " which must be carried in the observer's memory, and which changes with temperature. According to LeChatelier, Boudouard and Burgess,[47] as well as the experience of the Pyrometry Section, Bureau of Standards,[48] the Mesuré and Nouel instrument is of little value as a pyrometer. It would appear, *a priori*, highly probable that the leucoscope design would give much better accuracy than that of Mesuré and Nouel because of the more definite criterion it affords. Some data as to the accuracy of leucoscopic temperature measurements have been given just above; *but final judgment on the value of the leucoscope as a pyrometer should not be based on this evidence alone.* It is highly probable that a considerably higher degree of accuracy can be attained with a properly designed instrument (Cf. II–1 above), particularly in the range of industrial furnace temperatures. The author ventures to predict that, within this range, with such an instrument, by making and taking into account the readings for both the quality and brilliance match, temperature determinations from the mean of five observations may be made with an uncertainty of not more than 5 to 10° C. The leucoscope appears to be worthy of careful consideration and study as a pyrometer. If it should prove to possess sufficient accuracy for even a limited number of purposes, its convenience and portability would recommend its wide use. On this score, it is only necessary to say that a complete instrument could be inclosed in a rectangular carrying case not more than 10 × 10 × 30 cm, giving a total weight probably less than 2 or 3 pounds; and that absolutely no electrical or other accessory apparatus would be required for its use.

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[45] LeChatelier and Boudouard: " High Temperature Measurements," English translation by Burgess, New York, 1901, pages 158–160. Burgess and LeChatelier: " High Temperature Measurements," page 348, New York, 1912.

[46] In the only Mesuré and Nouel instrument which the author has examined, the thickness of quartz is about 11 mm. This would be *sufficient* to use in leucoscopic observations; but is probably not the most favorable thickness (about 20 mm.).

As a pyrometer, the leucoscope might with some reason be called the " duplex or double image sensitive tint pyrometer."

[47] *Loc. cit.*

[48] Information to the author from Dr. P. D. Foote.

Without attaching any special importance to them, the following leucoscopic determinations of temperature made by the author on the basis of the calibration curves in this paper are given for comparison with previous data:

Source	Leucoscope Reading (cir. degrees)	Temperature from Calibration	Previous Values	
Kerosene flame (flat)	76.1 (Mean of 20 observations)	2035° K (From Cal. Fig. 7)	2045	Hyde and Forsythe (Jour. Frank. Inst., 183, pp. 353–354).
Paraffin candle.....	73.9 (Mean of 10 observations)	1890 (From Cal. Fig. 7)	1920	
Crater, carbon arc..	89.6 (Mean of 5 observations)	3720 (From Cal. Fig. 6)	3690 3680 3720	Various determinations of black body temperature, by Waidner and Burgess (Bull. B.S., 1, p. 123).

The following possible fields for the use of the leucoscope as a pyrometer suggest themselves:

(1) Temperature measurements of industrial furnaces where the utmost accuracy is not required.

(2) The color grading of incandescent light sources in terms of color temperature.[49]

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[49] The author agrees with Hyde and Forsythe (*Jour. Frank. Inst.*, 183, pp. 353–354; 1917) that color temperature affords the most suitable basis for the color grading of illuminants.

(3) Field measurements of the quality of daylight. (Cf. Sec. VI, below.)

(4) The color grading of stars in terms of color temperature. (Subsequent to a more careful calibration for high temperatures.)

V. THE LAWS OF THE LEUCOSCOPE.

1. The Relation between the Leucoscope Reading and the Spectral Distribution of Light from the Source.

The first law derived from the observations recorded in this paper may be stated as follows:

The leucoscope reading is a linear function of the angle through which the plane of polarization of the wave-length of the spectral centre of gravity of the light from the source[50] is rotated by the quartz plate.

This relation has been arrived at by the following procedure:

(1) The spectral luminosity curves of a black body were drawn as shown in Fig. 9, on the basis of data as specified in the legend to that figure.

(2) The wave-lengths of centres of gravity of these curves were determined by a planimeter.

(3) These wave-lengths of centre of gravity were plotted against temperature as shown in Fig. 10.

(4) For selected temperatures (indicated in degrees K by numbers attached to circles in Fig. 11), the angular reading of the leucoscope was read from the calibration curve in Fig. 6 while the wave-length of centre of gravity for the same temperature was read from Fig. 10.

(5) The rotation of the plane of polarization for these latter wave-lengths by 20.000 mm. of quartz was then obtained from a very accurate large scale rotatory dispersion curve based on Lowry's data.[51]

(6) The angular readings of the leucoscope were then plotted against these rotations as shown in Fig. 11.

(7) The locus of these points is found to be accurately a straight line (heavy solid line, Fig. 11) throughout the range

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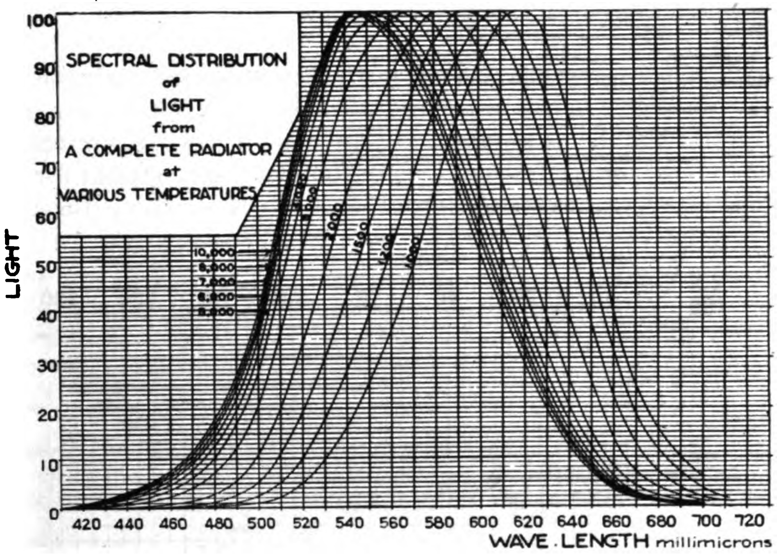
[50] Centre of gravity or so-called "centre of area" of "luminosity curve" of the source.

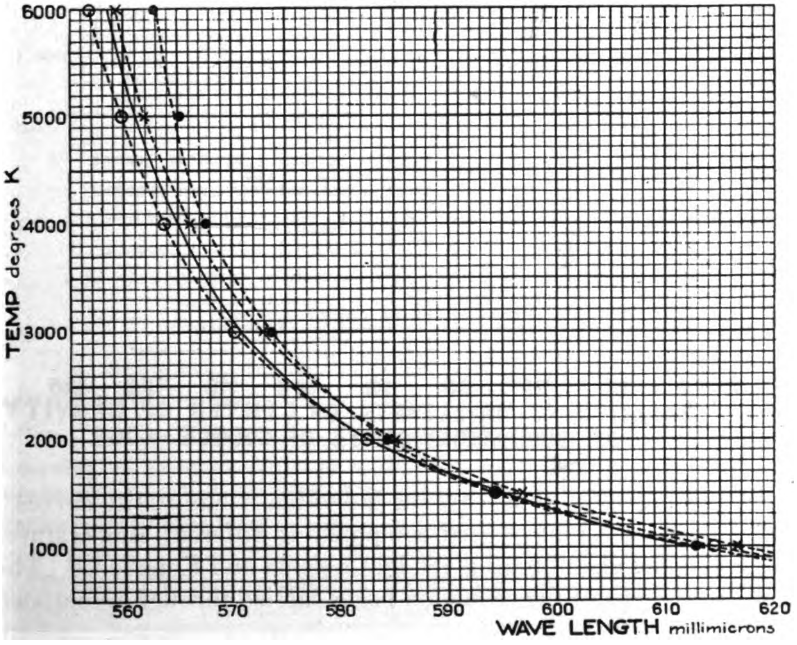
[51] *Phil. Trans. R. S.*, London, A, 212, pp. 288–289; year 1912–13.

FIG. 9.



Spectral Distribution of Light from a Complete Radiator at Various Temperatures. Energy by Planck's formula ($c_2 = 14350$).
Visibility: λ, 560–650, H. E. Ives, *Phil. Mag.*, Dec., 1912, p. 859. λ, 410–550 and 660–710, Hyde Forsythe & Cady, *Jour. Frank. Inst.* 48, p. 87.
Numbers attached to curves indicate temperatures in degrees K

FIG. 10.

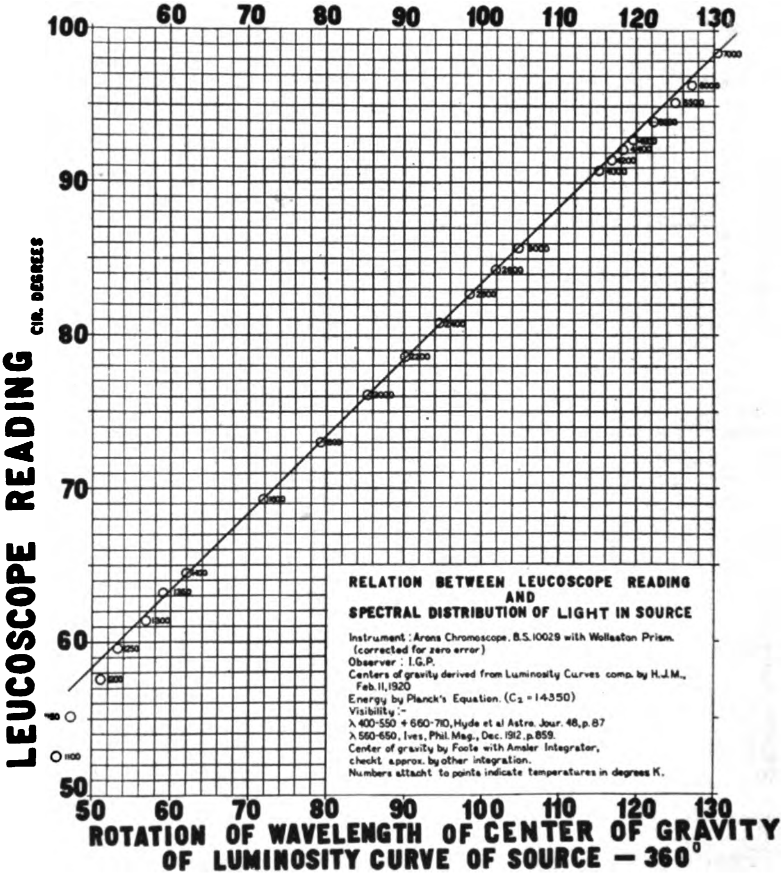


Relation of the Wave Length of Centre of Gravity of Spectral Distribution of Light to Temperature of the Source. Energy distribution by Planck's formula ($c_2 = 14350$).
Visibility:—Solid curve: Average data, 400–550 and 660–710 millimicrons, Hyde, Forsythe & Cady, *Astrop. J.* 48, p. 87. 560–650 millimicrons, Ives, *Phil. Mag.*, Dec., 1912, p. 859.

Solid circles:	Foote	Data by Coblentz
Crosses :	McNicholas	and Emerson,
Open circles:	Priest	*B. S. Sci. Pap.*, 303, pp. 184–185.

unaffected by great uncertainties of observation and computation. It is impossible to say at present whether the deviations for temperatures less than 1250° K and greater than 4000° K are real or due to these uncertainties. This relation has been retested for the limited temperature range 1700° to 3000°K by using the data

FIG. 11.



shown in Fig. 7. In this case the individual "luminosity curves" for each observer have been computed at different temperatures from *his own* visibility as given by Coblentz and Emerson.[52] The remaining steps of the reduction being carried through as outlined just above, we obtain the graphs shown in Fig. 12, which is plotted in the same way as Fig. 11. It will be noticed that the relation is linear for each observer, but that both the slope and the intercepts depend upon the individual.

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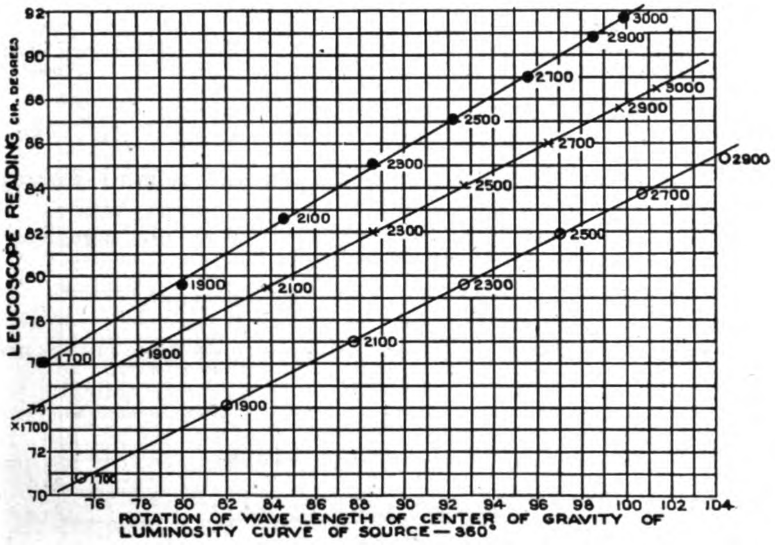
[52] *B. S. Sci. Pap., No. 303,* pp. 184–185.

FIG. 12.



Relation Between Leucoscope Reading and Spectral Distribution of Light from the Source for Three Observers.
Solid circles: Foote.
Crosses: McNicholas.
Open circles: Priest.
Numbers attached to the points indicate temperatures in degrees K.

2. *The Essential Condition That is Satisfied in Making the Leucoscope Setting.*

The second law derived from the observations recorded in this paper may be stated as follows:

The condition which is satisfied when the leucoscopic match is made according to Koenig's criterion is that the wave lengths of the spectral centres of gravity of the light of the source [33] and the two images in the leucoscope are coincident or nearly so.[34]

---

[33] Centre of gravity or so-called "centre of area" of "luminosity curve."

[34] Cf. Priest: "Relation Between Quality of Color and Spectral Distribution," *Jour. Op. Soc. Am.*, Sept., 1920.

This condition is illustrated for two particular temperatures, namely 2000° K and 3000° K in Figs. 13 and 14. The graphs in these figures represent, respectively:

1. *Solid curve,* spectral distributon of light (energy times visibility) for the given temperature, computed on the basis of Planck's equation (for $c_2 = 14350$) and visibility data as indicated in the legends of the figures.
2. *Dashed curve,* spectral distribution of light in the extraordinary image in the leucoscope, computed by the formula:

   $$(\text{Planck's Function})_\lambda\ (\text{Visibility})_\lambda\ \sin^2(\phi - 20\,\alpha_\lambda)$$

   where $\phi =$ leucoscope reading.

   $\alpha_\lambda =$ rotation of plane of polarization by one millimeter of quartz, for wave length, λ.
3. *Dotted curve,* spectral distribution of light in the ordinary image in the leucoscope, computed[55] by the formula:

   $$(\text{Planck's Function})_\lambda\ (\text{Visibility})_\lambda\ \cos^2(\phi - 20\alpha_\lambda).$$

The wave-lengths of the centres of gravity of these curves are as follows:

Temperature 2000° K.	
Extraordinary Image (relatively, slightly bluish)	581.5mμ
Black Body	582.6
Ordinary Image (relatively, slightly yellowish)	583.2
Temperature 3000° K.	
Extraordinary Image (relatively, slightly bluish)	568.9
Black Body	570.8
Ordinary Image (relatively, slightly yellowish)	571.9

VI. EXPERIMENTS ON LEUCOSCOPIC DETERMINATION OF THE QUALITY OF SUNLIGHT AND ITS DIURNAL VARIATION.

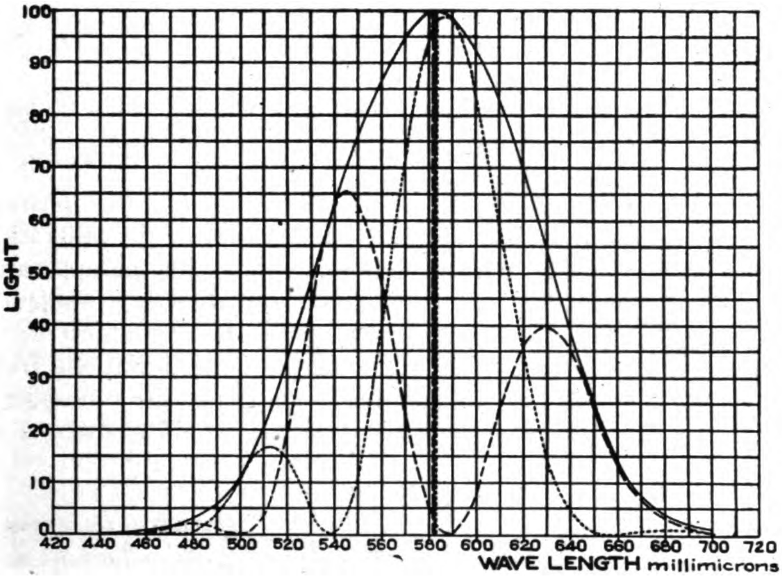
My average leucoscope reading for sunlight on a magnesium carbonate block, between 11 o'clock and 1 o'clock, February 20, 1920, at Washington, was about 94°+. Koenig gives 90.5° for his observation on sunlight,[56] but records neither the date nor the hour of observation. This is approximately equal to my readings at 8 A.M. and 4:30 P.M. The higher latitude of Koenig's station (Berlin, Germany) would make his reading lower than

---

[55] For computation of $\sin^2(\phi - 20\alpha_\lambda)$ and $\cos^2(\phi - 20\alpha_\lambda)$ see Appendix I.

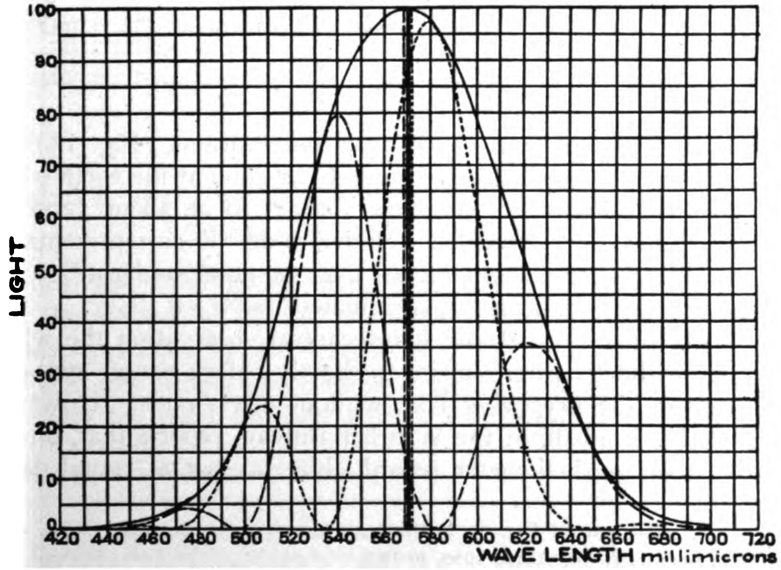
[56] *Ann. der Phy. und Chem.*, 17, p. 1004.

FIG. 13.



Spectral Distributions of Light in Leucoscope Images for Temperature 2000° K.
Energy by Planck's formula ($c_2=14350$).
Visibility: λ, 560–650, H. E. Ives, *Phil. Mag.*, Dec., 1912, p. 859. λ, 410–550 and 660 – 710, Hyde, Forsythe and Cady, *Astro. Jour.*, **48**, p. 87.
Solid curve: source, (Planck's function) × (visibility).
Dashed curve: extraordinary image, (Planck's function) × (visibility) $\sin^2(76.1° - 20\alpha_\lambda)$.
Dotted curve: ordinary image, (Planck's function) × (visibility) $\cos^2(76.1° - 20\alpha_\lambda)$.
Vertical lines at about λ = 582 indicate centres of gravity, solid, dashed and dotted having same significance as above.

FIG. 14.

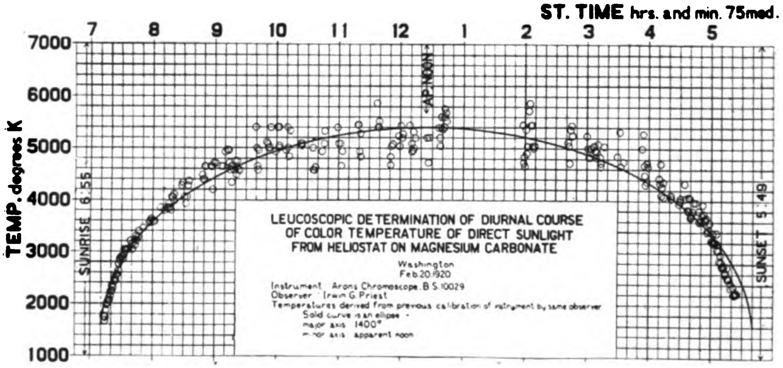


Spectral Distribution of Light in Leucoscope Images for Temperature 3000° K.
Energy by Planck's formula ($c_2=14350$).
Visibility: λ, 560–650, H. E. Ives, *Phil. Mag.*, Dec., 1912, p. 859. λ, 410–550 and 660–710, Hyde, Forsythe and Cady, *Astrop. Jour.* **48**, p. 87.
Solid curve: source, (Planck's function) × (visibility).
Dashed curve: extraordinary image, (Planck's function) × (visibility) $\sin^2(85.7° - 20\alpha_\lambda)$.
Dotted curve: ordinary image, (Planck's function) × (visibility) $\cos^2(85.7° - 20\alpha_\lambda)$.
Vertical lines at about λ = 570 indicate centres of gravity, solid, dashed and dotted having same significance as above.

mine because of increased air mass,[57] but there is really no basis for an *accurate* comparison because of the unknown relation of our personal equations.

In Fig. 15 are shown one day's observations on the apparent color temperature of sunlight derived by means of the calibration curve shown in Fig. 6 from determinations with the apparatus described above. *Each point plotted represents only a single observation.* The observations above 4000° K are subject to an uncertainty of 100° to 200°; but many of the deviations from a smooth curve may be real and due to changes in the atmosphere. Some of them were accompanied by large sudden changes in brightness. From these leucoscopic observations (Fig. 15) the apparent color temperature of sunlight at noon at the earth's surface in Washington in February appears to be about 5200° K to 5400° K. H. E. Ives has selected 5000° K as representative of considerable previous data for average noon sunlight,[58] while Abbot's data [59] indicates approximately 6000° K.

FIG. 15.



To ordinary casual and non-technical observation the whole day from shortly after sunrise to shortly before sunset appeared very clear, the sky being blue without any clouds. However, Dr. H. H. Kimball, of the Weather Bureau, reports that, on the basis of his pyrheliometer record, this was not a "good day."

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[57] Cf. discussion of Fig. 15, in third paragraph following this.

[58] *Trans. I. E. S.*, April, 1910, p. 193.

[59] Priest: *Phy. Rev.* (2), 11, p. 502, Fig. 1.

"There was a great deal of haze in the atmosphere which made the pyrheliometer readings quite irregular."[60]

Doctor Kimball has replotted the logarithms of these temperatures against air mass, and, extrapolating this curve to zero air mass, he finds 6500° K for the color temperature of the sun. The temperature of the sun hitherto derived by various methods appears to lie between 5800° K and 7000° K.[61] I would not attach any importance to the value found from my single day's observation; and give it merely to illustrate the use of the instrument.

Leucoscopic observations which I have made on daylight lead to the following conclusions:

(1) The quality of color of the densely overcast sky is very uniform in different directions; is subject to little diurnal variation (within 4 or 5 hours of noon); and is about the same as *sunlight outside the earth's atmosphere.* This conclusion is based upon the following values of color temperature derived from observations made by pointing the leucoscope (Arons chromoscope as described above) directly at the sky in Washington:

Date	Hour	Approximate part of sky	Condition of sky	Temp. degrees, K
Feb. 5, 1920	12:50 P.M.	60° South from Zenith	Uniform overcast. Mist.	6300
Feb. 5, 1920	1:00 P.M.	60° North from Zenith	Ditto	6500
Feb. 5, 1920	5:00 P.M.	60° South from Zenith	Ditto	6500
Feb. 6, 1920	9:00 A.M.	60° South from Zenith	Uniform overcast. Dense snowstorm.	6500

(2) The difference in color of a horizontal non-selective surface illuminated by sun and sky together and by the sky alone is strikingly shown by the leucoscope. The following are color temperatures found under these circumstances at about 4 P.M., February 11, 1920, the sky being blue with thin white clouds.

Sun and sky together	5700° K
Sky alone	7400

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[60] Letter, Kimball to Priest, Oct. 1, 1920.

[61] Smithsonian Physical Tables, 7th Ed. (year 1920). Table 549, p. 418.

In considering the applicability of the leucoscope to field investigations of the color of sunlight and daylight, it is perhaps worth while to emphasize again the following features:

(1) Its portability.

(2) The extreme rapidity with which observations can be made.

(3) The absence of any comparison source or accessory apparatus demanding the observer's attention.

VII. GENERAL SUMMARY AND CONCLUSION.

This paper has given an account of a new study of the leucoscope from the point of view of its pyrometric calibration. The complete calibration curve from about 1000° to 7000° K has been determined, at least approximately, for one observer; and more careful and accurate calibrations from 1700° to 2400° K have been determined for three observers. The questions of precision, personal equation, the reproducibility of observations at fixed temperatures and the practical application of the instrument to pyrometry and the color grading of light sources have been discussed rather fully. The possible utility of the instrument in industrial pyrometry, the color grading of light sources, and the study of the color of daylight has been pointed out and illustrations of its use given. Two new laws of the instrument have been formulated.

The applicability of the leucoscope to the study of color vision has been scarcely touched upon in this paper because the author believes it would be futile to discuss this phase of the subject without much more extensive statistical data on a large number of observers of known characteristics in regard to visibility, hue discrimination and color-blindness. It seems, however, that the data given in this paper should be of fundamental importance in elucidating the physiological theory of the leucoscope. It is not unlikely that the instrument would prove useful in the study of color vision, as was indeed long ago affirmed by Brodhun.[62] However, observations of several color-blind observers recently conducted here by Mr. C. M. Blackburn, show that these observers are quite unable to make *any* consistent observations, which appears to be entirely out of accord with Brodhun's findings.[63] This discrepancy may perhaps be due to our omission of making a brilliance match simultaneously with the color quality match. This question requires further investigation.

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[62] *Ann. der Phy. und Chem.*, **34**, p. 918.

[63] Brodhun: *Loc. cit.*, Appendix II, this paper.

It is planned to publish data upon a number of other observers in a subsequent paper.

NATIONAL BUREAU OF STANDARDS,
WASHINGTON, D. C.
November 2, 1920.

P. S. (Added to proof Dec. 3, 1920).—Since writing the above the author has devised a much more accurate method of setting up the color temperature scale from 3000° to 7000° K than that given in this paper (Sec. II, 2, and Figs. 3 and 4). This method will be described in a later number of this Journal under title: " A Method of Obtaining Radiant Energy Having the Spectral Distribution of a Complete Radiator at Very High Temperatures." Data on the actual spectral distribution of B. S. Lamp 1717 used in this paper as standard of color temperature at 2830° K will also be given in the forthcoming paper.—I. G. P.

APPENDIX I.

The Computation of Spectral Transmission by Rotatory Dispersion.

If a crystalline quartz plate, with plane surfaces perpendicular to the optic axis, be placed between nicol prisms so that the optic axis is coincident with the line of sight through the nicols, the system so constituted is, in effect, a selective light filter of adjustable spectral transmission. The transmission is a function of:

(1) the wave length, λ;
(2) the thickness of the quartz plate, b;
(3) the angle through which one of the nicols is rotated relative to the other, in the same direction as the rotation of the plane of polarization by the quartz and measured from zero for the position of extinction with the quartz removed, ϕ.

The relative transmission is given by the formula:

$$\sin^2(\phi - b\,\alpha_\lambda)$$

where α_λ = rotation by unit thickness of quartz for wave-length λ.

The computation chart, Fig. 16, has been prepared to facilitate the evaluation of this function for any value of ϕ and selected values of λ, for b = 1.000 mm. Values of ϕ are plotted as abscissa and values of $\sin^2(\phi - \alpha_\lambda)$ as ordinate.[64] The scale at the bottom is to be used in connection with the scale on the left; and the scale at the top, with that on the right. Each curve in these figures relates to a particular value of λ, indicated in the margins by numbers attached to the curves by arrows. By use of this chart, the spectral curve showing the relative transmission of quartz-nicol systems such as used in this investigation to obtain spectral energy distributions corresponding to high temperatures may be quickly plotted. Such plots for selected values of ϕ are shown in Fig. 17.

If a calcite rhomb or double image prism (Wollaston) be substituted for one of the nicols, as in the leucoscope, the relative transmissions corresponding to the two images will be:

extraordinary image, $\sin^2(\phi - b\alpha_\lambda)$
ordinary image, $\cos^2(\phi - b\alpha_\lambda)$

Figs. 18 to 21 (similar to Fig. 16) provide for evaluating these functions for the standard leucoscope quartz thickness of 20.000 mm. In order to avoid confusion of the curves, it has been necessary to prepare four separate figures each covering a limited range of wave-lengths.

The method of using these charts (Figs. 16, 18, 19, 20, 21) should be sufficiently obvious without further explanation.

Curves of relative spectral transmission derived from these computation curves for b = 20.000 mm. and selected values of ϕ (50, 60, 70, 80, 90°) covering nearly our complete range of leucoscope readings are shown in Fig. 22. They will be of interest to those who care to analyze critically the action of the leucoscope.

For further treatment of the above subject, reference may be made to the following:

1. Koenig: *Ann. der Phy. und Chem.*, **17**, pp. 994–995.
2. Brodhun: *Ann. der Phy. und Chem.*, **34**, pp. 897–918, particularly Fig. 3.
3. Arons: *Ein Chromoskop, Ann. der Phy.* (4), **33**, pp. 799–832; 1910.
4. Priest: *Phy. Rev.* (2), **10**, p. 208, Aug., 1917; *Phy. Rev.* (2), **11**, p. 502, June, 1918. The Application of Rotatory Dispersion to Colorimetry (forthcoming paper.)

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[64] Computed from Lowry's rotatory dispersion data, *Phil. Trans., Roy. Soc., Lon. A.*, **212**, pp. 288–289.

FIG. 16.

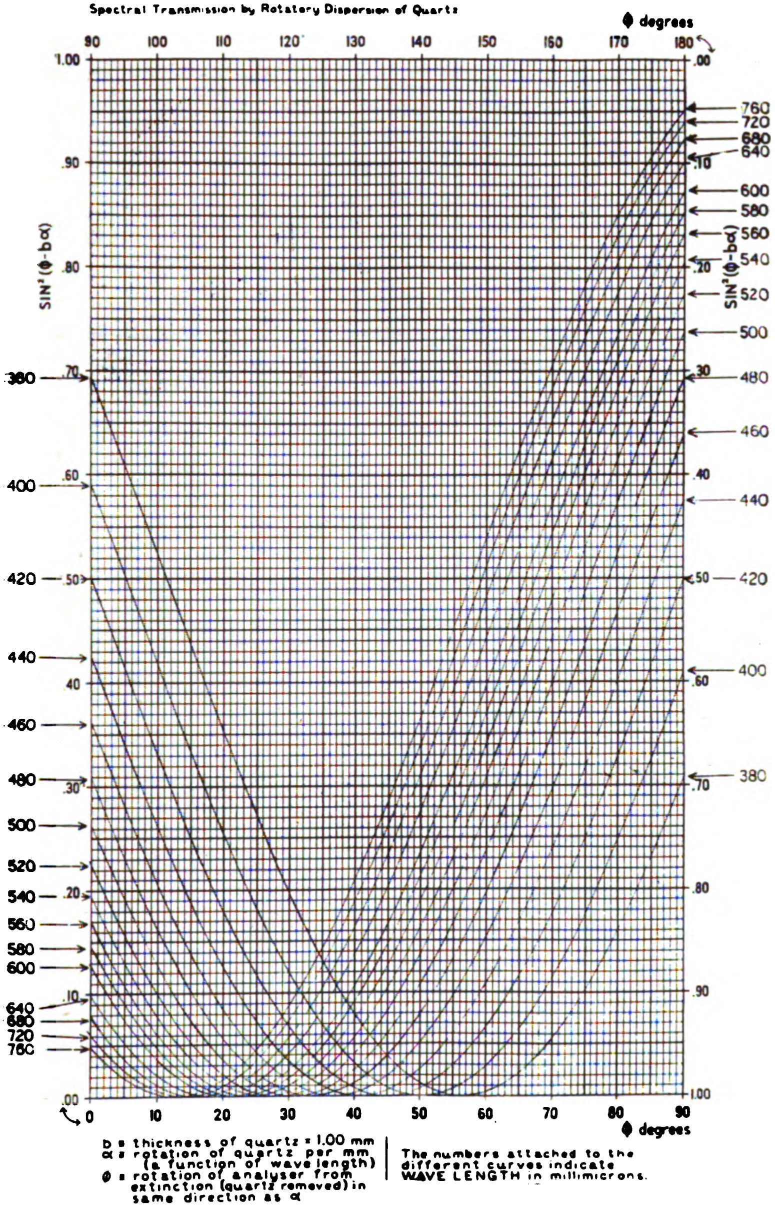
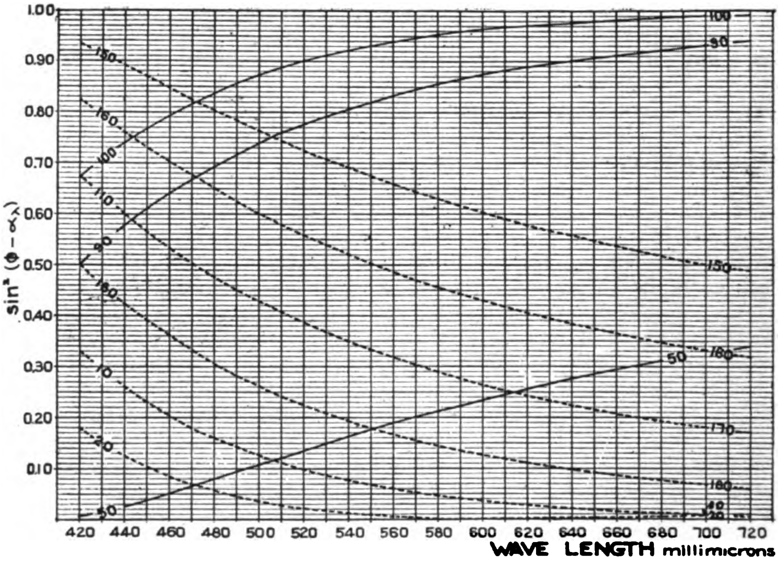
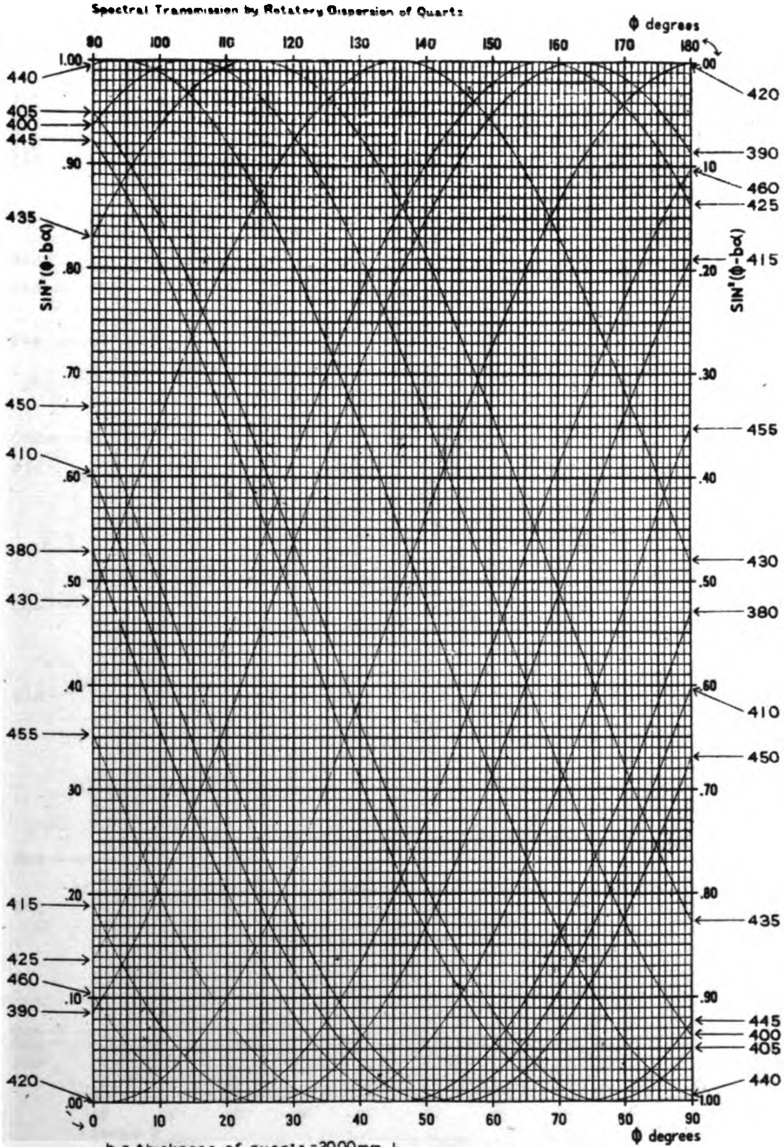

Spectral Transmission by Rotatory Dispersion of Quartz
ϕ degrees
90 100 110 120 130 140 150 160 170 180
1.00 .90 .80 .70 .60 .50 .40 .30 .20 .10 .00
SIN²(ϕ-bα)
.00 .10 .20 .30 .40 .50 .60 .70 .80 .90 1.00
760 720 680 640 600 580 560 540 520 500 480 460 440 420 400 380
380 400 420 440 460 480 500 520 540 560 580 600 640 680 720 760
0 10 20 30 40 50 60 70 80 90
ϕ degrees
b = thickness of quartz = 1.00 mm
α = rotation of quartz per mm (a function of wave length)
ϕ = rotation of analyser from extinction (quartz removed) in same direction as α
The numbers attached to the different curves indicate WAVE LENGTH in millimicrons.

FIG. 17



Relative Spectral Transmission of a One-millimeter Quartz plate between Nicol Prisms. Numbers attached to the curves indicate values of the angle ϕ in degrees. (Cf. Text, Appendix I.)

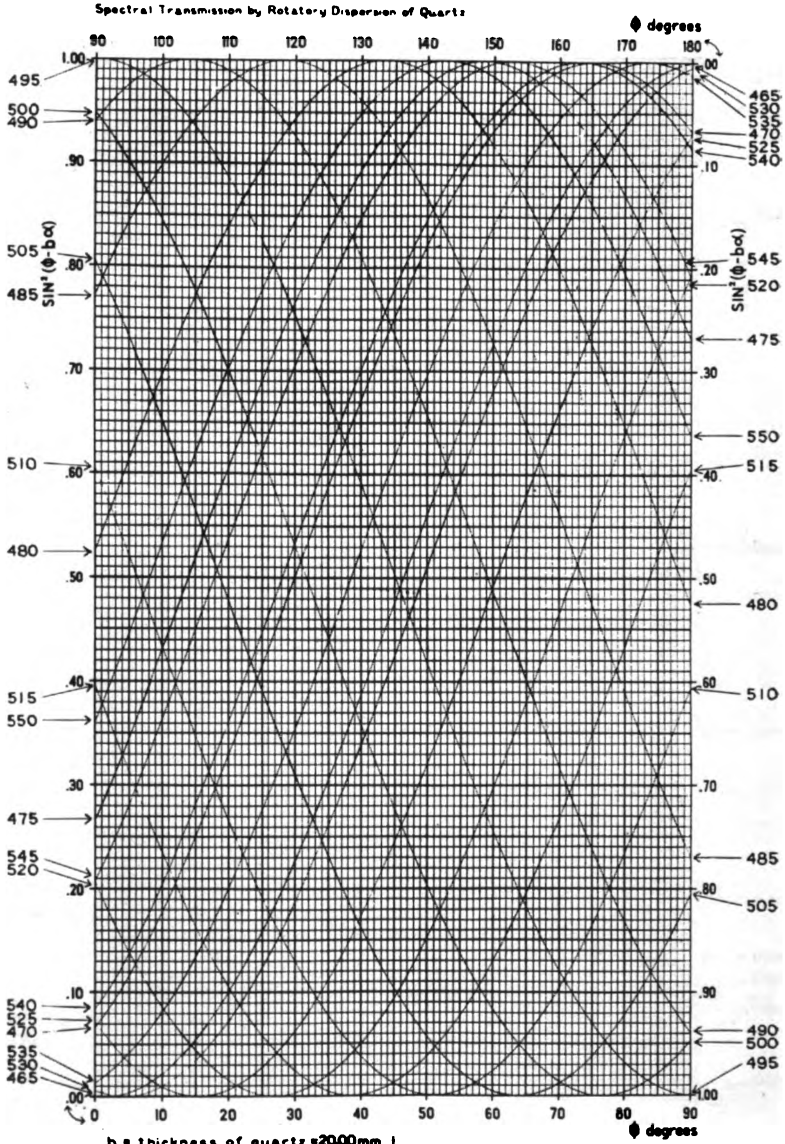
FIG. 18.



b = thickness of quartz = 20.00 mm
α = rotation of quartz per mm (a function of wave length)
φ = rotation of analyser from extinction (quartz removed) in same direction as α

The numbers attached to the different curves indicate WAVE LENGTH in millimicrons

FIG. 19.

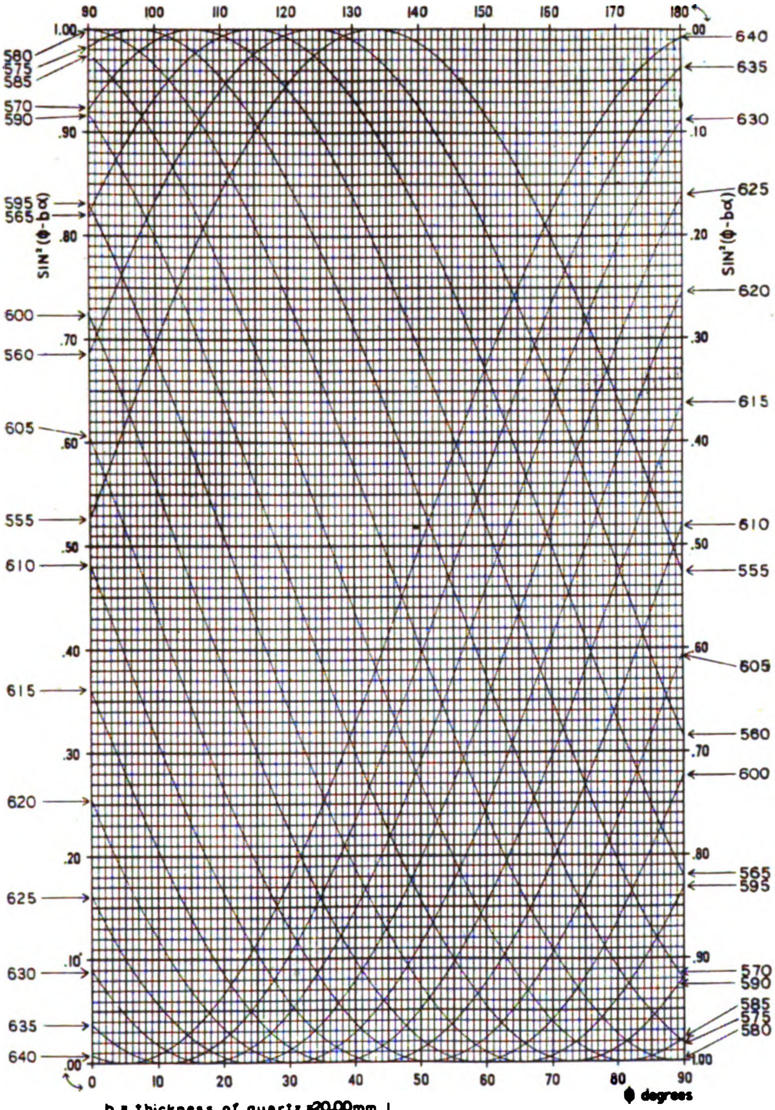


b = thickness of quartz = 20.00 mm
α = rotation of quartz per mm (a function of wave length)
ϕ = rotation of analyser from extinction (quartz removed) in same direction as α

The numbers attached to the different curves indicate WAVE LENGTH in millimicrons.

FIG. 20.

Spectral Transmission by Rotatory Dispersion of Quartz



b = thickness of quartz = 20.00 mm
α = rotation of quartz per mm (a function of wave length)
ϕ = rotation of analyser from extinction (quartz removed) in same direction as α

The numbers attached to the different curves indicate WAVE LENGTH in millimicrons.

FIG. 21.

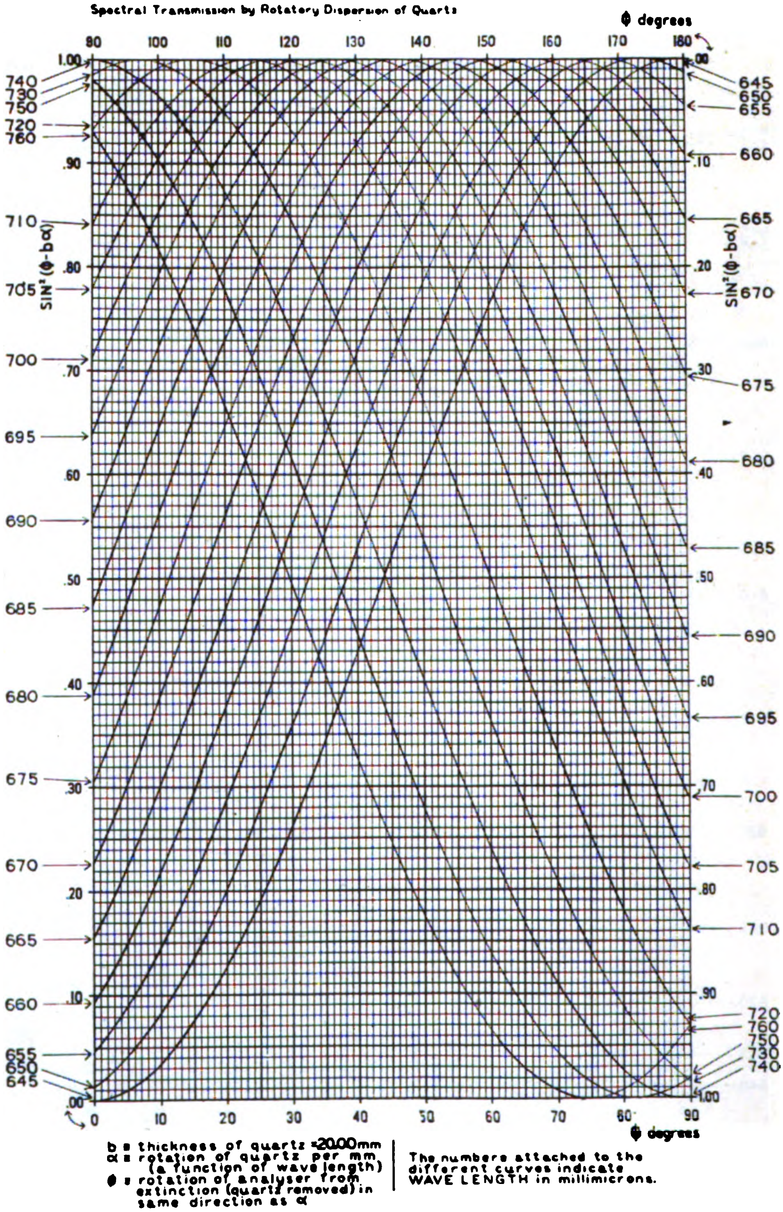
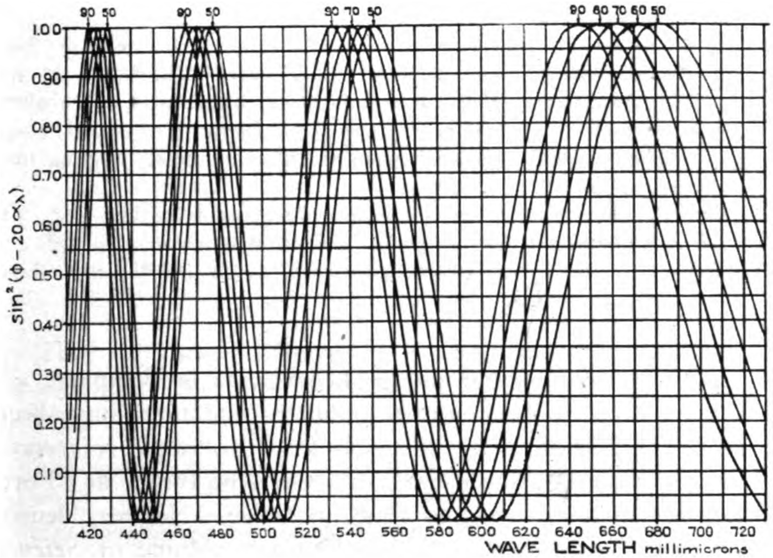

Spectral Transmission by Rotatory Dispersion of Quartz
ϕ degrees
80
100
110
120
130
140
150
160
170
180
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
SIN²(ϕ-bα)
740
730
750
720
760
710
705
700
695
690
685
680
675
670
665
660
655
650
645
.00
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
SIN²(ϕ-bα)
645
650
655
660
665
670
675
680
685
690
695
700
705
710
720
760
750
730
740
0
10
20
30
40
50
60
70
80
90
ϕ degrees
b = thickness of quartz = 20.00 mm
α = rotation of quartz per mm (a function of wave length)
ϕ = rotation of analyser from extinction (quartz removed) in same direction as α
The numbers attached to the different curves indicate WAVE LENGTH in millimicrons.

FIG. 22.



Relative Spectral Transmission of a Twenty-millimeter Quartz Plate between Nicol Prisms. Numbers attached to the curves indicate values of the angle φ in degrees. (Cf. Text, Appendix I.) These curves show the transmission for the extraordinary image in the leucoscope, φ being understood as the leucoscope reading. The transmission for the ordinary image may be obtained by reversing the ordinate scale.

APPENDIX II.

Bibliography.

(CHRONOLOGIC)

1863. ROSE, EDM.: *Archiv für path. Anat. und Physiol.* (Virchow's Archiv) (2), **8**, pp. 81–82, and 35–46.

Description of a " Farbenmesser " which must be regarded as the prototype of the leucoscope is given as an appendix to a paper entitled " Ueber die Hallucinationen im Santonrausch." (Cf. Norris and Oliver: " System of Diseases of the Eye," Vol. 2, p. 347.)

1878. KITAO, DIRO: " Zur Farbenlehre," *Inaug. Diss., Goettingen.* Also *Ann. der Phy.*, Beib, **4**, pp. 51–53; 1880.

1882. KOENIG, ARTHUR (and HELMHOLTZ, in discussion): " Ueber das Leucoscope." *Verh. der Phy. Gesel,* Berlin, Nr. 2, pp. 23–28, Feb. 3, 1882.

1882. KOENIG, ARTHUR: " Das Leucoscope und einige mit demselben gemachte Beobachtungen." *Ann. der Phy. und Chem.*, **17**, pp. 990–1008.

1882. KOENIG, ARTHUR: " Neue Beobachtungen mit dem Leucoscope." *Verh. der Phy. Gesel,* Berlin, Nr. 12, pp. 90–94.

1883. KOENIG, ARTHUR: "Das Leucoscope und seine Theorie." *Zeit. für Instk.*, 3, pp. 20–26.

1884. KOENIG, ARTHUR: "Ueber einen neuen Apparat zur Diagnose der Farbenblindheit." *Centralblatt für prak. Augenheilk.*, 8, pp. 375–377.

1884. KOENIG, ARTHUR: "Ueber ein verinfachtes Leucoscope. (Ophthalmo-Leucoscope.") *Verh. der Phy. Gesel,* Berlin, Nr. 11, pp. 41–43. (Also, Koenig's "Abhandlungen zur Physiol. Optik," Leipzig, 1903, pp. 34–36.)

1885. KITAO, DIRO: "Leucoscope seine Anwendung und seine Theorie." *Abhandlungen des Tokio Daigaku* (Univerität zu Tokio), No. 12,

1888. BRODHUN, EUGEN: "Ueber das Leucoscope." *Ann. der Phy. und Chem.*, 34, pp. 897–918.

The foregoing bibliography is believed to be complete. A thorough search of the literature indicates that there have been no further publications on this subject. No other references were found under "Farbensinn" or "Optische Inst.," in "Fortschritte der Physik," 1885 to 1898, inclusive. Neither "leucoscope" nor "leucoskop" is indexed in any volume of *Science Abstracts,* 1898 to date, under any of the headings where it might logically appear: viz., "Vision," "Photometry," "Apparatus and Instruments," "Measurements and Use of Instruments."

Professor Nagaoka, who was acquainted with Kitao, has informed the author that neither Kitao, who died about ten years ago, nor anyone else in Japan has published anything further on the leucoscope.

The leucoscope is not indexed and apparently not mentioned in any of the following books:

"Enc. Brit.," 1910–11.
"Intern. Enc.," 1918.
WINKELMANN: "Handbuch der Physik," 1906.
WUELLNER: "Experimentalphysik," 5th Ed., 1899.
KORSCHELT (and others): "Handwörterbuch der Naturwissenschaften," 1912.
ABNEY: "Colour Measurement and Mixture," 1891.
ABNEY: "Researches in Colour Vision," 1913.
EDRIDGE-GREEN: "Colour Blindness," 1909.
KOELLNER: "Stoerungen des Farbensinnes," 1912.
ROSMANIT: "Feststellungen der Farbentüchtigkeit," 1914.
LUCKIESH: "Color and its Applications," 1915.
PARSONS: "Introduction to the Study of Colour Vision," 1915.

Although its importance in this field is evident, it is not mentioned in any of the following papers on color grading of light sources:

IVES, H. E.: "Color Measurements of Illuminants—a Résumé." *Trans. I. E. S.*, April, 1910, p. 189.

BLOCK: "Kennzeichnung der Farbe des Lichtes." *Electrot. Zs. Heft*, **46**, p. 1306; 1913.

JONES: "The Color of Illuminants." *Trans. I. E. S.*, **9**, p. 687; 1914.

HYDE AND FORSYTHE: "The Quality of Light from an Illuminant as Indicated by its Color Temperature." *Jour. Frank. Inst.*, **183**, pp. 353-354; 1917.

That it is practically unknown in pyrometry is indicated by the fact that it is not indexed and apparently not mentioned in either:

(1) BURGESS AND LECHATELIER, "High Temperature Measurements," 1912,

or

(2) "Symposium on Pyrometry," published by *Am. Inst. of Min. and Metal. Eng.*, 1920.

NOTE ON A NEW METHOD OF JOINING GLASS.*

BY

C. O. FAIRCHILD.

Introduction.—The need for flat glass windows for incandescent lamp bulbs led the writer to seek a new method for accomplishing the fusion of glass joints without appreciable deformation of the surfaces. Such lamps were desirable for studying the effects of diffraction caused by the lamp filament in an optical pyrometer of the so-called disappearing filament, or Morse, type, because the diffraction effect was obscured by the imperfect surfaces of lamp bulbs of blown glass. Lamps have been constructed by the new method, as shown in Fig. 3, C having two flat windows, the transmission of which can be easily measured and which give an undistorted telescopic image of the filament.

Parker and Dalladay [1] have developed a precision method of uniting optical glass which depends upon optical contact and pressure, the glass being raised to a temperature well below its annealing range. Their method is particularly suitable for the construction of flat-sided cells in case most precise flatness is required. Its disadvantages lie in the necessity of obtaining perfect fit of the parts to be joined, by careful polishing and cleansing, and the time required (about one hour) for junction to be completed. Its application is restricted to glasses having annealing points which differ by not more than about 50°. The method devised by the writer fails only for glasses which have too widely different coefficients of thermal expansion.

Method.—Briefly the process consists in maintaining the glass at a temperature sufficiently high to avoid cracking while applying heat locally, the temperature of the main body of the glass being one at which undue distortion will not occur. The body temperature necessary is always near the annealing temperature and this depends, of course, primarily upon the kind of glass, but is also dependent to some extent upon the dimensions of the article. By this method complete fusion of the junction can be accomplished without propagating beyond a restricted portion near the joint the stresses occasioned by local heating; hence it is applicable where a slight deformation at the junction does not impair the usefulness of the article.

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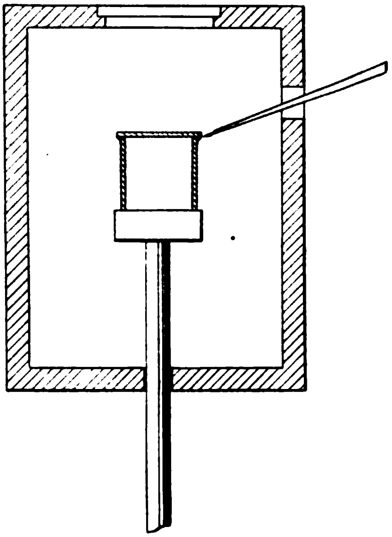
* Published by permission of the Director, Bureau of Standards.

[1] Parker and Dalladay: *Faraday Soc. Trans.*, 12, pp. 305–312. For short bibliography on thermal and other properties of glass see Tool and Valasek, *B. S. Sci. Papers*, No. 358, 1920.

For the purpose of maintaining the body of the glass at the proper temperature during local fusion of a joint an electric furnace was constructed, as shown in Fig. 1, having a mica window[2] above and a small hole in the side for introduction of a blowpipe. It is provided with a centrally located pedestal, which extends through a closely fitting hole in the bottom, upon which glass objects can be placed and rotated about a vertical axis. A sodium flame backed by an incandescent lamp is used for illuminating the interior—the sodium flame furnishing monochromatic illumination for interference fringes between polished surfaces nearly in contact and the white light furnishing the general illumination of the object. For measuring the furnace temperature a small platinum-rhodium thermocouple is inserted through

FIG. 1.



Cross section of furnace and glass tube and disc supported within on a pedestal.

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[2] Pyrex glass or fused quartz would be better because mica soon splits and becomes more opaque with continued heating.

the furnace cover and arranged so as to place its hot junction in contact either with the furnace wall or with the glass being treated. A blowpipe having a millimeter orifice is used with an oxy-gas flame about 1 cm long.

The furnace can be quickly heated to a required temperature and held there, so that a small glass article can be raised to temperatures above the annealing temperature, the junction completed by application of the flame, and cooling initiated, before the glass has sufficient time to appreciably change its shape.

Because the writer's primary interest lay in construction of incandescent lamps, as stated above, the first experiments were performed on short tubes ground and polished on one end and topped by a flat disc as shown in Fig. 1. The tube with the disc was placed on the pedestal, the temperature was rapidly raised to slightly above the annealing temperature, the flame inserted into the furnace so that it touched the edges of the disc and tube as the pedestal slowly rotated. The completion of the joint was thus accomplished in but a few seconds after inserting the blowpipe.

The first trial was satisfactory for the primary purpose in mind, the disc showing no distortion beyond two mm from the inner edge of the joint. The very slight application of the flame required in case the end of the tube was polished, indicated that fine grinding of the tube would be sufficient; also that flaring of the ends was unnecessary. After some practice quite satisfactory joints were obtained, using unflared tubes with the ends finished by grinding with wash emery. While the first experiments were tried having the disc of exactly the same glass as the tube, ordinary American plate was found quite satisfactory to use with common lead glass tubing. The annealing temperature of the plate glass used was approximately 565° C. and that of the tubing 460° C.[3] The coefficients of thermal expansion in the low range were respectively, 0.101×10^{-4} and 0.09×10^{-4}. There is quite an advantage in using glass for the disc which has a higher annealing temperature than that of the tube. Since the glass was thin, sudden application of heat to the disc at a temperature as low as 470° did not crack it and the unavoidable strains were not serious. They were probably greater than the allowable strains in a polarimeter tube, but were so slight that the incandescent lamps made could easily be heated over a bare flame during their exhaustion.

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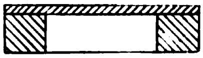
[3] Peters and Cragoe: *Jour. Opt. Soc. of Amer.*, 4, 3, pp. 105–145, 1920.

Discussion of the Principles Involved.—The combined effect of low thermal conductivity of glass and the rapid change in its mobility with temperature was the single important factor leading to the development of the simple process described. As a result of low thermal conductivity *large temperature gradients may be imposed with only slight flow of heat.* When glass is heated to its annealing temperature its mobility begins to increase at a rapid rate, *experiments showing the mobility to be approximately an exponential function of the temperature.* For example, some glasses become twice as soft for every rise in temperature of 8° or 10° C.[4] These two properties permit the attainment of such a temperature gradient that the variation in mobility in a very small distance is exceedingly great. If the relation of mobility to temperature persists over wide ranges of temperature, a gradient of 100° per mm. corresponds to a change of mobility per mm of over 1000 times. Thus a piece of glass at the annealing temperature may be suddenly heated superficially without distorting more than a small part of it. The glass being heated to its annealing temperature, or slightly above it, the application of the flame to an edge quickly raises the edge far above the temperature called by Tool[5] "upper limit of the annealing range." At this latter temperature the glass has become very nearly a true viscous substance, that is, minute stresses result in flow. It is due to this fact that union is so quickly started by the flame. The local distortion occurring under the flame does not extend far and, since the body of the glass is within its annealing range stresses cannot be propagated in any appreciable amount beyond the region overheated. In some cases it was possible by watching interference fringes during the process of joining, to observe very nicely the effect of sudden expansion of the part touched by the flame, the quick union occurring, the very rapid loss of heat by radiation, and the fairly rapid return to the original configura-

FIG. 2.



Cross section of glass ring and flat cover.

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[4] See ref. 1.

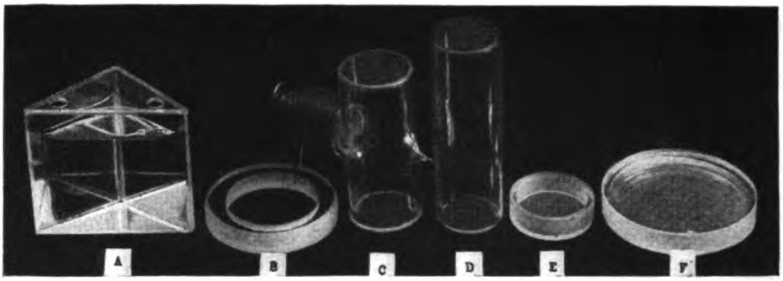
[5] Tool and Valasek: *B. S. Sci. Papers*, No. 358, 1920.

tion. Probably expansion forces the surfaces into contact before fusion and the force may be fairly large. Molecular forces are brought into play as soon as contact occurs. The surfaces become quite clean at the temperatures used because of the oxidation of organic matter (as dust) and the expulsion of occluded gases.

The change in the temperature coefficient of thermal expansion observed by Peters and Cragoe,[6] to take place at the annealing temperature, may play a part, and probably the best joints can be made above the annealing temperature; that is, above the temperature at which this coefficient changes from the value which persists from room temperature to the annealing temperature.

At the annealing temperature the flow of small glass articles due to their own weight when unevenly supported, is exceedingly slow and the quickness with which a joint can be made by this method allows very little distortion to occur. Hence, in nearly all cases it will be best to use the annealing temperature, in which event, after completing the joint the temperature can be allowed to fall slowly without any complications for the purpose of annealing.

FIG. 3.



A. Hollow 45° prism; B. glass cell; C. optical pyrometer lamp; D. tube closed at both ends; E. glass cell with thin rim; F. compound lens fused at periphery. Line of fusion is visible as dark band.

Applications of the Process.—A few articles of various forms have been made to determine the applicability of the process. Some of these are shown in Figs. 3 and 4. The method can also be employed for constructing absorption cells, colorimeter tubes, glass boxes, polarimeter tubes, Nessler tubes, glass-liquid prisms, lenses, etc., hæmacytometers, and even interferometer parts. In

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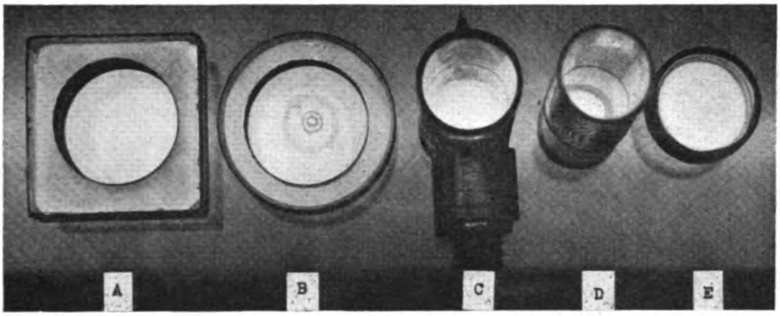
[6] See ref. 3.

Fig. 4 the articles have been placed on a glass plane and illuminated with a mercury lamp. The hollow 45° prism shown in Fig. 3 was not flat enough to photograph as in Fig. 4. The millimeter plate glass used had not been well annealed and merely heating to the annealing temperature distorted it. A compound lens was made by fusing the periphery with the inclusion of an air film. The joint is about 2 mm wide and the air film about 0.2 mm thick at the centre. The convex part was selected with a surface of greater radius than that of the concave. The two glasses[7] used were (1) borosilicate crown, annealing temperature 530°, coefficient of expansion (below 500°) 0.090×10^{-4}, and (2) medium flint, annealing temperature 450°, coefficient of expansion 0.097×10^{-4}. The lens is about 5 cms in diameter. Possibly such a lens is of little usefulness, but so far as distortion is concerned the result was satisfactory. If such a thing were of any utility, lenses could be made by carrying the weld through only a portion of the periphery, leaving an opening for the injection of Canada balsam. It may be found that glass-liquid lenses will be useful. A seeming disadvantage is the higher temperature coefficient of refraction of liquids. This would probably limit the use of such a lens to special laboratory problems. Such a lens could be easily made by welding the parts on to the two ends of a short tube, as shown in Fig. 5.

FIG. 4.



Articles placed on a glass plane and illuminated with a mercury lamp. A, B, E, glass cells. C. Optical pyrometer lamp. The end down is seen to be flat to about 1 micron.; D. simple tube. The end is seen to be flat to fraction of a wave-length within 1 mm. of edge.

Various details of the method of making hæmacytometers have been tried but will not be described. The writer is now constructing some total reflecting glass rhombs having fused-on protecting plates for the reflecting surfaces. These are for use in a colorimeter in which the rhombs are immersed in the liquid whose transmission is being studied. A preliminary experiment has shown that the plates can be fused on very easily without distortion of more than approximately a millimetre of the surfaces at the edges of the joints.

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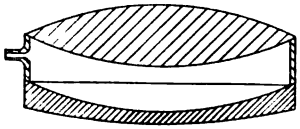
[7] See ref. 5.

The ordinary absorption cell with cemented sides has usually been made with a very thick rim for the purpose of cementing. This method does not require such heavy construction and for most ordinary purposes cells can be made with thin-walled tubing. Further experiments showed, however, that the thick rim was of considerable advantage in making cells with quite flat sides. It was found that a joint started by the flame at the outer edge in such case extended to the inner edge well beyond the parts heated above the annealing range. It became apparent that molecular attraction was brought into play, pulling the surfaces together when once molecular contact was started. In one case shown in cross section in Fig. 2, a flat disc about 5 cm in diameter and 8 mm thick, flat on one side, was bored out, affording a ring with a bearing surface 1 cm in width. The piece had not been properly annealed and cutting the hole resulted in a distortion which raised the inner edge of the ring so that a flat disc placed upon it touched only the inner edge and allowed an air film at the outer edge. Examination before the sodium flame exhibited six interference fringes. The result of joining a cover to this ring was quite unexpected. Quick fusion around the outer edge caused complete joining as far as the inner edge, pulling the periphery of the cover into contact and forcing the centre out about 10 microns. At first thought, such an effect might be assigned to fusion of the periphery and subsequent contraction.

FIG. 5.



Cross section of suggested form of a glass-liquid lens.

But such action would undoubtedly have resulted in a distortion of the cover in the opposite direction. Furthermore, the amount of distortion of the cover corresponded to the original condition of the ring. In this way it is emphasized that this new method of joining glass does not require a perfect fit of the articles to be joined and that the final result is measured quantitatively by the lack of fit. Thus, in the construction of various articles the fitting may be perfected to a degree governed by the precision desired. It is particularly to be noted that examination of the piece illustrated by Fig. 2, by placing a flat test plate upon it to obtain interference fringes, gave fringes which crossed the inner edge of the joint without any displacement. There remains little doubt that had the ring been quite flat the cover would have remained undistorted. There has not been opportunity to prove this, but the action described seems fairly conclusive. We may add that only for precise work, in which case fusion by the flame is kept at a minimum, is it necessary to exercise special precautions as to particles of dust caught between surfaces. A particle of dust lodged near the outer rim is no hindrance and is entrapped, becoming invisible. A particle near the inner edge obviously prevents complete junction.

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NOTICES.

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(Continued from page 242, Vol. IV, No. 4, July, 1920.)

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CORRECTION.

On pages 392 to 394, Volume IV, in the legends under Figs. 3 to 8 inclusive, for "sunlight" read "sun's energy."

PREVIOUS ISSUES OF THIS JOURNAL.

Beginning with Volume IV, the JOURNAL of the Optical Society of America has been published regularly, bimonthly.

Prior to this the issues were incomplete on account of war conditions and reorganization. Only six issues were prepared in the period 1917–1919, bearing the following numbers and dates. Libraries should bind these six numbers under a single cover designated by Volume I–III, 1917–1919. Succeeding volumes, one each year, will contain about 600 pages, and should be bound separately as Volume IV, 1920, etc.

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A REVIEW OF THE SERIES IN THE SPECTRA OF THE ELEMENTS.

BY

F. A. SAUNDERS.

THE analysis of the spectra of the elements into series and systems of series, and the study of the relations among these within the spectrum of one element, and among different elements, is a subject which has received but little attention in spite of its obvious interest. For thirty years or more it has been known that the vibrations of the atoms are not haphazard, but are often arranged in a regular order, suggesting a series of overtones in acoustics. In certain cases, such as hydrogen, or zinc, this arrangement is quite conspicuous, and was recognized as soon as the full spectrum was obtained; in others, the spectra are so complex that there is little reason to hope that we may ever work out their structure.

The interest in the problem lies in the information which these studies may yield as to the nature of the atom and the physical processes going on within and about it, and in these days no information about the atom can be branded as useless. The difficulties which prevent its immediate solution are very numerous. The eye is sensitive to but a small part of the spectrum. For the investigations in question, the entire spectrum must be studied, from the shortest ultra-violet waves to the longest ultra-red; and several different experimental methods and much elaborate apparatus have to be employed in order to yield such a wide sweep of spectrum. Moreover, we do not possess means of agitating the atom which are sufficiently vigorous to make it give its " complete " spectrum; in fact, such a word may have a very vague meaning. Some light-sources yield a little more than others, but, at the best, even with a variety of sources, we have to be content with a very imperfect view of the domain we wish to survey.

Certain comparatively recent discoveries will be of great assistance when applied to this problem. If the source of light is placed in a strong magnetic field, each line of the spectrum breaks up into a little group of lines (Zeeman effect), the arrangement of which appears to be characteristic of the series to which the line belongs. Something similar happens, at least in certain cases, with electric fields (Stark effect). But these effects cannot be observed for faint lines, and, unfortunately, the majority of the lines in all spectra seem to be faint.

The present state of the experimental study of spectrum series is one of flux, and whatever may be written about it now will probably be hopelessly out of date in half a dozen years. Something of the same sort may be said for the theoretical side of the question. Bohr, Sommerfeld, and others have built up a model of a simple atom whose unconventional actions would have shocked physicists of twenty years ago, but which seem to be capable of yielding the observed spectra. At the present writing, this very remarkable success has been obtained only for the simplest type of atoms, containing merely the nucleus, and one electron. This limits it to the spectrum of hydrogen and the spectrum of ionized helium. Other spectra, such as that of ordinary helium, are being attacked, but it is too early to say with what success.

The present article will not include any account of the theoretical investigations. These have been treated by Sommerfeld in a book, and are being made accessible to English readers in other ways. No comprehensive account of the experimental side of the problem exists. The beginnings of it may be found in Kayser's monumental " Handbook of Spectroscopy," but that book is now quite out of date on this branch of the subject. A book is to be published shortly by Professor A. Fowler, of London, which will give us an authoritative summary by one who has contributed largely to its development. In anticipation of this work, however, a brief attempt will be made in this article to outline our present knowledge of spectrum series. The matter will be presented by chemical families, in the order of the simplicity of their spectra, which is nearly the same as the historical order of their solution. The various laws of series will be found under the elements in whose spectra they first occur.

Hydrogen.—There are two spectra of hydrogen, the simple, or series spectrum, and the compound spectrum, recently shown by G. P. Thomson to be due to the hydrogen molecule. The simple or atomic spectrum consists of one series, occurring in at least three different parts of the spectrum, as though transplanted unchanged. The frequency-differences are what remain unchanged in such cases; such series are called parallel series. Any series consists in itself of an arrangement of lines whose spacing diminishes regularly, approaching a limit whose frequency is finite, and is sometimes called the "convergence-frequency" of the series. The intensities of the lines fade away the nearer the limit is approached, and usually so rapidly that only very few of the lines can be observed; in laboratory sources it is rare that more than 15 lines of a series can be observed; the maximum number is about 50, and was obtained by R. W. Wood from the absorption of sodium vapor.

It is usual to speak, not of wave-lengths, but of their reciprocals, called wave-numbers, these being the number of waves per cm; and it will be convenient to define the "term" of a series line as the difference in wave-number between the limit of the series and the individual line. Thus, the hydrogen series yields a sequence of terms, which occurs unchanged in each repetition of the series, though the limits in the three cases are very different.

The visible portion of the spectrum consists of four lines of the "Balmer" series, the rest of which is in the ultra-violet. This series can be represented by the formula

$$\frac{1}{\lambda} = N \left\{ \frac{1}{2^2} - \frac{1}{m^2} \right\}$$

where N is the so-called "universal series constant," having a value near 109678.2, and *m* takes in succession the integral values 3, 4, 5, etc. This formula yields the observed wave-numbers to quite an extraordinary degree of accuracy, though if the utmost refinement be demanded, there should be a correction term, of a nature indicated by theory. The lines are, moreover, actually very close doubles, whose frequency-differences are now known to remain approximately constant throughout the series.

In the extreme ultra-violet region, the series

$$\frac{1}{\lambda} = N \left\{ \frac{1}{1^2} - \frac{1}{m^2} \right\}$$

occurs, discovered by Lyman, and beginning with the line λ1216. Paschen has found three lines in the ultra-red which belong to the series

$$\frac{1}{\lambda} = N \left\{ \frac{1}{3^2} - \frac{1}{m^2} \right\}$$

Other series may exist in the deeper ultra-red, but these are very faint. The whole set of series, and therefore the whole spectrum of atomic hydrogen, can be represented by the formula

$$\frac{1}{\lambda} = N \left\{ \frac{1}{n^2} - \frac{1}{m^2} \right\}$$

where, in general, the lines are stronger the smaller are the integers n and m. The smallest value for n is 1, and for m is always the integer next larger than n. The Bohr theory has had its greatest success in the explanation of this spectrum.

The Sodium Family.—The elements of this family yield spectra which are closely similar to one another, and are one step less simple than that of hydrogen. Each consists of four chief series, and each line in each series is double. The four series are known as principal, diffuse or first subordinate, sharp or second subordinate, and fundamental. The latter are called the "Bergmann" series by German writers, though improperly, as Fowler was the first to find one of this type, and Runge the first to point out their true relationships. These four type-series have certain characteristics which are worth noting. The *principal* series contain the chief line of the spectrum (the D line in the case of Na), which is produced in flames, and readily reversed in hotter sources, due to self-absorption. The separation of the pairs is not constant in this series, but they shrink rapidly in passing out toward the limit. The lines of a principal series pair have slightly unequal intensities, the stronger one being on the side of the shorter wave-length. The *diffuse* and *sharp* series have constant wave-number differences in each pair, and equal to that for the first pair of the principal series; their lines have an aspect well indicated by their names; and these two series converge to a common limit. The stronger line in each pair is here the one of *longer* wave-length. They require higher temperatures or more vigorous excitation for their production than does the principal series. The *fundamental* series is a faint series of narrow pairs, lying in these cases in the ultra-red; its name is somewhat misleading.

Any of these series may be represented approximately by a series formula, of which there are several types. The general type is

$$\frac{1}{\lambda} = \frac{N}{[n+a+f(n)]^2} - \frac{N}{[m+b+f(m)]^2}$$

where N is the universal series constant, as before, though there are indications that its value should change slightly; *n* and *m* are variable integers, *n* having a single value for the whole of a series, while *m* takes a set of integral values in succession; *a* and *b* are constants, and the functions $f(n)$ and $f(m)$ may take many forms, none of them quite satisfactory. Fair success is obtained by giving the functions the form c/n^2 and d/m^2, *c* and *d* being new constants, but in many cases no simple form can possibly serve for these functions. The portion of the denominator written as $a+f(n)$ is sometimes referred to for brevity as the "residual"; it should settle down to a constant value as we proceed toward the limit of the series, and usually does so with rapidity. These formulæ have as yet no very sound theoretical basis, nor any established and universal shape; hence it has proved convenient to dodge the question of how they should be written by using abbreviations. In the above general formula the right-hand half of the expression for the wave-number is what we have already called the term, and we indicate it by the symbol (m,b), no matter how complex the formula actually used might be. The letter bracketed with *m* in this symbol is usually chosen to indicate the series to which it refers; thus, the letters *p,s,d* and *f* refer respectively to the principal, sharp, diffuse and fundamental series; for example, (1,p) stands for the first term of the principal series, (3,s) for the third term of the sharp. In the case of such elements as have series of other sorts beside pairs, the letters indicate the sort; thus in such elements, for example in Ca, small letters are used for series of triplets, large letters for single-line ("singlet") series, and Greek letters for pair series, except that German authors have so far used old Gothic letters instead of Greek. These distinctions are, naturally, not to be made, and have no meaning in the case of the elements of the sodium family, where all series are pairs.

The variable integer *m* has its lowest value for the first line of the series. In no cases but those of hydrogen and ionized helium can this integer be definitely settled as yet; in fact, since *m* is always associated with an added constant whose value is anything we please, the value of *m* becomes equally adjustable. German writers use for the first line of the series the value $m=2$ in the case of the principal series, 3 for the diffuse and 4 for the fundamental, while for the sharp they use values intermediate between integers, namely 1.5, 2.5, 3.5, etc. Some English-speaking writers prefer to put $m = 1$ for all first lines; such theoretical indications as there are rather favor the German custom.

A principal series may be written, in the symbolic way explained above, $1/\lambda=(1,s)-(m,p)$. The limit of the series is the term $(1,s)$, which is the first term of the sharp series, as the letter indicates. This connection between the principal and sharp series amounts to saying that they have a first member in common. On this idea, the first line of the principal series has the wave-number $(1,s)-(1,p)$, while the first line of the sharp series is $(1,p)-(1,s)$. The latter is negative, and it might well be objected that a negative wave-number is meaningless. The Bohr theory offers a possible explanation of this curious case, and others of the same sort, as indicated below. The *Rydberg-Schuster law* says that the wave-number-difference between the limit of the principal series and the limit of the sharp and the diffuse series (which they share in common) is equal to the wave-number of the first line of the principal series. If this is otherwise expressed it amounts to saying that the limit of the sharp series is the first term of the principal. This fact was used above in writing the first line of the sharp series as $(1,p)-(1,s)$. The whole of this series would be indicated by $1/\lambda=(1,p)-(m,s)$.

The diffuse series is given by the formula $1/\lambda=(1,p)-(m.d)$. The terms (m,d) and (m,p) are each double in the case of pair series, and in reality the diffuse pair contains three lines, the first and stronger line having a faint satellite. This is noticeable only in the case of wide pairs (*e.g.*, Rb and Cs), and even then only in the first few series members, for the reason that the satellite and the major line close up together.

The fundamental series is given by $1/\lambda=(1.d)-(m,f)$. Its limit is the first term of the diffuse series, a law due to Runge. The terms (m,f) are also double, though very close in most cases. Since the two values of (1,d) are not far apart, this series consists of narrow pairs, of which each line is itself a close double. While this complexity has been found only for Cs, there is every reason to believe that it is general. We are dealing here with what is generally called the "fine structure" of the lines, and a great many series lines prove to be complex when examined under the highest resolving powers. In most cases this complexity is so difficult to observe that we may safely ignore it.

Ritz formulated a so-called "combination law" to the effect that any two terms, chosen from among the lists of those in the four type-series, might be combined, and their difference would give a wave-number which might correspond to a real line. Combinations which occur more or less commonly are as follows: (1,p)–(m,p); (2,s)–(m,p); (2,p)–(m,d); (2,p)–(m,s); (2,d)–(m,f); etc. Of these the first, but only the first, are similar to difference tones in acoustics. In such elements as have systems of triplet and singlet series (*e.g.*, Zn, Ca, etc.), there also occur inter-system combinations, of which the most important is (1,S)–(m,p_2). Here the (m,p) terms are triple, but the combination occurs only with the middle value, indicated by the subscript.

On the basis of the modern theory, the Ritz combination principle can be given a simple physical interpretation. If the outer electrons in the atom can exist in a number of different stationary states, in each of which their energy has a definite value, then the radiation is supposed to occur on the transfer of an electron from one of these states to another. The change of energy which is involved in this event is proportional to the frequency of the light, and the light is supposed to be emitted during this process and at no other time. Thus, the "terms" should be proportional to the energy in the corresponding states, and their differences, which by the combination principle give the frequency of observed spectrum lines, are proportional to the changes in energy associated with the transfer of an electron from one state to another. On this theory, if a negative wave-number occurs, it indicates merely that the electron passes from one state to the other in a direction opposite to that which we might expect on spectroscopic grounds, and this idea involves no special difficulty.

The entire spectrum of Li,Na,K,Rb or Cs could then be written as $1/\lambda=(1,s)-(m,p)$; $1/\lambda=(1,p)-(m,s)$; $1/\lambda=(1,p)-(m,d)$;

$1/\lambda=(1,d)-(m,f)$, together with combinations formed from these terms. Most of the combinations which one might test are in the ultra-red and are not observed. All combination lines are likely to be faint. Exactly the same combinations have not been observed in all these spectra, but it seems hardly worth while to give the details in the separate cases.

In comparing the spectra of the elements in this group it is interesting to note that the widths of the pairs is proportional approximately, though not exactly, to the square root of the atomic number of the element. It might also be remarked that the spectrum as a whole shifts toward the slower vibrations with increasing atomic weight, as one might expect from mechanical considerations. These laws appear to be general.

All these elements probably possess another spectrum as yet practically unknown. Traces of it have been found in the heavier elements. This has been called the spark, or enhanced spectrum, but it might better be called the spectrum of the ionized element. It is particularly easy with the elements of this family to detach one electron from the atom. The new spectrum would appear when *two* electrons were detached temporarily, and one of these returned, thus giving us the optical vibrations of the atom minus one electron. As the atom in this condition would probably have a structure resembling the inert element to the left in Mendelejeff's table, *i.e.*, to Neon, Argon, etc., it is not surprising that it is difficult to tear off a second electron, and obtain the spectrum in question. It is probable that the " Grundspektra " of Goldstein are mixtures of this spectrum with the ordinary one.

The Calcium-Zinc Family.—Passing to the next column of Mendelejeff's table of the elements, we find more complicated spectra, which are not as yet quite completely understood. Each contains at least four type series in each of two systems, in one of which the lines are all triplets, in the other singlets. In addition, the spectrum of the ionized element is rather readily produced, giving rise to an independent but similar system, this time of pairs, as one might expect, since, with one electron gone, the atomic structure of this family should resemble that of the sodium family. In each system combination series occur, and they are apparently more important with the heavier elements, which give the richer spectra. Inter-system combination series are here first met with, as mentioned above. There are other features of these spectra as yet unexplained, and in some cases our knowledge is far from complete. For instance, well known as the spectrum of Hg is supposed to be, its pair system has not yet been studied.

The Inert Gases, He, Ne, etc.—The spectrum of *Helium* likewise contains two related systems of series, in this case observed as pairs and singlets, but perhaps really more complex, as well as an unrelated system (pairs) of lines produced by ionized helium. The atom of He probably possesses two external electrons, or perhaps only two altogether. On this account its spectrum ought to have a form similar to that of the other elements which have this structural feature, *i.e.*, to Mg, etc. This is the case. If one of these electrons were removed, the atomic structure should bear an external resemblance to that of hydrogen, and in fact the spectrum of ionized He consists of one series, following the Balmer formula, and repeated, exactly as is that of H, in two other parts of the spectrum, with fair prospects of the discovery, eventually, of still more of these parallel series. There is a difference, however, shared by the series of ionized Mg, Ca, etc., that 4N must be used instead of N itself in the series formula, and Bohr's theory explains this as due to the presence of the extra, unbalanced charge on the nucleus of the atom.

When we pass to Neon, however, we are dealing, following the fruitful atomic theory of Lewis and Langmuir, with a cubical atom, whose vibrations would be expected to be very complex. As a matter of fact this is the case, and only recently has this spectrum been successfully attacked, in a masterly memoir by Paschen. He finds one series system, but each "term" of each series is really very complex, consisting perhaps of ten widely separated lines, shrinking, however, on passing out along the series, though not in any simple manner. The spectrum of ionized Neon has not yet been identified, and the complexity of its ordinary spectrum is such that it is doubtful if we yet know all about it. Argon is equally rich in lines, and the study of its spectrum is apparently yielding results somewhat similar to those for Ne.

The Cu, Ag, Au Group.—These spectra combine the features of the Na group (pair system) with some of the characteristics of Ne and A, and they have not yet been adequately studied. The spectra of the ionized elements have not been clearly isolated.

The Al, In, Tl Group.—Passing next to the Al group, we return to pair systems as in Na, but the spectra of the ionized elements contain systems of triplets and singlets, which one would expect by analogy to resemble the ordinary (un-ionized) spectra of the Ca group. The pair systems are fairly well known, the others practically unknown; they lie among the short wavelengths, as one might anticipate from the fact that the forces in an unbalanced system should be greater, and therefore the rates of vibration more rapid.

The Elements of the Carbon Group.—The spectra of C and Si show definite signs of series, but none have as yet been published. Analogy indicates that the ordinary spectra should contain two related systems of series, perhaps of singlets and triplets, and that the ionized element should give a single system, perhaps of pairs. The latter spectrum should be easy to obtain, and therefore difficult to separate from the former.

The other elements in this column show parallel groups of lines, scattered at what seem to be random intervals. The analysis of the spectrum of Neon, which seemed to contain such groups, but was shown to be made up of series, leads us to hope that a similar result will be found to hold in these cases also, but the work has not yet been carried out. These groups constitute what has been called the " second type of regularity " in spectra.

The Elements of the Nitrogen Group.—The same remarks may be made in regard to this family of elements as apply to the preceding, except that one would expect the arrangement of the ordinary spectrum and the spectrum of the ionized element to be interchanged. No series have as yet been worked out here, but obvious triplets and pairs occur in N. The heavier elements contain parallel groups.

The Elements of the Oxygen Group.—The Oxygen family contains two systems of series, pairs and triplets, which seem both to come from the ordinary atom. Apparently the spectrum of the ionized element is the " spark " spectrum, which is well known, but rather complex, and in which no regularities have as yet been found.

The Elements of the Chlorine Group.—These give complex spectra most of which have not yet shown any systematic arrangement, though Paschen has very recently shown that ionized Cl gives series of triplets.

The Fe-Pt Elements.—These elements also give very complex spectra, though a patient analysis of a few of them has yielded parallel groups. It seems at present quite hopeless to attempt to arrange these spectra into series.

This brief survey of our knowledge of spectrum series serves to show quite clearly how limited that knowledge is, and how urgent is the need for more workers in this important field. Every new source of light, or mode of excitation, at once opens up an extensive field of experimental investigation, as it should be tried out for all possible elements in all possible parts of the spectrum. Last year at least two such modes of excitation were developed which have great interest: the high-vacuum spark used by Millikan to break new ground among the shortest wavelengths, and the method of explosion of very thin wires used by Anderson. Each of these promises to yield interesting new spectra, and should be most carefully studied from a series standpoint. Other methods will doubtless be developed in the future.

A general review of the series spectra of the elements yields two new laws, recently stated by Kossel and Sommerfeld on the basis of evidence still incomplete, though their probable validity has long been recognized.

The first of these is that the spectrum of an ionized element (lacking one electron) is like that of the element in the next column to the left in Mendelejeff's table, though shifted toward higher frequencies; for example, ionized Ca (Ca^+) gives a spectrum very like that of K; and this would be expected since each has but one external electron.

The second law states that the spectra alternate in character in passing across Mendelejeff's table. Thus the spectra of Na and Al consist of pair series and resemble each other closely; N is probably similar. Ca, Si and S have many features in common, and each probably furnishes two sets of series, singlets and triplets. This could not be foreseen from considerations of atomic structure, but indicates that when electrons are added to the outside of an atom, as occurs in passing from left to right across the table, the electrons have a strong tendency to unite into rather inactive pairs; thus the spectrum of Al appears to be that due to a single external electron, though there must be three present; but two of these are so closely bound together that they do not take part in the emission of light. Langmuir has recently called attention to this tendency of electrons to form pairs, on the basis of quite different evidence. That other laws of general interest will come out of such study cannot be doubted, and it is earnestly hoped that the near future will witness a vigorous development of this subject.

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NOTE ON A FORMULA ASSOCIATED WITH THE PATH OF A RAY THROUGH A PRISM.

BY

H. S. UHLER.

PROFESSOR SOUTHALL has recently published a trigonometrical formula for the calculation of the angle of incidence of a ray which traverses a prism in a principal section and emerges under a prescribed angle of deviation.[1] With regard to this formula he makes the following comment: "It is possible that it may be arranged in a form more convenient for logarithmic computation." More than seven years ago the present writer published the solution of this problem for the case of a ray traversing a prism obliquely.[2] Accordingly it may be of practical interest to present, in this place, the logarithmic formulæ as simplified for a principal plane.

The angle α_0 is first calculated from the equation

$$\sin \alpha_0 = n \sin \frac{1}{2}\beta,$$

where α_0 and β denote respectively the angle of incidence corresponding to minimum deviation, and the refracting angle of the prism.

An auxiliary angle ϕ is next computed from the equation

$$\tan \phi = \tan c. \sqrt{\left\{\sin\left[\frac{1}{2}(\epsilon + \beta) + \alpha_0\right] \sin\left[\frac{1}{2}(\epsilon + \beta) - \alpha_0\right]\right\}},$$

where c and ϵ symbolize respectively the critical angle, and the prescribed angle of deviation. If more convenient, $\tan c$ may be replaced by

$$1 \Big/ \sqrt{(n + 1)(n - 1)}\ .$$

The two values of the angle of refraction, α', may now be obtained by the aid of the following formula:

$$\tan\left(\alpha' - \frac{1}{2}\beta\right) = \pm \sin \phi \cot \frac{1}{2}(\epsilon + \beta).$$

---

[1] James P. C. Southall: "Note on the path of a ray through a prism in a principal section." *Jour. Opt. Soc.*, **4**, p. 283, 1920.

[2] H. S. Uhler: "On the deviation produced by prisms." *Am. Jour. Sci.*, **35**, p. 396, 1913.

Finally

$$\sin\alpha = n\sin\alpha'.$$

In the illustrative case, taken by Professor Southall, where $n = 1.5$, $\beta = 60°$, and $\epsilon = 40°$ we find

$$\begin{aligned}
\alpha_0 &= 48° \ 35' \ 25.4'' \\
\phi &= 7° \ 56' \ 28.7'' \\
\alpha' - \tfrac{1}{2}\beta &= \pm\,(6° \ 36' \ 45.8'') \\
(\alpha')_+ &= 36° \ 36' \ 45.8'' \\
(\alpha')_- &= 23° \ 23' \ 14.2'' \\
(\alpha)_+ &= 63° \ 27' \ 27.7'' \\
(\alpha)_- &= 36° \ 32' \ 32.3''
\end{aligned}$$

The numerical values of α just given confirm those of Professor Southall absolutely.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
Nov. 29, 1920.

NOTE ON MR. T. SMITH'S METHOD OF TRACING RAYS THROUGH AN OPTICAL SYSTEM.

BY

JAMES P. C. SOUTHALL.

In the *Proceedings of the Physical Society of London* and partly also in the *Transactions of the Optical Society* Mr. T. Smith of the National Physical Laboratory at Teddington, England, has recently contributed a series of valuable papers "On tracing rays through an optical system" which I am sure will be of special interest to such readers of this JOURNAL who, like myself, are particularly concerned with the theory and design of optical instruments. In all the various schemes for the trigonometrical computation of the path of a ray refracted at a spherical surface the ray itself is determined by two arbitrary parameters, sometimes called the "ray-coördinates"; and if one method possesses any decided advantage over another, it will nearly always be found to be due chiefly to a more convenient choice of these controlling factors. Onc of these parameters must be a linear magnitude, for example, the intercept of the ray on the optical axis or on some other prescribed line; whereas the other parameter may be, and indeed usually is, an angular magnitude; the angle which is most often used for this purpose being the slope of the ray or its inclination to the axis (which is denoted in the following by the symbol θ). In the scheme which Mr. Smith recommends the ray-parameters are this angle θ and the length of the perpendicular let fall from the vertex A of the spherical refracting surface on to the ray in question. One advantage of using this perpendicular is that the formulæ obtained apply equally well to a plane refracting surface; and it may be remarked in passing that other writers have made use of this parameter also, among whom may be mentioned A. Kerber, who has proposed several systems of formulæ for tracing rays through an optical instrument. If the feet of the perpendiculars let fall from A on to the incident ray and its corresponding refracted ray are designated by T, T′, and if the lengths of these perpendiculars are denoted by g, g', then $g = \mathrm{AT}$, $g' = \mathrm{AT}'$. Let the slopes of these rays be denoted by θ, θ' and the angles of incidence and refraction by α, α'; and moreover let R denote the curvature of the spherical refracting surface; and n, n′ the indices of refraction of the two media; then Mr. Smith's formulæ, as published by him originally in the *Proceedings of the Physical Society*, xxx (1918), 221–223, written, however, in terms of the above symbols, may be given as follows:

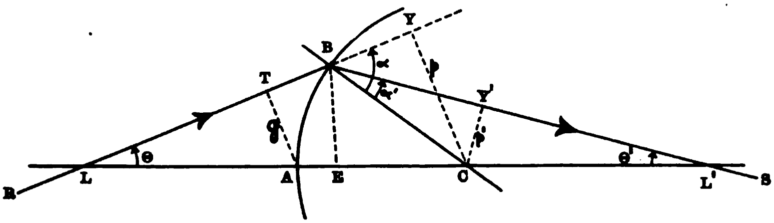
$$\sin\alpha = \sin\theta + g\cdot R,$$

$$\sin\alpha' = \frac{\mathrm{n}}{\mathrm{n}'}\sin\alpha,\ \theta' = \theta + \alpha' - \alpha,$$

$$g' = \frac{\cos\alpha' + \cos\theta'}{\cos\alpha + \cos\theta}\,g.$$

Now in practically all these schemes of calculation the process involves, as here, the application of the law of refraction as expressed by the equation $\mathrm{n}'.\sin\alpha' = \mathrm{n}.\sin\alpha$ at each surface of the lens-system in succession, whereby the angle of refraction (α') may be ascertained, and consequently the slope (θ') of the refracted ray can be found by use of the invariant-relation $\theta' - \alpha' = \theta - \alpha$. And in fact it is right at this stage of affairs that much of the labor of such calculations occurs, inasmuch as tables have to be employed and interpolations made in order to obtain the values (in degrees, minutes and seconds or preferably in radians) of the angles denoted by α, α' and θ. In his last paper on this subject (*Proc. Phys. Soc.*, xxxii (1920), 252–264) Mr. Smith seeks to find some substitution for the third step ($\theta' = \theta + \alpha' - \alpha$) in order to avoid the tedious procedure of evaluating these angles and in the hope of thereby effecting a material

FIG. 1.



shortening of the entire process, as is devoutly to be wished by all who have much of this work to do. Whether he has succeeded in this latter purpose appears to me doubtful; in fact, I myself have long been of the opinion that it was almost a waste of effort to try to devise essentially new systems of ray-calculation, that any really satisfactory method is on the whole about as economical as another, and that in all of them there is a certain limiting number of actual and necessary arithmetical operations which cannot be made less by any algebraic artifice. Of course, some individuals will work faster than others, and particular types of calculating machines will be better adapted than others. I am even old-fashioned enough to raise the question whether a calculating machine is more expeditious in the long run than the in-and-out process of logarithms; and my own experience, after trying out many schemes of ray-tracing, is that just when one fondly imagines he has found a promising short cut here or there, almost invariably he encounters a compensating amount of difficulty or delay at some other quite unexpected stage of the proceedings.

However, about these things opinions may differ; and so let us examine what Mr. Smith actually does. The main feature of his new method consists essentially in obtaining a convenient trigonometrical expression for the ratio $g' : g$, such that, the sines and cosines of the angles α, α' and θ having been ascertained, it is no longer necessary to know the magnitudes of the angles themselves. Mr. Smith's way of doing this is most ingenious and interesting in every way, as the readers of his paper will see for themselves; but the same result can be obtained also by a comparatively simple trigonometrical transformation as follows:

$$\frac{g'}{g} = \frac{\sin\alpha' - \sin\theta'}{\sin\alpha - \sin\theta} = \frac{\sin\frac{\alpha' - \theta'}{2}\cdot\cos\frac{\alpha' + \theta'}{2}}{\sin\frac{\alpha - \theta}{2}\cdot\cos\frac{\alpha + \theta}{2}} = \frac{\cos\frac{2\alpha' + \theta - \alpha}{2}}{\cos\frac{\alpha + \theta}{2}},$$

since $\alpha' - \theta' = \alpha - \theta$. Now if numerator and denominator of the last fraction are multiplied, first, by $\cos\frac{1}{2}(2\alpha' - \theta + \alpha)$, and, second, by $\cos\frac{1}{2}(\alpha + \theta)$, we shall obtain thus two expressions for the ratio $g' : g$, which may evidently be written as follows:

$$\frac{g'}{g} = \frac{\cos 2\alpha' + \cos(\theta - \alpha)}{\cos(\alpha + \alpha') + \cos(\theta - \alpha')} = \frac{\cos(\alpha - \alpha') + \cos(\alpha' + \theta)}{1 + \cos(\alpha + \theta)};$$

and, hence, also:

$$\frac{g'}{g} = \frac{\cos 2\alpha' + \cos(\theta - \alpha) + \cos(\alpha - \alpha') + \cos(\alpha' + \theta)}{1 + \cos(\alpha + \theta) + \cos(\alpha + \alpha') + \cos(\theta - \alpha')}$$

$$= \frac{1 + \cos(\alpha + \alpha') + \cos(\theta - \alpha') + \cos(\theta + \alpha) + 2(\sin\alpha - \sin\alpha')(\sin\alpha' + \sin\theta)}{1 + \cos(\alpha + \alpha') + \cos(\theta - \alpha') + \cos(\alpha + \theta)}$$

$$= 1 + \frac{2(\sin\alpha - \sin\alpha')(\sin\alpha' + \sin\theta)}{1 + \cos(\alpha + \alpha') + \cos(\theta - \alpha') + \cos(\alpha + \theta)}.$$

It is easy to pass now to Mr. Smith's final form of this expression, as follows:

$$g' = g + \frac{4\,g(\sin\alpha - \sin\alpha')(\sin\alpha' + \sin\theta)}{(\sin\theta - \sin\alpha + \sin\alpha')^2 + (\cos\theta + \cos\alpha + \cos\alpha')^2 - 1}$$

wherein the only trigonometrical functions that occur are the sines and cosines of the angles α, α' and θ.

In a very interesting discussion of Mr. Smith's paper in the same number of the *Proceedings of the Physical Society* Mr. F. J. W. Whipple has communicated an elegant geometrical method of obtaining this result.

Thus, the system of formulæ for tracing the ray at the kth surface, given in the order of the operations, is as follows:

$$\sin\alpha_k = \sin\theta_k + g_k \,.\, R_k,$$

$$n_{k+1} \,.\, \sin\alpha'_k = n_k \,.\, \sin\alpha_k,$$

$$g'_k = g_k + \frac{4\,g_k(\sin\alpha_k - \sin\alpha'_k)(\sin\alpha'_k + \sin\theta_k)}{(\sin\theta_k - \sin\alpha_k + \sin\alpha'_k)^2 + (\cos\theta_k + \cos\alpha_k + \cos\alpha'_k)^2 - 1}$$

$$\sin\theta_{k+1} = \sin\alpha'_k - g'_k \cdot R_k,$$

$$g_{k+1} = g'_k + d_k \cdot \sin\theta_{k+1} \quad (\text{where } d_k = A_kA_{k+1}).$$

(For a paraxial ray we have merely to write the angles themselves in place of the sines of the angles and put each of the cosines equal to unity.)

In order to avoid as much as possible all unnecessary reference to trigonometrical tables, all that is needed here is a simple table giving the cosine of the angle corresponding to a given value of the sine, and for this purpose a four-place table which can be printed all on one single sheet is quite sufficient. The accompanying table is reproduced from Mr. Smith's paper. The interpolations, as he points out, can be made mentally without difficulty.

COSINES IN TERMS OF SINES

(Copied from *Proc. Phys. Soc.*, xxxii, opposite p. 258)

	0	1	2	3	4	5	6	7	8	9	D	D^{-1}
.70	.7141	.7132	.7122	.7112	.7102	.7092	.7082	.7072	.7062	.7052	−10	−10
1	.7042	.7032	.7022	.7012	.7001	.6991	.6981	.6971	.6960	.6950	−10	−10
2	.6940	.6929	.6919	.6908	.6898	.6887	.6877	.6866	.6856	.6845	−11	− 9
3	.6834	.6824	.6813	.6802	.6791	.6781	.6770	.6759	.6748	.6737	−11	− 9
4	.6726	.6715	.6704	.6693	.6682	.6671	.6659	.6648	.6637	.6626	−11	− 9
5	.6614	.6603	.6592	.6580	.6569	.6557	.6546	.6534	.6523	.6511	−11	− 9
6	.6499	.6488	.6476	.6464	.6452	.6440	.6428	.6416	.6404	.6392	−12	− 9
7	.6380	.6368	.6356	.6344	.6332	.6320	.6307	.6295	.6283	.6270	−12	− 8
8	.6258	.6245	.6233	.6220	.6208	.6195	.6182	.6170	.6157	.6144	−13	− 8
9	.6131	.6118	.6105	.6092	.6079	.6066	.6053	.6040	.6027	.6013	−13	− 8
.80	.6000	.5987	.5973	.5960	.5946	.5933	.5919	.5906	.5892	.5878	−14	− 7
1	.5864	.5851	.5837	.5823	.5809	.5795	.5781	.5766	.5752	.5738	−14	− 7
2	.5724	.5709	.5695	.5680	.5666	.5651	.5637	.5622	.5607	.5592	−15	− 7
3	.5578	.5563	.5548	.5533	.5518	.5502	.5487	.5472	.5457	.5441	−15	− 7
4	.5426	.5410	.5395	.5379	.5363	.5348	.5332	.5316	.5300	.5284	−16	− 6
5	.5268	.5252	.5235	.5219	.5203	.5186	.5170	.5153	.5136	.5120	−16	− 6
6	.5103	.5086	.5069	.5052	.5035	.5018	.5000	.4983	.4966	.4948	−17	− 6
7	.4931	.4913	.4895	.4877	.4859	.4841	.4823	.4805	.4787	.4768	−18	− 6
8	.4750	.4731	.4712	.4694	.4675	.4656	.4637	.4618	.4598	.4579	−19	− 5
9	.4560	.4540	.4520	.4501	.4481	.4461	.4441	.4420	.4400	.4379	−20	− 5
.90	.4359	.4338	.4317	.4296	.4275	.4254	.4233	.4211	.4190	.4168	−21	− 5
1	.4146	.4124	.4102	.4080	.4057	.4035	.4012	.3989	.3966	.3943	−23	− 4
2	.3919	.3896	.3872	.3848	.3824	.3800	.3775	.3751	.3726	.3701	−24	− 4
3	.3676	.3650	.3625	.3599	.3573	.3546	.3520	.3493	.3466	.3439	−26	− 4
4	.3412	.3384	.3356	.3328	.3299	.3271	.3242	.3212	.3183	.3153	−29	− 3
5	.3122	.3092	.3061	.3030	.2998	.2966	.2934	.2901	.2868	.2834	−32	− 3
6	.2800	.2765	.2730	.2695	.2659	.2622	.2585	.2548	.2510	.2471	−37	− 3
7	.2431	.2391	.2350	.2308	.2265	.2222	.2178	.2132	.2086	.2039	−44	− 2
8	.1990	.1940	.1889	.1836	.1782	.1726	.1667	.1607	.1545	.1479	−53	− 2
9	.1411	.1339	.1262	.1181	.1094	.0999	.0894	.0774	.0632	.0447	....	....
.990	.1411	.1404	.1397	.1389	.1382	.1375	.1368	.1361	.1353	.1346	....	....
1	.1339	.1331	.1324	.1316	.1309	.1301	.1293	.1286	.1278	.1270	....	....
2	.1262	.1254	.1247	.1239	.1231	.1222	.1214	.1206	.1198	.1190	....	....
3	.1181	.1173	.1164	.1156	.1147	.1138	.1130	.1121	.1112	.1103	....	....
4	.1094	.1085	.1075	.1066	.1057	.1047	.1038	.1028	.1018	.1009	....	....
5	.0999	.0989	.0979	.0968	.0958	.0948	.0937	.0926	.0916	.0905	....	....
6	.0894	.0882	.0871	.0859	.0848	.0836	.0824	.0812	.0799	.0787	....	....
7	.0774	.0761	.0748	.0734	.0721	.0707	.0692	.0678	.0663	.0648	....	....
8	.0632	.0616	.0600	.0583	.0565	.0548	.0529	.0510	.0490	.0469	....	....
9	.0447	.0424	.0400	.0374	.0346	.0316	.0283	.0245	.0200	.0141	....	....

As an actual numerical illustration will doubtless be of interest, I give below the results of the calculation of the path of a ray through a double cemented achromatic microscope objective composed of a symmetric convex (or equibiconvex) crown glass lens and a planoconcave flint glass lens, with numerical data as follows:

Indices: $n_1 = n_4 = 1$ (air); $n_2 = 1.5166$ (crown); $n_3 = 1.6256$ (flint).

Radii and curvatures: $r_1 = -r_2 = +0.35$ inch; $r_3 = \infty$.

$R_1 = -R_2 = +2.85714$; $R_3 = 0$.

Thicknesses: $d_1 = d_2 = +0.04$ inch.

The ray which is here calculated emanates from a point L_1 six inches from the vertex of the first surface ($A_1L_1 = v_1 = -6$), and meets the first surface at a point which is 0.1 inch above the axis ($h_1 = +0.1$). The initial values of the ray-parameters are found from the formulæ:

$$\tan\theta_1 = -\frac{h_1}{v_1 - r_1 + \sqrt{r_1^2 - h_1^2}}, \quad g_1 = -v_1 \,.\, \sin\theta_1.$$

The table below shows the values of the sines and cosines of the angles and the magnitudes of the linear parameters g, g' as found by Mr. Smith's system of formulæ:

	k = 1	k = 2	k = 3
$\sin\theta_k$	+0.016624	−0.08943	−0.06313
$\sin\alpha_k$	+0.30160	−0.36749	−0.06313
$\sin\alpha'_k$	+0.19887	−0.34285	−0.10262
$\cos\theta_k$	+0.9999	+0.9960	+0.9980
$\cos\alpha_k$	+0.9535	+0.9300	+0.9980
$\cos\alpha'_k$	+0.9800	+0.9394	+0.9947
g_k	+0.099744	+0.097327	+0.095377
g'_k	+0.100904	+0.097902	+0.095063

If the emergent ray crosses the axis at L_4, and if we put $v'_3 = A_3L_4$, then $v'_3 = -g'_3/\sin\theta_4$, so that we find here:

$$v'_3 = +0\;92636 \text{ inch.}$$

These calculations were made on a machine, and I found that it requires a rather large number of operations to obtain the values of g' for each surface.

It might be well to compare Mr. Smith's formulæ with the following system in which the magnitudes denoted by p and p' are the central perpendiculars on the incident and refracted rays:

$$p' = \frac{n}{n'}p, \quad \sin\alpha = pR, \quad \sin\alpha' = p'R',$$

$$\theta' = \theta + \alpha' - \alpha;$$

and in passing from the kth to the (k + 1)th surface:

$$p_{k+1} = p'_k + a_k \cdot \sin\theta_{k+1},$$

where

$$a_k = d_k + r_{k+1} - r_k.$$

For the plane surface we shall need in this case a special set of formulæ, as follows:

$$v_3 = v'_2 - d_2, \quad \sin\theta_4 = n_3 \sin\theta_3 / n_4, \quad v'_3 = v_3 \cdot \tan\theta_3 / \tan\theta_4.$$

The initial value of the central perpendicular is given by $p_1 = (r_1 - v_1)\sin\theta_1$. In the above formulæ $v_k = A_kL_k$, $v'_k = A_kL_{k+1}$, where L_k and L_{k+1} designate the points where the ray crosses the axis before and after refraction at the kth surface.

Before leaving this subject there is at least one other matter in Mr. Smith's paper which seems to me of more or less theoretical interest, and to which I think special attention should be called. In the accompanying diagram the centre of the spherical refracting surface is at the point marked C and the vertex at A, so that the optical axis is represented by the straight line AC and the step from A to C is equal to the radius ($r = AC$). A ray RB, incident on the surface at B, crosses the axis at the point marked L, and the corresponding refracted ray BS crosses the axis at L′; and in order that the following statements may be quite general and applicable to all cases, it should be noted that R is used here to designate any point in the first medium (of index n) which lies on the incident ray; and, similarly, S designates any point in the second medium (of index n′) through which the corresponding refracted ray passes. The abscissæ of L, L′ with respect to the centre C may be denoted by c, c', that is, $c = CL$, $c' = CL'$; and the "ray-lengths" measured from the point of incidence may be denoted by l, l' ($l = BL$, $l' = BL'$). These magnitudes are connected by the well-known invariant relation:

$$\frac{n'c'}{l'} = \frac{nc}{l}.$$

Draw the bisector of the angle RBS, and let E designate the point where it meets the optical axis, and put $EL = x$, $EL' = x'$; then, evidently, since BE is the bisector of the interior or exterior angle at B in the triangle LBL′, we must have:

$$x' : x = l' : l,$$

and hence also:

$$\frac{n'c'}{x'} = \frac{nc}{x}.$$

Now $c = x - e$, $c' = x' - e$, where $e = EC$, and if we substitute these expressions for c and c' in the last equation, we may write:

$$\frac{n'}{x'} = \frac{n}{x} + \frac{n' - n}{e}.$$

The similarity between this formula and the familiar abscissa-equation for the refraction of paraxial rays at a spherical surface, namely,

$$\frac{n'}{u'} = \frac{n}{u} + \frac{n' - n}{r},$$

where u, u' denote the abscissæ, with respect to the vertex A, of the pair of conjugate axial points M, M′ ($u = AM$, $u' = AM'$), is immediately apparent. The difference between the two cases is that whereas in the latter formula the abscissæ u, u' and r are measured from the fixed point A, the abscissæ x, x' and e in the former equation are all measured from a variable point E, whose actual position on the axis depends on the slope (θ) of the incident ray. On account of this analogy, Mr. Smith speaks of the point E as "the equivalent vertex" of the spherical refracting surface. The "equivalent radius" e may easily be computed in terms of the angles α, α' and θ, θ' and the actual radius r; thus, for example, in the case shown in the figure, where BE is the bisector of the interior angle at B in the triangle LBL′, we may write:

$$EC : BC = \sin \underline{/EBC} : \sin \underline{/CGB},$$

and since

$$\underline{/EBC} = \frac{\pi - (\alpha + \alpha')}{2} \text{ and } \underline{/CGB} = \frac{\pi + (\theta + \theta')}{2},$$

we obtain:

$$e = \frac{\cos \dfrac{\alpha + \alpha'}{2}}{\cos \dfrac{\theta + \theta'}{2}} \, r\,.$$

It is also easy to see how the formula which Professor Woodworth published some years ago in a communication to *Science* may be derived from the above. This formula, written in the notation used here, is as follows:

$$\frac{n}{c'} - \frac{n'}{c} = \frac{n' - n}{r} \cdot \frac{\cos \dfrac{\theta + \theta'}{2}}{\cos \dfrac{\alpha + \alpha'}{2}}$$

This formula may be quickly derived also (without reference to the abscissa e and the bisector of the angle at B in the triangle LBL′) from equation (7) in my paper entitled "Note on Calculation of Path of Ray through a Symmetrical Optical Instrument," published in a previous number of this JOURNAL (iv, 294–299, 1920).

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NOV. 10, 1920.

DIOPTRICS OF THE EYE.

BY

A. AMES, JR., and C. A. PROCTOR.

PREFACE.

THE researches described in the following paper were undertaken for the purpose of determining the exact nature of the image received by the human eye. It was believed that a knowledge of the nature of this image would be of aid in suggesting how the various parts of a picture should be painted to give a pleasing and artistic effect. The steps which led to this belief are as follows:

About 1910 Mr. Ames and his sister, Mrs. Oakes Ames of North Easton, Mass., who were painting together, believed that if they could make an exact reproduction in form and color of a scene they would have a technically satisfactory work of art. To accomplish this they made a very complete set of colors consisting of about thirty-five hundred different colored cards. With these they could match up every shade of color in the scene they were depicting and by making very careful drawings could reproduce very exactly both in color and form any scene they desired. The result, however, from an artistic point of view was not satisfactory. It was in the vernacular "tight" and "hard" and "photographic," and lacked "atmosphere." They were then led to believe that to be technically satisfactory their paintings must be similar to the image received by the human eye.

In the early part of 1913 they painted various pictures introducing in a rough way the distortion which exists in the eye, loss of detail towards the sides of the picture and also a relative increase of blue towards the sides of the picture. Though encouraging, none of these attempts were satisfactory. Believing this lack of success was due to insufficient knowledge of the exact nature of the image the eye receives, Mr. Ames went to Clark University, Worcester, to see the late Dr. J. W. Baird, who was an authority on the physiology of vision. As the information he desired had not yet been ascertained, he went to Worcester to do the necessary research under Dr. Baird early in 1914.

The first problem he undertook was a quantitative determination of the color sensitivity of the peripheral retina; *i.e.,* just what shade a color would appear at various degrees out on the retina. A great many observations were made by a large number of observers. It became obvious from lack of agreement in the readings that certain factors were not being controlled. From various suggestions in the literature[1] and from the fact that a gray spot appeared to the peripheral retina very reddish, it was decided that the uncontrolled factors were the oblique aberrations of the crystalline lens and cornea. Accordingly, early in 1914 a very thorough study of the literature was made and steps taken to determine the aberrations of the human eye. The questions in advanced optics and lens design which arose necessitated the assistance of some one trained in that line. This was first given by a Mr. Herman Beck, then by Dr. A. K. Chapman. The work was stopped when Dr. Chapman and Mr. Ames went into service. After the war, Dr. Chapman went to the Eastman Kodak Company and Mr. Ames went with the work to Dartmouth College, Hanover, N. H., to get the assistance of Dr. C. A. Proctor of the Physics Department, who has collaborated with him in the work since that time.

The false starts, development and discarding of unsatisfactory apparatus and the slow perfecting of the apparatus used, and other vicissitudes common to all research work, were gone through. The work is by no means concluded. It would be nearer the truth to say it has just been begun. Only the eyes of Dr. Chapman and Mr. Ames have been measured and those only for relaxed accommodation, and in one or two meridians. The aberrations determined have been, 1st, Spherical aberrations for different zones; 2nd, Axial chromatic aberration; 3rd, Oblique astigmatism, *i.e.,* the position of the primary and secondary astigmatic fields. It is hoped that the lateral chromatism can also be determined.

It is recognized, of course, that the results of testing one or two eyes cannot be accepted as conclusive. The average readings of many eyes must be found and also under various conditions of accommodation, illuminations, etc.

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[1] Especially an article by Pietsch, "Die Ausdehnung des Gesichtfeldes für weisse und farbige Objekte bei Verschiedenen Refraktionszustanden." Breslau, 1896.

It is believed, however, that the description of the apparatus, method of procedure, and such readings as have been obtained are of sufficient interest to warrant their being published at this time.

Special thanks are due to Dr. Baird and to Clark University and its library, which rendered the greatest assistance in bibliographical work, and also to Dr. Chapman. Thanks are also extended to Dr. Kellner of Bausch and Lomb Optical Company, and to individuals in the Eastman Kodak Company.

SPHERICAL ABERRATION.

Spherical aberration is a spreading of rays along the axis near the image plane due to different zones of the lens not having the same focal length.

Considerable work has been done on the determination of the spherical aberration of the human eye. A very good summary is given in Tscherning's "Physiologic Optics," which has been translated into English. Various methods have been used which are described there. It was the purpose of our research to obtain more exact quantitative results than had been gotten heretofore.

To determine quantitatively the spherical aberration of a lens, it is necessary to ascertain the difference in inclination between rays which pass through the axis of the lens and those that pass through zones at various distances from the axis.

Described in general terms, the method used in this investigation was to pass beams of light of small cross section through different zones of the eye and determine the inclination of the beams when their image coincided with the image formed by an axial ray.[2]

DESCRIPTION OF APPARATUS.

The image of a filament in bulb (A), see Fig. 1, after passing through a collimator (B) and prism (C) was focused by lens (D) on a hole in the forty-five degree mirror (E).

Bulb (A) could be adjusted in any position by screws a,a,a, (see photograph, Fig. 3).

The whole system (ABC) could be adjusted in any position by means of the tangent screw at (b) the elevating device at (c) and the tipping screws at (d,d) (see photograph, Fig. 3). The forty-five-degree mirror (E) could also be adjusted in any position by means of the lateral screws at (e,e) the elevating screw at (f) and the tangent screw at (g) (see photograph, Fig. 3).

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[2] We are indebted to Dr. Kellner for the idea of passing beams of small cross section through the different zones of the eye and the use of strips as described on page 26 to obtain such beams.

The light passing through the small hole in (E) strikes a concave mirror (F) which is mounted in a slide that moves back and forth along the bed (GG), Fig. 2. By means of the screws (hh) this mirror can be either tipped or moved laterally (see photograph, Fig. 3).

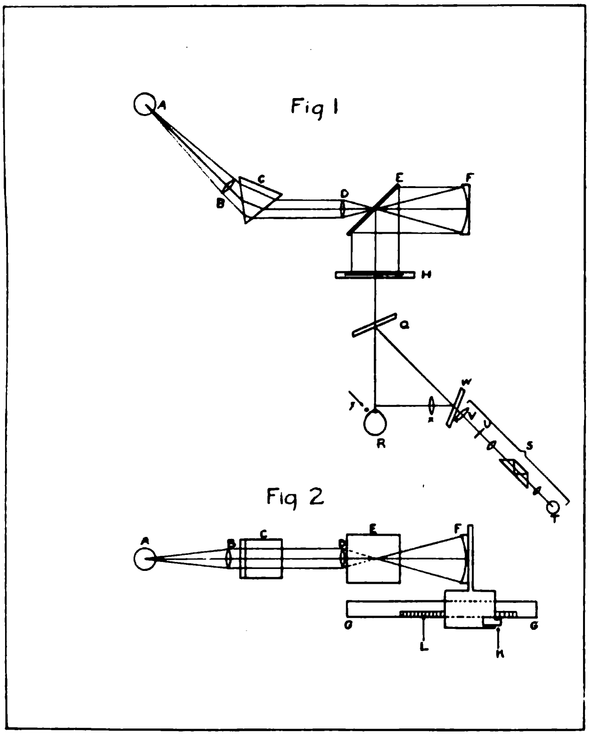
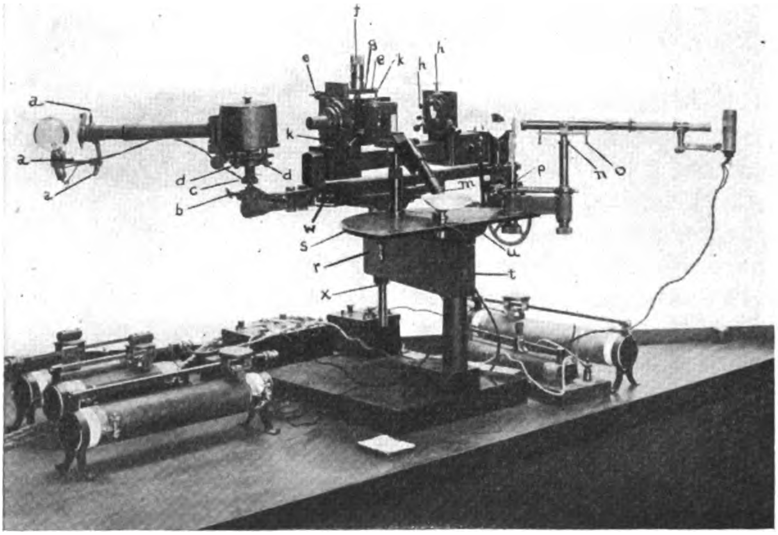


FIG. 1.—Diagram of apparatus, plan.
FIG. 2.—Diagram of apparatus, elevation.

The light is reflected by this concave mirror (F) back to the forty-five-degree mirror and thence out in the direction (EH). It will be seen that when the mirror (F) is at a distance from (E) equal to its focal length the light reflected in the direction (E) (H) will be parallel. If it is at a less distance, the light will be divergent; if at a greater distance, convergent. An indicator (K), Fig. 2, attached to (F) gives on the scale (L) marked on the bed the exact position of (F) from which the inclination of the reflected light can be calculated.

At (H) Fig. 1, there is a block with a hole in it, see Fig. 4. In this block two slots are arranged at right angles to each other into which metal strips with slits cut as indicated can be shoved.

Fig. 3.

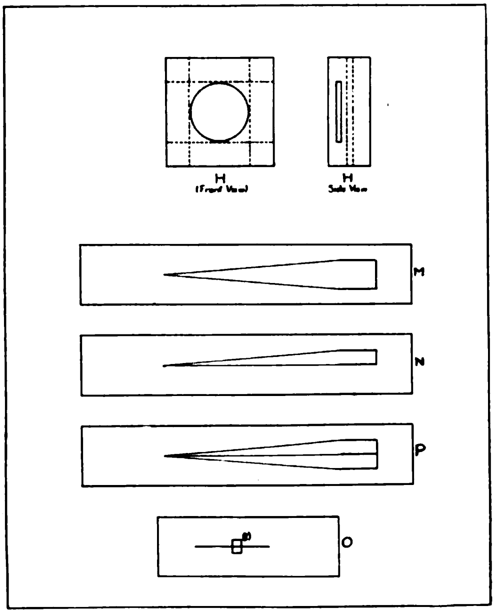


Photograph of apparatus.

This block can be adjusted up or down or sideways by the push screws (k) (k), (see photograph, Fig. 3). It can also be rotated in its mount around an axis perpendicular to the plane in which it lies. The width of the slits in the strips can be adjusted as desired. It was found that if they were too narrow, diffraction bands became too prominent. After trying varying widths slits of .5 mm. in width were used.

It will be seen that if strip (O), Fig. 4, is shoved into block (H) all the light will be shut off except a narrow band which will come through the slit in (O). If strip (M) is then shoved into block (H) at right angles to strip (O) all the light on the narrow band will be cut off except where the slits in strip (M) coincide with the slit in (O), which will leave two beams whose cross section is determined by the width of the slits in (M) and (O). By shoving (M) forward and back, these two beams will move together or apart as desired. If strip (N) is used, two similar beams are obtained. But upon moving (N) forward and back, one of the beams will remain stationary and the other will move relative to it. Similar results are obtained with strip (P).

FIG. 4.



Detailed drawing of block and strips.

In this way two or more beams of light of any desired cross section are obtained which can be given any desired inclination to each other, or whose inclination with any setting of the mirror (F) can be determined. Their separation can also be controlled and light of any wave-length can be obtained by changing the position of (A) and (B), Fig. 1, relative to that of the prism (C). We have therefore all the means necessary to determine the spherical aberration of a lens, granting that the lens can be held stationary. This was one of the most difficult problems to solve in connection with the eye and was accomplished as follows:

On a plane parallel placed at (Q), Fig. 1, a small white speck was placed. The plane parallel is mounted with optical wax on a small table which can be elevated or lowered, moved laterally, and tilted by screws (see photograph, Fig. 3). This speck was illuminated by focusing upon it the image of a small light placed at (m), (see photograph, Fig. 3). This illuminating system is fixed to the small table so that when the white speck is adjusted, the light which illuminates it moves with it. The head was positioned so that the eye of the observer came at (R), Fig. 1, the chin resting on a chin rest. With the eye accommodated for distant vision the speck images behind the retina. It appears on the retina where the retina cuts the cone of light as a circular disk of light, the border of which is determined by the edge of the pupil. When the speck is placed upon the optical axis of the system, a ray of light from (E) passing through the slits in, say (O) and (N) along the axis of the system will pass through the centre of the pupil, if the eye is so positioned that the image of the ray falls in the middle of the above described disk of light. The shadow of the radiating cellular structure of the crystalline lens thrown on the retina by the speck at (Q) assists in centring in the middle of the disk. The slightest lateral movement of the head and eye will cause the image formed by the ray to move across the light disk.

To hold the accommodation constant, which is absolutely imperative, the system (S), Fig. 1, was used. It consisted of a light source (T), Fig. 1, the light from which after passing through a direct vision prism was focused on a vertical line, scratched on a silvered glass disk (u, Fig. 1) which is at the focus of the lens (V), Fig. 1. Monochromatic light was used to make the image sharper and to obviate any inclination to focus for different colors that exist in white light. The system was so placed that with the eye properly positioned the monochromatic line which appeared to be at infinity was visible by reflection from the plane parallel (Q). The system could be raised, lowered and tipped by adjustments (n and o), (see photograph, Fig. 3), and could be turned laterally by a fine tangent screw at (p). The line (U) can be placed at any apparent distance from the eye by changing its distance from (V). In all readings made to date the apparent position of (U) was at infinity.

For directing rays into the eye obliquely that part of the apparatus designated by (A), (B), (C), (D), (E), (F), (G), and (H) is mounted on an arm, see (r), Fig. 3, which revolves about a vertical axis the centre of which lies directly below the nodal point of the eye when it is positioned. The semicircular plate, see (s) Fig. 3, is marked in degrees and an indicator attached to the above-mentioned arm shows in degrees the obliquity at which the rays strike the eye. A set screw at (t), Fig. 3, enables this movable system to be locked in any desired position.

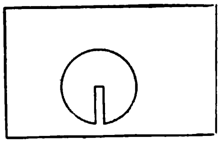
When rays are put into the eye obliquely it is necessary that the position of the eye be also fixed along the axis of vision, *i.e.*, that there shall be no forward and backward movement of the eye. This is accomplished by putting a half silvered surface at (W), Fig. 1. The monochromatic accommodation line from system (S) will be visible through the half silvered surface. An eye positioned at (R) will see reflected from the front surface of the plane parallel (Q) and the half silvered plate (W) a profile image of itself. A lens (x) is placed at its focal distance from the eye so that this profile image of the eye will be at infinity and can be seen sharply when the eye is focused on the monochromatic line from the system (S). It was found more convenient to put an illuminated spot at (y), Fig 1, close to the nose on the farther side of the cornea from (W). This illuminated spot is seen reflected from the surface of the cornea, appearing like a new moon. The slightest forward or backward movement of the head will cause this moon-shaped image to move across the circle of light formed on the retina by the illuminated speck on (Q).

A revolving drum is attached directly under the bed (GG), see (u), Fig. 3. A sheet of paper can be placed around and attached to the drum by a metal strip. The indicator (K) is extended down so that when it is pressed, a knife-edge point will make a mark on the paper.

Before taking readings, the concave mirror (F), Fig. 1, is set to give parallel light and the knife-edge point is lightly pressed on the paper and the drum revolved. This leaves a line on the paper. When readings are made by pressing the knife-edge point into the paper—the drum being turned slightly between each reading—the position of the mirror in front or behind its focal position is given. This saves considerable time, and makes it unnecessary to turn on a light while making observations.

It was believed that in measuring oblique astigmatism it would be of advantage to have another zonal ray whose inclination could be varied independently of the zonal ray from mirror (F). To accomplish this a mirror similar to (F) was made with a slot cut in it as shown in Fig. 5. This allowed the light from the hole in (E) to pass through it and strike a mirror with a longer focus which travelled on the bed (GG) in the rear of (F). The light was reflected from this rear mirror back through the slot in (F) to the eye. Up to the present no use has been made of this elaboration except in attempts to perfect our methods of measuring spherical aberration where it was not found to be advantageous.

FIG. 5.



Detail of front mirror used in obtaining two rays with independently variable inclination.

MEASUREMENTS AND DIMENSIONS.

The following more specific measurements and dimensions are given to facilitate, if it is desired, the duplication of the apparatus.

The various parts will be taken up as they are lettered in Figs 1 and 2.

A	28 volt 25 w. nitrogen bulb with tightly wound filament. A bulb with a double "V" filament was used and black paint put on the glass of the bulb to blank off all but a straight section of the "V." The brightness of the illumination was controlled by a slide resistance 290 ohm. 1. amp.
B	corrected lens 25 cm. focal length.
C	60° prism; 42 mm. high, 32 mm. across base.
D	corrected lens about 60 mm. focal length.
E	silvered or platinized optical flat 25 mm. in diameter, 2 mm. thick. The hole scratched in silver or platinum was about .03 x .05 mm. Smaller holes were tried but given up on account of objectionable diffraction effects. To prevent light coming through the hole from

internal reflections of the glass plate, all of the back is covered with black balsam paint except a small portion behind the hole in the silver.

F	concave spherical surface of about 72 mm. focal length, silvered or platinized, 24 mm. diameter. The spherical aberration in this mirror at 3 mm. from its axis amounts to only 1/109 diopters, so can be neglected.
G G	bed made of circular steel rod 4 cm. in diameter, with one side flattened off and a mm. scale marked on flat side. The slide on which mirror (F) is mounted is a block turned to fit the rod with an adjustable key to take up wear which fits against flattened side of rod.
H	built up block 45 mm. x 45 mm. slots to take metal strips 25 mm. x 1.5 mm. hole in centre 24 mm. in diameter.
M N & P	steel or brass strips 1.5 mm. thick, 25 mm. wide, 15 cm. long. The "Vs" in the centre are adjustable back and forth, which gives control of the width of the slits. The sides of the slits are bevelled back to prevent any chance of reflection from the edges of the metal.
O	same as above except its length is 7.5 cm.
Q	plane parallel 13 mm. wide, 30 mm. long, 3.5 mm. thick, with edges bevelled as shown in (Q), Fig. 1. The white speck was made by putting on the centre of the glass a small drop of white ink and scratching it down to a round disk of about .05 mm. in diameter. This speck is illuminated by focusing upon it the light from a small 2.7 V bulb with a small lens of about 2.5 cm. focal length. The brightness of the light is controlled by a sliding resistance of 67 ohms 2.5 amp.
S	This system was built up of a small telescope with an objective of 20 mm. diameter and about 13 cm. focal length, a silvered or painted disk with a cross made by a long vertical line with a short horizontal cross line, scratched on it in the focus of the objective and a system to give monochromatic light, consisting of a direct vision prism, the necessary lenses and a small 6 volt bulb with a "V" shaped filament, which was mounted so it could be placed in any position. Change in wave-length of image was accomplished by moving the filament. The brightness of the light was controlled by a slide resistance of 67 ohms 2.5 amp.
W	half silvered plate 25 mm. x 25 mm., whose inclination was adjusted by tipping screws, such as are used to control the inclination of a plane parallel in a Michelson Interferometer.
X	a spectacle lens of the proper focal length.
Y	The illuminated spot was produced by focusing the light from a small 6 V bulb into the end of a small solid glass tube which has been silvered on the outside. The end of the tube is ground off at an angle of 45° and polished, and opposite this face the circular side

of the tube is ground down to a flat surface. The light is reflected by the 45° face on this ground surface, the brightness of which is controlled by a slide resistance. The tube is mounted on a stand which is placed on the table (S), Fig. 3, and moved so that the spot of light can be put in the desired position.

ADJUSTING APPARATUS.

First align the apparatus and adjust (A) (B) (C) (D) (E) and (F), Fig. 1, in a rough way so that the eye positioned approximately at (R) will see a yellow light of bright intensity.

Adjust (A) (B) (C) and (D) so that the light when viewed through a spectroscope will be monochromatic. Tip the mirror (F) until the light from it falls upon the hole in (E). This can best be seen by interposing a ground-glass surface between (Q) and the eye. As the mirror (F) is moved back and forth, a round yellow disk will be seen on the ground glass, changing in size according to whether the light is focused on it or in front or behind. When the light is focused approximately on the 45° mirror (E), a yellow disk of considerable size will appear on the ground glass. If any part of the beam is falling on the hole in (E), a dark spot will appear in the yellow disk. By adjusting (F), or if more convenient (E), this spot will fall in the middle of the yellow disk and when (F) is moved to its focal distance from the hole in (E), all the light will pass back through the hole again and the yellow disk will become dark all over. When this takes place, we know that the axis of the mirror (F) and the hole in (E) are in the same line.

Plate (Q) with the small white speck on it is then adjusted so that when the light is focused on the speck the yellow disk on the ground glass becomes dark as described before. This is due to the opacity of the white speck preventing the light from passing any farther. We now know that the hole in (E) the axis of (F) and the white speck on (Q) are all on the same line.

Two strips similar to (O) are then shoved into (H). The only light coming beyond (H) will be that which comes through a square hole the size of the width of the slits in the two strips (O) (O). It will appear as a small square on the ground glass except when the light is focused in the hole itself when it will appear as a small round disk. (H) is then adjusted sidewise and up and down until when (F) is moved forward and backward the light on the ground glass remains stationary and opens out evenly from a square to a circle and *vice versa.* We then know that the centre of the hole formed by the cross slits (O) (O) is on the axis of the system and that the inclination of the beam of light that comes through it remains the same, no matter what the position of (F).

The system (S) is then adjusted to give an image at infinity of monochromatic light. The slits (O) (O) are removed from (H); a telescope focused for infinity is placed at (R), and (F) is set to give parallel light. In the telescope will be seen a sharp image of the hole in (E) and the image of the line from system (S). System (S) is then adjusted so that the centre of the line falls exactly in the middle of the hole in (E).

The apparatus is now adjusted for taking measurements of spherical or axial chromatic aberration.

Further adjustments have to be made if rays are to strike the eye obliquely.

First, a small rod ground to a fine point, is inserted in a hole centred in the bearing, about which the arm (r), Fig. 3, and the system (A) (B) (C) (D) (E) (F) (H) revolve. This small rod is moved up and down until the top of the fine point is in the plane of the axis of the system (A) (B) (C), etc. This system is then turned about its other bearing (X), Fig. 3, until the focused light falls on the point of the small rod. The system is then revolved to a position at right angles to its former position and adjusted by turning about its bearing (X), Fig. 3, and tipping with screws at (W), Fig. 3, until the light can be again focused on the point of the small rod. By swinging the system (A) (B) (C), etc., to its extreme positions it can be adjusted so that in any position the focused light will fall on the same spot. We know then that this spot is in the centre of the system and that if the nodal point of the eye is placed here an axial ray will always pass through it, no matter what its obliquity.

To arrange (W) (X) and (Y), Fig. 1, so that the nodal point of the eye can be put in this position a thread is hung in front of the fine point on the small rod, at a distance equal to that between the front of the cornea and the nodal point of the eye. This is about 7.07 mm., (W) (X) and (Y) are then so positioned and adjusted that on looking through the telescope we see the shadow of the thread fall exactly upon the vertical line from system (S). With the eye so placed at (R) that the shine from the surface of the cornea from the light (Y) falls on this line, the nodal point of eye will be at the centre of the system. As we have said before, the slightest forward or backward movement of the eye will cause this shine to move away from that position.

METHOD OF TAKING READINGS OF SPHERICAL ABERRATION.

Various methods of taking readings were tried out.

The apparently simplest one was to use strips (N) and (O), Fig. 4, in the block (H) with a monochromatic light source. These would give an axial ray and another ray which would strike the eye at any desired distance from the axis. When the eye was properly positioned and the mirror (F) set for parallel light,

Table I.

Spherical Aberration. (Dr. Chapman.)

Observer, Dr. Chapman. Right eye, vision normal. No corneal astigmatism. Both eyes treated with homatropine, pupil dilated to about 7.5 mm. Accommodation relaxed. Focal length of mirror (F) = 71.9 mm scale reading 16.40. Hole in 45° plate (E) is at 92.1 scale reading. Distance from eye to mirror (F) = 277 mm.

Wave-length	Zone distance from axis, mm	Average scale readings, mm	Calculated object distance in front of eye, V′ mm	Calculated distances behind retina where incident parallel rays cut axis, mm
Red, 6560	1.0	167.3	−1361	0.22
	1.5	165.8	−2667	0.12
	2.0	165.1	−4495	0.06
	2.5	166.4	−1949	0.16
	3.0	167.4	−1315	0.22
	3.5	167.5	−1272	0.23
Yellow, 5890	1.0	164.0	∞	0.00
	1.5	163.0	+5375	−0.05
	2.0	163.5	+10547	−0.02
	2.5	164.3	−17023	0.02
	3.0	166.6	−1783	0.17
	3.5	166.9	−1578	0.19
Blue, 4860	1.0	161.8	+2555	−0.11
	1.5	159.6	+1380	−0.20
	2.0	161.1	+1988	−0.14
	2.5	162.4	+3436	−0.08
	3.0	163.7	+17437	−0.01

For calculation of V′ equation $1/V = 1/71.9 - 1/U$ was used. U = scale reading minus 921, and $V' = 277 - V$.

A typical set of readings from which one of the above averages was determined contained sixteen readings. Their average was 16.43. The largest reading was 166.0, the smallest 161.9.

the images from these two rays would not coincide on account of spherical aberration. Mirror (F) could then be moved until they did coincide and the inclination of the ray determined, which would give the spherical aberration for the zone tested.

This method was used by Dr. Chapman in making the readings given in Table I. Though good it lacked accuracy from the fact that the images formed by the rays are relatively large, due to the diffraction bands occasioned by the small dimension of the source and of the holes from the combined slits. The displacement of the two spots on the retina was very evident, but it was difficult to judge when they were exactly superimposed. The holes could not be made larger without making the cross section of the rays too great to measure the small separation in zones which was desired.

It was thought it might be easier to make this exact superimposition if one of the images was one color and the other another. This was accomplished by focusing another source of different color on the hole in (E) by reflection from the face of the prism (C) and putting small pieces of different monochromatic filters over the holes in the slits. This gave slightly better results.

Another method tried was to vary the widths of the slits so that a rectangular instead of a square hole was formed, the long dimension of one hole being at right angles to that of the other. The short dimension was made so small that considerable extension of the image was produced by diffraction. The result was two line images at right angles to each other which when superimposed appeared like a cross. This method was better, and together with different colored spots, was used by Dr. Chapman in observations given in Table II.

The method finally adopted was as follows: Instead of obtaining our axial image from a ray of small cross section passing along the axis, the image of the fine fixation line from system (S), Fig. 1, was used. Its image, which was formed by light coming through the entire pupil, was much smaller than the image formed by the ray of small cross section and enabled much more accurate settings to be made. The centre of the fixation line was carefully adjusted in the middle of the hole in (E) while being observed in a telescope as described on page 33. Strips (M) and (O), Fig. 4, were used with this method. A small plate, see (I) (O), Fig. 4, was placed on strip (O). By sliding strip (O) this small plate would cut off either side of the "V" in (M) or leave both sides open as desired. This enabled the zones on both sides of the eye to be tested separately or together. Light of only one color was used. The observations taken in this way are given in Tables III, IV, and V.

TABLE II.

Spherical Aberration. (*Dr. Chapman.*)

Conditions were the same as in Table I except that Dr. Chapman's pupils were not dilated and he used light of different wave-lengths in the two beams, and apertures of slits were ½ x ¼ mm with the long dimension of one at right angles to that of the other.

Wave-length	Zone distance from axis, mm	Average scale readings, mm	Calculated object distance in front of eye, V′ mm	Calculated distances behind retina where incident parallel rays cut axis, mm
Red, 6560	0.50	170.8	− 620	0.48
	0.75	168.6	− 986	0.30
	1.00	167.1	−1530	0.20
	1.50	165.5	−3308	0.09
Yellow, 5890	0.50	166.1	− 2324	0.13
	1.00	164.3	−17090	0.02
	1.50	163.1	+ 5882	−0.04
Blue, 4860	0.50	164.2	−25710	0.02
	1.00	161.8	+ 2488	−0.12
	1.50	159.7	+ 1340	−0.21

$V' = 212 - V$.

A typical set of readings from which one of the above averages was determined contained five readings. Their average was 164.3. The largest reading was 164.7, the smallest 163.7.

Dr. Chapman made the following settings with the slits removed, *i.e.*, with his pupil filled with light. Accommodation relaxed.

Wave-length	Average scale readings, mm	Calculated object distance in front of eye, mm	Calculated distance behind retina where incident parallel rays cut axis, mm
Red, 6560	166.3	−2000	0.144
Yellow, 5890	164.9	−5600	0.052
Blue, 4860	162.8	4600	−0.059

$V' = 277 - V$.

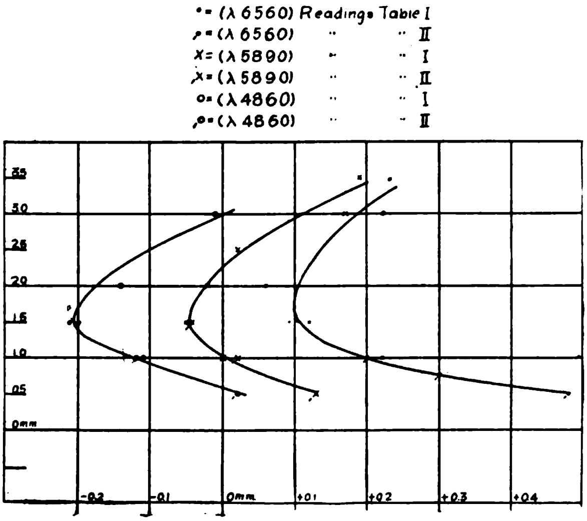
MEASUREMENTS.

The first measurements were made by Dr. Chapman, whose vision is normal, at Clark University, Worcester, Mass., in July,

1917. He measured the differences in aberration of the zones directly above the axis. He used two beams, one on the axis and a movable one above. Instead of a plane parallel at (Q), Fig. 1, he used a very thin cover glass and in place of a monochromatic source he focused on very small point sources at infinity illuminated by white light.

In making the readings given in Table I, he dilated his pupil with homatropine and used beams .5 mm in cross section and the same colored light in both beams. He made the readings given in Table II with normal pupil, using light of different color in the two beams with beams which were narrower in one cross section than in the other, giving the effect of two short lines at right angles to one another.

FIG. 6.



Spherical aberration curves from measurements of Dr. Chapman's eye. Abscissæ represent 0.1 millimeters from retina. Ordinates represent distance in millimeters of zone from axis.

Fig. 6 shows in diagrammatic form the distances in front and behind the retina at which light of the different wave-lengths is focused by the different zones.

Fig. 10 shows the actual form of the ray bundles due to spherical aberration for the three colors. In making the figures it was assumed that the aberrations on the lower half of the vertical meridian were the same as those on the upper half. This accounts for the symmetry of the bundles.

The displacement along the axis of the curves in Fig. 6 and the bundles in Fig. 10 for different wave-lengths is due to the axial chromatic aberration of the eye which will be treated in more detail later.

The measurements by Mr. Ames were made in 1919 at Dartmouth College, Hanover, N. H. He had considerable trouble getting consistent readings, especially for the zones near the axis.

TABLE III.

Spherical Aberration. (*Mr. Ames.*)

(Horizontal Meridian.)

Observer Mr. Ames, right eye, slight myopia and astigmatism. Pupil normal. Accommodation relaxed. Focal length of mirror (F) = 71.9 mm; scale reading when set for parallel light, 17.62 cm. Distance eye to mirror (F) = 205 mm.

Wave-length	Zone distance from axis, mm	Average readings (distances from focus of mirror), mm		Calculated object (distances in front of eye), mm		Calculated distance behind retina where incident parallel rays cut axis, mm	
		Temp.	Nasal	Temp.	Nasal	Temp.	Nasal
Red, 6563	0.50	1.71	1.56	− 2800	− 3100	0.103	0.093
	0.75	1.58	1.38	− 3070	− 3570	0.095	0.081
	1.00	1.98	1.03	− 2360	− 4850	0.124	0.059
	1.50	0.71	−0.12	− 7150	∞	0.040	0.000
	2.00	0.39	+0.03	−13100	∞	0.022	0.000
	2.50	0.31	1.05	−16700	− 4750	0.017	0.060
	3.00	0.60	0.87	− 8400	− 5800	0.034	0.049
Yellow, 5893	0.50	+0.65	+0.40	− 7750	−12750	0.037	0.022
	0.75	1.26	0.12	− 3950	∞	0.073	0.000
	1.00	−0.08	−1.04	∞	5300	0.000	−0.055
	1.50	−1.58	−2.04	3500	2700	−0.083	−0.108
	2.00	−1.05	−1.56	5250	3550	−0.056	−0.080
	2.50	−0.76	−0.79	7070	6850	−0.042	−0.043
	3.00	−0.54	−0.46	9750	11450	−0.031	−0.026
Green, 5461	0.50	−1.02	−0.78	5400	6900	−0.054	−0.042
	0.75	−0.99	−0.72	5550	7450	−0.052	−0.039
	1.00	−2.39	−2.62	2350	2170	−0.123	−0.134
	1.50	−3.72	−3.95	1600	1510	−0.180	−0.191
	2.00	−3.90	−3.56	1530	1670	−0.188	−0.173
	2.50	−3.16	−2.59	1850	2200	−0.156	−0.132
	3.00	−3.37	−2.68	1750	2130	−0.165	−0.136
Blue, 4861	0.50	......	......	2000	2252	−0.145	−0.128
	0.75	......	......	1754	1949	−0.166	−0.149
	1.00	−6.26	−6.72	1035	978	−0.276	−0.292
	1.50	−6.72	−6.73	978	977	−0.292	−0.292
	2.00	−6.20	−5.97	1043	1075	−0.273	−0.266
	2.50	−5.50	−4.85	1147	1273	−0.250	−0.225
	3.00	−5.98	−5.79	1073	1100	−0.267	−0.260

After much experimentation it was found that this was due to failure to get the axis of his eye on the axis of the optical system of the apparatus. This was in turn due to the decentration of his pupil. After the axis of his eye had been determined, his readings showed very little variation. The method of determining this was as follows:

Two beams with the separation of 1 mm. were obtained with slits (M) and (O), Fig. 4. The mirror (F) was pushed slightly in front of its focus position so that the images from the two beams were just separated. A lateral movement of the eye caused the images to move relative to each other. A position was found where movement towards either side caused the images to come together. In this middle position the beams were coming into the

TABLE IV.

Spherical Aberration. (*Mr. Ames.*)

(Vertical Meridian.)

(Conditions the same as in Table III.)

Wave-length	Zone distance from axis, mm	Average readings (distances from focus of mirror), mm		Calculated object distances in front of eye, mm		Calculated distances behind retina where incident parallel rays cut axis, mm	
		Lower	Upper	Lower	Upper	Lower	Upper
Red, 6563	0.5	+1.07	+1.76	−4670	−2730	0.062	0.107
	1.0	−0.53	+0.15	9900	∞	−0.030	0.000
	1.5	−1.03	−1.27	5350	4370	−0.054	−0.067
	2.0	−0.82	−1.56	6580	3540	−0.045	−0.080
	2.5	−1.36	−2.04	4070	2690	−0.072	−0.106
	3.0	−2.12	−3.02	2600	1925	−0.112	−0.151
Yellow, 5893	0.5	−0.53	−0.41	9900	13000	−0.030	−0.022
	1.0	−2.82	−1.32	2040	4200	−0.142	−0.070
	1.5	−3.01	−2.66	1930	2150	−0.150	−0.135
	2.0	−3.31	−4.17	1775	1440	−0.163	−0.200
	2.5	−3.14	−3.57	1860	1660	−0.156	−0.174
	3.0	−4.63	......	1320	......	−0.217	....
Green, 5461	0.5	−1.59	−1.09	3470	5050	−0.084	−0.058
	1.0	−3.22	−3.44	1820	1715	−0.158	−0.168
	1.5	−4.82	−4.82	1275	1275	−0.225	−0.225
	2.0	−4.81	−5.35	1280	1175	−0.224	−0.244
	2.5	−4.92	−5.47	1255	1155	−0.228	−0.248
	3.0	−5.99	......	1075	......	−0.266	......
Blue, 4861	0.5	−5.24	−5.88	1193	1087	−0.240	−0.263
	1.0	−7.05	−6.31	940	1027	−0.303	−0.277
	1.5	−8.61	−8.04	807	848	−0.352	−0.335
	2.0	−9.09	−8.60	775	808	−0.367	−0.352
	2.5	−9.26	−7.70	763	877	−0.373	−0.325

eye at equal distances from the axis and the point half way between is the axis of the lens system. When the eye is so placed that the monochromatic line from system (S), Fig. 1, falls on this point and is carefully kept there, very consistent readings could be obtained.

Table III shows Mr. Ames' readings for the different wave-lengths for the different zones, both temporal and nasal. Table IV shows his readings for the zones in the vertical meridian. Table V shows his readings for the temporal and nasal zones combined and was made as follows:

TABLE V.

Spherical Aberration. (*Mr. Ames.*)

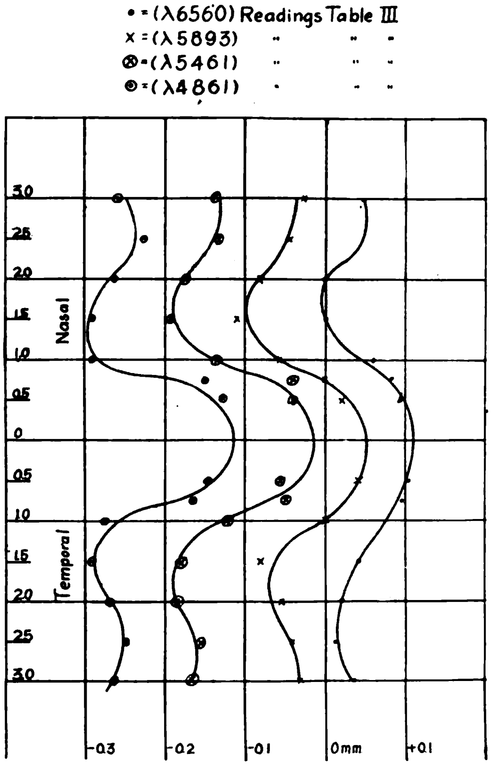
(Horizontal Meridian.)

(Conditions the same as in Table III. Readings for corresponding nasal and temporal zones combined as described on page 41.)

Wave-length	Zone distance from axis, mm	Average readings (distances from focus of mirror), mm	Calculated object distances in front of eye, mm	Calculated distances behind retina where incident parallel rays cut axis, mm
Red, 6563	0.50	+0.21	∞	0.000
	0.75	+0.35	−14750	0.018
	1.00	−0.51	10300	−0.028
	1.50	−0.96	5660	−0.051
	2.00	−1.09	5050	−0.058
	2.50	−0.02 nasal only	∞	0.000
Yellow, 5893	0.50	−1.08	5080	−0.058
	0.75	−0.81	6660	−0.044
	1.00	−1.61	3420	−0.086
	1.50	−2.60	2190	−0.133
	2.00	−2.35	2390	−0.122
	2.50	−1.27 nasal only	4350	−0.068
Green, 5461	0.50	−2.76	2080	−0.140
	0.75	−2.77	2070	−0.140
	1.00	−3.70	1610	−0.180
	1.50	−4.28	1415	−0.023
	2.00	−4.32	1400	−0.205
	2.50	−2.95 nasal only	1960	−0.148
Yellow, 5893 (later reading)	0.50	−0.50	10500	−0.027
	0.75	−0.85	6400	−0.046
	1.00	−1.75	3150	−0.093
	1.50	−2.66	2130	−0.137
	2.00	−2.49	2270	−0.128
	2.50	−1.78	3090	−0.095

Slits (M) and (O), Fig. 4, were used, both sides of the " V " in (M) being left open. Mirror (F) was moved until both spots superimposed. To insure they were superimposed, slit (O) was pushed back and forward so that plates (1) (O), Fig. 4, cut off first one image and then the other. If one image came exactly where the other had been, the setting was correct. If there was the slightest jump in one way or the other, a more careful setting was made.

FIG. 7.



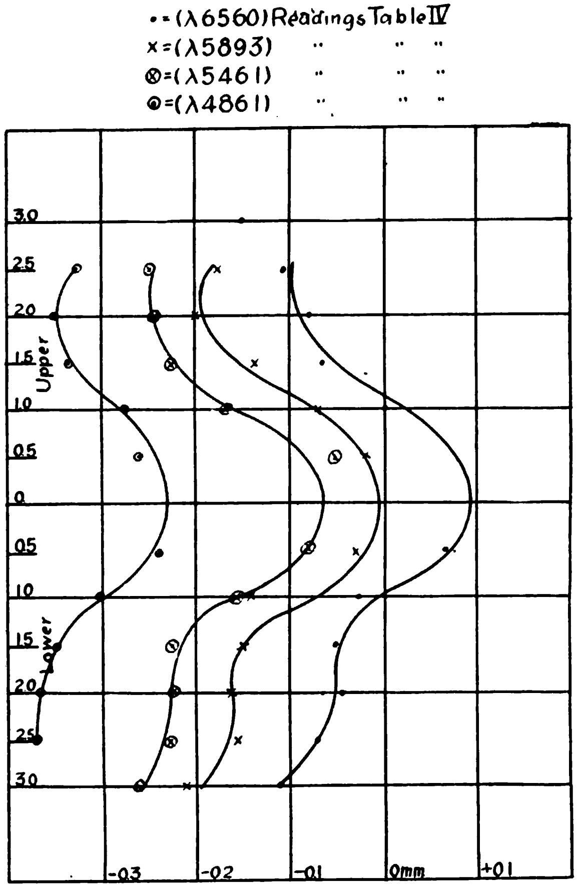
Spherical aberration curves from measurement of horizontal meridian of Mr. Ames' eye.

In all readings the wave-length of the vertical monochromatic accommodation line at infinity was (λ6563). Owing to his slight myopia, Mr. Ames was unable to get a sharp image if a light of a shorter wave-length was used.

The curves in Figs. 7, 8, and 9 show these measurements in graphic form.

It will be seen that the part of the curves showing the spherical aberration in the outer zones in the vertical meridian (Fig. 8) are displaced further forward than those in the horizontal meridian (Fig. 7). Difference in shape of the curves in the two meridians is probably due to asymmetry in shape of the cornea similar to that found by Gullstrand[3] caused by corneal astigmatism.

FIG. 8.



Spherical aberration curves from measurements of vertical meridian of Mr. Ames' eye.

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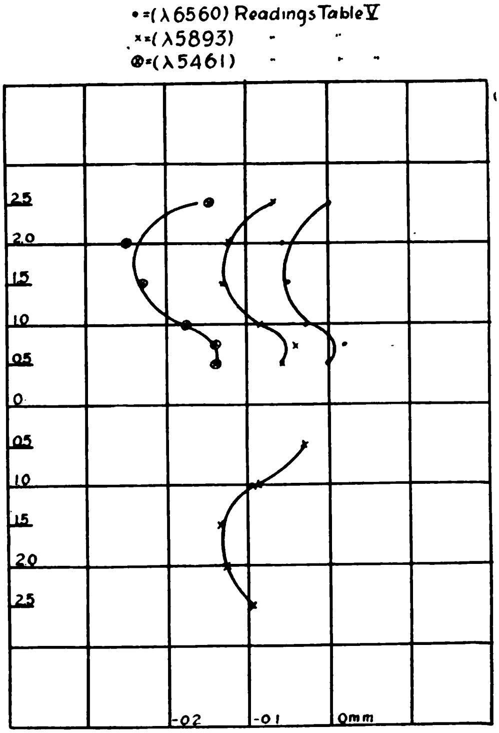
[3] Helmholtz Physiologischen Optik, 3rd Ed., pp. 268 and 269, Vol. 1.

This corneal astigmatism, or the difference in refraction for Mr. Ames' eye in the horizontal and vertical meridian, is shown

FIG. 9.



Spherical aberration curves from measurement of horizontal meridian of Mr. Ames' eye, the aberration from corresponding zones on both sides of the axis being determined at the same time.

by the difference in the position of sharp focus of a vertical and horizontal line source which is as follows:

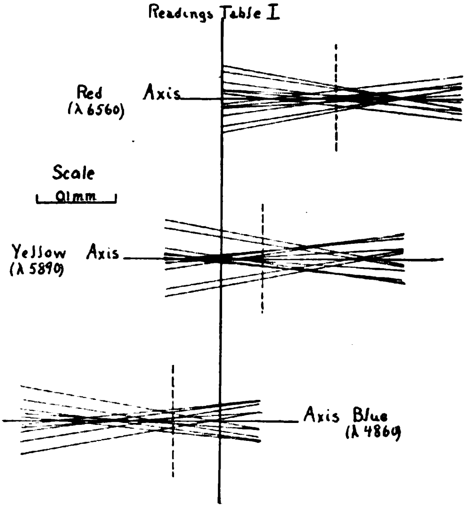
Wave-length	Vertical line, calculated object distance, mm	Horizontal line, calculated object distance, mm	Vertical line, calculated image distance from retina, mm	Horizontal line, calculated image distance from retina, mm	Difference, mm
Red, 6563	286000	4030	0.001	0.072	0.071
Yellow, 5893	3000	1670	0.095	0.172	0.077
Blue, 4861	1100	843	0.260	0.342	0.102

It is also shown in the difference in readings made with a horizontal and vertical beam of light extending across the whole pupil. The following readings were made with strip (O), Fig. 4, placed in a horizontal and vertical position. Accommodation relaxed.

Wave-length	Horizontal slit calculated object distance, mm	Vertical slit calculated object distance, mm	Horizontal slit calculated image distance from retina, mm	Vertical slit calculated image distance from retina, mm.	Difference, mm
Red, 6563	19000	4100	0.015	0.059	0.044
Yellow, 5893	2300	1585	0.125	0.181	0.056
Blue, 4861	1010	822	0.287	0.347	0.060

The curves in Fig. 9, though similar in shape and separation to those in Fig. 7, are displaced toward the left. It is believed this is due to a slight difference in accommodation; these measurements being taken about six months later than those shown in Fig. 7.

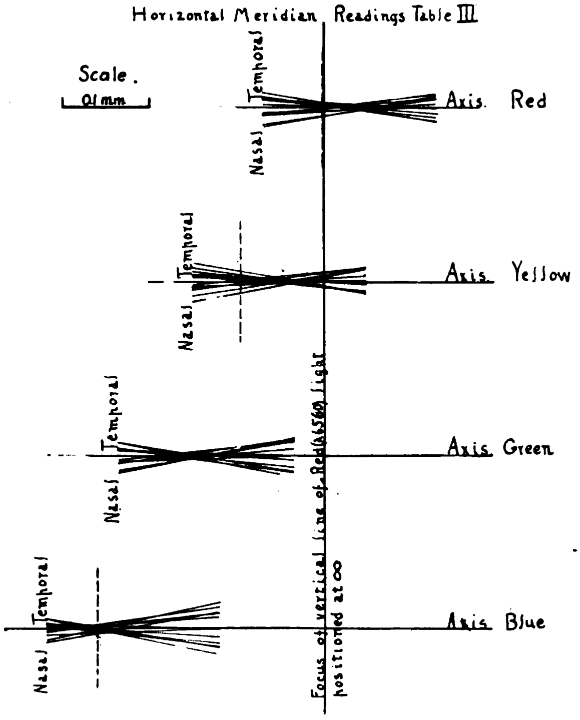
FIG. 10.



Actual shape of spherical aberration bundles as constructed from measurements of Dr. Chapman's eye. The solid line shows the focus of a spot of white light positioned at infinity. The dash lines show the position of the retina in the bundles when the eye focused on infinity sees most sharply a spot of the given color.

Figs. 10 (a), (b), (c) show the actual form of the ray bundles due to spherical aberration. Fig. 10 (a) shows those formed by the horizontal meridian, Fig. 10 (b), those by the vertical meridian and Fig. 10 (c), those formed by the combined horizontal and vertical meridian. It will be noted that the bundles for the different colors in the horizontal meridian are very similar to each other and also that those in the vertical meridian, while similar to each other, are different from those in the horizontal meridian. The displacement of the bundles formed by the two meridians is due to the corneal astigmatism which has been mentioned.

FIG. 10a.

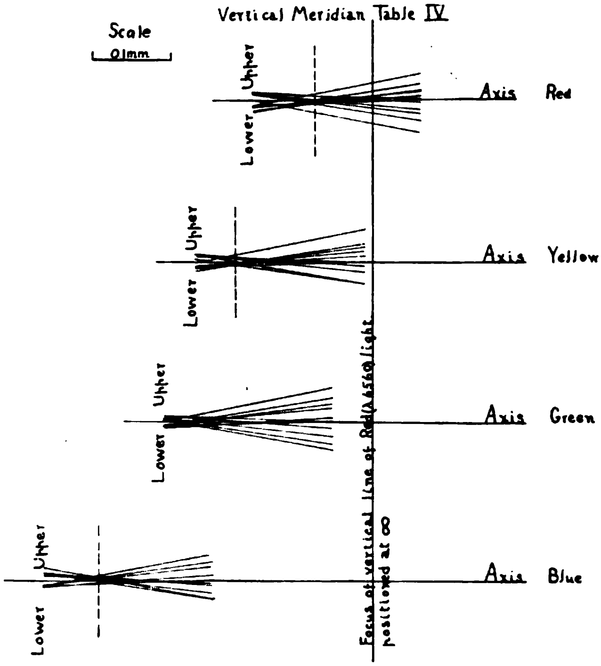

Horizontal Meridian Readings Table III
Scale.
.01 mm
Temporal
Nasal
Axis. Red
Axis. Yellow
Axis. Green
Axis. Blue
Focus of vertical line of Red(.6560) light positioned at ∞

Actual shape of spherical aberration bundles as constructed from measurements in the horizontal meridian of Mr. Ames' eye. The dash lines show the position of the retina in the bundles when the eye focused on infinity sees most sharply a vertical line of the given color.

It will be seen that while both Dr. Chapman's and Mr. Ames' curves are in general form similar, Dr. Chapman's curves differ from Mr. Ames'. That the spherical aberration should be different in the eyes of different individuals is in conformity with the measurements made by other methods, see Tscherning's "Physiological Optics" referred to above.

The results in general form are similar to the results found by other methods. The aberration is, however, considerably less than that calculated by Gullstrand, see p. 363, Vol. I, Helmholz, "Phy. Opti.," 3rd Ed.

FIG. 10b.


Vertical Meridian Table IV
Scale
.01mm
Upper
Lower
Axis
Red
Yellow
Green
Blue
Focus of vertical line of Red (λ 656) light
positioned at ∞

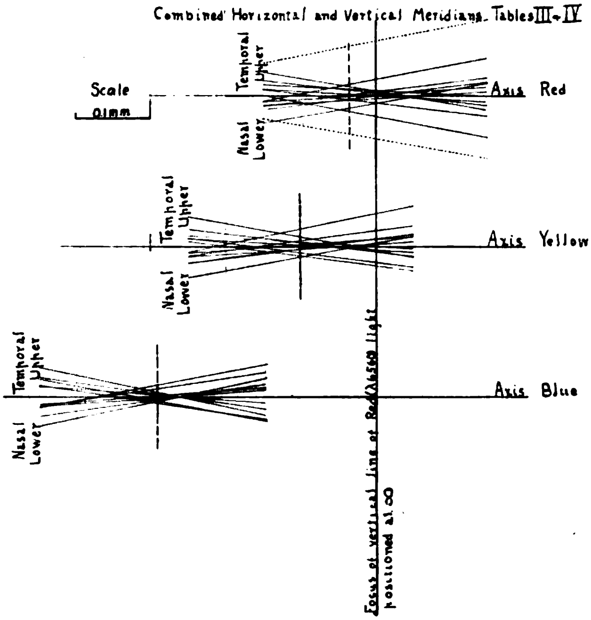
Actual shape of spherical aberration bundles as constructed from measurements in the vertical meridian of Mr. Ames' eye. The dash lines show the position of the retina in the bundles when the eye focused on infinity sees most sharply a horizontal line of the given color.

There is a much mooted question as to whether the eye so focuses that that part of the cone which has the smallest diameter falls upon the retina or that part where the section of the caustic due to spherical aberration is smallest. Both Tscherning and Gullstrand believe that it is that part of the bundle where the caustic is smallest and not where the cone has the least diameter. The exact nature of the results obtained in this method of research make it possible to answer this question very definitely. For

Dr. Chapman's and Mr. Ames' eyes there seems to be no question but that they focus approximately on that part of the bundle where the cone has the least diameter.

The long vertical line in Fig. 10 shows the position of Dr. Chapman's retina when he saw most sharply a white source positioned at infinity. The long vertical lines in Figs. 10a, 10b, and 10c show the position of Mr. Ames' retina when he saw most sharply a vertical red line positioned at infinity. The bundles show the position of the image cones of point sources of different wave-length positioned at infinity, their distance from the vertical lines giving the chromatic aberration. The short vertical broken lines show the position of the image for the corresponding wave-lengths of an object placed at infinity. In the case of Fig. 10 this object was a point source; 10a a vertical line; 10b a horizontal line; 10c a point source. These positions are given for Dr. Chap-

FIG. 10C.


Combined Horizontal and Vertical Meridians. Tables III + IV
Scale
0.1mm
Temporal
Upper
Nasal
Lower
Axis Red
Axis Yellow
Axis Blue
Focus of vertical line at Red (λ6560) light
positioned at ∞

Actual shape of spherical aberration bundles as constructed from the combined measurements in the horizontal and vertical meridians of Mr. Ames' eye. The dash lines show the position of the retina in the bundles when the eye focused on infinity sees most sharply a spot of the given color. The dotted lines show the outside rays of a blue (λ4860) bundle when the eye sees most sharply a red (λ6560) spot.

man in the tables on page 36, for Mr. Ames in the tables pages 43 and 50.

Figs. 10, a, b, and c, show that the retina shifts its position in the image bundle according to whether Mr. Ames is looking at a vertical or horizontal line or a point source. This is to be expected in view of the difference in the spherical aberration curve in the horizontal and vertical meridian. The sharpness of an image from a vertical line source is in the main governed by the aberrations in the horizontal meridian, that from a horizontal line source by those of the vertical meridian and that of a point by a combination of both. It will be noted in all cases, however, that the retina as indicated by the vertical dash lines falls approximately on that part of the image bundle where its diameter is smallest.

That this is usually the case would seem to be confirmed by the following observations by Dr. Proctor and Mr. Ames. With light entering the whole pupil the brightness of the source was made so low that it was just visible. It was then made to focus as sharply as possible by moving the mirror (F) while the accommodation of the eye was kept constant by observing the monochromatic fixation cross in system (S). The brightness of the source was then increased until it became bright enough to give a glare. No change in accommodation was perceptible, *i.e.*, the fixation line continued to be equally sharp and clear. Nor at any time while increasing the brightness of the spot could it be made to appear sharper or smaller by changing the focus of (F). It is believed that this could only be true if the eye was focused on the smallest diameter of the cone. If it was focused on the smallest part of the caustic, there would be a halo around it, invisible with very low illumination but very bright with high. This halo would be larger than the smallest diameter of the cone and with a very bright source it would be expected that a smaller image could be obtained by a slight shift in focus which would bring that part of the cone which has the smallest diameter upon the pupil.

In view of the definite results it seemed possible that a lens could be made which would correct both the spherical and chromatic aberration resulting in a smaller image bundle. Such a lens is being designed. While it is realized it could not improve vision in an eye like Mr. Ames' where the corneal astigmatism is relatively large, it might correct the aberrations in one meridian. In an eye with no corneal astigmatism it might possibly result in improved vision.

As has been stated, the readings that have been made are scarcely more than preliminary. The spherical aberration of a large number of eyes should be measured. This should be done for near and intermediate and distant accommodation. The measurements should also be made over at least four different meridians of the eye. The angle between the visual and optical axis (angle alpha) of each observer should be determined as well as the corneal astigmatism and the relationship between these factors and the spherical aberration noted.

AXIAL CHROMATIC ABERRATION.

Introductory.

The literature on axial chromatism of the human eye is very voluminous. Tscherning gives a short chapter to it in his "Physiologic Optics."

One method to determine quantitatively the axial chromatism of a lens is to ascertain the difference in inclination of rays of varying wave-length which come to the same focus. The chromatism may be determined for the lens as a whole by passing a large bundle through its full aperture, or for its various zones by passing small bundles through them.

The first method is, as far as is known, the only one that has been used up to date in measuring the eye. Nutting[4] was the last to use this method and found the mean axial chromatism for seven eyes between the C and F lines to amount to 0.99 D. Young estimated it to be 1.3 D. Fraunhofer found it 1.5 to 3 D for the range of the visible spectrum.

Method of Making Measurements.

The apparatus above described is admirably fitted to make such measurements. The method is as follows: With the slits removed from (H), Fig. 1, so that the whole pupil will be filled with light, the eye is accommodated on the monochromatic fixation line in system (S), Fig. 1. Monochromatic light of a given wave-length is focused through the hole in (E), Fig. 1. Mirror

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[4] Proceedings of the Royal Society A, Vol. 90 (1914).

(F), Fig. 1, is then moved until the sharpest image is formed on the retina. The reading will give the inclination of the rays necessary for light of that wave-length to come to focus on the retina. The wave-length of the light coming through the hole in (E) is then changed and a new reading taken. In this way the axial chromatic aberration curve for the different wave-lengths can be determined.

The table, page 36, gives the curve at three points for Dr. Chapman's eye; those on pages 43 and 44 for three points for Mr. Ames' eye. The amount of the aberrations varying from 0.72 D to 0.98 D correspond with the figures found by others as given above.

The method followed by Nutting is practically the same as given above, except that he did not use a monochromatic accommodation control. He also went further in using a greater number of different wave-lengths which enabled him to plot more accurately the entire curve. He finds this curve in some cases considerably flattened through its central portion, *i.e.,* on both sides of the D line and assumes that this is due to the eye being more or less corrected.

TABLE VI.

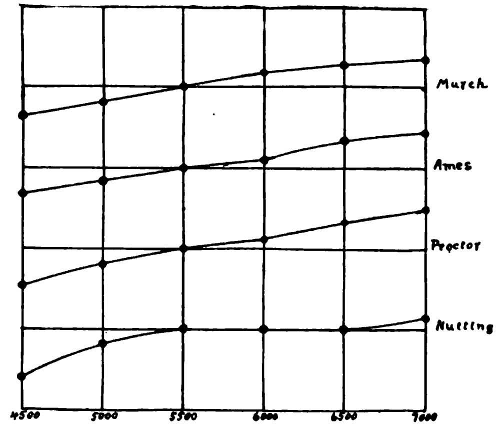
Chromatic Aberration, Accommodation Relaxed. Monochromatic Fixation.

Wave-length	Mr. Ames Fixation, $\lambda=6560$		Dr. Proctor Fixation, $\lambda=5600$		Mr. Murch Fixation, $\lambda=6560$	
	Average scale readings	Retina to focus, mm	Average scale readings	Retina to focus, mm	Average scale readings	Retina to focus, mm
7000	+0.04	0.000	3.46	−0.226	− 2.24	0.127
6500	−0.71	0.038	2.30	−0.147	− 3.60	0.174
6000	−1.55	0.081	0.64	−0.037	− 6.70	0.291
5500	−3.5	0.170	−0.31	+0.017	− 8.30	0.345
5000	−5.8	0.260	−1.90	+0.103	−11.20	0.425
4650	−8.45	0.347	−4.81	+0.248	−14.70	0.508

Dr. Proctor, Mr. Murch and Mr. Ames made readings for a similar set of wave-lengths, the results of which are given in Table VI and Fig. 11. It will be seen that they did not find any flattening of the curve. It is suggested that the flattening found by Nutting was due to his using a chromatic, *i.e.,* white light accommodation object. With an object illuminated by white light the eye normally accommodates so that the yellow light which is of the greatest intensity focuses on the retina. A slight change of accommodation which would cause a slightly more reddish or more greenish light to focus on the retina would not perceptibly change the visibility of the object. There would be a tendency to make such a shift where the eye was trying to focus a monochromatic source of a wave-length near the D line. This would explain the flattening in this region which Nutting found. The fact that neither Dr. Proctor or Mr. Ames or Mr. Murch found such a flattening when they used a monochromatic accommodation object is confirmation of this.

The determination of the axial chromatism for different zones

FIG. 11.



Chromatic aberration curves for eyes of four individuals; ordinates are relative focal distances, $\delta v/v$.

was found when the spherical aberration of the eyes for different colors was measured. It is shown in Tables I, II, III, IV, and V and in Figs. 6, 7, 8, 9, 10, and 10 (a), (b), and (c). In making the observations shown in Table I and Fig. 6, Dr. Chapman used an accommodation object illuminated by white light. But in making the readings given in Table II and Fig. 6, his axial ray was of one wave-length and his zonal ray of another. With a blue zonal ray he used a yellow axial ray; with a yellow zonal ray, a red axial ray; with a red zonal ray, a green axial ray.

Figs. 10 and 10 (a), (b), and (c) give a very clear idea of the chromatic aberration in the eye. They also give a very definite measure of its size. In Fig. 10 (c), top figure, the dotted lines represent the outside rays of the blue bundle when the eye is focused on red. The question of the actual size of spherical and chromatic aberration image bundles is of considerable interest. It is our intention to treat the matter in a later paper.

As was said in respect to spherical aberration the work that has been done in axial chromatism is hardly more than preliminary. Readings should be taken on a large number of eyes with different accommodation and with dilated and normal pupil. And measurements should be made with a large number of different wave-lengths.

Two special problems of interest are: 1st, a more positive determination of whether or not a flattening of the chromatic curve, such as Nutting found, exists; 2nd, a more definite determination of the chromatic aberration for the different zones.

OBLIQUE ASTIGMATISM.

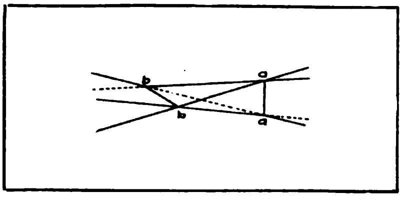
Introductory.

Oblique astigmatism causes the rays of light from a point source not on the axis to focus into a ray bundle of complex form. The ray bundle from a point source on the axis focuses in the form of a cone to a point where the rays cross to spread out into another cone. The ray bundle from a point source not on the axis forms a more complicated figure. In its pure form in a simple lens it focuses first to a line, see aa, Fig. 12, which lies in a position tangential to a circle about the axis of the lens. It then crosses and narrows in its long dimension and lengthens in its short one until it becomes a line again, see bb, Fig. 12, which is perpendicular to the first line.

The image of every point source not on the axis has this peculiar form. The farther the source from the axis the greater the separation between the two parts of the image which have the form of lines. If a sensitive plate or ground glass screen is placed behind the lens the form of the image that is apparent depends upon the position of the screen. If it is placed far back, *i.e.*, to the left of bb, Fig. 12, the image will be in the form of a radial oval; if at bb, in the form of a radial line; if half way between aa and bb, the form of a circle; if at aa, the form of a tangential line; if still nearer the lens, in the form of a tangential oval. If the screen is held stationary relative to the lens, a similar change in form of image can be noted by moving the point source, while keeping it at the same angular obliquity, from a distance to a position near the lens.

If instead of a point a line source is used, a similar imaging takes place. Every point on the line source is stretched tangentially or radially, depending upon the position of the line source. It can be seen that if the stretching of the various parts of the line source are in the same direction as that of the line source itself, its image will appear perfectly sharp and slightly elongated. That is, if a line source is put in a position tangential to the axis, its image will be sharp when the source is so positioned that the part of the image that forms a tangential line, *i.e.*, aa, Fig. 12, falls on the screen. If it is put in a position radial to the axis, its image will be sharp when that part of its image that forms a radial line falls on the screen.

Fig. 12.



Diagrammatic form of astigmatic image bundle.

In all simple lenses this characteristic image formation is more or less confused by coma, a one-sided blur.

Bibliographical.

Oblique astigmatism has been recognized to exist in the human eye for a long time. As far as can be found, however, no comprehensive collection of the literature upon the subject has been made. A list and brief summary of all important articles that could be found will be given.

(1) Young, in 1801, remarks on the small extent of the angle of perfect vision. Imperfections which begin a degree or two from the axis are due, he claims, partly to aberration of oblique rays but primarily to insensibility of the retina. He calculates the position of the primary and secondary astigmatic image fields of a circle ten inches from the eye. He uses a homo-

geneous lens and considers rays passing through its anterior vertex. He gives curves showing first and second astigmatic image fields formed by cornea alone, by cornea and anterior surface of lens, and by cornea and whole lens. In each case he shows a curve of the circles of least confusion and says this curve would fall on the retina if there existed a diminution of density in lateral parts of the lens.

Astigmatism, Oblique Rays, Eye.

Investigators	Date	Nature of work	Reference
(1) Young	1801	Theo.	Phil. Trans. 1801, Vol. XCIII, p. 23.
(2) Stammerhaus	1874	Exp.	Arch. für Ophth. Vol. XX, p. 146.
(3) Peschel	1878	Exp.	Pflüg Arch. Vol. XVIII, p. 504.
(4) Hermann	1878–9	Theo.	Pflüg Arch. Vol. XVIII, p. 443.
(5) Fick	1879	Theo.	Pflüg Arch. Vol. XIX, p. 145.
(6) Matthiessen	1879	Theo.	Pflüg Arch. Vol. XIX, p. 481.
(7) Rasmus & Wauer	1879	Theo.	Pflüg Arch. Vol. XX, p. 264.
(8) Matthiessen	1879	Theo.	Arch. für Ophth. Vol. XXV, p. 257.
(9) Peschel	1879	Exp.	Pflüg Arch. Vol. XX, p. 338.
(10) Hermann	1879	Theo.	Pflüg Arch. Vol. XX, p. 370.
(11) Schoen	1879	Theo.& Exp.	Du Bois-Raymond, Arch. für Physiol., p. 146.
(12) Hermann	1882	Theo.	Pflüg Arch. Vol. XXVII, p. 291.
(13) Parent	1882	Exp.	Rec. D'Ophthal., 1882, p. 216.
(14) Schoen	1884	Theo.	Beitrage zur Dioptick des Auges, Leipzig, Engleman, 1884.
(15) Matthiessen	1884	Exp.	Arch. für Ophthal. Vol. XXX, p. 91.
(16) Schmidt Rimpler	1884	Exp.	Congres International des Sciences Medicals, Compte Rendue, III, p. 73.
(17) Albini	1886	Exp.	Giornale de R. Acad. di Torino, 49, p. 657.
(18) Staub	1887	Exp.	Graefe Arch. für Ophthal, 33, p. 84.
(19) Wertheim	1894	Exp.	Zeit f. Psy. & Phy. der Sinn, 7, p. 172.
(20) Heinrich	1895	Exp.	Zeit f. Psy. & Phy. der Sinn, 9, p. 353.
(21) Heinrich	1896	Exp.	Zeit f. Psy. & Phy. der Sinn, 11, p. 410.
(22) Drualt	1900	Exp.	Arch. d'Ophth, 20, p. 21.
(23) Loria	1906	Exp.	Zeit f. Psy. & Phy. der Sinn, 40, p. 160.
(24) Heinrich	1909	Exp.	Brit. Journ. Psy., Vol. 3, p. 66.

(2) Stammerhaus, in 1874, was the first to attempt to determine experimentally the position of the astigmatic fields. He used an ophthalmoscope and observed radial and tangential blood-vessels in the retina and determined the aberration by the strength of lens necessary to give a clear image.

In normal eyes he found radial blood-vessels (secondary astigmatic focus) were clearly visible out to about 43°. From there on the retina becomes hypermetropic until at a distance of about 54° it becomes so to the extent of 3.71 diopters. In most eyes the change took place gradually but in some suddenly. At about 54° there is always hypermetropia of not less than 2.46 diopters.

He found that with tangential blood-vessels (primary astigmatic focus) he could see them clearly with the same lens out to the equator. He also gives measurements of abnormal cases. With a case of central hypermetropia of 3.5 D he found the hypermetropia at periphery increased very little. With a case of central myopia of 6 D at 50° he found the radial vessels 1.5 D myopic, which would show that as the axial diameter increased the other diameters increased also. With another case of central myopia of 5 D at 50° radial vessels were 3.5 D hypermetropic, which would show that the bulbus was stretched in its axial diameter.

(3) and (9) Peschel, in 1878, was the first to try to use radial and tangential lines outside of the eye to determine the position of the astigmatic fields. Unfortunately his observations were limited to his own left eye which was myopic 6.6 diopters. He used black lines on white paper, and placing them first in a tangential and then radial position, moved them back and forth until the lines appeared clear. He measured from 15° out and found that the secondary astigmatic field fell on the retina, the primary field falling in front of it. His results are inconsistent with those of Stammerhaus. This could be explained if his myopia was of the kind measured by Stammerhaus and mentioned above as the second abnormal case. In 1879 Peschel (9) contributed measurements on the extirpated eyes of animals and man. He used the method employed in testing a lens on a lens bench. At 70° he found the distance between the first and second astigmatic image to be 1.5 mm.

(4), (10), and (12) Hermann, in 1878 and 1879 and 1882, published long papers in which he discusses at great length the refraction of rays of oblique incidence with special reference to the eye, taking consideration of its laminated structure. These articles are so technical that no attempt will be made to summarize them.

(5) Fick, in 1879, using Hermann's formulas, figured out the position of a primary and secondary astigmatic image and found the retina near the secondary focus. The contribution is purely theoretical and is questioned by Matthiessen in his articles in the *Arch. für Ophth.*, Vol. 25.

(6), (8), and (15) Matthiessen, in 1879, in an article entitled "The Differential Equations of the Dioptrics of the Human Eye," gives an elaborate theoretical dissertation on the subject. In this same year he published another article entitled "Uber der Geometrische Gestalt der theoretical Retina des periskopischen schematischen Auges," (8) in which he says the work of Rasmus and Wauer and his own (see (6)) show that the retina lies between the primary and secondary astigmatic image fields and criticizes Fick (5). In 1884 (15) he investigates the position in which black letters on a white screen must be placed to focus on the peripheral retina but does not get any very conclusive results.

(7) Rasmus and Wauer, in 1879, were the first to calculate the position of the retina relative to the primary and secondary astigmatic image fields for more than one angle. They made calculations for every 10° from 0° to 90° for both the accommodated and unaccommodated eyes of Helmholtz. They find the retina lies approximately half way between the two fields. They mention Stammerhaus, Schoen, Peschel, Fick and Matthiessen. Their charts for both the accommodated and unaccommodated eye with the retina as calculated by Matthiessen are shown in Fig. 15.

(11) and (14) Schoen, in 1879, investigated the position of the retina relative to the primary and secondary astigmatic image fields at 60°, by observing in an eye dilated with atrophine the image of radial and tangential lines which were thrown on the retina. The apparatus is described in another article by him in *Arch. für Ophth.*, xxiv, 4, p. 91, 1877. He found that in ten out of sixteen cases the retina lay between the primary and secondary image fields. The average distance between the fields was 2.85 D. He does not mention Stammerhaus, though their results agree. He shows that Peschel got 2.1 D between the two image fields. He also gives some calculated results which agree closely with those of Matthiessen. In his book (14) Schoen deals at some length with the theory of formation of images by oblique rays.

(13) Parent, in 1882, attempted to determine the position of the primary and secondary astigmatic fields by means of the ophthalmoscope but found it was difficult to hold the ophthalmoscope and the observed person sufficiently still and to find radial and tangential blood-vessels to observe. He, therefore, tried the

skiascopic method, using the keratoscope of M. Cuignet. The refraction for the primary and secondary fields for an emmetropic eye are given in a table and figure in the summary.

(16) and (18) Schmidt Rimpler, in 1884, and M. Staub, in 1887, investigated the difference in refraction between the fovea and papilla (blind spot). Schmidt Rimpler found that with marked ametropia it varied greatly, *i.e.*, from 1 D to 5 D, where the subject had myopia of 5 D. Staub found the average difference about 1.5 D. The ophthalmatic method was used.

(17) Albini, in 1886, believed that the poor vision in the peripheral parts of the retina was due to the aberration of oblique rays. His method of attempting to correct this was by interposing lenses of different strength in the path of the oblique ray until the paraxial object under observation became most evident. He used black spots on white background and colored objects. He found that by interposing the proper lenses his objects could be seen much further out on the periphery, and that all the colors could be made equally perceptible almost out to the full field of vision. Nothing in his work shows that he was cognizant of the nature of the image formed by oblique rays. An analysis of his results, however, shows that he was probably causing the secondary astigmatic image field to focus on the retina. From the strength of the lenses he used the position of this field was calculated and is shown in Fig. 13.

Although mentioned by Baird in "The Color Sensitivity of the Peripheral Retina," Carnegie Inst., May, 1905, little consideration has been given Albini's work by researchers in the color sensitivity of the peripheral retina. The possible importance of it will be discussed in our conclusion.

(19) Wertheim, in 1894, attempts to test the visual acuity of different parts of the retina with a wire grating which could be placed in a radial and tangential position. He could find no evidence of astigmatism on the peripheral retina. He mentions Hermann, Matthiessen and Albini.

(20), (21) and (24) Heinrich, in 1895, 1896 and 1909, makes three most important contributions. In 1895 he investigated the changing of curvature of the lens and variation of pupillary opening under different conditions of attention. He used an ophthalmometer to notice changes of curvature in the crystalline lens and then fixed attention on the peripheral object.

He found that the size of the pupil increased as attention object was moved outward up to 40° or 60°; it then decreased. The curvature of the lens also changed, becoming flatter, *i.e.*, hypermetropic. He mentions oblique astigmatism and raises question as to which image falls on the retina. He also treats in this article of the effect on pupil and lens of giving the attention to cumputing.

In 1896 he investigated the amount of peripheral accommodation that is possible. He found that accommodation for a paraxial object depended on its distance away and on the distance of the central fixation object. He finds it not exact, only approximate. He considers the primary and secondary astigmatic image fields for oblique rays. He speaks of the great importance from a physiological standpoint of determining where the retina lies relative to primary and secondary astigmatic fields; size of focal lines; size of diffusion circle or ellipse. He mentions Hermann, Rasmus, Wauer, Peschel, Schoen, Parent and Butz and discusses their findings. He concludes that

1. Eye has power to accommodate on paraxial objects.
2. Such accommodation depends upon location of axial fixation point.
3. Such accommodation causes coincidence of primary astigmatic image plane with retina.
4. Depth of such paraxial accommodation is smaller than axial accommodation and decreases up to 40° or 60° and then increases.
5. Length of astigmatic focal lines increases with angle of incidence.
6. Length of astigmatic focal lines increases with increase of radius of curvature.
7. The angle at which the refracted rays converge decreases with the increasing radius of curvature and increases with increasing angle of incidence. The first fact favors paraxial accommodation.

His contribution in 1909 (24) will be taken up after Loria's work (23) has been considered.

(22) Drault, in 1900, determined the position of the primary and secondary astigmatic field by skiascopy, the method used by Parent (13) and whose work he mentions. He made readings for every 10° on both nasal and temporal side of lens out to 50°. He gives the average result of twenty-two observations. He says the correction for hypermetropic eyes was from +2.5 to +3.4 D in the vertical meridian. The readings of the abnormal eyes are included in his average, and he gives no average for normal eyes. His results are shown in a table and figure in the summary.

He gives size and shape of the images he found on different parts of the retina, stating it is round or oblong according as to whether it is between or nearer one or the other of the astigmatic fields.

He concludes that

1. The astigmatism of rays in the eye "is a little less than would be calculated."
2. That it is a little less for the temporal part of the visual field than the nasal, due to the inclination of the visual and optical axis.
3. That the retina is situated between the two fields.

(23) Loria, in 1906, gives a most valuable contribution. He reviews Heinrich's work and determines experimentally, 1st, the extent of the field in which tangential lines can be focused on the retina without keeping the axial accommodation fixed; 2nd, the shape of the field in which tangential lines appear most sharp when the axial accommodation is kept fixed. He did his work most thoroughly, taking a great many observations and using the atrophined and normal eye. He used for a stimulus parallel lines 2 mm. wide, 2 mm. apart and 2 cm. long. The shape of the fields which he found with a fixed accommodation are the position in space of the primary astigmatic object field with the given accommodation. The position of the corresponding image fields was calculated and is shown in Fig. 14. Loria was the first to expound the theory that with a given accommodation there is a field in space upon which all points are seen distinctly.

He concludes

1. The accommodation of the eye for paraxial vision is determined by the distance of the stimuli and is independent of the distance of the axial fixation point.
2. The eye is paraxially myopic, increasing with the paraxial angle.
3. The paraxial accommodation zone decreases with increasing angle.
4. It is possible to determine with a fixed curvature of lens a surface in space where the conjugate focus of the retina lies. The surfaces corresponding with the least and greatest curvature of the lens are the limits of the paraxial accommodation space.
5. Objects placed on this surface will be seen most distinctly.

Though working with the primary astigmatic image and object fields, he makes no mention of that fact which he probably did not recognize, nor does he make any mention of the secondary astigmatic image and object fields.

(24) Heinrich, in 1909, investigated a fact noticed in Loria's

experiments, that, when objects lay in the primary astigmatic object field, they all appeared to the observer to be the same distance away. He used fine threads .3 mm. thick in a vertical position and found the positions in space out to an angular distance of 5° where the paraxial thread appeared to be at the same distance as the axial thread upon which the eye was fixed. He obtained very definite readings which he shows in graphs of the object fields. The image field corresponding to the object field shown in Fig. 1, page 70, is shown in Fig. 15. When Mr. Kurtz, the observer, who was myopic 2.5 D, fixated an axial point 60 and 80 cm. distant, a periodical displacement of the equidistant areas took place. Mr. Kurtz described these oscillations in an article following on page 75 of the *British Journal of Psychology*, Vol. 3. Heinrich suggests no explanation. He states that a series of similar experiments were begun on other individuals which are not described. He also states that if his curves are prolonged, they will fit into the curves found by Loria. He concludes that monocular space perception is tri-dimensional. He says on page 74,

> "Points in space, which in a given state of accommodation of the lens in monocular vision form well-defined retinal images, represent a surface which serves as a fundamental surface in relation to which the position of other points in the third dimension is determined. The points outside such a surface are regarded as more distant, those within it as lying nearer.
>
> "The images of the points outside the surface are diffusion circles. With these diffusion circles such factors as the degree of diffusion, light-intensity, etc., will become of importance; and these factors contribute to the appearance of depth. In other language, the image formed on the retina by means of the human dioptric apparatus must include all those factors which correspond to the report of depth. An analogy may make this clearer. It is well known that, in a picture representing objects in spatial order, every object must have its local tone. It is this local tone, and not the geometrical perspective, which determines the position of the object in space. Any object not in the projected line does not fit into the space of the picture. Something analogous may be supposed to occur in the case of the image formed on the retina. Yet another example. The images of objects formed by means of a badly constructed system of lenses appear on a screen as ill-defined. Yet the whole image is characterized by an exaggerated effect of depth."

It seems without question that he was determining the position of the primary astigmatic field, although he does not mention the fact. Nor does he make any suggestion of trying to determine the position of the secondary astigmatic field in a similar manner.

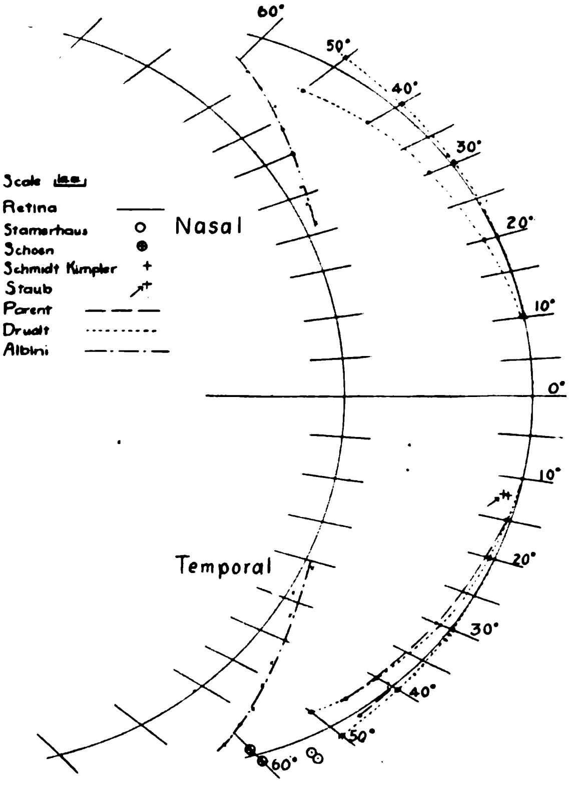
SUMMARY.

The results of the experimental work that has been done are on the whole very consistent.

Three different methods were used:

1st. Ophthalmoscopic—Stammerhaus, Schoen, Schmidt Rimpler, Staub.

FIG. 13.



Position of the primary and secondary astigmatic image fields relative to retina as computed from data by authors indicated.

2nd. Skiascopic—Parent, Drualt.

3rd. Subjective determination of position of sharp focus of paraxial stimuli—Peschel, Albini, Wertheim, Heinrich, Loria.

In consideration of the results due allowance has to be made for the distorted shape of the bulbus which is apt to be found in ametropic cases.

The result of the ophthalmoscopic measurements are put in the following table for comparison:

Angle	Stammerhaus		Schoen		Schmidt Rimpler		Staub	
	Prim. field	Second. field	Prim. field	Second. field	Prim. field	Second. field	Prim. field	Second. field
12.5°	0	0	......	....	−1.0 to −1.5D	0	−1.5D	0
43°	0	0	......	....	....	......	......	......
54°	0	+2.46 D to +3.71 D	......	....	....	......	......	......
60°	0	....	Difference between fields = 2.85 D		....	......	......	......

Calculated Corresponding Primary and Secondary Image Distances from Retina in mm

Angle	Stammerhaus Prim. field	Stammerhaus Second. field	Schoen Prim. field	Schoen Second. field	Schmidt Rimpler Prim. field	Schmidt Rimpler Second. field	Staub Prim. field	Staub Second. field
12.5°	0	0	....		.3 mm to .44 mm	....	.44 mm	......
43°	0	0	....		....	......	......	......
54°	0	.54 to .85 mm	....		....	......	......	......
60°	0	....	Difference between fields = .51 mm		....	......	......	......

These results which agree in general are also shown in Fig. 13 for comparison with results obtained by other methods.

The comparative results of the skiascopic method are as follows:

Angle	Parent		Drualt			
			Nasal		Temporal	
	Prim. field	Second. field	Prim. field	Second. field	Prim. field	Second. field
10°	......	......	−0.3 D	+0.1 D	−0.1 D	0.0 D
15°	−0.50 D	0.0 D	......	......	......	......
20°	−0.75	0.0	−1.4	+0.1	−0.4	+0.1
30°	−1.75	0.0	−2.9	+0.3	−1.2	+0.2
40°	......	......	−4.6	+1.0	−2.3	+0.8
45°	−2.75	+0.75	......	......	......	......
50°	......	......	−6.7	+2.1	−3.9	+2.0

Calculated Corresponding Primary and Secondary Image Distances from Retina in mm

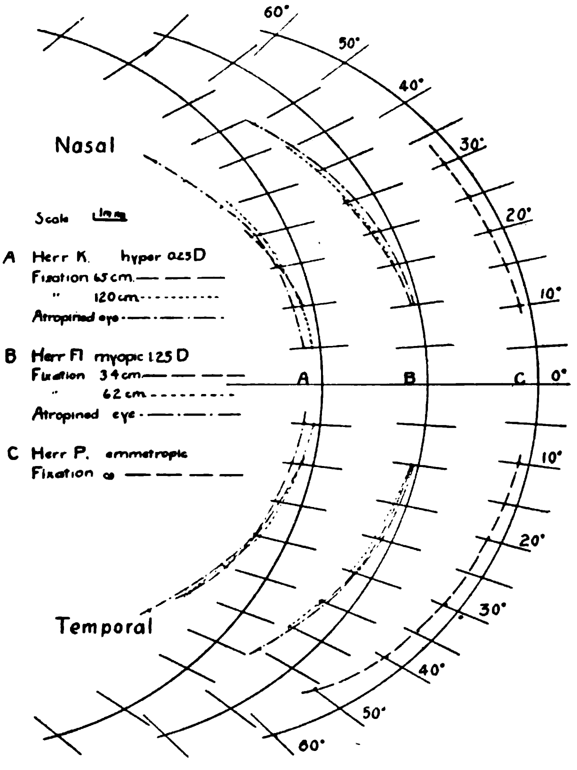
Angle	Parent Prim. field	Parent Second. field	Drualt Nasal Prim. field	Drualt Nasal Second. field	Drualt Temporal Prim. field	Drualt Temporal Second. field
10°	......	......	−0.09	+0.03	−0.03	0.0
15°	−0.15	0.0				
20°	−0.22	0.0	−0.40	+0.03	−0.12	+0.03
30°	−0.46	0.0	−0.73	+0.07	−0.32	+0.045
40°	......	......	−1.0	+0.25	−0.53	+0.20
45°	−0.59	+0.17				
50°	......	......	−1.23	+0.49	−0.77	+0.46

These results agree very well except for the difference of the findings on the nasal side. This is probably due to the inclination of the visual to the optical axis as Drualt concludes.

These results are shown in graphic form in Fig. 13.

In considering the results of the subjective method, Peschel's eye was so myopic (6.6 D) that the position of his fields relative to his retina is of little value in comparison with the measurements on emetropic eyes. Wertheim got no results. In view of the results found by others and their substantial agreement, it must be concluded that he was in error.

FIG. 14.



Positions of the primary and secondary astigmatic image fields relative to retina as computed from Loria's data.

The following tables give the results of Albini, Loria and Heinrich:

Albini's Results, Averaged.

Angle	Correcting lens		Calculated distance of secondary image from retina	
	Temporal	Nasal	Temporal	Nasal
18°	....	+0.25 D	....	+0.11 mm
21°	+0.37 D	+0.62	+0.31 mm	0.44
24°	+0.50	+0.75	0.15	0.45
27°	+0.87	+1.0	0.47	0.58
30°	+1.3	+1.4	0.85	0.93
33°	+1.6	+1.8	1.00	1.20
36°	+2.1	+2.4	1.37	1.50
39°	+2.7	+2.6	1.66	1.59
42°	+3.0	+3.0	1.75	1.77
45°	+3.5	....	2.06	....

Heinrich's Results.

Angle	Distance of vertical thread from eye		Calculated distance of primary image from retina	
	Nasal	Temporal	Nasal	Temporal
0°	350 mm	350 mm	0	0
1°	348	349	−0.02	−0.02
2°	341	343	−0.05	−0.05
3°	331	332	−0.09	−0.09
4°	315	320	−0.16	−0.14
5°	294	202	−0.24	−0.21

Loria's Results.

Object Distances in mm.

Angle	Herr K hypermetropic 0.25 D						Herr Fl. Myopic 1.25 D						Herr P emetropic	
	Fixation 65 cm.		Fixation 120 cm.		Eye treated with atropine		Fixation 34 cm.		Fixation 62 cm.		Eye treated with atropine		Fixation infinity	
	Nasal	Temp.	Nasal	Temp.	Nasal	Temp.	Nasal	Temp.	Nasal	Temp.	Nasal	Temp.	Nasal	Temp.
5°	...	...	625	650	675	675	...	...	....	....	...	...	...	...
10°	365	375	480	480	500	500	300	300	480	475	700	700	1000	1000
15°	250	...	325	...	...	...	250	...	385	...	550	...	...	...
20°	175	190	250	250	275	275	200	250	325	335	450	550	650	650
25°	...	152.5	175	160	175	175	175	225	237.5	287.5	350	400	600	...
30°	...	110	...	120	125	125	...	180	200	235	250	300	500	600
35°	...	...	...	...	100	100	...	...	170	182.5	200	225	...	...
40°	...	...	...	...	...	...	...	...	....	....	150	175	...	400
50°	...	...	...	...	...	...	...	...	....	....	...	...	...	200

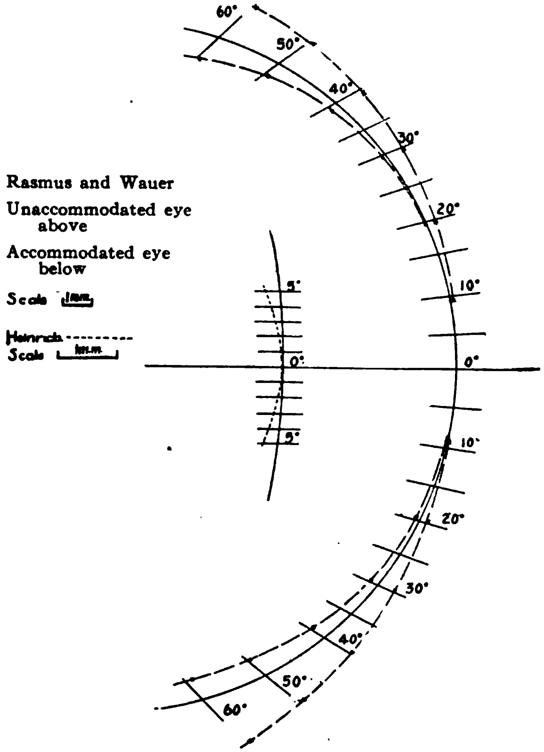
Calculated image distances in mm.

5°	...	...	0.23	0.21	0.50	0.50	...	...	....	....	...	...	...	...
10°	0.35	0.33	0.37	0.37	0.67	0.67	0.20	0.20	0.22	0.23	0.13	0.13	0.31	0.31
15°	0.71	...	0.65	...	...	...	0.42	...	0.40	....	0.28	...	...	...
20°	1.14	1.02	0.88	0.88	1.07	1.07	0.71	0.44	0.55	0.53	0.41	0.30	0.45	0.45
25°	...	1.30	1.27	1.40	1.53	1.53	0.91	0.57	0.86	0.67	0.60	0.50	0.47	...
30°	...	1.83	...	1.82	1.99	1.99	...	0.86	1.05	0.87	0.90	0.73	0.53	0.47
35°	...	...	...	...	2.30	2.30	...	...	1.24	1.15	1.12	0.99	...	...
40°	...	...	...	...	...	...	...	...	....	....	1.46	1.21	...	0.60
50°	...	...	...	...	...	...	...	...	....	....	...	...	...	1.02

The results are all shown graphically in Figs. 13, 14, and 15.

The theoretical work is too long and technical to attempt to summarize. The investigators all agree, however, that the retina lies between the primary and secondary astigmatic fields, approximately half way.

FIG. 15.



Positions of the astigmatic image fields as computed from data given by Rasmus, Wauer and Heinrich.

Fig. 15 shows Rasmus' and Wauer's calculated position of the primary and secondary fields relative to the retina for the accommodated and unaccommodated eye. Their findings are in agreement with those of Mattheissen.

CONCLUSION.

By comparison it will be seen that the results obtained by the different experimental methods in general agree. The separation of the primary and secondary astigmatic image fields as found experimentally is somewhat less than that found by calculation.

The conclusion from all work to date is:

First, that the image of oblique rays formed by the lens system of the eye is astigmatic;

Second, that the retina lies between the primary and secondary astigmatic image fields formed by a surface in space, all parts of which are equidistant from the eye;

Third, that those parts of the astigmatic image bundle which are in the form of tangential and radial lines are sufficiently sharp and the peripheral retina is sufficiently sensitive to make it possible to determine when the primary or secondary astigmatic focus falls upon the retina;

Fourth, that monocular space perception is tridimensional.

PURPOSE OF RESEARCH.

The facts which had been determined relative to oblique astigmatism were not sufficient for the purposes for which this work was undertaken for the following reasons:

1st. The position of the secondary astigmatic image field had not been sufficiently determined. Peschel's results were vitiated by his abnormal myopia. The ophthalmoscopic and skiascopic results were not sufficiently definite and no measurements had been made close to the axis. Albini's results are questionable in that he used a number of dots for a stimulus instead of a radial line, nor did he make measurements close to the axis.

2nd. The position of the primary astigmatic field had not been sufficiently determined near the axis. Loria's readings only go in to 10°, and Heinrich's readings from 0° to 5° are by only one observer who had 2.5 D of myopia.

3rd. No determination has been made of the position of the astigmatic fields for light of different wave-length. The paraxial stimuli used have all been illuminated by white light in both the objective and subjective method. This excepts Albini's attempts, which were made valueless, as he used disks of colored paper which were round in shape.

METHOD OF MAKING MEASUREMENTS.

In designing the apparatus it was thought that it might be possible to determine the inclination of rays which constitute the primary and secondary astigmatic image focus by means of three monochromatic rays of small cross section whose inclination could be controlled as described on page 30.

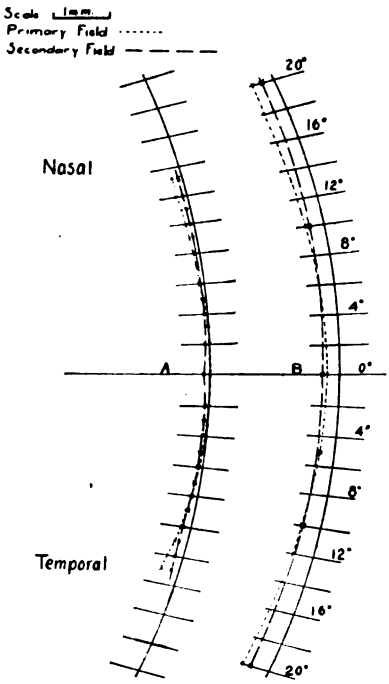
One of the rays would strike the eye at any desired obliquity and pass through its axis. A second ray whose inclination and separation from ray (1) could be controlled would strike the eye in say a horizontal plane to the left or right of ray (1). A third ray whose inclination could be controlled independently from rays (1) and (2) would strike the eye in a vertical plane above or below ray (1). The position of the primary astigmatic focus for a given obliquity and zone could be determined by the inclination of ray (2) when the image of rays (1) and (2) were superimposed. In the same way the position of the secondary astigmatic focus could be determined. The apparatus was adjusted for such measurements and a few attempts to use it made. The method was discarded because of the apparent difficulty of setting and because of the great length of time it would take to get the desired measurements by such a method. It has, however, not been sufficiently tested to prove that it cannot be used successfully. It was decided to use in its place the method by which Loria and Heinrich got such definite results. That is, by line sources the rays from which could be caused to fall on the eye at any desired obliquity; the reading where the line focuses most sharply when in a tangential position giving the primary astigmatic field; that where it focuses when in a radial position giving the secondary astigmatic field.

The apparently simplest method of getting a tangential and radial source was to substitute a fine line, which could be placed in a vertical and horizontal position, for the hole in Mirror (E), Fig. 1. This, however, could not be used because due to the 45° inclination of Mirror (E) the two ends of the line source when in a horizontal position would be in different focus.

A tangential and radial line source was first obtained as follows: With a point source in (E), Fig. 1, very narrow horizontal and vertical slits were placed at (H), Fig. 1. The effect of a very narrow horizontal slit was to spread the point source into a vertical line by diffraction effect. The apparent position of this line in space is controlled by moving Mirror (F), Fig. 1. Similarly a very narrow vertical slit gives a horizontal line.

The method of making observations was first to properly position and accommodate the eye. With the rotating system set to give zero obliquity, readings were made. The system was then rotated to give any desired obliquity and locked, and another set of readings taken. In making the zero readings, allowance has to be made for the lateral displacement of the beam to the right by the plane parallel (Q), Fig. 1. To do this the displacement of the beam was first calculated. Another small white speck is placed on the front of (Q), the calculated distance to the left of the speck already there. In making zero obliquity readings the fixation line is centred in the diffusion disk from the right-hand speck. In making all other readings it is centred on the diffusion disk from the left-hand speck.

FIG. 16.



(*A*) shows positions of primary and secondary astigmatic image fields relative to retina from measurements of Mr. Ames' eye using a red (λ6560) point source and the diffraction effect of very narrow horizontal and vertical slits to obtain vertical and horizontal line stimuli.

(*B*) shows positions of primary and secondary astigmatic image fields relative to retina from measurements of Mr. Ames' eye using a green (λ5461) source and the improved apparatus.

A set of readings, see Table VII, was taken in September, 1919, by this method. The hole in (E) was illuminated by red (λ 6563) light. The readings were obtained from the settings of (F), Fig. 1, where the horizontal and vertical lines appeared to be most sharp. Both the nasal and temporal side of the lens were measured at the angular obliquity shown.

TABLE VII.

Oblique Astigmatism. (*Mr. Ames, Red Stimulus.*)

Observer Mr. Ames, right eye, accommodation relaxed. Color of paraxial stimuli red (λ6563). Focal length of mirror = 71.9 mm.

Angular obliquity of source	Primary astigmatic field			Secondary astigmatic field		
	Average readings (distance from focus of mirror), mm	Calculated object distance, mm	Calculated image distance from retina, mm	Average readings (distance from focus of mirror), mm	Calculated object distance, mm	Calculated image distance from retina, mm
Temporal						
0°	+0.32	−1600	−0.018	−1.99	+2770	0.104
2°	−0.18	∞	0.000	−1.06	+5180	0.057
3°	−0.55	+9600	+0.030	−1.27	+4350	0.066
4°	−0.58	9100	0.032	−1.57	+3530	0.082
5°	−0.66	8060	0.036	−1.83	+3000	0.096
6°	−1.06	5200	0.055	−1.44	+3840	0.075
7°	−1.62	3400	0.082	−1.57	+3530	0.082
8°	−2.38	2350	0.123	−1.49	+3700	0.078
9°	−2.99	1940	0.150	−1.75	+3140	0.092
10°	−3.96	1510	0.176	−2.55	+2210	0.129
11°	−4.53	1345	0.213	−2.37	+2370	0.122
12°	−6.26	1035	0.277	−2.82	+2030	0.142
13°	−7.00	946	0.302	−3.08	+1890	0.152
Nasal						
0°	+0.24	−22500	−0.014	−1.99	+2770	0.104
2°	+0.58	− 7400	−0.039	−1.82	+3030	0.095
3°	−0.40	+13300	+0.022	−1.33	+4170	0.069
4°	−0.77	7000	0.041	−1.36	+4070	0.071
5°	−1.08	5080	0.059	−1.40	+3950	0.073
6°	−2.02	2650	0.110	−1.43	+3870	0.075
7°	−1.51	3650	0.081	−1.49	+3700	0.078
8°	−2.21	2500	0.115	−1.53	+3600	0.080
9°	−2.34	2380	0.121	−0.99	+5540	0.052
10°	−2.56	2200	0.131	−1.00	+5480	0.053
11°	−3.34	1760	0.163	−1.07	+5140	0.056
12°	−4.49	1380	0.211	−1.33	+4170	0.069
13°	−4.05	1480	0.194	−1.16	+4750	0.061

Fig. 16 (A) shows the position of the primary and secondary image fields as determined relative to the retina.

Analysis of these findings will be deferred until later. Great care must be taken to hold the accommodation constant and practice is required in making the observations. Loria also found this necessary, see (23).

It was thought desirable to make similar readings, using actual line sources. Accordingly, a lens of 125.6 mm focal length was substituted in the place of mirror (F), Fig. 1, and a slit .03 to .05 mm wide and 4.0 mm long was placed behind it. The slit was arranged so that it could be placed in either a horizontal or vertical position and illuminated by monochromatic light. The method of making the measurements was the same as before. The measurements which were made with light of four different wave-lengths are given in Table VIII and Fig. 17.

It was thought that there might be a tendency of the eye to accommodate for these peripheral stimuli (see Heinrich (21)); that the crystalline lens might flatten to bring the first astigmatic focus on the retina and become more convex to see the second focus more clearly. It was known that this could not be of large amount as the monochromatic central fixation cross remained

TABLE VIII.

Oblique Astigmatism. (*Mr. Ames, Stimuli of Various Wave-lengths.*)
Observer Mr. Ames, right eye, accommodation relaxed. Focal length of lens = 125.6 mm.

(Primary Astigmatic Fields.)

Angular obliquity of source	Red (λ6563)			Yellow (λ5893)			Green (λ5461)			Blue (λ4861)		
	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm
Temporal												
0°	−0.045	286000	0.001	−5.70	3000	0.095	−8.93	2000	0.144	−18.40	1100	0.260
2°	1.39	11500	0.026	......	....	....	10.11	1800	0.161	21.45	980	0.291
3°	.....	.....	....	6.03	2840	0.101	....	....	....	....	....	....
4°	1.60	10100	0.029	6.90	2520	0.115	10.62	1730	0.167	23.15	928	0.308
6°	5.97	2880	0.101	8.45	2090	0.138	12.04	1550	0.185	25.10	877	0.325
8°	8.98	1990	0.144	11.76	1570	0.184	16.40	1200	0.239	28.55	805	0.353
10°	11.0	1660	0.173	14.75	1310	0.221	21.52	978	0.292	33.20	732	0.388
12°	15.4	1260	0.226	17.31	1150	0.249	27.62	824	0.346	41.50	669	0.426
14°												
16°		Readin	gs pre	vented	by b	lind s	pot.					
18°												
20°	21.8	967	0.294	22.75	939	0.304	40.60	656	0.432	46.90	644	0.430
Nasal												
0°	−0.045	286000	0.001	−5.70	3000	0.095	−8.93	2000	0.144	−18.40	1100	0.260
2°	0.92	17100	0.017	......	....	....	......	....	....	19.7	1040	0.274
3°	.....	......	....	6.35	2720	0.107	10.26	1780	0.162			
4°	2.93	5590	0.052	7.23	2420	0.119	10.61	1730	0.167	20.7	1010	0.285
6°	3.76	4410	0.066	9.03	1970	0.147	13.40	1420	0.202	23.6	914	0.302
8°	4.65	3610	0.081	9.48	1890	0.153	14.54	1320	0.216	25.0	880	0.324
10°	7.00	2490	0.115	11.01	1660	0.173	15.87	1230	0.233	25.3	872	0.327
12°	8.36	2110	0.135	12.11	1530	0.187	17.17	1160	0.247	29.3	792	0.359
14°	11.52	1590	0.182	15.02	1290	0.224	21.71	964	0.296	30.9	765	0.372
16°	13.13	1430	0.201	17.95	1120	0.255	25.65	864	0.330	37.9	678	0.428
18°	16.05	1220	0.235	17.19	1160	0.247	29.43	792	0.359	38.3	674	0.430
20°	18.10	1110	0.258	24.63	888	0.322	28.95	800	0.356	45.0	652	0.434

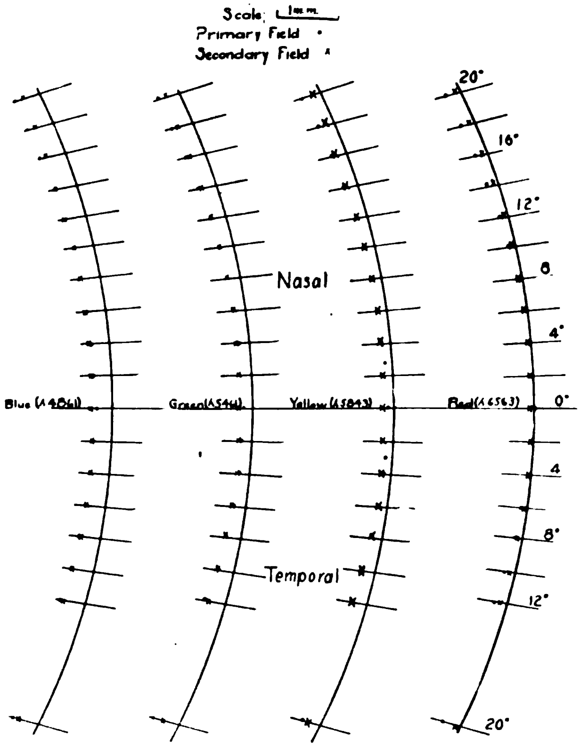
TABLE VIII—(*Continued*).
(Secondary Astigmatic Fields.)

Angular obliquity of source	Red (λ6563)			Yellow (λ5893)			Green (λ5461)			Blue (λ4961)		
	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm
Temporal												
0°	−4.11	4030	0.072	−10.93	1670	0.172	−15.12	1310	0.218	−27.10	834	0.342
2°	2.52	6450	0.045	9.64	1860	0.161	12.99	1440	0.199	23.95	906	0.315
4°	2.42	6710	0.043	9.86	1830	0.158	13.97	1360	0.210	23.80	909	0.314
6°	3.47	4760	0.061	9.78	1840	0.156	14.91	1290	0.221	23.52	917	0.311
8°	5.11	3300	0.087	11 54	1590	0.180	17.11	1160	0.247	25.25	875	0.326
10°	6.17	2790	0.104	14.17	1350	0.213	19.67	1040	0.274	31.92	750	0.379
12°	7.50	2340	0.123	17.01	1170	0.246	21.84	967	0.298	31.75	753	0.378
14° 16° 18°		Readin	gs pre	vented	by bli	nd sp	ot.					
20°	3.97	4180	0.069	13.61	1390	0.206	19.57	1050	0.272	27.05	835	0.341
Nasal												
0°	4.11	4030	0.072	−10.93	1670	0.172	−15.12	1310	0.218	−27.1	834	0.342
2°	2.94	5590	0.052	9.86	1830	0.158	13.38	1410	0.203	25.9	860	0.331
4°	2.84	5750	0.050	10.03	1800	0.160	14.20	1340	0.214	25.1	877	0.325
6°	3.64	4550	0.064	9.73	1850	0.156	14.01	1360	0.211	26.5	846	0.337
8°	3.30	4980	0.058	10.15	1780	0.162	13.92	1370	0.210	25.2	875	0.324
10°	3.38	4890	0.059	11.61	1580	0.181	15.10	1280	0.224	24.9	881	0.321
12°	4.27	3920	0.074	11.90	1550	0.185	15.33	1260	0.227	26.9	838	0.340
14°	4.42	3790	0.076	12.53	1490	0.193	17.39	1150	0.250	27.3	830	0.343
16°	5.37	3150	0.093	12.52	1490	0.193	17.94	1120	0.255	24.5	890	0.320
18°	5.74	2990	0.097	8.15	2160	0.133	16.67	1180	0.242	22.9	935	0.305
20°	4.28	3890	0.074	7.39	2370	0.122	14.06	1360	0.212	21.1	992	0.288

sharp. To avoid any such possibility, however, an apparatus was constructed which presented both a tangential and radial line source at the same time, the apparent distance of each source being controlled separately. This enabled the focus of both the first and second astigmatic fields to be determined at the same time, thereby giving the absolute separation between the two independent of accommodation.

A drawing of the optics of this apparatus is given in Fig. 18. Units A and B are made up of two 6 V lights (a′) and (b′), which illuminate plates (a) and (b), back of which are placed monochromatic filters. Plates (a) and (b) are silvered and on plate (a) a series of vertical lines is scratched, and on plate (b) a series of horizontal lines. The units A and B can be moved forward and back independently of one another. The light from A and B is made parallel by the two lenses L and L and after passing through the prism system C, with its half-silvered surface (c) follow the same path HE. The eye is placed at (E), about which centres the units AB and C turn. The obliquity at which the ray HE strikes the eye is given by the position of the pointer P on the scale SS. D is a plane parallel which reflects into the eye the monochromatic fixation cross from the system S, which is similar to the system described in the other apparatus. F is a very small glass bead illuminated by a light placed at K. It serves to centre the eye in the same manner as the speck on the plane parallel described in the other apparatus. DFK and system S remain stationary when the units A, B and C are turned about the centre E. To keep the eye fixed in a forward and back position, a mouth grip and forehead rest were used instead of the method described on page 29. The mouthpiece was an indentation of the observer's teeth made in sealing wax on a brass plate. This

FIG. 17.



Positions of the primary and secondary astigmatic image fields relative to the retina from measurements on Mr. Ames' eye using vertical and horizontal line stimuli of the wave-length given.

and the forehead rest were so positioned that the eye came in the proper place. This is a possible arrangement where the beam used is larger than the diameter of the pupil. It has the advantage of being somewhat simpler but is not as exact as the other method.

With this apparatus both vertical and horizontal lines of the same obliquity are made to fall upon the eye at the same time. As the apparent position of each can be controlled independently, it is possible to get a sharp image on the retina of both sets of lines at once.

FIG. 18.

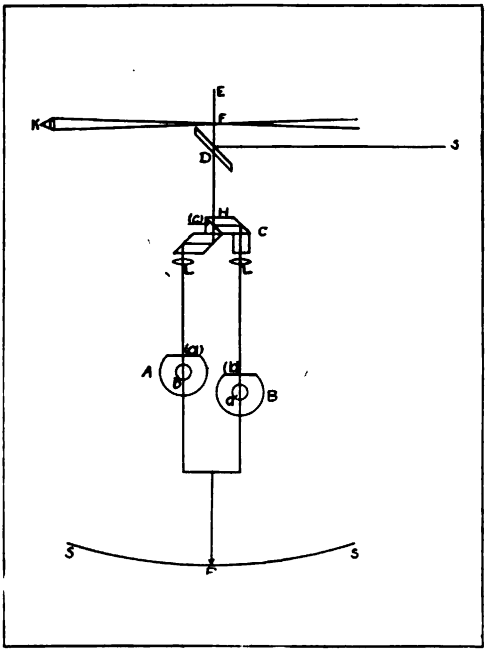


Diagram of improved apparatus for measuring oblique or field astigmatism.

Up to date only a set of test readings has been made with this apparatus, see Table IX and Fig. 16B. These readings were made to see how measurements with this method checked up with former measurements. Readings were taken in which the vertical and horizontal lines were used separately, that is, in the same method which was followed with the old apparatus, and also where the vertical and horizontal lines were both judged to be sharp at the same time. No difference was found.

It will be seen by comparing the curves in the figures that the results obtained by both methods are substantially the same.

Table IX.

Oblique Astigmatism. (Mr. Ames, with Improved Apparatus.)

Observer Mr. Ames, right eye, accommodation relaxed. Color of paraxial stimuli Green (λ5461).

Angular obliquity of source	Primary astigmatic field			Secondary astigmatic field		
	Average readings, mm	Object distance, mm	Image distance, mm	Average readings, mm	Object distance, mm	Image distance, mm
Temporal						
0°	−11.9	1326	0.216	−14.7	1070	0.269
5°	13.3	1190	0.229	15.4	1026	0.252
10°	20.2	791	0.360	17.7	890	0.320
20°	26.1	606	0.476	18.6	847	0.336
Nasal						
0°	11.9	1326	0.216	14.7	1070	0.269
5°						
10°	17.8	895	0.319	14.25	1102	0.259
20°	20.8	758	0.375	11.6	1348	0.213

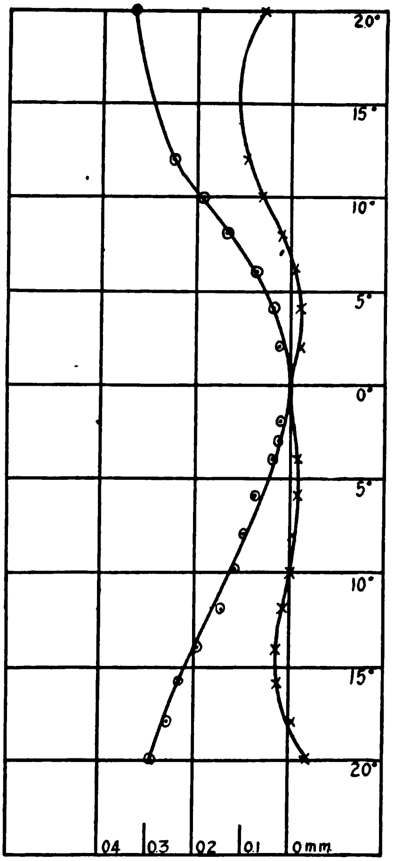
ANALYSIS OF RESULTS.

The similarity of the curves showing the position of the two astigmatic image fields relative to the retina found by the different methods, see Figs. 16 and 17, is very conclusive proof that they are substantially correct measurement of the conditions that exist in Mr. Ames' eye. This is further confirmed by the similarity of the curves for different wave-lengths, and the conformity of these curves with the difference in the spherical aberration measurements in the horizontal and vertical meridian which is explained in the next paragraph.

At first sight one is struck by two features of these curves:

First, that the two fields do not meet at the axis. On consideration, however, it will be seen that this was to be expected in view of the corneal astigmatism in Mr. Ames' eye, which was determined very accurately and is shown in the difference of the measurements for spherical aberration on the horizontal and vertical meridians. If allowance for this separation is made and the two fields for one wave-length moved relatively to each other until they coincide at the axis, they will take the position shown in Fig. 19. In this figure the scale of the image distance has been increased relative to that of the angular obliquity to show more clearly the characteristic form of the curves, especially that of the secondary field which was found in all measurements. The two fields were brought together at the axis and show the position of the fields in an eye free from corneal astigmatism.

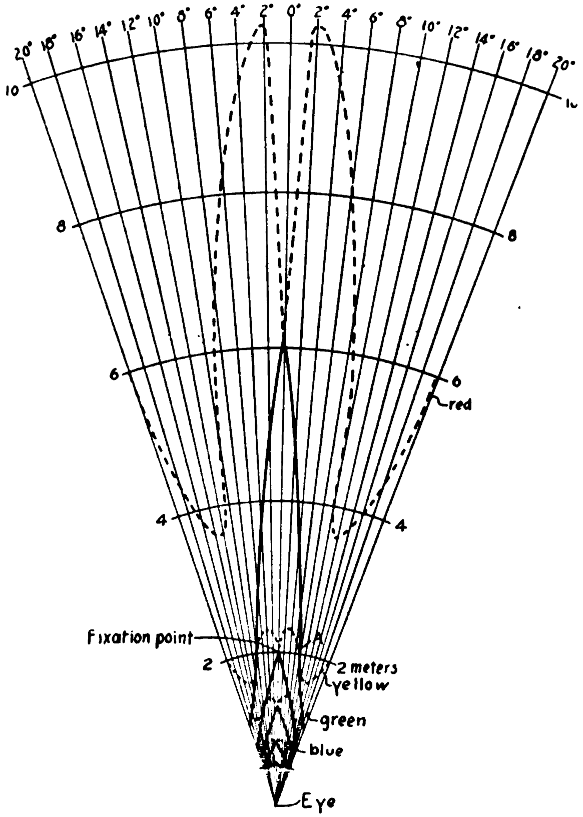
FIG. 19.



Curves showing the position of the primary and secondary astigmatic fields relative to the retina from the combined results of Mr. Ames' readings for all wave-lengths, correction having been made for chromatic aberration. Correction has also been made for Mr. Ames' corneal astigmatism so that the two fields coincide with the retina on the axis.

Second, that the fields on the nasal and temporal side do not lie in the same position relative to the retina. This is due either to the difference between the visual and optical axis (angle alpha) in Mr. Ames' eye or to a lopsideness in the shape of his lens or retina or both. The angle alpha in Mr. Ames' eye amounts to 3° on the horizontal meridian, and the amount of asymmetry of the astigmatic curves suggests that it may be due to the tipping of the lens system relative to the bulbus of the eye. The relationship between the angle alpha and the position of the astigmatic fields would be an interesting matter to determine. It would require, however, the readings from a number of eyes.

FIG. 20.



Curves showing the position in space of the primary and secondary astigmatic object fields; that is where tangential and radial lines of the colors given will focus on the retina when the eye is focused for yellow at a point two meters distant. These curves are constructed from the averages of all Mr. Ames' readings, corrections being made for his corneal astigmatism so that the primary and secondary image fields will coincide on the axis.

So far we have been considering only the position of the primary and secondary astigmatic image fields. Corresponding to these different colored image fields there are object fields in space. They are shown in Fig. 20. The position of the primary astigmatic object field for one color is such that with any given accommodation of the eye all point sources of the same color in that field will be imaged as tangential lines on the retina. The position of the secondary astigmatic object field is such that with a given accommodation all point sources in that field will be imaged as radial lines on the retina. The position of these fields for light of different wave-length is different. See Fig. 20 where the position of these object fields for red, yellow, green and blue is given. With a change of accommodation the position of all these fields shifts.

Consider the nature of the image of a white light point source situated at A, Fig. 20. Being on the first astigmatic object field for red, the red light in it will be stretched in a tangential direction. Being in the second astigmatic image field for yellow, the yellow light will be stretched in a radial direction. Being beyond the second astigmatic field for green and blue, the green and blue light in it will be in the form of a radial diffusion circle. The images formed by light of intermediate wave-lengths will take intermediate shapes.

Images of a similar nature will be formed by points lying in a circle formed by revolving point A about the visual axis. With the same accommodation, however, there is no other position in space, either nearer or farther or at a lesser or greater angle of obliquity where a point source will form a similar image. The image of a point source in any other position in space, though different, will have its characteristic form. It follows that:

1st. With any given accommodation every object in space has its characteristic retinal image form due to its position in space relative to the fixation point.

2nd. This characteristic image form is governed both by the shape and color of the object.

A very definite basis is thus given for monocular tridimensional space perception. Heinrich (24), page 60, was the first to recognize this possibility, but had not carried his experimentation far enough to discover the effect of the secondary astigmatic fields and that of color.

A photographic lens with the same achromatism and oblique astigmatism as have been found in Mr. Ames' eye is being constructed. It is hoped that with this the subjective nature of monocular depth perception as based on the above described characteristic imaging can be more definitely ascertained.

CONCLUSION.

It is recognized that this work on oblique astigmatism, as is the case with that on spherical and chromatic aberration, is hardly more than preliminary.

Measurements should be made on a large number of eyes both normal and abnormal. These should be made for various accommodations and with various sizes of pupil. The relationship between angle alpha and the position of the astigmatic fields relative to the retina should be determined.

There is another very important collateral research that should be undertaken in view of the above findings. That is, the determination of the color sensitivity of the peripheral retina.

In the voluminous research that has been done on this subject great stress has been laid upon the necessity of equating the intensity of the different colors used for objective stimuli. No attention (with one exception, Albini (17)) has been given the nature of the image that falls on the peripheral retina. To make any comparison of the relative sensitivity of the peripheral retina to different colors it is necessary that the images of the different colors shall have the same intensity.

As far as is known in making measurements of peripheral color sensitivity, 1st, the accommodation of the observer has always been kept constant; 2nd, the distance of the different color stimuli from the observer has always been kept constant (with the exception of Albini's work); 3rd, the position of the fixation or accommodation object and the distance of the peripheral color stimuli was the same. This distance for the sake of convenience was not great, usually less than one metre.

From Fig. 20 it can be seen that with a given accommodation the nature of the images on the retina from stimuli of different colors all the same distance away would be very different both in shape and in intensity.

If, for example, a stimulus 1 cm in diameter and one metre distance were used, and the eye were accommodated for red, the mean intensity of the image on the fovea of a red stimulus would be approximately twice that of a blue. If the eye were accommodated for blue, this difference would be reversed. Similar differences in general greater in amount would exist in the case of peripheral stimuli.

In view of these facts all past measurements on peripheral color sensitivity may be of no value. At least their validity is put in question until measurements are made in which the intensity of the retinal image formed by the different colored stimuli are equated. It may be of interest that the late Dr. J. W. Baird, an authority on the subject, shared this view.

The simplest way to do this would be with an apparatus as described on page 24 and following. The primary or secondary astigmatic object field of the observer would be first determined for the different colors; the brightness of the different color sources equated; then either a radial or tangential line source could be used, its apparent position in space being so controlled by a lens that the first or second astigmatic image for the different obliquities would fall on the retina. A change in the length and width of the line source would have to be made for the different colors to compensate for the change in size of image in its different positions in space.

DISTORTION.

Distortion is that characteristic of a lens which causes variation in distance between points in the image plane which in the object plane are equidistant. The human eye is subject to so-called barrel distortion, which means that in the image plane distances between points which are equidistant in the object plane keep decreasing from the centre outward.

The existence of barrel distortion in the eye has been recognized for a long time. The work that has already been done both theoretically and experimentally is of such a nature that it was believed that it could not be improved upon. It was therefore decided to carefully tabulate the results and use the findings.

The articles considered will be taken up chronologically.

(1) Helmholz[5] described a series of experiments which show how this distortion can be seen. It is apparent in causing a straight line at one side of the axis of vision to bow out in its central portions. A strip 8 to 10 centimeters wide when its centre is fixated will appear wider in its middle than at its ends. He

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[5] "Handbuch der Phys. Opt."

also constructed a hyperbolic chess board on a theoretical basis, which when held twenty centimeters from the eye assumes the appearance of straight lines when fixated in the centre.

(2) Matthiessen,[6] in his profound article "Uber die geometrische Gestalt des theoretischen Retina der periskopschen Schematischen Auges," gives theoretical data from which the distortion of the eye can be theoretically calculated. The corresponding external and internal angles have been calculated and are as follows:

External angle	Internal angle	External angle	Internal angle
0°	0°	50.32°	40.16°
3.20°	2.40°	53.25°	42.26°
6.40°	5.21°	56.15°	44.35°
9.58°	8.02°	59.05°	46.42°
13.16°	10.44°	61.54°	48.47°
16.33°	13.24°	64.37°	50.47°
19.53°	16.04°	67.17°	52.45°
23.09°	18.41°	69.55°	54.38°
26.21°	21.15°	72.29°	56.30°
29.29°	23.46°	75.00°	58.19°
32.35°	26.14°	77.29°	60.04°
35.38°	28.40°	79.56°	61.47°
38.40°	31.03°	82.22°	63.28°
41.41°	33.25°	84.46°	65.07°
44.41°	35.44°	87.12°	66.45°
47.39°	38.02°		

Curve (A), Fig. 21, shows this distortion in graphic form.

(3) Donders[7] made measurements on a patient affected with exophthalmia, *i.e.*, protruding of the eyeball. He projected a light into the eye at a known angle and measured the distance from the corneal margin to the point on the schlera where the image was formed. The following is a set of measurements he made on an eye which, except for the exophthalmia, was normal.

Angular obliquity of light	Distance of image from the corneal margin	
	Temporal	Nasal
50°	15.3 mm	15.7 mm
60°	13.2	13.5
70°	11.2	11.6
80°	9.3	9.5
90°	8.0	8.0

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[6] Arch. für Oph. 25, p. 257 (1879).

[7] "Die Grenzen des Gesichtsfeldes in Beziehung zu denen der Netzhaut," Arch. für Ophth. 23, p. 255 (1877).

(4) Drualt[8] reviews the work of Groenouw and Donders. He finds their measurements substantially similar to his and to those found theoretically. They all show that for paraxial objects the posterior nodal point of the eye changes.

He gives theoretical determinations and measurements on dead and living eyes. His methods of measurement are the same as Donders.

In his calculations he uses an eye having the form given by Merkel with refractive surfaces having the shape and position given by Tscherning.[9] Diameter of bulbus 24.4 mm, that of corneal, 12 mm.

Angular obliquity of light	Distance of image from corneal margin
20°	20.4 mm
40°	17.2
60°	13.4
80°	9.7
100°	7.3

His measurements on dead eyes which he said were "very fresh" were as follows:

Angular obliquity of light		Distance of image from corneal margin	
		Temporal	Nasal
I.	30°	18.2 mm.	....
	40°	16.8	....
	50°	15.2	14.5 mm.
	60°	13.5	13.0
	70°	11.4	11.4
	80°	9.7	10.
	90°	8.4	8.8
II.	30°	19.6	....
	40°	17.6	18.3
	50°	15.8	16.4
	60°	13.0	14.4
	70°	10.6	12.0

The diameter of the bulbus in Case I was 23 mm, that of cornea 10.3 mm; in Case II, 25 mm and 11 mm.

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[8] "Note sur la Situation des Image retiennes formées par les Rayons tres obliques sur l'Axe optique," Arch. d'Ophth., 18, p. 685 (1898).

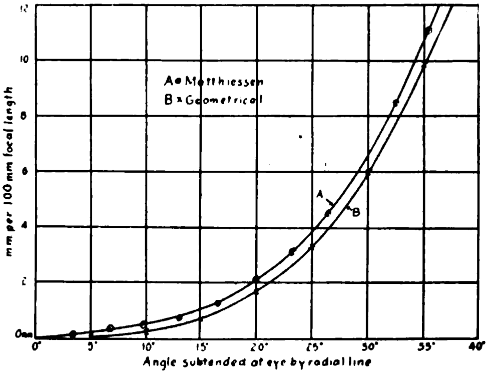
[9] Oeuvres de Young, 1896, p. 134.

His measurements on live eyes were on two patients affected with exophthalmia. He measured both eyes of each, and as they were all approximately the same, he gives the average results of the two eyes. They are as follows:

Angular obliquity of light	Distance of image from corneal margin	
	Temporal	Nasal
III. 50°	15.2 mm.	....
60°	13.5	....
65°	....	14.0 mm.
70°	11.6	13.0
80°	9.7	11.1
85°	9.2	....
90°	....	9.7
100°	....	8.7
IV. 50°	16.0	....
60°	14.1	....
70°	11.8	14.2
80°	10.0	12.7
90°	8.5	11.2
100°	....	9.9
105°	....	9.3

The diameter of the bulbus in Case III was 26.5 mm, the horizontal diameter of the cornea 11.7 mm.

FIG. 21.



Distortion, curves show apparent shortening of radial line from axis to designated angle. Curve *B* shows distortion due to spherical shape of retina. Curve *A* shows distortion due to both spherical shape of retina and optical effect of lens system as calculated by Matthiessen.

Drualt made measurements on two other people and found similar results.

(5) Tscherning[10] describes optical distortion and gives an account of his experiments with an artificial eye, in which he finds a distortion similar to that described by Helmholtz.

SUMMARY.

The results of Donders and Drualt on the experimental side are substantially similar to those calculated by Matthiessen. The difference on the temporal and nasal side is probably due to the existence of the angle alpha in the eyes measured. If we take the average of all the readings we get the following table:

Angular obliquity of light	Distance of image from corneal margin	Corresponding internal angle
30°	18.9 mm	24.5°
40°	17.6	32.3°
50°	15.5	39.4°
60°	13.5	47.8°
70°	11.8	54.4°
80°	10.2	61.0°
90°	8.9	67.0°

The internal angles which correspond with the above distances of images from the corneal margin have been calculated as accurately as possible and are given in the right-hand column. It will be noted by comparing the external and internal angles with those of Matthiessen on page 80 that their results agree very closely.

The computed results of Drualt are very similar. (See tables, page 81.)

It may be of interest to consider the cause of this distortion. Curve (B), Fig. 21, shows the distortion which would exist in the human eye due solely to the effect of the spherical shape of the retina. This might be called geometrical distortion. By comparison with curve (A) it is seen that this geometrical distortion alone is not as great as that which exists in the eye. It can be concluded therefore that the distortion caused by the optical system is also barrel in its nature and increases that caused by the spherical shape of the retina.

Fig. 22 shows the distortion of the eye as found in another form. The barrel-shape grid shows the appearance to the eye which a rectilinear grid would have if its center was fixated and if it was viewed at such a distance that the points on the lower right-hand diagonal subtended angles to the visual axis as indicated.

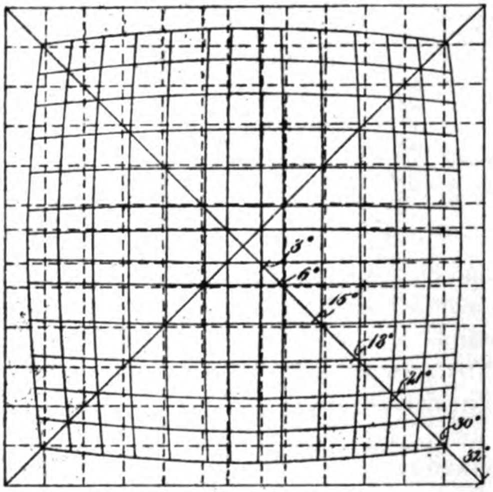
---

[10] "Physiologic Optics," published by The Keystone Press, 1904. Chapter xiv.

FINAL CONCLUSION.

As was stated in the preface the researches described were undertaken for the purpose of determining the exact nature of the image received by the human eye in the belief that a knowledge of its nature would be of aid in suggesting how the various parts of a picture should be painted to give a technically pleasing and artistic effect.

FIG. 22.



Grids showing distortion in eye. The barrel-shaped grid shows the amount of distortion the rectilinear grid suffers when viewed by the eye.

As has been shown, the most important characteristics have been approximately determined, *i.e.*, spherical aberration, chromatic aberration, oblique astigmatism and distortion. A photographic lens embodying these characteristics is being constructed. It is believed that in connection with the three-color process a substantial reproduction of the retinal picture can be obtained.

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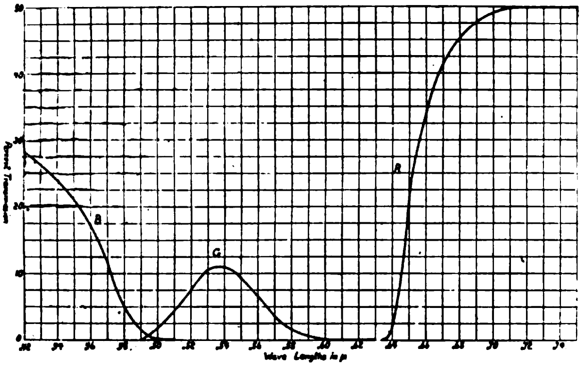
A COMPARISON OF MONOCHROMATIC SCREENS FOR OPTICAL PYROMETRY.

BY

W. E. FORSYTHE.

In working with any optical pyrometer it is generally sufficient to use a so-called monochromatic screen between the eye and the pyrometer filament or the comparison source in order that brightness comparisons can be made without trouble due to color differences. For the most part red glass screens have been used in the past, and the question is often asked—Why not use screens of other colors, as, for instance, a green or a blue screen? In connection with some other work quite a number of readings have been made with particular red, green and blue screens before the eyepiece of a disappearing-filament optical pyrometer. Since the above readings with the different glasses extended over a wide range of temperatures, it was thought worth while to make a study of these readings to see if settings made with any one of these glasses as the monochromatic screen were more accurate than with either of the others.

FIG. 1.



Spectral transmission of different screens.
Curve *B*, two thicknesses blue uviol glass, total thickness 3.9 mm.
Curve *G*, two thicknesses green glass, total thickness 5.2 mm.
Curve *R*, two thicknesses Jena red glass, total thickness 6.8 mm.

The spectral transmissions of the three glass screens used are shown in Fig. 1. The red glass consisted of two thicknesses of

Jena Glass (total thickness 6.8 mm). This is the glass that has been used in Nela Research Laboratory as a standard red screen for pyrometric work for the past six or seven years. The green glass consists of two thicknesses (total thickness 5.2 mm) of an unsaturated green glass that was obtained from Mr. Gage of the Corning Glass Works. The blue glass consisted of two pieces of blue uviol glass (total thickness 3.9 mm).

In Table I is given the percentage of the total light from a black body at different temperatures that is transmitted by each of these glasses. These values were obtained by calculation, using the following formula:

$$T_R = \frac{\int_0^\infty J(\lambda T)\, V_\lambda\, T' d\lambda}{\int_0^\infty J(\lambda T)\, V_\lambda\, d\lambda}$$

where $J(\lambda T)\, d\lambda$ equals black-body energy as given by Wien's equation for the interval λ to $\lambda + d\lambda$, V_λ = visibility and T' the spectral transmission of the glass used. This integral can be calculated by the step-by-step method with sufficient accuracy for this purpose.

TABLE I.

Percentage of the Total Light from a Black Body at Different Temperatures Transmitted by Different Glasses.

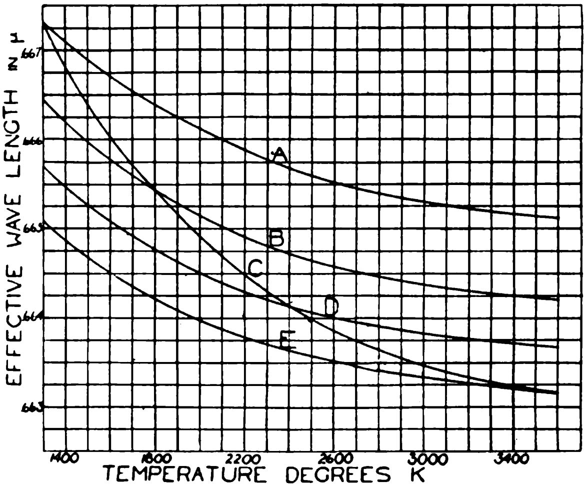
Temperature of black body	Kind of glass		
	Jena red two thicknesses	Green two thicknesses	Blue uviol two thicknesses
	per cent.	*per cent.*	*per cent.*
1000°K	6.6	1.4	0.004
1800	2.1	3.2	.06
3000	1.0	4.2	.2

The values given in Table I show very well one reason for the preference for the red glass at low temperature. It will be noticed that at 1000° K, which is about as low as readings are generally made with the optical pyrometer, the red glass transmits much more of the light than does the green or blue glass. The table also shows why it is practically impossible to make readings through the two blue glasses at the very low temperature. At high temperatures the green glass transmits much more than does the red glass, but even the red glass as used on the pyrometer for temperature ranges, roughly 1500° to 1800° K, transmits enough light to very greatly tire the eye.

If two red glasses such as described are being used in the eyepiece to observe a source at a temperature of 2000° K and one is removed, about twice as much light will reach the eye as before. If for the same condition, the two green glasses are being used and one of them removed, about 4.3 times as much light will reach the eye as before; while about 5.5 times as much light is transmitted by one thickness of blue glass as is transmitted by two thicknesses. These ratios do not vary but a small amount for different temperatures of the source observed. Corning high transmission red (50 per cent.) is a somewhat better glass than the Jena Glass here used. There may be better green or blue glasses than the ones used here, but these are the best we have found. With any one of the three glasses here used, a single thickness will not give a color match when there is an intensity match between the pyrometer filament and the source being studied for a large temperature difference between them, that is, when a rotating sector of small transmission is used. It is, therefore, necessary to use two thicknesses of glass. Even with two thicknesses of the green glass a good color match is not obtained when using a sector of small transmission. However, it is possible to repeat the setting quite well with two of these green glasses. For low temperatures it does not make much difference in the readings whether one or two glasses are used. This difference between the readings for two red glasses and for one red glass amounts to about one degree near 1800° K and to somewhat less at 1500° K, the reading being lower for one glass, for a tungsten pyrometer filament compared with a black body. For the blue glass for the same conditions the difference is somewhat greater, corresponding to about two degrees at 1800° K and to somewhat less at 1500° K. At lower temperatures this difference for either glass is much less. The accuracy gained by having the greater amount of light far outweighs the small error due to the change in effective wave-length.

Effective Wave-length of Monochromatic Screen.—An optical pyrometer can be calibrated and so used as to make unnecessary a knowledge of the extent to which the screen is monochromatic. To do this requires a black-body furnace that can be operated at various temperatures up to the highest temperature for which the pyrometer is to be used. However, to use Wien's equation to extend the temperature scale, either above or below that of the standard furnace, by the use of a rotating sector disk or an absorbing glass, that is, to find the temperature of a black body having a brightness of, say, ten times (assuming a sector or absorbing glass transmission of one-tenth) that of a black body whose temperature can be measured directly, a knowledge of what wave-length to use, or the effective wave-length,* is necessary. The effective wave-length also must be known if the pyrometer is used to measure the temperature of non-black bodies. In using the pyrometer, it is the integral luminosities through the monochromatic screen that are compared, for which reason the effective wave-length of the screen corresponding to a certain temperature interval has been defined as the wave-length for that temperature interval of a black body, such that the ratio of its radiation intensities equals the ratio of the integral luminosities through the screeen used.

FIG. 2.



Effective wave-length for Jena red glass.
Curve *A*, effective wave-length from 1300°K to other temperatures.
Curve *B*, effective wave-length from 1800 to other temperatures.
Curve *D*, effective wave-length from 2400 to other temperatures.
Curve *E*, effective wave-length from 3600 to other temperatures.
Curve *C*, limiting effective wave-length.

---

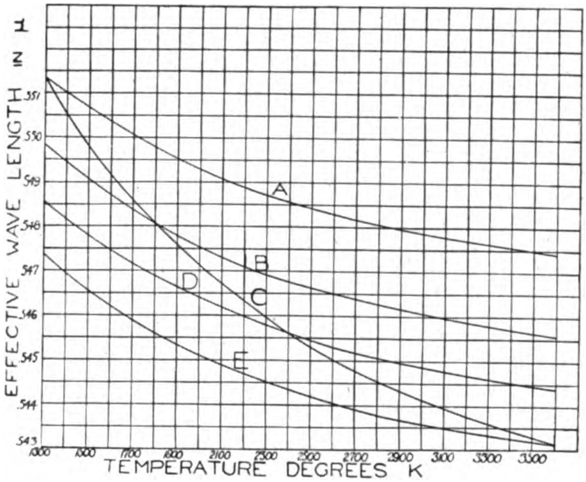
* *Astrophys. Jour.*, **42**, p. 294, 1915.

Knowing the spectral transmission of the glass, it is possible to calculate the effective wave-length λ_e for any temperature interval by means of the following equation:

$$\left[\frac{J(\lambda T_1)}{J(\lambda T_2)}\right]_{\lambda e} = \frac{\int_0^\infty J(\lambda T_1)\, V_\lambda\, t'\, d\lambda}{\int_0^\infty J(\lambda T_2)\, V_\lambda\, t'\, d\lambda}\ .$$

where t′ is the spectral transmission of the glass and V_λ is the visibility. These integrals also can be computed by the step-by-step method with sufficient accuracy for this purpose. Using this equation, the effective wave-length was calculated for each of the glasses whose spectral transmission is shown in Fig. 1 for a number of temperature intervals and the results plotted as shown in Figs. 2, 3 and 4. These curves in addition to those in Fig. 1 show the characteristics of the three glass screens. It will be noted in Fig. 2 that the effective wave-length for the red glass for the temperature interval 1800° K to 3000° K is 0.6659μ. The effective wave-length for the green and the blue glass screens for the same interval (Figs. 3 and 4) are respectively 0.5478μ and

FIG. 3.

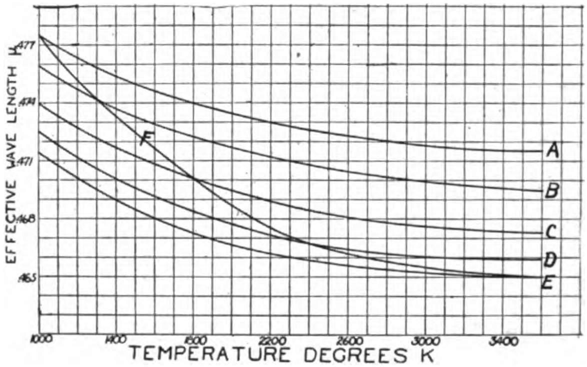


Effective wave-length of green glass.
Curve *A*, effective wave-length from 1300°K to other temperatures.
Curve *B*, effective wave-length from 1800 to other temperatures.
Curve *D*, effective wave-length from 2400 to other temperatures.
Curve *E*, effective wave-length from 3000 to other temperatures.
Curve *C*, limiting effective wave-length.

0.4677μ. If it is desired to find the effective wave-length for some other temperature interval—as 2000° K to 2900° K—for which the curve is not drawn, the curve can be imagined drawn parallel to the 1800° K curve, one point being determined where the curve for the limiting effective wave-length crosses the ordinate for this temperature. The desired effective wave-length will be given by the point where this curve would cross the ordinate for the other temperature, in this case, the 2900° K ordinate.

When making a set of readings with the optical pyrometer the method used is first with the pyrometer filament darker than the background to get a brightness match by increasing the pyrometer current. This is called a dark reading. The next brightness match is made starting with the pyrometer filament brighter than the background. This setting is called a bright reading. The average of a dark and a bright is what is called a reading. Three or four such averages constitute a set. The readings that are used to study the accuracy consist of two or three such sets where each set consists of the average of six separate brightness matches. The different sets were obtained by making first a set with the red glass, then one with the green glass, and finally a set with the blue glass. These were then repeated in the same order. A study of the readings thus obtained showed that for brightnesses

FIG. 4.



Effective wave-length for blue glass.
Curve *A*, effective wave-length from 1000°K to other temperatures.
Curve *B*, effective wave-length from 1300 to other temperatures.
Curve *C*, effective wave-length from 1800 to other temperatures.
Curve *D*, effective wave-length from 2400 to other temperatures.
Curve *E*, effective wave-length from 3600 to other temperatures.
Curve *F*, limiting effective wave-length.

corresponding to temperatures above 1400° K or 1500° K the range in the averages of the currents for the condition of balance was about the same, whether a red, green or blue glass was used. The percentage range in the readings are given in Table II.

TABLE II.

Percentage Range in Current Through Pyrometer Filament for Brightness Matches with the Background Under Various Conditions and for Different Colored Screens in the Eyepiece.

	Screen		
	Red	Green	Blue
	per cent.	*per cent.*	*per cent.*
No. 1. Average range in pyrometer current for forty different determinations of temperature, 1200°K to 3000°K, each consisting of two or three different groups of six settings each......	0.12	0.15	0.19
No. 2. Range for a single reading consisting of a dark and bright setting for forty determinations, temperature range 1200°K to 3000°K.........	.47	.42	.51
Same as No. 1, excepting for temperatures below 1500° K. 9 determinations..................	.12	.13	.36
Same as No. 2, excepting for the nine readings below 1500° K..............................	.35	.28	.90

For the temperature range covered by this investigation, the relation between the percentage variation of the current through the pyrometer filament and the corresponding percentage variation in the temperature of the source being investigated for a brightness match is very nearly a constant independent of the temperature. It was also found that the constant was about the same for readings made with either of the three glasses. The result was that $\frac{dI}{I}=\frac{1.8\ dT}{T}$ where the constant 1.8 does not vary more than a few per cent. in going from the red to the blue. Thus, the per cent. variation in current divided by 1.8 will give the per cent. variation in the temperature. As can be seen from the data in Table II, the average range for the pyrometer currents for all temperature measured was very nearly the same for the readings with each of the three glasses and amounted to about 0.15 per cent. This makes the average range in the temperature measured less than one-tenth of a per cent., or less than 2° C for a temperature of 2000° K.

The data in Table II show that for temperatures below 1500° K, the readings with the blue glass had a range about three times as great as the readings with the red or green glass.

In the table is given also the average range due to a single measurement consisting of the average of a dark and a bright reading. The average range in pyrometer current for the forty temperature determinations was about 0.5 per cent., which makes the temperature range due to a single measurement about 5° at 2000°. Some of the readings, of course, showed a range somewhat greater than this. It must be remembered that this range corresponds to the greatest difference between any two readings (consisting of the average of a dark and a bright brightness match) for eight or ten readings. These results are thought to be pretty good and to indicate about the accuracy to be expected in this class of work. The results on the green glass show that the average for a single reading is less for the low temperatures than for the average of all the readings taken. This is due to the fact that many of the readings at high temperature were taken with a rotating sector and trouble due to poor color match was experienced. When a sector was used the current through the pyrometer was such that it corresponded to a temperature about 1500° K or higher.

To find out how different observers would agree in their readings, using the different colored screens, a number of readings were made by different observers. The brightness temperature of a tungsten lamp filament was measured at four different temperatures, using in turn each of the three glasses. The use of a tungsten lamp gives a greater brightness for the green and blue than would be obtained if a black body had been used at a temperature the same as this red brightness temperature. The brightness as observed at the blue end was increased by using a larger opening before the eyepiece telescope of the pyrometer, since it was found that readings could be made with this larger opening. The opening used with the red and green glasses was 9 mm in diameter, while for the blue glass the opening was 12 mm in diameter. Seven observers made readings for the test. The first four, as they are arranged in Table III, had had much experience with the optical pyrometer, using different colored screens. Observer No. 5 had had considerable experience with a commercial form of the disappearing-filament pyrometer using red glass. Observer No. 6 had had a small amount of experience, having read once before in a test of this kind. Observer No. 7 had had no previous experience with this kind of work. The first readings were made on the lamp at a temperature of 1774° K, then at 2154° K, and finally at 1456° K. At a later date the apparatus was again set up and most of the observers made readings with the tungsten lamp at a brightness temperature of 1245° K. The temperature 2154° K was measured, using a 2° rotating sector disk between the pyrometer lamp and the lamp filament being measured. This made the current readings for the 2154° K point of the same order as for the 1456° K.

TABLE III.

Settings by Individual Observers Using Different Monochromatic Glass Screens.

Observer	Screen used	Current through pyrometer filament for apparent brightness match with tungsten filaments at the following brightness temperatures			
		1245°K direct	1456°K direct	1774°K direct	2154°K 2° sector
WEF	Red glass...........	2669	3353	4773	3197
AGW	Red glass...........	....	3358	4775	3193
CHS	Red glass...........	2665	3362	4780	3188
IAV	Red glass...........	2671	3357	4778	3190
FH	Red glass...........	2667	3356	4774	3212
FG	Red glass...........	2672	3363	4780	3185
HP	Red glass...........	2670	3351	4770	3199
	Mean............	2669	3358	4776	3195
WEF	Green glass.........	2696	3393	4855	3619
AGW	Green glass.........	....	3394	4852	3609
CHS	Green glass.........	2687	3393	4863	3615
IAV	Green glass.........	2693	3398	4854	3617
FH	Green glass.........	2692	3398	4858	3631
FG	Green glass.........	2710	3384	4858	3605
HP	Green glass.........	2691	3410	4841	3615
	Mean............	2695	3394	4854	3616
WEF	Blue glass...........	2720	3407	4882	3974
AGW	Blue glass...........	....	3413	4884	3969
CHS	Blue glass...........	2695	3414	4888	3982
IAV	Blue glass...........	2710	3412	4888	3966
FH	Blue glass...........	2707	3418	4894	3961
FG	Blue glass...........	2795	3429	4893	3979
HP	Blue glass...........	2722	3406	4766	3901
	Mean............	2725	3414	4885	3971

These readings were made by the method described above. Each observer made but a single set of readings, consisting of six brightness matches at a particular temperature. The average of the current obtained by each observer is given in Table III. It will be noticed that the agreement is very good for the three highest temperatures and about the same for each of the screens used. For the lower temperatures the best agreement among the different observers is found in the case where the red glass was used; also the agreement for the green glass is much better than for the blue glass.

For the readings on the 1240° K point a change of about 0.0003 amp. in the pyrometer current corresponds to one degree in temperature and for the 1456° K point a change of 0.0004 amp. in pyrometer current corresponds to about one degree in temperature. For the 1774° K point a change of one degree in temperature requires a change of 0.0005 amp. in pyrometer current, while for the 2154° K point a change of 0.0004 amp. corresponds to a change of about 3°. The reason for this greater change is due to the use of the rotating sector with small transmission.

From the results of the tests shown for all readings except at low temperatures it would be pretty hard to decide whether the green or red glass screen was the better, but the data show very definitely that the readings with the blue glass are not so good as those with the red or green. For very low temperatures it was almost impossible to make readings with the blue glass. For temperatures below 1200° K, and thus very low brightnesses, there is no question but that the red glass is the best to use. For temperatures in the neighborhood of 1400° K, the readings with the red and green glass have about the same range and this is somewhat greater than 0.1 of one per cent. The readings for the blue glass are very erratic and the range in some cases is several times this amount, the average being about three or four times this amount. Excepting at very low temperature (*i.e.*, below say 1200° K) there is no question of the amount of light obtained since either the red or green glass transmitted sufficient light for brightness matching. Personal preference will have considerable influence on this decision. The observers who made readings on these tests were about equally divided in their preference for the red or green glass screens. To the author, the fact that good color matches are not obtained under all conditions with the green glass would be against this glass.

There are four reasons for choosing the red glass rather than the green. In the first place, at low temperatures, the red radiation first becomes visible and thus readings may be made at a somewhat lower temperature with the red glass. This is well shown in Table I. In the second place, the color change for a change in wave-length is much less in the red part of the spectrum than in the green, which makes the red the better part of the spectrum for this work. This is shown by the fact that for the green glass here used, a good color match is not obtained when a sector of small transmission is used, while no such trouble is found with the red glass. The third reason for using red glasses as monochromatic screens is that better red screens can be obtained, that is, the spectral transmission of the red screens ends with a steeper slope than does the green screen. This is illustrated very well by the relative slopes of the curves showing the spectral transmission of the red and green glass shown in Fig. 1. These curves show that the transmission of the red glass is very steep on the short wave-length side and the other side being at the limit of the visible spectrum, the very rapid falling off of the visibility curve in this region makes the luminosity curve steep on that side.

The fourth advantage of the red screen is that when measuring a temperature so high that a rotating sector or absorbing glass is necessary, the transmission of the sector or absorbing glass that must be used is larger for the longer wave-length than it is for the short wave-length. From Wien's equation the following relation can be derived between the wave-length and the transmission (t) of the sector used for extrapolating to a higher temperature.

$$\frac{\log t_1}{\log t_2}=\frac{\lambda_1}{\lambda_2}$$

Suppose that it is necessary to measure a temperature of 3100° K, using a pyrometer lamp or other comparison source that cannot be heated to a higher brightness than that corresponding to 1800° K. If the red glass described above is used, the transmission of the absorbing glass or sector that must be used is about 0.0054. For the green glass this same temperature range requires a sector or absorbing glass with a transmission of about 0.0017, while a sector or absorbing glass with a transmission of about 0.00061 is required for the blue glass described above. Such small transmissions are very hard to measure and thus it is better to use the red glass when measuring very high temperatures.

NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO,
December, 1920.

THE USE OF THE ULBRICHT SPHERE IN MEASURING REFLECTION AND TRANSMISSION FACTORS.*

BY

ENOCH KARRER.

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I. INTRODUCTION.

THE purpose of this article is to present a general survey of the theory of the methods of measurement of the reflection factor of substances, and to describe in detail some new pieces of apparatus for the measurement of this quantity and for the measurement of the transmission factor.[1]

The apparatus described in greatest detail is a reflectometer consisting of a photometric sphere and the Martens polarization photometer. The novelty in this suggestion lies not in the use of this particular photometer in the study of the transmission and reflection by bodies, nor in the use of the sphere in this connection, but in the use of the sphere in a way that is in accord with the simple theory of the sphere, and that, in conjunction with a photometer such as the Martens, allows an absolute deter-

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* Published by permission of the Director, Bureau of Standards.

[1] The reflection and the transmission factors for light are formally defined by the Illuminating Engineering Society (Rep. of Com. on Nomenclature and Standards, 1918). The terms are here used in accordance with these definitions, with the understanding, however, that they are determined under conditions of diffused light. Other symbols used in this article conform also with the I. E. S. nomenclature.

mination of either the transmission or reflection factor in one observation.

II. HISTORICAL.

The measurement of reflection factors for surfaces that reflect more or less diffusely has received considerable attention. Thaler[2] made a thorough test of the behavior of a few substances in this respect. Although he was not interested in the total diffuse reflection factors, his data[3] for a surface smoked with magnesium oxide determine values between .89 and .90 for such a surface.

Sumpner[4] made use of the increased brightness of the walls resulting when a lamp was introduced within an inclosure to study reflection and transmission by substances. Rosa and Taylor[5] used the same phenomenon with the sphere to obtain the reflection factor of their sphere paint. The method depends upon the comparison of illumination at a particular point from a given source when the direct flux only is considered and when this flux is altered by enclosing the source. Since the enclosure affects the flux in all directions from the source, the direct flux, with which comparison is made, must be the average direct flux from the source.

Gardner[6] gives data obtained in the Electrical Testing Laboratories by the use of the sphere in a manner suggested by Little, on the reflection factors of colored surfaces. This method is relative, consisting of the measurement of the brightness of the sphere walls when illuminated by the test or standard surface placed successively inside the sphere. The test or standard surface only is illuminated directly, by a source that is most conveniently placed outside the sphere.

Nutting[7] has made some interesting and important considerations on an instrument for determining the reflecting power of opaque bodies. This instrument was a very portable reflectometer, depending upon no standards and had the unique property of requiring no light source as a necessary adjunct. Any general illumination at the place where measurements were made might suffice. Because this reflectometer bears some semblance to that described in this article, some parts of the underlying theory are given below. The sources of error that exist in the Nutting reflectometer will not be dwelt upon here. They have been pointed out by others.[9, 11]

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[2] *Ann. d. Phys.*, 316, p. 996; 1903. Good for reference to earlier work pertaining to Lambert's law. See also in this connection Mascart Traite D'Optique, Vol. 3, p. 213.

[3] *Loc. cit.*, p. 1006, Table I. (There appears to be a typographical error here in the record of angles.)

[4] "The Diffusion of Light," *Phil. Mag.*, 35, p. 81; 1893.

[5] "The Integrating Photometric Sphere, Its Construction and Use," *Tr. I. E. S.*, 2, p. 453; 1916.

[6] "The Light Reflecting Values of White and Colored Paints," *J. F. I.*, 181, p. 99; 1916.

[7] *Tr. I. E. S.*, Vol. 7, p. 412; 1912.

III. THE USE OF THE SPHERE.

Luckiesh[8] has more recently called attention to the unsatisfactory state of the measurement of reflection factors, pointing out the desirability of arriving at some standard methods, and suggesting the use of the sphere in the following manner. Around a translucent sphere are distributed several light sources, and the whole surrounded by white-walled box. The surface whose reflection factor is sought fills a small aperture in the sphere. The brightness of the test surface is measured by matching the brightness of it with that of the field in an auxiliary photometer. The proposed method is good, but it involves the use of all the photometric accessories required in the ordinary use of the sphere, and requires further a standard reflecting surface. The means employed of illuminating the sphere walls should have to be altered somewhat in order that full advantage may be taken of the theory of the sphere. In any case, however, the use of the principle of the sphere has decided advantages over the laborious method of measuring the reflection factor for various directions, then by integration, graphically or otherwise, derive the total reflection factor.

Much credit is due Taylor[9] for pointing out several ways by which the sphere may be used to determine the absolute reflection factor. The most obvious ways (including those of Mr. Taylor and others) of using the sphere to measure the reflection factor may be enumerated as follows:

(1) Comparison of the illumination due to reflected flux alone with that due to both direct and reflected flux. If E_1 is the

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[8] "Measurement of the Reflection Factor," *Elec. World,* Vol. 69, p. 958; 1917.

[9] "Measurement of Diffuse Reflection Factors," JOUR. AM. OPT. SOC., 4, p. 9; 1920. Also *B. S. Sci. Papers, No. 391* and *No. 405.*

illumination due to the average direct flux and E_2 that due to the reflected flux then

$$\frac{E_2}{E_1 + E_2} = \rho$$

This is the method used by Sumpner and Rosa and Taylor as described above to obtain the absorption factor (α) directly and derive therefrom the reflection factor. In this case

$$\frac{E_1}{E_1 + E_2} = \alpha = 1 - \rho$$

Sumpner, however, used an enclosure in the form of a hollow cylinder rather than in the form of a hollow sphere. Sumpner also used this method to obtain the transmission factor of substances. It is applicable only to materials that can be made into the form of a sphere, or that may be applied to the inner sphere walls.

(2) Sphere walls illuminated by illuminating a small portion of them. Brightness of the walls is measured when complete and when a known portion of the sphere is removed. This is the basis of a proposed reflectometer by Taylor[10] to obtain the reflection factor of painted sphere walls. The reflection factor of the sphere wall is given by the equation

$$\rho^2\left[q(p+q-1) + \frac{b}{b_0}p(1-q)\right] + \rho(1-q)\left[q + (p+q-1)\left(1-\frac{b}{b_0}\right)\right] + (1-q)^2\left(1-\frac{b}{b_0}\right) = 0$$

where q is per cent of sphere surface cut off by the hole, p, the per cent of area added to the sphere wall when the hole is covered by a plane surface; b and b_0 are brightnesses of sphere wall due to reflected light when test surface has a reflection factor of ρ and 0 respectively. A particular application of this method was made by Benford[11] for the determination of the reflection factor of magnesium carbonate as follows. The brightness (b_1 and b_2) of the sphere wall was measured after the removal in succession of two known fractional parts ($1 - p_1$) and ($1 - p_2$) of the sphere. The reflection factor is given by

$$\rho = \frac{b_1 - b_2}{b_1 p_1 - b_2 p_2}$$

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[10] *Loc. cit.*, p. 16.

[11] *G. E. Rev.*, 23, p. 72; 1920.

This method, as the previous one, is only applicable when the test substance can be made into a sphere or can be applied to the inner sphere walls. It has one very desirable feature, in that not only is the illumination diffuse but the reflected flux which is measured is also, and the whole of it is directly contributory to the results. The degree to which the flux is diffused depends upon the test substance itself, for the latter constitutes the whole of the sphere wall. It must be applied with caution when the test substances are any other than highly-diffusing.

(3) The method (2) above may be modified so that the specimen need not be the whole of the sphere wall but merely a small known portion of it. In this case the reflection factor ρ_x of the test surface is given by

$$\rho_x = \frac{1 - \frac{b_0}{b_x}}{\frac{q}{1-q} \cdot \frac{b_0}{b_x} + \frac{\rho p}{\rho p + (1-q)(1-\rho)}}$$

where p, q, ρ, and b_0 have the same meaning as in the method (2), and b_x is the brightness of sphere wall when the test surface is in place. The equation given under method (2) is derived from the present one where $\rho_x = \rho$. This method retains the desirable diffuseness in both incident and reflected light and obviates the restrictions of method (2) above, but the sensitivity of it depends upon the value of the reflection factor measured.

(4) Part Sphere.—Test (or standard) surface constituting a known portion of the sphere wall is illuminated directly. The standard is a specimen having the reflection factor of the sphere wall. The brightness of the sphere wall is measured first when illuminated by the specimen, then when illuminated by the standard. The reflection factor (ρ_x) of the test surface is given by [10]

$$\frac{\rho \frac{b_x}{b}}{1 - \frac{\rho^2 p \left(1 - \frac{b_x}{b}\right)}{1 - q + \rho(p + q - 1)}}$$

b_x and b are the values of the brightness of the sphere wall when the aperture is covered by the test surface and by the standard surface respectively. This method was one adopted by Taylor in preference to the previous one, because it is more accurate and more sensitive.

(5) The sphere is complete. The specimen placed inside and illuminated directly (preferably by a narrow beam of light projected through a window). The standard is a specimen of known reflection factor. Two measurements of brightness of the sphere wall are necessary—first, when illuminated by the specimen, then when illuminated by the standard. This method was suggested and used by Little[6] whose standard was a magnesium carbonate surface. The ratio of these gives the reflection factor of the sample in terms of the standard. This method may be improved by eliminating the standard surface. This may be accomplished by screening the test surface from the observation window; then comparing the brightness of the sphere wall first, when illuminated by the sample, and second, when illuminated by illuminating a small portion of the wall. In the latter case no screen is interposed.

(6) The sphere is complete. The specimen placed inside and illuminated by the sphere wall. The brightness of the sample and of the sphere wall compared by one observation. The sphere wall may be illuminated by illuminating a small portion of it. The reflection factors are in terms of that of the sphere wall. If the test surface only is screened from the illuminated portion, the method becomes an absolute one.

(7) The specimen constitutes a negligible portion of the sphere wall, and is illuminated directly. The walls are illuminated by the specimen. The brightness of the sphere wall at a point screened from the test surface is measured first as resulting from the illumination of the test surface, second as resulting from an equal illumination of a portion of the sphere walls unscreened from the observed spot. The ratio of these is the reflection factor of the surface. This is the method embodied in the reflectometer more recently designed by Taylor.[12] No standard is required.

(8) Specimen constituting a negligible portion of the sphere is illuminated by sphere walls, which are illuminated by a narrow

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[12] "A Simple Portable Reflectometer," A. H. Taylor, *Tr., I. E. S.* Paper read at Fourteenth Annual Convention of *I. E. S.*, 1920. Also *B. Sci. Papers, No. 405.*

beam admitted through a small window. The specimen is screened from the illuminated spot. No standard is required. This method was used by Sharp and Little.[26] These authors showed that the reflection factor of a mirror as well as that of other surfaces may be correctly measured by this method. It may also be pointed out that all movements of the photometer in making the measurements required (viz., of the brightness of first the specimen and second the sphere wall) may be avoided by employing a mirror of known reflection factor. In some experimental work with this method by the author a glass prism was used by the rotation of which either the specimen or the comparison spot could be brought into view.

(9) The method just described may be simplified by the use of a Martens Polarization Photometer. One measurement, comparing the brightness of the specimen with that of the sphere wall, will then suffice. This is the method to which special attention is called in this article. It is the method embodied in the reflectometer described in detail below. It is suggestive of the best combination (for certain photometric work) in which the sphere is used in a manner that takes advantage of its properties and at the same time affords simplicity of parts, compactness and directness.

Still other methods may be designed. For example:

(10) Specimen may constitute a known portion of sphere wall. Another known portion removed. Two measurements of brightness of sphere wall are required, first when sphere is complete, and second, when the known portion is removed. This has the advantage that both the incident and the reflected light are diffused. The degree to which the light is diffused depends in this case and in method (3) above not upon the specimen as in methods (1) and (2) above, but upon the sphere wall.

(11) Most of the above methods may be modified so that only one measurement is necessary by employing two spheres simultaneously.

(12) A new method of obtaining the reflection factor of a substance that may be applied to the walls of the sphere, or that may be made into a sphere is suggested by the equation given below under the theory of the sphere. The relation between the brightness (b_n) of the sphere wall resulting when a portion (p)

of it is made luminous and the brightness (b) of the luminous portion is given by

$$b_s = pb \cdot \frac{\rho}{1-\rho}$$

The reflection factor $\rho = \frac{r}{p+r}$, where $r = \frac{b_s}{b}$. The quantities p, b, and b_s may be measured. In particular, if p were changed until $b_s = b$, then $\rho = \frac{1}{1+p}$. It seems that this method is simpler than method (2) where portions of the sphere are removed, and simpler than method (1) where it is required to know the mean spherical candles of the source.

One simple way which suggests itself, whereby no beam of light need be projected into the sphere; viz., a plate of diffusing glass is illuminated on one side so that the brightness of the reverse side is b. The plate with the latter side toward the sphere is then placed against the sphere aperture of known dimensions. Finally the brightness of the sphere walls produced by this luminous plate is measured. This arrangement would readily allow of a variable aperture into the sphere.

Furthermore, the method of illuminating either the specimen or the sphere wall in some instances may be different from that stated above, for example, in method (8) above, the sphere wall might be illuminated by transmitted light as indicated by Luckiesh.[8]

IV. THE THEORY AND USE OF INFINITE PLANES.

The Nutting instrument is based upon the principle of the infinite luminous plane. The illumination on a plane exposed to the flux [13] from an infinite parallel plane source of uniform brightness b (Lambert's) is πb, provided every element of the infinite plane radiates according to the cosine law. If a small reflecting surface be exposed to this illumination, it will reflect per unit area at the rate of $\pi\rho b$ lumens, where ρ is the reflection factor of the surface. Its brightness therefore is ρb. Hence the ratio of the brightness of the small plane to that of the infinite plane is the reflection factor sought. The surface under test should be small enough not to change the brightness of the luminous plane. Such simple conditions are not realized, however, in practice. In case of Nutting's reflectometer, two planes of the same size constitute the radiating and the receiving planes. In such a case, if we assume them infinite, their resulting brightness is readily calculable.

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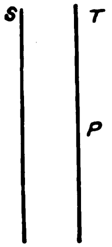
[13] *B. S. Bulletin*, Vol. 7, p. 553; 1910.

The brightness of the planes is increased by the multiple reflection of light between them. Let S (Fig. 1) be the infinite plane of brightness b, illuminating the infinite plane T, and let the reflection factors of these planes be ρ_2 and ρ_1 respectively. As a result of the direct illumination from S, T will be of uniform brightness $\rho_1 b$, and will reradiate onto S at the rate $\pi\rho_1 b$. As a result of this S will have its brightness increased by an amount $\rho_1\rho_2 b$. The plane S will in turn increase its illumination on T by an amount, $\pi\rho_1\rho_2 b$. The total illumination E_T of surface T will be

FIG. 1.



Infinite planes.

$$E_T = \pi b + \pi\rho_1\rho_2 b + \pi\rho_1^2\rho_2^2 b + \cdot\cdot\cdot + \text{etc.}$$

The brightness of T is therefore,

$$b_T = \rho_1 b\,(1 + \rho_1\rho_2 + \rho_1^2\rho_2^2 + \cdot\cdot\cdot + \rho_1^{n-1}\rho_2^{n-1} + \cdot\cdot\cdot +) = \rho_1 b/(1 - \rho_1\rho_2).$$

For the surface S we obtain the brightness

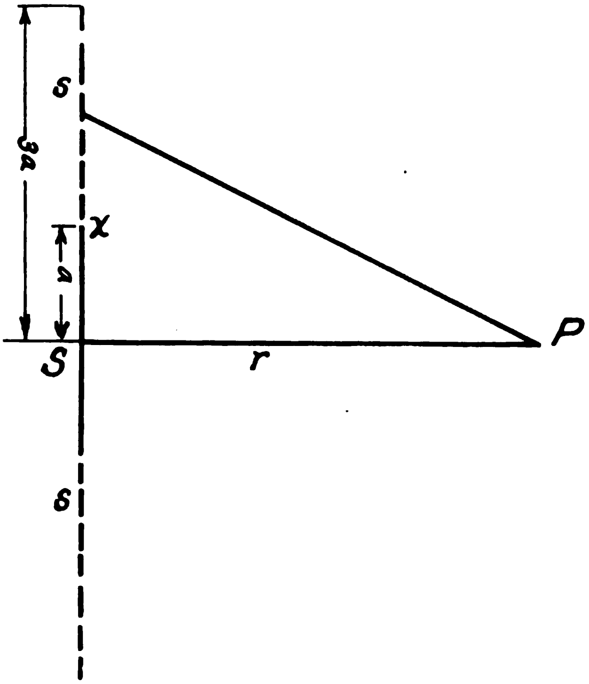
$$b_S = b\,(1 + \rho_1\rho_2 + \rho_1^2\rho_2^2 + \cdot\cdot\cdot + \rho_1^{n+1}\rho_2^{n+1} + \cdot\cdot +) = b/(1 - \rho_1\rho_2).$$

$$b_T/b_S = \rho_1.$$

In order to approach the condition of infinite planes, and at the same time retain limits consistent with a portable instrument, Nutting bounded the planes with a mirror.

The effect of the mirror is virtually to extend by multiple reflection the planes indefinitely. If the mirror were non-absorbing the finite planes bounded by it would give effects similar to infinite planes. The theory of them then would be also as simple as that of infinite planes. The mirror, however, will reflect only a certain proportion ρ_m of the flux falling upon it, and as a consequence, instead of an infinite plane with uniform brightness, we have an infinite plane composed of concentric zones of varying brightness. It becomes more difficult to calculate the illumination due to such a luminous plane at any point in space. This will be seen by considering, for sake of simplicity, the illumination at a point P which lies on the line perpendicular to the plane at the common centre of all the zones. Let S, Fig. 2, be a luminous disk of diameter 2a and of uniform brightness b, surrounded by an annular area of uniform brightness ρ_mb. For the present the annular area may be regarded as the first reflection of the disk in a surrounding circular mirror which reflects an amount ρ_m.

FIG. 2.



A disk S surrounded by annular areas ss of lesser brightness.

The width of the annular area is therefore 2a; its brightness $\rho_m b$. The illumination due to the disk S at the point P is

$$E_d = \pi b \int_0^a \frac{2\,r^2\,x\,dx}{(r^2+x^2)^2} = \frac{\pi\,ba^2}{r^2+a^2}$$

where r is the distance of the point P from the disk.[14] Similarly the illumination at P due to the annular portion *ss* of the disk of brightness b_1 is

$$E_1 = \pi\,b_1 \int_a^{3a} \frac{2r^2\,x\,dx}{(r^2+x^2)^2} = \frac{8\,\pi\,b_1\,a^2\,r^2}{(r^2+9a^2)\;(r^2+a^2)}.$$

So that the total illumination at P is the sum of these

$$E_t = \frac{\pi\,ba^2}{(r^2+a^2)}\left(1+\frac{8\,\rho_m r^2}{r^2+9a^2}\right).$$

where $\rho_m = b_1/b$.

If the disks were of uniform brightness, *i.e.*, $\rho_m = 1.00$,

$$E_t = 2\pi\,br^2 \int_0^{3a} \frac{xdx}{(r^2+x^2)^2} = \frac{9\,\pi\,ba^2}{(r^2+9a^2)}.$$

In the actual case there is an infinite number of concentric zones which contribute to the illumination at P and the resultant effect at P may be represented by a summation of such integrals as that given above for the annular zone *ss* due to the first order of reflection. The effect of the nth order of reflection may be written after noting that the position and boundary of the nth image are determined by its radii, which are $a(2n-1)$ and $a(2n+1)$. The illumination at P due to the nth reflection is then

$$E_n = \pi\,b\rho_m^n \int_{a(2n-1)}^{a(2n+1)} \frac{2r^2\,x\,dx}{(r^2+x^2)^2} = \frac{8\,\pi\,ba^2r^2n\,\rho_m^n}{[r^2+(2n-1)^2a^2]\;[r^2+(2n+1)^2a^2]}$$

The illumination due to all zones is

$$E_a = \sum_{n=1}^{n=\infty} \pi\,b\rho_m^n \int_{a(2n-1)}^{a(2n+1)} \frac{2r^2\,x\,dx}{(r^2+x^2)^2} = \sum_{n=1}^{n=\infty} \frac{8\,\pi\,ba^2r^2n\,\rho_m^n}{[r^2+(2n-1)^2a^2]\;[r^2+(2n+1)^2a^2]}.$$

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[14] *B. S. Bulletin,* Vol. 6, p. 551; 1910.

And the total illumination becomes

$$E_t = E_d + E_a = \pi ba^2 \left\{ \frac{1}{r^2 + a^2} + 8r^2 \sum_{n=1}^{n=\infty} \frac{n\rho_m^n}{[r^2+(2n-1)^2a^2][r^2+(2n+1)^2a^2]} \right\}.$$

A few terms of this expression may be written out for the sake of the discussion.

$$E_t = \pi ba^2 \Bigg[\frac{1}{a^2+r^2} + 8r^2 \left\{ \frac{\rho_m}{(r^2+9a^2)(r^2+a^2)} + \frac{2\rho_m{}^2}{(r^2+25a^2)(r^2+9a^2)} + - - - + \right\}$$

$$+ 8r^2 \left\{ \frac{n\rho_m^n}{[r^2+(2n-1)^2a^2]\ [r^2+(2n+1)^2a^2]} \right.$$

$$\left. + \frac{(n+1)\rho_m^{n+1}}{[r^2+(2n+3)^2a^2]\,[r^2+(2n+1)^2a^2]} + - - - + \right\} \Bigg].$$

This series is in general very rapidly converging. The ratio of the $(n+1)^{th}$ to the n^{th} term being,

$$\rho_m \cdot \frac{n+1}{n} \left\{ 1 - \frac{(8+16n)a^2}{r^2+(2n+3)^2a^2} \right\},$$

which approaches the value ρ_m for large values of n, but is always less than ρ_m if a is large compared with r (say $a = 3r$ or more). The value of this ratio for the first and second terms is ρ_m if $a = 2r$. The effect of the illumination due to the n^{th} reflection compared with that of the disk alone is

$$e = \frac{8r^2 n\, \rho_m^n\, (a^2 + r^2)}{[r^2 + (2n+1)^2a^2]\ [r^2 + (2n-1)^2a^2]}.$$

If a/r is assumed to be 5, and ρ_m to be 100, then the effect of the first and second images is about 5 per cent. of that of the disk and the effect of the first 10 images is slightly less than 6 per cent. of that of the disk. It is evident that as the reflection factor of the mirror approaches unity the disk as extended by its images becomes more like an infinite plane.

From the equation for E_t it is also seen that as r becomes very small the illumination at P approaches that given by an infinite plane of uniform brightness, $E_t = \pi b$.

The foregoing considerations show that the theoretically simple conditions are not at all attained in the use of infinite planes as in the Nutting instrument. An improvement could be effected by making the test surface only a small portion of one of the planes. This would also be the case if one were to consider the instrument from the point of view of a finite enclosure rather than from the viewpoint of infinite planes.

A very good approach to two infinite planes can be had by two concentric spherical surfaces. Either one of the surfaces may be made luminous with uniformity of brightness. The test surface may be made a part or the whole of the non-luminous sphere. A sphere uniformly bright illuminates all points on any given concentric spherical surface equally, and more nearly like an infinite plane as the points approach the surface of the luminous sphere.[15] Of course the last condition would be impossible to realize in a practical instrument. Yet, this interposes no hindrance to the practical use of two concentric spherical surfaces as described. For when the surfaces are separated any finite distance the relative distribution of the illumination from the one on to the other remains unaltered. The magnitude only of the illumination changes, becoming less than that which is given by an infinite plane in the ratio of the square of the radius of the sphere to the square of the distance from the centre of the sphere to the concentric spherical surface. (The latter statement applies when the inner sphere is the luminous one.) Just what other difficulties would arise in the actual execution of this arrangement cannot be foretold. It is of interest to note that the flux between two infinite parallel planes depends upon the reflection factors of them in a manner quite different from that in which the flux in a sphere depends upon the reflection factor of the sphere wall.

V. THE THEORY OF THE SPHERE.

For an enclosure we find the most satisfactory case to consider is that of the hollow sphere, for although the flux in any enclosure is everywhere the same,[16] if the walls of the enclosure are uniformly bright, it is only in case of the sphere where this latter condition of uniformity of brightness is easily obtainable.

In the sphere, whose walls reflect according to Lambert's law, every element of surface illuminates every other element to the same degree,[17] no matter how the wall of the sphere or any portion of it is directly illuminated. This property of the spherical enclosure was first discovered by Sumpner.[4] It was rediscovered by Ulbricht,[18] who first demonstrated its great practical importance to photometry. So that, if the surface of the sphere has a uniform reflection factor and, if a small portion of it is made luminous, the whole sphere will be uniformly bright, except for this spot which may be of negligible proportions. The brightness may be calculated as follows:

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[15] *B. S. Bulletin*, Vol. 6, p. 558; 1910.

[16] *B. S. Bulletin*, Vol. 6, No. 4, p. 562.

Let the illuminated spot of area A have an average brightness of b. It radiates πbA lumens. Owing to multiple reflections this flux will accumulate to a value F until the rate of loss αF equals the rate of input πbA. Where α is the absorption coefficient,

$$\alpha F = \pi bA.$$

$$F = \frac{\pi bA}{\alpha} = \frac{\pi bA}{(1 - \rho)}$$

This was first derived by Mascart.[19] This reasoning may be employed for any enclosure whose walls obey Lambert's cosine law of emission. When the walls are not uniformly bright, b refers to the average brightness.

The illumination on every other element of the surface of the sphere will be

$$E = \frac{bA}{4r^2} \cdot \frac{1}{1 - \rho}.$$

Hence the brightness of every element of the surface of the sphere except A is

$$\frac{\rho bA}{4\pi r^2} \cdot \frac{1}{1 - \rho} = b_s.$$

The ratio of the area of the luminous spot to the area of the sphere is

$$\frac{A}{4\pi r^2} = p$$

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[17] Liebenthal, *Praktische Photometrie*, p. 301.

[18] *Electroteknische Zeit.*, 21, p. 595; 1900, and 26, p. 512; 1905.

[19] Palaz, "Traite de Photométrie Industrièlle," Trans. by the Pattersons, p. 297.

So that

$$b_s = \frac{pb\rho}{1-\rho}.$$

If now for a small portion of the sphere surface there is substituted another non-luminous surface whose reflection factor is ρ_x; the brightness b_x of the latter will be

$$b_x = \frac{\rho_x bp}{1-\rho} = \frac{\rho_x b_s}{\rho}.$$

$$b_x / b_s = \rho_x / \rho.$$

The reflection factor of any specimen is given in terms of that of the sphere wall. To make it possible to obtain the reflection factor of the specimen absolutely, certain conditions must be imposed upon the manner in which the specimen is illuminated. If all of the flux illuminating the specimen is reduced by an amount ρ, the brightness of the specimen will also be reduced in the same proportion.

Then b_x becomes $b_x' = \rho b_x = \rho_x b_s$.
and

$$\frac{b'_x}{b_s} = \rho_x.$$

This result is achieved by interposing a screen between the illuminated spot and the test surface. The screen is just sufficiently, large to enable it to intercept all of the flux reflected from the former in the direction of the latter. The illuminated spot must be small in order that no appreciable illumination due to the second, and higher order of reflections would come to the test surface, if the screen were not present. The new surface also is assumed to be so small that its presence does not alter the brightness of the sphere walls.

All of these theoretical conditions can be attained to a satisfactory degree in practice. This is the idea underlying several uses of the sphere in reflectometry listed above and in particular the reflectometer[20] depicted in Fig. 3.

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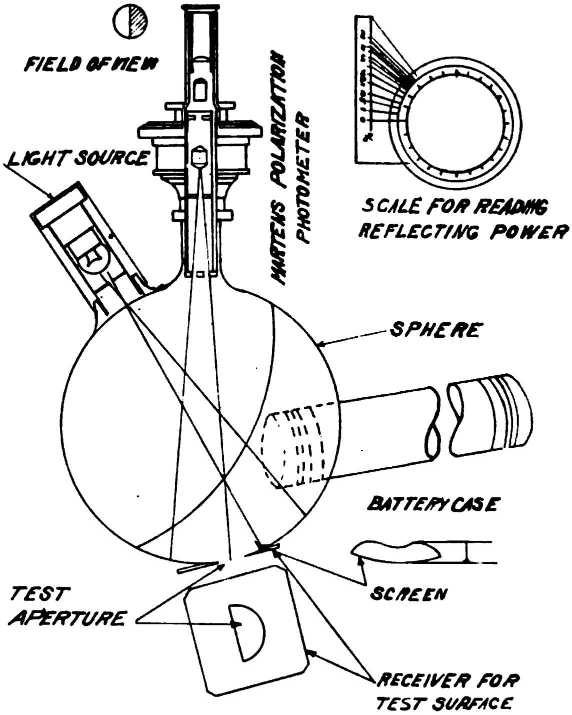
[20] This reflectometer was shown before the Illuminating Engineering Society at Cleveland, Oct. 5, 1920. A general description of it will appear in *Trans. I. E. S.*, as discussion of the papers: "Measurement of Reflection Factor," C. H. Sharp and W. F. Little; and "A Simple Portable Reflectometer of the Absolute Type," A. H. Taylor,

VI. DESCRIPTION OF THE PRESENT REFLECTOMETER.

The reflectometer sketched in Fig. 3 is essentially a sphere, having, firstly, an aperture which may be closed by any surface whose reflecting factor is sought; secondly, a means for comparing directly the brightness of the test surface with that of the sphere surface such as a Martens[21] photometer; and thirdly, a lamp for illuminating a small portion of the sphere.

Fig. 3.


FIELD OF VIEW
LIGHT SOURCE
MARTENS POLARIZATION PHOTOMETER
SCALE FOR READING REFLECTING POWER
SPHERE
BATTERY CASE
SCREEN
TEST APERTURE
RECEIVER FOR TEST SURFACE

The sphere reflectometer using the Koenig-Martens polarization photometer.

The most satisfactory photometer for comparing the brightness of two surfaces not far removed from each other is the Martens Polarization Photometer. It is very compact, accurate, and has an extremely good photometric field.[22] Its optical parts are in order of position in the optical path, Fig. 3, two apertures, lens, Wollaston prism, biprism, Nicol prism, eyepiece containing lens and aperture. A scale reading directly the absolute reflection factor is attached to the circular scale already a part of the Martens Photometer. The lamp is a small mazda flashlight lamp operated at 3.8 V. by a small 3-cell flashlight battery. It is not difficult to get sufficient flux into the sphere and at the same time illuminate a rather small spot, say 3 cm in diameter, when proper lenses are chosen. For the first instrument constructed no lenses were used. For ease and accuracy of setting, however, the resulting illumination was insufficient.

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[21] *Physikalische Zeitschrift,* Vol. 1, pp. 299–303; 1900.

[22] It is to be regretted that this instrument is not more readily procurable.

The spot illuminated by the beam is screened from the test aperture. This screen is made no larger than is necessary. Its dimensions and shape were determined graphically. The size and shape and location of it are shown in Fig. 3.

The battery is placed in an ordinary flashlamp tube whose end is soldered to the sphere. Thus a very convenient handle is afforded at the same time.

The sphere is a copper-covered steel float, procurable at plumbers' supply houses. Its diameter is 6 inches. The copper coat was actually found to be undesirable, when the sphere is enamelled, and was removed. The screen between the test aperture and illuminated spot was riveted in place and enamelled as the rest of the surface. A coat of white enamel facilitates greatly the attainment of a satisfactory surface, where a matt white surface of high reflection factor is desired. The enamelled surface is easy to paint and requires a much thinner coat of paint than otherwise. The paint used was that described by Mr. A. H. Taylor [23] of this Bureau.

A few tests are recorded here to indicate the behavior of an instrument constructed along these lines. These tests were made with the first instrument, constructed rather hastily and imperfectly. No lenses were used with the lamp; no adequate holder for the battery attached; and no provision made for readily turning the Martens Photometer through 180° to eliminate any asymmetric errors. Settings were made in each of the four quadrants.

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[23] *B. S. Sci. Papers,* in press, "The Integrating Sphere," Rosa and Taylor.

Magnesium Carbonate Block No. 1 (A. H. T.).

		Quadrant			
	First	Seccnd	Third	Fourth	Reflection Factor.
	43.9	134.5	223.2	313.6	
	43.3	134.2	223.9	313.9	
	43.9	133.9	223.3	314.1	
	43.7	134.4	223.1	314.2	
	43.8	133.8	223.2	314.3	
Mean	43.7	134.1	223.3	314.0	.982

Magnesium Carbonate Block No. 2 (A. H. T.).

		Quadrant			
	First	Second	Third	Fourth	Reflection Factor.
	43.1	133.8	223.6	314.8	
	43.2	133.9	223.2	314.7	
	44.0	134.1	223.2	314.3	
	43.5	134.2	223.1	314.5	
	43.2	134.3	223.3	313.7	
				314.2	
Mean	43.4	134.1	223.3	314.3	.972

Magnesium Carbonate Block No. 1(E. K.).

		Quadrant			
	First	Second	Third	Fourth	Reflection Factor.
	44.0	134.1	223.0	313.5	
	44.1	134.6	223.5	314.1	
	43.8	133.9	223.2	314.0	
	43.3	134.5	223.5	313.9	
	43.2	134.0	223.2	313.7	
	43.5		223.2		
Mean	43.6	134.2	223.3	313.8	.987

The first two blocks of magnesium carbonate were loaned by Mr. Taylor, who has determined the reflection factor of them by various methods, arriving at .99 for No. 1 and .982 for No 2. There seems to be a consistent difference, yet it is small and almost within experimental errors.

Other tests were made on a series of surfaces which had been studied by Mr. Taylor using the point-by-point method. Differences in the factor were usually obtained (although not always), which indicated that the above reflectometer was reading slightly low. The value obtained here for the magnesium carbonate agrees with those found first by Mr. Taylor[24] and later by Mr. Benford[25] and others.[26]

The sample needs to be neither opaque nor diffusing. It may be a clear piece of glass or mirror. This condition follows on the assumption that the hole in the sphere over which the specimen is placed is so small that its area is negligible in comparison with that of the whole sphere. In the one described above, the area of the hole is about 3 square cm, or less than one-half per cent. of that of the sphere. In the case of a mirror, however, precautions must be taken not to have the second reflecting surface too far removed from the opening. The angle subtended by the image of the test aperture must always be sufficiently large to fill the field of view of the photometer.

VII. GENERAL COMMENTS ON METHODS OF MEASURING REFLECTION AND TRANSMISSION FACTORS, AND ON STANDARDS.

Many aspects of the problem of measuring the diffuse reflection factor call for consideration, some of which may or may not turn out to be of any moment. For instance, in case of the reflection factor, the surface may be diffusely illuminated, but in general can be observed from one direction only. Or it may be illuminated from one direction only, and the diffusely reflected flux measured. In either case the question of direction enters, concerning which nothing much can be said until more work is done; the use of the sphere is, however, a step toward uniformity and standardization. In case of the transmission factor the illumination on the specimen may be entirely diffuse and the whole of the transmitted flux measured. This can be accomplished by the use of two spheres, and comparing brightness of the walls of them. So that the question of direction may be easily eliminated in case of the transmission factor.

Four of the methods enumerated above for determining the reflection factor, namely (2), (3), (9), and (12), afford diffusion in both the incident and the measured flux. The first and last of these places very serious limitations upon the nature and the form of the surface to be tested. The other two are insensitive for certain values of the reflection factor, their sensitivity depending upon this factor in a rather complicated manner. Method (1) virtually allows diffuseness in both incident flux and measured flux, but the extent to which this condition actually prevails depends upon the relative importance of the secondary flux (E_2), and therefore upon the reflection factor itself.

---

[24] A. H. Taylor: "Measurement of Diffuse Reflection Factors and a New Absolute Reflectometer," JOUR. AM. OP. SOC., Vol. 4, No. 1, p. 9; 1920.

[25] F. A. Benford: "An Absolute Method for Determining Coefficients of Diffuse Reflection," *Gen. Elec. Rev.*, Vol. 23, p. 72; 1920.

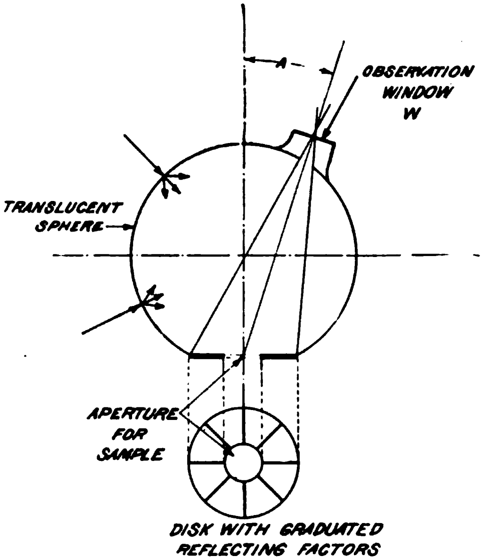
[26] C. H. Sharp and W. F. Little: "Measurement of Reflection Factors," *Trans. I. E. S.* Paper read at Fourteenth Annual Convention, *I. E. S.*, 1920. See also paper by Mr. A. H. Taylor, presented at the same time; and discussion by Karrer.

At all events, so far as a working standard is concerned, the procuring of a magnesium carbonate block at any market is to be discouraged so far as preliminary observations go. Magnesium carbonate blocks purchased at different times may have reflection factors varying by at least several per cent. It would be an admirable thing for some scientific supply house to have in stock magnesium blocks of guaranteed texture and purity. Yet those who have the facilities of a photometric sphere need not rely on even such standards.

VIII. A SIMPLER INSTRUMENT FOR COMMERCIAL USE.

The Nutting reflectometer requires neither lamp nor battery, a great advantage in a portable instrument. This advantage could possibly be obtained with other reflectometers by sacrificing accuracy for serviceability. It might, for example, be possible to accomplish this by the use of a translucent sphere or one in part transparent. In the reflectometer shown in Fig. 3 a mirror mounted so as to have sufficient freedom of rotation was substituted for the lamp to obviate the necessity of maintaining a light source. Unless, however, an intense external light source is available, the brightness is too low. With sunshine it works admirably. For practical uses, however, even simpler instruments might be used; for example, the one shown in Fig. 4. In this an opal glass sphere has a rather large opening covered with a flat disk. The disk has a circular aperture against which the test surface is placed, surrounded by an annular area divided into sectors, having different known reflection factors. The test surface and sectors are equally illuminated by the sphere wall, and are viewed through an aperture, W. The brightness of the test surface is taken to be that of the sector whose brightness most nearly matches it. The use of a simple disk with graduated reflection factors was suggested many years ago. It is not satisfactory where any great amount of specular reflection exists. In the present instrument, such is not the case. Furthermore, a definite direction determined by angle A, Fig. 4, for observing is given. This direction might be chosen most favorably for many common materials met with in practice. Several workers[27] seem to find that at some angle of emergence for normal illumination many surfaces have a reflection factor nearly equal to that of a perfectly diffusing surface of the same diffuse reflection factor.

FIG. 4.



A simple and inexpensive reflectometer for commercial use.

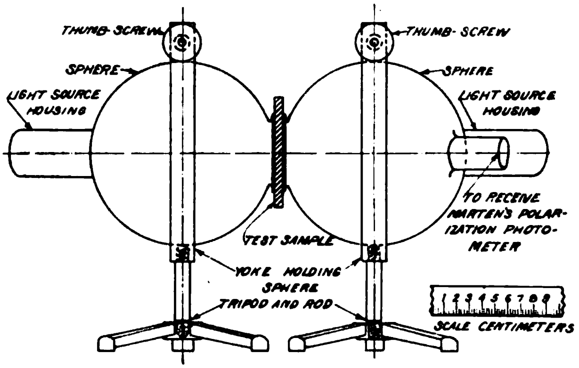
IX. TRANSMISSOMETER.

The diffuse transmission factor is sometimes more important than the reflection factor, as of photographic negatives and in diffusing glassware for illumination purposes. It is therefore desirable to have at hand a means of readily measuring this quantity with a fair degree of accuracy. While there are several ways in which the properties of the Ulbricht Sphere may be taken advantage of for this purpose, let it suffice to point out in detail here only one such way which is merely an extension of that involved in the reflectometer described above. Another similar sphere with an aperture equal to that in the reflectometer sphere, as is shown in Fig. 5, is added. The second sphere is illuminated by the same lamp or by one similar to, and in series with, the one illuminating the first sphere. For a stationary piece of apparatus where no restrictions are placed on size, the same lamp could illuminate both spheres. The flux into the second sphere is controlled by means of a diaphragm, with a variable aperture, such as an iris, slit, or wedge (see Fig. 6). No exacting specifications are necessary. The aperture of the second sphere is screened from the direct flux, as in the reflectometer sphere. In this case, as in the reflectometer, we assume that the area of the aperture is so small that it will not effect the brightness of the sphere walls. This can probably always be sufficiently well attained. It is also assumed that the illuminated area is small.

---

[27] A. H. Taylor finds this angle to be about 50° for many surfaces. Prof. W. J. Drisko informs me that he has found the angle to be about 46° for many surfaces.

FIG. 5.



An absolute transmissometer using two spheres.

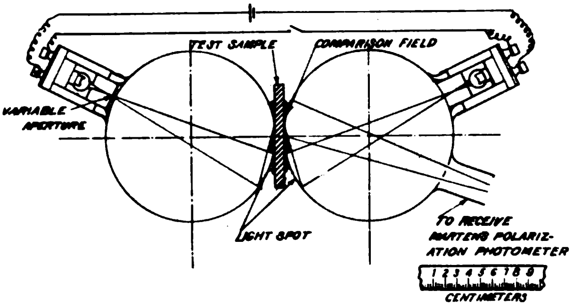
The use of this apparatus as a transmissometer is as follows: The transmission sample is placed over the aperture of the reflectometer as usual and its reflection factor ρ measured. The auxiliary sphere is then brought up so that the reverse side of the specimen covers its aperture. The brightness b_x of the sample now is due to two factors: (1) the flux reflected from its front surface (which is illuminated by the walls of the first sphere); and (2) the flux transmitted by the specimen in consequence of the illumination by the walls of the second sphere. Let the walls of these spheres have a brightness of b and b_1, respectively, and let the transmission factor of the sample be τ. Then $b_x = \rho b + \tau b_1$. The quantity b_1 may be measured in terms of b once for all, and may have any value, but for the sake of simplicity, we shall make $b_1 = b$.

Then $b_x = b(\rho + \tau)$.

$$b_x/b = \rho + \tau.$$

In this equation ρ has been determined; b_x is measured in terms of the brightness of the walls of the reflectometer sphere; τ is the factor desired; and $\frac{b_x}{b}$ is measured and read off directly by means of the reflectometer. We see therefore that it is just as simple so far as manipulation is concerned to obtain the value of the combined reflection and transmission factors as it is to obtain the reflection factor alone. The difference of these two quantities gives the transmission factor. In this way one piece of laboratory apparatus enables us to obtain both the reflection factor and the transmission factor in an absolute manner, involving no calibration of a standard of transmission or reflection, and no light standards, no measurement of distances, no manipulation of the specimen, no electrical measuring instruments. One placing of the sample against the reflectometer is all.

FIG. 6.

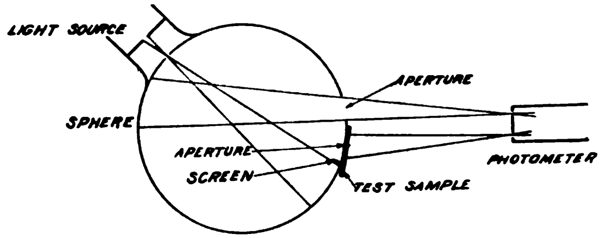


The transmissometer in horizontal section; showing the position of the light source, test sample, photometer and screens.

Although the arrangement described appears to have certain advantages for laboratory purposes where duplication of costly photometric or spectrophotometric apparatus is not desired, or where it is desirable that such apparatus when once set up shall not be disturbed, it may be of interest to describe a few other arrangements that may be had. A simple method would be to insert the sample between the end of the photometer and sphere in such a way that one of the apertures only of the photometer is covered by it. The other aperture of course is illuminated by the sphere wall, which in turn is illuminated by projecting a narrow beam into the sphere. Precautions must be taken to screen the test sample properly as has been noted in the description of the reflectometer. Narrow limits are set to the size and shape of the sample that is to be tested.

A modification of this is shown in Fig. 7, where the photometer is removed from the sphere to a distance that allows greater freedom in the size and shape of the sample. In this case a comparison is made of the brightness b of the sphere wall with the brightness τ b of the sample. The Martens polarization photometer measures τ directly.

FIG. 7.



An absolute transmissometer using one sphere only and the Koenig-Martens polarization photometer.

In all these arrangements nothing novel can be claimed except in the use of the sphere rather than of an irregular inclosure or of superficial standards. The sphere affords two important essentials: (1) the kind of diffuse illumination desired in such measurements, and (2) the conditions for absolute determination. Neither of these can be so closely approximated by any other means.

The combination of the sphere with the Martens Photometer has been suggested and used by Goldberg[28] in measuring the density of photographic plates. His suggestions differ from those given in this article regarding the general use of the sphere in measuring the transmission and reflection factors of substances in that in accordance with the latter the sphere is to be used not only to secure a good diffuseness of the light but to obtain these factors in an absolute manner.

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[28] Photographische Korrespondenz, May–June, 1910. Densograph, ein Registrierapparat zur Messung der Schwarrzung photographischer Platten.

X. USE OF REFLECTOMETER AND TRANSMISSOMETER WITH MONOCHROMATIC LIGHT.

Although the instruments discussed above were primarily intended for mixed light, obviously they may also be used in determining the diffuse reflection and transmission factors of substances for spectral light. For this purpose the Koenig-Martens spectrophotometer is admirably adapted. In this connection, too, the use of the sphere, in ways described above, would make for definiteness and standardization, and afford absolute methods.

Most of this work was carried out with the coöperation of the Searchlight Investigation Section of the Engineers, U. S. Army, who also made the executing of it possible by providing for it. Gratitude therefore is sincerely expressed to them. I am indebted to Mr. U. M. Smith for the illustrations of the text.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

NOTICES.

As the result of the recent election, the following members of the Optical Society of America have been elected officers for the year 1921:

President, J. P. C. Southall.
Vice-President, C. E. Mendenhall.
Secretary, Irwin G. Priest.
Treasurer, Adolph Lomb.
Members at large of the Council:
P. G. Nutting,
C. E. K. Mees,
L. A. Jones,
W. E. Forsythe.

(Signed) F. L. Mohler, *Teller.*
Paul D. Foote, *Secretary.*

The Council has appointed F. K. Richtmyer as the official representative of the Optical Society in the American Association for the Advancement of Science.

Members of the Optical Society should notify the Secretary of change in address. Examine the address on the envelope containing the Journal, and if incorrect please advise.

The membership of the Optical Society is now more than twice as great as that of one year ago and the subscription list to the Journal has increased by three times. In order to continue this rate of growth it is necessary for each member to secure at least one additional member during the next few months. Please make special effort to secure new members so that the success of the society and of its Journal may be insured.

RECENTLY ELECTED MEMBERS OF THE OPTICAL SOCIETY.

(Continued from Page 504, Vol. IV, No. 6, Nov., 1920.)

A. H. Taylor, Nela Research Laboratories, Nela Park, Cleveland, O.
Walter Finnall Cross Ferguson, Navy Yard, Washington, D. C.
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Fred M. Bishop, 95 Troup St., Rochester, N. Y.
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William J. Whiting, care of Naval Inspector of Ordnance Optical Mat., Bausch & Lomb Optical Co., Rochester, N. Y.
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J. H. Powrie,* Hotel Pennsylvania, New York, N. Y.
Emery Huse, 3 Kodak Park, Rochester, N. Y.
Robert Thomas Morrison,* 3 Kodak Park, Rochester, N. Y.
H. E. Buck,* 3 Kodak Park, Rochester, N. Y.
F. R. Fraprie, 428 Newbury St., Boston, Mass.
G. V. Wendell, Physics Dept., Columbia University, New York, N. Y.
Milton Franklin Fillius,* 3 Kodak Park, Rochester, N. Y.
William Edward Story, Jr., General Electric Co., Research Laboratory, Schenectady, N. Y.
Herman W. Farwell, Physics Dept., Columbia University, New York, N. Y.

---

* Associate membership.

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Paul Anderson, R. R. 3, Anderson, Mo.
A. J. Newton, 343 State St., Eastman Kodak Co., Rochester, N. Y.
Heber D. Curtis, Allegheny Observatory, Pittsburgh, Pa.
W. Ewart Williams, University of London, King's College, Physics Dept., Strand W.C. 2, London, England.
Frank Schlesinger, Director Yale University Observatory, New Haven, Conn.
Hibbard S. Busby, P. O. Box 841, South Manchester, Conn.

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Devoted to all Branches of Optics

Vol. V MARCH, 1921 Number 2

A WIDE-ANGLE ASTRONOMICAL DOUBLET.*

BY

FRANK E. ROSS.

For numerous special lines of work in astronomy there appears to be a need for photographic lenses having properties not to be found in existing lenses.

The degree of perfection of a lens or of an optical system is measured in terms of tolerances, absolute perfection being unattainable. The difficulty facing the astronomer in particular in the matter of lens equipment is due to his requiring smaller tolerances than those which govern the lens designer in the commercial development of the photographic lens. In general, the needs of the greatest number of photographic workers are met in a high-speed lens covering a wide field with moderate definition. The astronomer may or may not want a high-speed lens, but it is absolutely necessary that the definition be what is called critical from the centre to the edge of the plate. In general, the *central* definition of a well-made commercial lens, especially if retouched, is sufficiently good to meet the astronomer's exacting needs. But on passing outward from the axial or central region, the definition in the case of all lenses will "break" before many degrees have been passed over, defects of coma, astigmatism, and field curvature appearing, which are more or less serious, depending on the particular use to which the lens is put. In the case of several existing lens-types, these errors do not reach a serious amount until a point

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* Communication No. 107 from the Research Laboratory of the Eastman Kodak Company.

5 degrees from the axis is reached. From 5 degrees to 20 degrees, or even to 30 degrees or more in some cases, the definition retains a certain average quality which is sufficiently good for the purpose for which the lens was designed. In short, the object of the designer has been to secure a large field of moderate definition rather than a small field of critical definition. So far as the writer is aware, in no case has a lens been designed which gives critical definition as much as 10 degrees from the axis, a flat photographic plate being presupposed.

The errors which arise in photographing an extended field are of two kinds: (1) errors in position; (2) errors in intensity, or photometric errors. Errors in position are due to general distortion of the image-field and to lack of symmetry in the image-cone where it cuts the plane of the photographic plate. This asymmetry is due to the defects of coma and astigmatism. Photometric field errors are due to asymmetry of image and field curvature. There is an additional photometric error due to vignetting by the mount of the lens, if the stop is not sufficiently small compared with the clear aperture, and to foreshortening of the stop itself. If the field of the lens is not flat, vignetting produces an error in position as well.

The lens about to be described is a by-product of the war-time activity of the Eastman Kodak Company in lens design and manufacture. It became necessary to design for the aerial service a lens of 6 inches aperture and 48 inches focal length, having a 15-degree field of absolute flatness. The field of this lens is shown in Fig. 2 (Hawk-Eye 48 inch). One of these lenses was constructed in 1918, and passed the most rigid test. An attempt to reduce the small outstanding astigmatism in this lens was successful to such an extent that it was felt that a lens had been designed which would meet the exacting needs of the astronomer in several lines of research where a wide field is of advantage. The aperture-ratio, or ratio of opening to focal length, is not as great as could be wished for certain lines of work, as, for example, in photographing nebulæ and comets. To what extent an extended field having perfect definition can be combined with a large aperture in a single lens can only be conjectured. The problem is one of enormous difficulty. It is, however, certain that the design of the present lens can be altered to produce a lens working at an aperture ratio as high as 1/7 without materially affecting the definition over an extended field. But as there is a large program of work calling for an aperture ratio not greater than 1/14, it was deemed best to select this value as the limiting aperture, aiming with it as a foundation to produce a field as nearly perfect as possible. A further advantage in choosing the small opening is the greater thinness of the component lens-elements, giving a higher relative light-transmission and a shorter lens. It will be noted below that there are but four different radii, thus minimizing the labor of manufacture.

Specifications for Wide-Angle Astronomical Lens.

Glass (Catalogue of Chance Bros., January, 1920):

Cat. No.	μ_D	ν	μ_F	μ_G		
580	1.60650	58.0	1.61388	1.61984	Positives	A and D
458	1.54720	45.8	1.55568	1.56275	Negatives	B and C

The radii are, counting from the object side:

$R_1 = 47.70$ mm.; $R_2 = 186.639$ mm.; Biconvex Lens A
$R_3 = R_4 = R_5 = R_6 = 71.5925$ mm.; Biconcave Lens B and C
$R_7 = 186.639$ mm.; $R_8 = 45.62$ mm.; Biconvex Lens D

Thickness of lens elements:

$t_A = t_D = 2.3$ mm. $t_C = t_D = 1.0$ mm.

Separation of lens elements:

S_1 (axial distance between adjacent surfaces of A and B) = 6.2 mm.
S_2 (axial distance between adjacent surfaces of B and C) = 5.9 mm.
S_3 (axial distance between adjacent surfaces of C and D) = 6.6 mm.

Focal Length (G′ Line) .... 204.380 mm.
Back Focus (distance of focal point from rear surface) 193.674
Distance of first Gauss point within lens (G′ Line) 16.620
Distance of second Gauss point within lens (G′ Line) .. 10.706
Focal Length (A + B) .... 407.37 mm.
Focal Length (C + D) .... 310.13 mm.

Color correction:

Back Focus (F — G′ line) = +.015 mm.

A lens of any desired focal length can of course be constructed by multiplying all dimensions given by the appropriate factor.

Longitudinal Spherical Aberration and Sine Condition:

Aperture	Spherical Aberration mm.	Sine C mm.	Coma mm.
1/21	—.005	—.010	—.005
1/18	+.005	+.004	—.001
1/14	+.049	+.052	+.003

Field aberrations:

Angle	Distortion mm.	Secondary Image mm.	Primary Image mm.	Best Image mm.
5°	.0000	—.013	+.028	+.008
7½	—.0007	—.043	+.047	+.002
10	—.0024	—.025	+.018	—.004
12	—.0035	+.054	—.017	+.019
14	—.0057	+.209	—.157	+.026

The distortion, secondary and primary image in this table are for pencils pointed at the first Gauss point at the angles given. Let D be the distance at which any pencil, pointed at the first Gauss point at an angle α cuts the focal plane, measured from the intersection of focal plane and axis. If F is the focal length, the distortion in the above table is calculated from the formula

$$\text{Distortion} = D - F \tan \alpha .$$

The diameter of stop, placed centrally, giving a speed ratio of F/14 is 12.7 mm. The clear aperture of the lens elements is 20 mm. With a clear aperture of this amount there is no loss of intensity from the lens mount up to 10° from the axis. The loss of intensity from the foreshortening of the stop is small, of the order of 1½ per cent. at 10°.

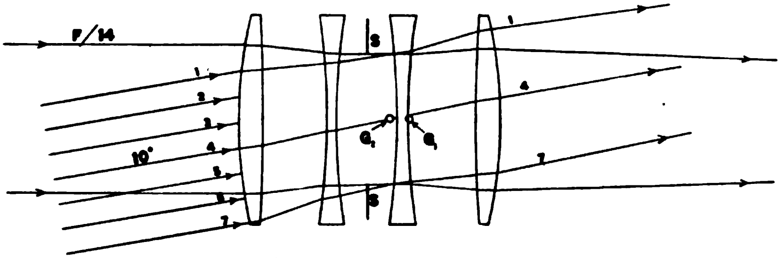
Coma: Freedom from coma for small field angles is secured when the curves of spherical aberration and of sine condition are identical. From the values given above, this condition is seen to be fulfilled. For the larger angles, however, it is not always a true criterion for the absence of coma. It is accordingly necessary to trace trigonometrically a number of rays lying in the meridional plane parallel to each other and intersecting the stop at various distances from its centre. This has been done, with results given in the following table. The common inclination of the rays is 10°:

TABLE I.

Ray	D mm.	V		
1	36.037	7°	57′	22″
2	36.035	8	37	51
3	36.035	9	18	26
4	36.035	9	59	6
5	36.035	10	39	51
6	36.038	11	20	44
7	36.041	12	1	43

The rays and their course through the lens are shown to scale in Fig. 1. The calculations are for the G′ line. The central ray, No. 4, is aimed at the first Gauss point. The rays are equally spaced, the perpendicular distance between consecutive rays being 2.457 mm. D is the distance at which the emergent ray cuts the focal plane, measured from the axis; V is the angle which the corresponding ray makes with the axis. A very slight amount (.003 mm.) of residual coma is seen to be present, agreeing with the result already found at F/14. Ray No. 7 is seen to be unsymmetrical as compared with the others. But reference to Fig. 1 shows that this ray cannot traverse the lens unless the stop is in contact with the second negative element. The best position of the stop will be close to the first negative element. In the figure it is shown in a central location. It is very easy to eliminate the small residual coma just found. It does not seem worth while, however, to make a recomputation. For coma is controlled by the ratio of the radii of the first and last surfaces, or R_1/R_8. It is found by calculation that upon diminishing R_1 by one-tenth of one per cent. and increasing R_8 by the same amount, the effect on coma is –.003 mm., or just sufficient to eliminate the amount now outstanding. But it is doubtful if these surfaces can be ground to this degree of accuracy, so that further elimination of coma would be but a refinement of calculation impossible to realize in practice. If when constructed a lens is found to have coma present, the coma can be progressively "worked out" by small radii changes in R_1 and R_8. If positive coma, for example, is present as in our case (edge zones focusing further from the axis), R_1 should be decreased and R_8 increased equally; if negative coma is present, R_1 is to be increased and R_8 decreased.

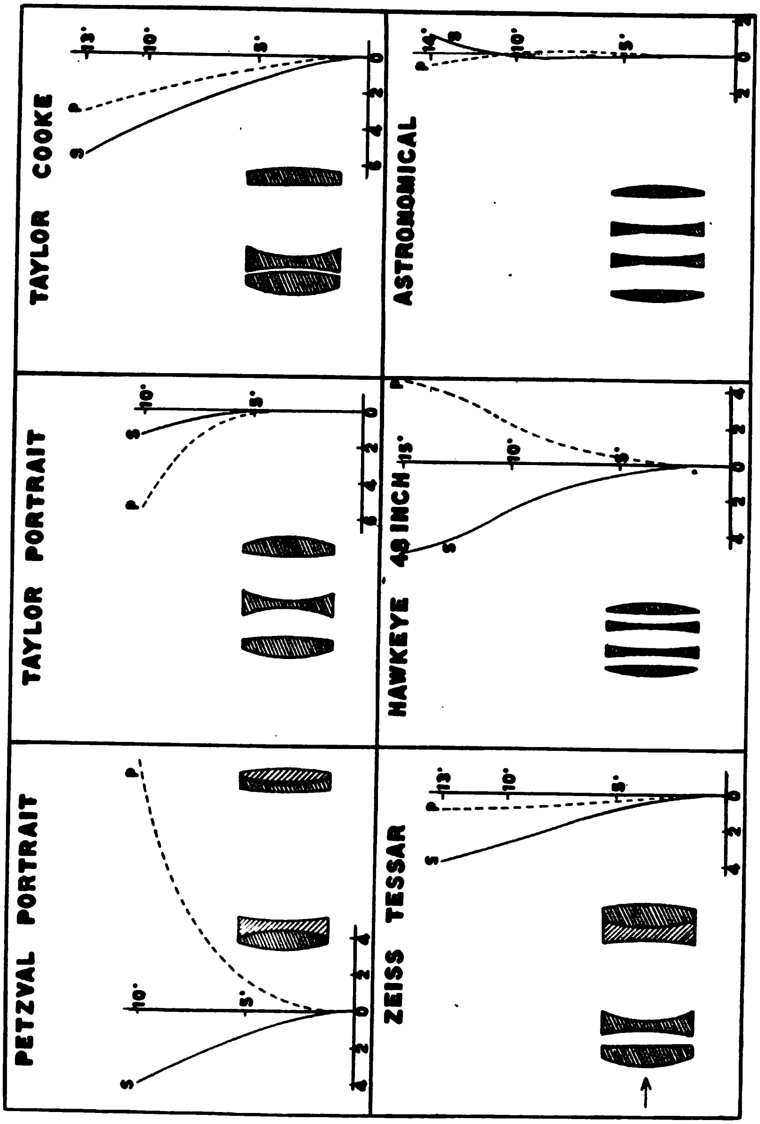
FIG. 1.



Wide-angle astronomical doublet.

Astigmatism.—There is no question but that the amount of residual astigmatism and field curvature in the lens as calculated,

FIG. 2.



FOCAL LENGTH 1000

Field curves of lenses.

within 12° of the axis at least, is much smaller than can be detected by the most rigid tests. But in manufacturing, astigmatism and accompanying it field curvature are sometimes introduced. They can be very simply and effectively removed by altering the central separation S_2. If, for example, the primary focus is found to be nearer the lens than the secondary focus by one-tenth of one per cent., at a field angle of 10°, it can be corrected by increasing the central separation S_2 by 6 per cent.

Distortion.—Distortion has been corrected by making S_1 and S_3 unequal, or bodily moving the two negative lenses forward by the same amount. Negative or barrel distortion is corrected by moving the negatives to the rear; positive or pin-cushion distortion is corrected by moving them forward. The lens separations and thicknesses can be measured so accurately that there appears to be no danger of introducing any distortion that can be detected by any but the severest tests. In the above lens a forward movement of the negative elements of 0.1 mm. produces a negative distortion of .01 mm. at 10°.

Color.—The lens has been calculated to bring the F and G′ rays into coincidence. Any error due to faulty determination of the dispersion of the glasses or to other causes can be corrected by altering S_1 and S_2 by equal amounts. For example, in the lens as given an undercorrection of 0.1 mm. can be corrected by decreasing S_1 and S_3 by 0.25 mm. There is, however, in this case a considerable change introduced into the focal length, amounting to +4 mm., or 2 per cent. Since, however, the effect of any color error is symmetrical, of the same nature as spherical aberration or unsteadiness in seeing, it is not serious, unless of large amount. The correction should, therefore, not be undertaken.

It will be of interest to compare the field characteristics of the present lens with those of lenses already in use. These are collected in Fig. 2. Only lenses are included which have some claim to be considered wide-angle lenses in the astronomical sense. In the case of the Petzval Portrait and the two Taylor lenses, the data are taken from Von Rohr's "Theorie und Geschichte des photographischen Objective." The Tessar curves are from calculations of the writer. Those of the Hawk-Eye 48-inch are from measured curves. The curves of the "Astronomical," which is the designation of the present lens, are from numerical data given above. It will be noticed that in the case of the Petzval

Portrait and the Hawk-Eye 48-inch, there is considerable astigmatism, but very small field curvature. The Petzval Portrait has proved very satisfactory in astronomical photography over a total field of about 8°. The Taylor Portrait is seen to be nearly perfect over a half-field of 5°, but "breaks" very rapidly from this point on. The Cooke and Tessar are of the same general type, with inward curving field. It is quite likely that the Hawk-Eye 48-inch would give critical definition over a half-angle of 8°. The definition of the "Astronomical" should be critical over a half-angle of 14° or 15°.

ROCHESTER, N. Y.,
November 26, 1920.

THE PRESENT STATUS OF THE CONSTANTS AND VERIFICATION OF THE LAWS OF THERMAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.*

BY

W. W. COBLENTZ.

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I. INTRODUCTION.

THIS seems to be an appropriate time for a retrospect of past accomplishments and for a new starting point in the determination of the constants of thermal radiation, which investigations, everywhere, have suffered much from interruption during the past six years.

The various determinations of the numerical values of the constants of radiation have been hampered by the lack of exact knowledge of numerous contributory constants which enter into their evaluation.

Among these factors are the temperature scale, the absorptive coefficients and refractive indices of the prisms used, the correction for atmospheric absorption, and the correction for reflection from the radiometer receivers.

When the writer began studying this subject, about fourteen years ago, it appeared desirable to determine, more exactly, the numerical values of the contributory constants, first of all, and then take up the determination of the constants of thermal radiation. At this day it is quite evident that such a procedure would have been the most efficient in obtaining the desired results. While such a program could not be fully carried out, some progress was made by determining the diffuse reflection of various substances [1]† (*e.g.*, platinum black), the absorption constants of quartz,[2] and the correction for atmospheric absorption.[3, 4] Somewhat similar experiments have been made by Warburg[5] on the absorptivity of quartz and by Gerlach[6] on the atmospheric absorption.

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* Section of 1920 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

Instead of a unified program of research, we find that while one set of experimenters was engaged in establishing the temperature scale, another experimenter was determining the optical constants of the prism material and a third group, using both the temperature scale and the optical constants, was attempting to determine the radiation constants. In turn, the first set of experimenters had to apply the results of those working on the radiation constants in order to verify and extrapolate the temperature scale beyond that attainable with the gas thermometer. In the meantime, the second experimenter improved his measurements of the optical constants (refractive indices) of the prism material (fluorite and quartz), while a fourth entered the field and added refinements by determining the temperature coefficients of these refractive indices. This in turn necessitated the recalculation of the prism calibration and the numerical values of the spectral radiation constants. Added to these difficulties is the constant change in personnel, as is apparent from perusal of the title pages of the published papers. The foregoing is an impersonal statement of the difficulties under which experimenters have been laboring. Fortunately, the spectral radiation observations are in terms of the scale of the spectrometer circle, and if the necessity arises, as described on a subsequent page, one can revise the corresponding wave-lengths and recalculate the constants.

As a result of these various difficulties there is scarcely a determination of the constant, σ, of total radiation, and of the constant, C, of spectral radiation, which does not require corrections, some of which are obvious and could have been made by the experimenter.

It seems appropriate to point out specifically some of the difficulties experienced by the two national laboratories.

The Reichsanstalt Investigations.—The determinations of the radiation constants, made at the German Reichsanstalt during the past two decades, involved measurements extending to about 1600° C., or even higher, and hence they have suffered from both the lack of knowledge of the temperature scale and of the refractive indices of the prisms used. In the later work, the determination of the temperature scale was rightfully combined with the measurements of the radiation constants, instead of having the former measurements made in a separate laboratory.

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† References will be found at end of text.

The published papers do not state whether an attempt was made to harmonize the various determinations of the refractive indices of fluorite. The most recent work was done with quartz prisms and the determination of the indices of refraction of quartz was undertaken,[7] but as yet no data have been published.

The Bureau of Standards' Investigations.—The determinations of the radiation constants, especially the more recent ones, made at the Bureau of Standards, have suffered but little from a lack of knowledge of the temperature scale, owing to the fact that the radiators were not operated at a temperature much above 1200° C., and usually near the melting point of copper—1083° C. The chief difficulty has been the lack of exact knowledge of the refractive indices of fluorite; or, as it appears from the present inquiry into the matter, the lack of exact knowledge of the refractive index for the yellow sodium line, and especially the yellow helium line, which was used as a reference point in calculating the infra-red calibration of the prism.

The size of the apparatus used in determining the refractive indices is an important element in the accuracy attained. Langley[8] used a spectrometer, having an image-forming mirror of over 4.5 metres focal length, in an enclosure which could be kept at a constant temperature. Paschen's[9, 10, 11, 12] apparatus had an image-forming mirror of only 1/10 to 1/12 that of Langley's and hence could not compete with it in accuracy. Hence, from the beginning, more weight should have been given to Langley's measurements. On applying Micheli's[13] temperature coefficients of refraction, Paschen's various determinations are found in good agreement with those published by Langley,[8] whose reference point is the solar absorption line (the "A line") $\lambda = 0.7604\mu$; refractive index $n = 1.431020$ for fluorite at 20° C. From this point into the infra-red, the various determinations of the infra-red refractive indices of fluorite are in excellent agreement.

However, the fiducial line in this Bureau's spectroradiometry is the yellow helium line. From the present study of the data it appears that prior to 1913 the refractive indices of fluorite, for the visible spectrum, and in particular that of the yellow helium line $\lambda = 0.58758\mu$, were not known with sufficient accuracy to meet the present-day demands for refinements in the radiation constants. As a result, erroneous wave-lengths were assigned to the infra-red (minimum deviation) settings of the spectiometer circle. This error is only 3″ (true value, 6″) in the Wadsworth system of spectrometer settings and hence scarcely observable experimentally. However, it enters as a constant error of 0.3 to 0.4 per cent. in the calculations of the calibration curve. Errors of such small magnitude were considered insufficient eight years ago, when there was a disagreement of 1 to 2 per cent., or even more, in the various determinations of the constant of spectral radiation.

As will be shown on a subsequent page, using Paschen's[12] recent measurements of the refractive index of fluorite for the yellow helium line (which measurements were made with a new and very accurately graduated spectrometer), all the apparent errors seem to be eliminated from this Bureau's determination of the constant of spectral radiation which is now in good agreement with the value that is to be expected from this Bureau's determination of the constant of total radiation[15] and from theoretical considerations.[4]

Object of the Present Paper.—In previous papers[15, 4] corrections were made to the determinations of the radiation constants applied by various experimenters. The object of the present paper is to make an up-to-date examination into the various instruments and methods employed in these various determinations, and to give an estimate of the results therewith obtained.

At first glance, the various determinations of these constants appear to cover a wide range of numerical values. This is especially true of the values of the coefficient of total radiation, in which all sorts of ill-considered methods have been used. Moreover, no corrections were made for atmospheric absorption of the thermal radiation in its passage from the radiation to the receiver. As will be shown presently, most of these older determinations are in markedly close agreement with the newer ones after making obvious corrections for atmospheric absorption and for reflection of the incident radiation from the receiver.

This paper does not include a discussion of the attempts which have been made to determine the constant, *C*, by means of an optical pyrometer, which at least in its ordinary form cannot be considered an instrument of high precision. There are many difficulties to be overcome before one can conclude that the data so obtained are trustworthy. One of the uncertainties is the effective wave-length of the red glass used in the eyepiece of the pyrometer. Other difficulties are encountered when using a spectral pyrometer. Also the temperature scale is in doubt. Hence recent experimenters have not attempted to determine C, but have reversed the process and, working on the assumption that C is a certain value, say $C = 14350$, have determined the melting point of palladium. This seems to be the preferable procedure.

II. DISCUSSION OF BLACK BODY RADIATION, RADIATORS, SHUTTERS AND REDUCTION FORMULAS.

In order to arrive at an intelligent understanding of what has been accomplished in the determination of the constants of radiation, and in order to indicate the way to future progress in this subject, it is important to consider very briefly some of the conceptions, as well as some of the most recent instruments and methods used in the production and measurement of the radiation from a uniformly heated enclosure or so-called black body.

Every substance has a characteristic emission spectrum. On the other hand, the emission spectrum of a uniformly heated cavity is independent of the composition of the material forming the walls of the enclosure. It is a function only of the temperature.

The emission of thermal radiation from, and the absorption of thermal radiant energy of, all wave-lengths entering into the cavity is complete. In a material substance this can occur only on the short wave-length side of the region of anomalous dispersion where the absorptivity is high but where the reflecting power is practically nil; for example, in quartz,[29] selenite and aluminum oxide, in the region of 7μ to 10μ ($\mu = 0.001$ mm.).

Modern conceptions of black body radiation are the result of slow development from such simple experiments as those of Draper,[19] who found that the interior of a rifle barrel became luminous at a certain temperature; of Christiansen,[21] who observed that the scratches and conical-shaped holes in an incandescent metal surface were brighter than the smooth surface; and of St. John,[22] who found that the selective emission, commonly observed in the rare oxides, disappeared when these substances were heated in a uniformly heated porcelain tube.

Radiators.—The modern uniformly heated cavity or so-called black body is the result of such observations and facts as just enumerated. It is the invention of Wien and Lummer [28] and, for attaining fairly high temperatures, such as are used in many of the investigations mentioned in a subsequent part of this paper, consists of a diaphragmed porcelain tube wound with platinum ribbon, which is electrically heated.

In the modification used by Coblentz [30] the radiator consisted of two concentric tubes suitably wound with platinum ribbon in order to obtain a uniform temperature throughout a great length of the interior of the radiator, as shown by Waidner and Burgess.[31] The use of platinum ribbon instead of thick platinum wire eliminates local non-uniformity of temperature in the porcelain tube.

In radiators of this type the porcelain tubes sag on heating to 1200° C. To overcome this difficulty, in the determination of the radiation constants, to be described presently, Coblentz [30] placed a wedge-shaped fragment of porcelain, with a sharp edge, about 2 mm. long, about 25 to 30 mm. to the rear of the radiating diaphragm. This support causes no local cooling, and it preserves the life of the radiator. Further improvements were made in the completeness of the emission of the radiation by painting the inside walls and the front side of the radiating diaphragm with a mixture of chromium oxide and cobalt oxide. Since these oxides become electrically conducting at 1200°, the front thermocouple was completely inclosed in a porcelain insulating tube. The short piece of porcelain which lies across the radiating wall of the black body was covered with cobalt oxide. Since the cobalt oxide did not adhere well to the porcelain tube, the latter was first covered with a thin layer of iron oxide, obtained by wetting the porcelain tube with writing ink, which was burned into the tube. The cobalt oxide paint was then applied and also burned upon the tube by means of a blast lamp. After replacing the thermocouple, inclosed in this short tube within the radiator, the whole interior was again painted with a mixture of cobalt and chromium oxides.

The cobalt oxide has a high temperature coefficient of absorption so that it appears black on slight heating. Its emission spectrum [29] is continuous, so that there is less difficulty in producing a perfect radiator than when the radiating inclosure is of white porcelain. However, porcelains having a low melting point when heated above 1000° emit a far more continuous spectrum than the "Marquardt porcelain" from which are made the black bodies ordinarily used.

Water-cooled Shutters and Diaphragms.—In discussing the relative merits of the various determinations of the radiation constants, and in particular the coefficient of total radiation, the writer will have occasion to refer frequently to the use, or misuse, of shields for preventing the receiver from being heated by the radiator, and of shutters for exposing the receiver to radiation. It is, therefore, important to consider briefly the functions of this apparatus.

The determination of the coefficient of total radiation usually consists in noting the heat interchange when the receiver is exposed to two radiators which are at widely differing temperatures, say 0° C. and 100° C., *i.e.*, ice and boiling water. The radiator at the lower temperature may be, and usually is, used as a shutter. If the temperature of the shutter is lower than that of the receiver, the latter radiates to the shutter. Hence, it is important to have the receiver face a large (say, water-cooled) diaphragm, maintained at a constant temperature, back of which is placed the shutter and the radiator. In this manner the surrounding conditions, facing the receiver, are not changed when the shutter is moved in order to expose the receiver to radiation.

There is no doubt that some of the unusual results obtained in the determination of the radiation constants are owing to the operation of the shutter under such conditions that the receiver did not face uniform conditions when the shutter was operated in making the measurements.

Experience shows that the arrangement of the shutter should be similar to that used[3] (*loc. cit.*, p. 516, Fig. 5) in the determination of the coefficient of total radiation. In this arrangement the water-cooled shield employed consisted of a tank, 35 cm. in diameter and 1.5 cm. in thickness, which faced the radiator, and a second tank, 20 cm. in diameter and 3 cm. in thickness, which faced the radiometer. The water-cooled shutter, consisting of a thin metal box 3.5 by 3.5 by 0.8 cm., was operated in vertical ways between these two shields. A mercurial thermometer was used to measure the temperature of the water flowing through this shutter. The side of the shutter facing the radiometer was smoked in a sperm candle, and in connection with the conical-shaped opening (painted black and smoked) in the water-cooled diaphragm which faced the radiator it formed a miniature black body, the temperature of which remained constant throughout a series of measurements.

Statement of the Laws of Radiation.—The law of total radiation, generally called the Stefan-Boltzmann law,[17] states that the total emission of thermal radiation, R, of all wave-lengths passing from a uniformly heated enclosure, at temperature T_1 to another body at a temperature T_0 is proportional to the difference of their absolute temperatures to the fourth power, or

$$R = \sigma(T_1^4 - T_0^4) \qquad \text{(1)}$$

In this formula, σ is the coefficient or constant of total radiation under discussion. The numerical value of this coefficient is of the order $\sigma = 5.7 \times 10^{-5}$ erg per cm.^{2} per deg.^{4} In this paper only the first three significant figures will be mentioned, *e.g.*, $\sigma = 5.32$.

The law of spectral radiation, which specifies the distribution of thermal emission intensities in the spectrum of the radiation emitted by a uniformly heated enclosure, or so-called black body, is best represented by Planck's [18] formula

$$E_\lambda = c_1 \lambda^{-5} (e^{C/\lambda T} - 1)^{-1} \qquad \text{(2)}$$

In this formula the constant C determines to a great extent the shape of the isothermal spectral energy curve. Suffice it to say that these laws are based on well-grounded theoretical foundations, and after almost two decades of discussion they remain unchanged.

Recently, Nernst and Wulf [78] have arbitrarily modified the coefficient in the Planck equation, changing C_1 to $C_1(1 + \alpha)$ where α is a variable. The whole procedure is based upon the assumption that the numerical value of the constant is $C = 14300$, as observed from isochromatic observations. Hence, in view of the fact that the older values of C obtained from isothermals are somewhat higher, they must be reduced by a factor $(1 - \alpha)$, depending upon the temperature of the radiator. They point out explicitly that the whole procedure depends upon the truth or falsity of the value of $C = 14300$; or rather, as will be seen presently, upon the truth or falsity of the older values of $C = 14400$ (about) obtained from isothermal measurements. Their deductions lead to a value of $\sigma = 6.04$, which, from a consideration of the whole subject, appears to be much higher than the true value. Until we have more reliable experimental data using isothermals, it does not appear necessary to consider a modification of the Planck equation.

The isothermal spectral energy curves published by Coblentz[76] (*loc. cit.*, pp. 474 and 476, Figs. 1 and 2) show that, in the region free from atmospheric absorption to 6μ, the agreement between the observed and the computed curves agree to within the errors of observation, *viz.*, 0.5 to 1 per cent.

Rubens and Kurlbaum[77] have made isochromatic observations, using the residual rays of fluorite in the spectral region of 24μ and 52μ. Their data are also in exact agreement with those computed by the Planck equation. From these data it appears that the Planck equation may be considered as representing within the errors of observation the energy distribution of a black body in the spectrum extending from 0.5μ to 50μ.

Spectral Radiation Formulas.—For computing the constant C from an *isochromatic* energy curve, at any wave-length, Planck's equation is used in the following form:[30]

$$C = \frac{(\log E_2 - \log E_1)\lambda T_1 T_2}{\log e(T_2 - T_1)} - \frac{(e^{-c/\lambda T_2} e^{-c/\lambda T_1})\lambda T_1 T_2}{T_2 - T_1} \quad \text{.........(3)}$$

where E_1 and E_2 refer to the emissivities corresponding to the temperature T_1 and T_2, respectively. In this equation the terms $\log(1 - e^{-c/\lambda T_1})$, etc., are expanded in series and only the first term of the expansion $(-e^{-c/\lambda T_1} \log e)$ is used. An approximate value of $C = 14300$ is used in applying the second term correction. For wave-lengths up to 1μ this correction term (to the Wien equation) is small, but beyond 1μ these corrections increase very rapidly with wave-length. This explains the rise in the value of C with wave-length as found by Lummer and Primgsheim,[71] who did not use the second term correction. Energy curves which coincide closely with the Wien equation give uniform values of C with increase in wave-length, when computed by using only the first term in equation, as was found by Paschen.

Using an *isothermal* spectral energy curve, the constant of spectral radiation C (eq. 2) may be derived from the value of the wave-length of maximum emission, λ_m, by means of the equation:

$$C = a\,\lambda_m T \quad \text{..(4)}$$

in which the value of $\alpha = 4.9651$. This equation makes use of the Wien displacement law, which is the mathematical expression for the shifting of the wave-length of the maximum emission toward the shorter wave-lengths, with rise in temperature, *viz.*,

$$\lambda_m T = A \text{ (a constant)} \quad \text{..............................(5)}$$

The value of the constant is close to A = 2885 micron degrees.

In case one cannot observe the complete energy curves, as, for example, when using a quartz prism which absorbs heavily beyond 2.2μ, it is possible to compute the value of λ_m from the Planck equation by reducing the observed energy curve to the normal spectrum and taking from the energy curve the values of E_1 at λ_1 and E_m. The proper equation is:

$$\frac{E}{E_m} = \left(\frac{\lambda_m}{\lambda}\right)^5 \frac{(e^{4.9651} - 1)}{(e^{4.9651\lambda m/\lambda} - 1)} \qquad (6)$$

This equation, which has been extensively used by Warburg[86] and his collaborators, appears to be well adapted when using a quartz prism; but the temperatures must be sufficiently high so that the E_m does not fall in the region beyond 2.2μ, where quartz begins to absorb heavily. This equation is not well adapted to a fluorite prism owing to the small dispersion of the prism, and hence the steepness of the energy curve on the short wave-length side where one would want to use it, and also owing to the fact that, at the temperatures usually employed, the E_m falls in the region of atmospheric absorption bands or in the region of 1.6μ where the dispersion curve of fluorite has a point of inflection. The latter makes it quite difficult to determine accurately the factors for reducing the observations from the prismatic to the normal spectrum.

Another method of determining the values of λ_m from an isothermal spectral energy curve is by the "method of equal ordinates." For this purpose the normal spectral energy curve is drawn on accurately ruled coördinate paper and at two points where the intensities (the galvanometer deflections) are equal, $E_1 = E_2$, the corresponding wave-lengths λ_1 and λ_2 are read from the curve. The value is easily derived[30] from equation (2) by equating $E_{\lambda_1} = E_{\lambda_2}$ and for the complete solution it is:

$$\lambda_m = \frac{5(\log\lambda_2 - \log\lambda_1)\lambda_1\lambda_2}{a(\lambda_2 - \lambda_1)\log e} - \frac{\lambda_1\lambda_2[\log(1 - e^{-c/\lambda_1 T}) - \log(1 - e^{-c/\lambda_2 T})]}{a(\lambda_2 - \lambda_1)\log e} \qquad (7)$$

The second term in this equation can usually be abbreviated, since terms involving λ_1 are usually negligible. For values of λ_2, which are less than 4μ, the term $\log(1 - e^{-c/\lambda_2 T})$ may be expanded into a series and (by dropping all terms but the first) may be used in the form $-e^{-c/\lambda_2 T}\log e$. In this equation $\alpha = 4.9651$. It may be noticed that an approximate value of C is necessary in order to obtain a solution for the second-term factors in equation (7). This may be obtained by solving for λ_m by using only the first term in equation (7), which is the solution of the Wien equation as used by Paschen.[69] It is sufficiently accurate [30] to use $C = 14300$ in making this computation. A decrease of 100 units in C (say, $C = 14200$) decreases the mean value of λ_m by 0.0012μ.

The method of equal ordinates was extensively used by Paschen [69] and by Coblentz.[30, 76] It has several commendable features because it is possible to utilize values of λ_1 and λ_2, which correspond with values of E_{λ_1} and E_{λ_2}, which are closely the same in magnitude as originally observed, and hence contain the same errors of observation; also the slit widths are closely the same. This method does not require the most accurate wave-lengths and refractive indices (at least not for values of λ_2 greater than 4.5μ) and it permits the selection of parts of the spectral energy curves which are free from atmospheric absorption bands.

The method of equal ordinates necessitates reducing the prismatic spectral energy measurements to a normal spectrum, plotting the data upon coördinate paper and drawing in a smooth curve from which the values of λ_1 and λ_2 are read, corresponding with the equal ordinates E_{λ_1} and E_{λ_2}. Although it requires but little additional time to plot the data after having made the observations, if one is certain that the observations lie close to the curve, the obviously logical procedure is to compute the spectral radiations constants from any two observed points E_{λ_1} and E_{λ_2}. This, however, does not shorten the observational work, for, as in previous investigations, it will be necessary to make observations at three adjacent spectrometer settings (separated by the width of the radiometer receiver) in order to determine the spectral purity factor which is required in order to reduce the data to a normal spectrum. After spending months in obtaining the data, the question of saving time in their reduction is of minor importance. It is desirable to draw the complete curve and preserve it for future reference. If the observations do not lie close to the smooth curve, then this method is slightly arbitrary as to the manner of combining the observations so as to obtain an integration of the whole curve without making more computations than would be used in the "method of equal ordinates."

The appropriate formula for computing the radiation constants from any two observed points E_{λ_1} and E_{λ_2} on an isothermal spectral energy curve is easily deduced[76] from Planck's equation, and in its complete form, as shown by Dellinger,[80] is:

$$C = \frac{\lambda_1 \lambda_2 T}{(\lambda_2 - \lambda_1) \log e} \left[\log \frac{E_{\lambda_2}}{E_{\lambda_1}} + 5 \log \frac{\lambda_2}{\lambda_1} - \log \left(\frac{1 - e^{-c/\lambda_1 T}}{1 - e^{-c/\lambda_2 T}} \right) \right] \ldots\ldots (8)$$

The term $\log (1 - e^{-c/\lambda T})$ may be expanded into a series and usually all the factors can be neglected, except one which is $\log_e \cdot e^{-c/\lambda_2 T}$. It is therefore necessary to know the approximate value of C, as was required in Coblentz's[30, 76] method of computation.

In concluding this discussion we may notice a calculation of the constant C by an extension of the theory of least squares by Roeser.[81] His calculation gave a value of $C = 14342$, as compared with $C = 14339$ computed by Coblentz[76] (*loc. cit.*, p. 462), by the method of equal ordinates.

A spectrophotometric method may also be used in determining the constant C. It consists in determining the ratio of brightness of a black body at, say, the melting point of gold (1063° C.) and at some higher temperature, say the melting point of palladium (about 1556° C.). This requires a knowledge of the wave-length ($\lambda = 0.65\mu$ is usually taken) of the light photometered and the absolute temperatures T_1 and T_2 of the radiator. The appropriate formula is:

$$\log \frac{E_2}{E_1} = C \frac{\log e}{\lambda} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The intensities E_1 and E_2 are usually measured with an optical pyrometer. If the constant C is known the temperature T_2, say 2000° C., may be computed. Conversely using known temperatures T_1 and T_2 the constant may be determined. As a matter of fact, the method has never been used very successfully.

III. DISCUSSION OF VARIOUS DETERMINATIONS OF THE COEFFICIENT OF TOTAL RADIATION.

It has been shown by Stefan,[16] Schneebeli,[25] Wilson,[26] Lummer and Pringsheim,[24] Valentiner,[27] and others,[3] that the total radiation emitted from a uniformly heated cavity is proportional to the fourth power of the absolute temperature. The measurements extend over a temperature range of perhaps 1500° C. They verify the fourth power law within the accuracy of the temperature scale, and the general errors of observation which are small owing to the fact that the measurements are relative.

The evaluation of the coefficient of thermal radiation in absolute measure is an extremely difficult task. It has been attempted by numerous experimenters, whose data are fully considered in a paper by Coblentz,[10] an abstract of which is given in the following pages, and summarized in Table I.

The early determinations of the coefficient of emission of a substance (*e.g.*, blackened plates or balls of glass or copper) by Lehnebach,[34] Christiansen[35] and others are of great historical interest, but they cannot be considered in connection with present-day determinations of the coefficient, or constant σ, of total radiation of a uniformly heated enclosure.

1. *Bolometric Methods of Electric Compensation.*

The principle of the method is as follows: Three branches of a Wheatstone bridge (bolometer) consist of thick manganin wires, which are not affected by a change in current through the bridge. The fourth branch consists of thin bolometer platinum, the resistance of which is affected by a change in current in the bridge. Starting with the bridge balanced, the bolometer branch of thin platinum is exposed to the radiation from a black body and the change in resistance (or galvanometer deflection) is noted. With the bolometer branch shielded from radiation, the bridge current is varied until the change in resistance in the bolometer branch is equal to that attained when exposed to radiation. In other words, the bridge current is increased by a sufficient amount to produce the same galvanometer deflection as was caused by radiation falling on the bolometer strip. From a knowledge of the change in the bridge current, the bolometer resistance, etc., the observer is able to compute the energy input.

Observations by this method have been made by Kurlbaum[36] and by Valentiner.[27] The value of $\sigma = 5.32$, first obtained by Kurlbaum, has been corrected[3] for reflection, giving a value of $\sigma = 5.45$. Valentiner[38] corrected his measurements for reflection from the receiver and for lack of blackness of the radiator, raising his original value from $\sigma = 5.36$ to $\sigma = 5.58$. Coblentz[107] adds corrections for atmospheric absorption, thus increasing the value to $\sigma = 5.68$, with a possibility of the value being $\sigma = 5.75$.

Observations have been made also by Gerlach,[42, 48, 106] by Coblentz,[3, 15, 4] and by Kahanowicz,[54] using modified forms of the Ångström pyrheliometer.[43]

In the instrument used by Gerlach and by Coblentz there is but one strip of manganin which is about 0.01 mm. in thickness, and 2 to 8 mm. wide (length about 3 cm.). At the back of this manganin strip and close to it is a thermopile (joined through a galvanometer), which is heated by radiation from the manganin strip, thus differing from Ångström's device in which a single thermojunction was heated by conduction from the manganin strip. Coblentz attached the potential terminals directly to the receiver at a sufficient distance from the ends to avoid the question of heat conduction to the electrodes. These potential wires, which are from 0.003 to 0.025 mm. in diameter, accurately define the length of the central part of the receiver which was utilized in the measurements. By exposing the whole length of the receiver to radiation conduction losses from the end of the receiver do not enter the problem.

Gerlach's original value was $\sigma = 5.9$. Subsequently he repeated the work and obtained a value of $\sigma = 5.85$, which is now revised [106] to $\sigma = 5.80$. Coblentz [4] has recalculated his data, the original value [15] remaining unchanged at $\sigma = 5.72$ (see Table I).

Kahanowicz [54] used a receiver at the centre of a spherical mirror and obtained a value of $\sigma = 5.61$. Coblentz has corrected [4] this value for atmospheric absorption and obtained a value of $\sigma = 5.69$ to 5.73.

Puccianti [53] determined the constant, σ, by means of a very ingeniously constructed black body bolometer, one branch of which was exposed to a receiver which was cooled by carbon dioxide snow or liquid air and the other was protected from it. These bolometer branches consisted of vessels of thin sheet copper having the form of a cone and a frustrum of a cone united at the bases. The internal surface was smoked. The external surface was polished and upon it was wound two thin insulated wires. One of these wires, of iron, formed the bolometric branch, and the other, of manganin, was used as a heating resistance. The other two branches of the bolometer bridge were formed of resistance coils and the whole was connected with a galvanometer and storage battery as in any ordinary bolometer.

Puccianti measured the (electrical) energy necessary for compensation to prevent the bridge from being unbalanced when one branch was exposed to the receiver. He obtained a value of $\sigma = .596$.

2. *Thermometric Methods with Black Receivers.*

Numerous determinations of the coefficient of total radiations have been made by means of conical or spherical-shaped receivers formed into thermometer bulbs. The space surrounding the receiver was filled with a liquid, *e.g.*, aniline, which served as a thermometric substance whose expansion could be observed in a capillary tube. The method of operation consists in exposing the receiver to radiation and noting the thermometer reading. Then a coil of wire is heated electrically to give the same thermometer reading.

The unsatisfactory part of the arrangement is the heating coil. In some cases the heating coil was wound on the outside of the conical receiver, and, as shown in the critique by Coblentz,[107] an erroneous (too high) value of the radiation constant is obtained. This is the chief criticism against the measurements of Féry [55] ($\sigma = 6.30$), of Féry and Drecq [46] ($\sigma = 6.51$) and Puccianti [61] ($\sigma = 6.15$). When the thermometer is calibrated by placing the heating coil within the (spherical-shaped) receiver, as used by Keene [62] ($\sigma = 5.89$) the arrangement appears to be free from criticism.

Keene's receiver and the radiator being close together, and the time for attaining temperature equilibrium being rather long, there is a possibility of diffusion of hot gases into the receiver when the shutter is raised for observations. This would tend to give a high value. It would have been interesting to determine whether it would have required less electric power in the heating coil, provided the opening in the receiver had been closed with a reflecting cover to prevent escape of heated gases, while calibrating the device.

3. *Indirect and Substitution Methods.*

Shakespear [63] determined the constant of total radiation from a series of experiments in which a plate of metal with a silvered surface was heated electrically to 100° C. and close to it was another plate, blackened with soot, which was cooled with water. Between the plates was air at atmospheric pressure. Shakespear measured the energy input in order to maintain the plate at 100° when (1) the surface of the plate was highly polished, and (2) when it was blackened. He measured also the ratio of the emissivities of the plate under these two conditions, using a radiomicrometer for the purpose. He obtained a value of $\sigma = 5.67$.

Todd [64] obtained indirectly a value ($\sigma = 5.48$) of the coefficient of total radiation from his experiments on the thermal conductivity of gases, in which he used a thin layer of air, enclosed between two horizontal, parallel, good conducting plates, which were maintained at different temperatures.

It is interesting to find that these data, which were obtained by widely different methods, and which appear so discordant on first perusal of the original papers, are in excellent agreement. The various values range about the number $\sigma = 5.7$. In fact, two-thirds of the total number (12) of determinations are close to $\sigma = 5.7$. The mean value of all the data which are free from obvious errors is $\sigma = 5.72$ to 5.73. If we neglected Todd's low value, determined from gas conduction experiments, and Puccianti's high value, which is no doubt too high, the mean value remains unchanged. As we shall see presently, experimental evidence on ionization potential, X-rays, and photoelectric work shows that the value of the coefficient of total radiation is of the order $\sigma = 5.7 \times 10^{-5}$ erg cm.$^{-2}$ deg.$^{-4}$ sec.$^{-1}$.

TABLE I.

Observed Value and the Most Probable Value of the Constant of Total Radiation After Correcting for Reflection, Atmospheric Absorption, etc.

Observer.	Date.	$\sigma \times 10^5$	Probable value of $\times 10^5$	Method.
Kurlbaum	1898	5.45	?	Bolometer.
Féry	1909	6.3	?	Thermometer.
Bauer and Moulin.	1909	5.30	5.7	Thermopile.
	1910	5.7	5.7	Pyrheliometer.
Todd	1909	5.48	5.48	Gas conduction expts.
Valentiner	1910	5.58	5.68 to 5.75	Bolometer.
Féry and Drecq	1911	6.51	?	Thermometer.
Féry and Drecq	1912	6.2	5.68	Calibrated pyrometer.
Féry and Drecq	1912	5.57		
Shakespear	1912	5.67	5.67	Ratio of emissivities, metal; black body.
Gerlach	1916	5.85	....	Modified Ångström.
	1920	5.80	5.80	Pyrheliometer.
Puccianti	1912	5.96	5.96	Bolometer.
		6.15	?	Thermometer.
Westphal	1916	5.67	5.67	Ratio of emissivities, metal; black body.
Keene	1913	5.89	5.89	Thermometer.
Coblentz	1915	5.72	5.73	Modified Ångström pyrheliometer.
Kahanowicz	1919	5.61	5.69 to 5.73	Modified Ångström pyrheliometer.
Foote	1918	5.70	5.70	Marvin pyrheliometer.

Mean value, $\sigma = 5.72$ to 5.73×10^{-5} erg cm^{-2} deg^{-4} sec^{-1}

IV. DISCUSSION OF VARIOUS DETERMINATIONS OF THE CONSTANT OF SPECTRAL RADIATION.

Planck's [18] equation is, no doubt, the closest representation yet formulated of the observed energy distribution in the spectrum of a black body. The constant *C* (often written in the more cumbersome form C_2) determines, to a great extent, the shape of the isothermal spectral energy curve.

The formula is based on theoretical considerations, and after almost two decades of discussion it remains unchanged.

From the experimental data of Coblentz,[76] and of Rubens and Kurlbaum,[77] it appears that the Planck equation may be considered as representing, within the errors of observation, the energy distribution of a black body in the spectrum extending from 0.5μ to 50μ.

In spite of all that has been published on the partition of energy in the spectrum of a black body, there are but few data at hand which are more than qualitative in value.

It seems that all these early data should be considered from the standpoint of historical interest, since it seems impossible to give them much weight in connection with the results obtainable at the present day.

The pioneering work in spectral radiation measurements and constants was inaugurated by Paschen,[69, 70] who observed an extensive series of spectral energy curves.

In a recent critique [107] it is shown that Paschen's original data fall within the range of the recent and more accurately determined values, being of the order $C = 14360$.

In this same critique,[107] it is shown that the observations of Lummer and Pringsheim,[71, 73, 83] after corrections are applied to their calibration of the prism, yield a value of $C = 14300$.

Observations of Warburg and collaborators: [86-95] During the past twenty years the determination of the constants of radiation and their application to the temperature scale has been relentlessly pursued by the *Phys. Tech. Reichsanstalt*,[83] at Berlin.

During the past ten years these investigations have been pursued by Warburg and his collaborators. They used fluorite and quartz prisms, and vacuum radiators. The sodium line was used as a zero setting of the spectrometer. The radiometer was a vacuum bolometer, which was operated by a null method. In this manner the galvanometer acted merely as an indicator. The temperature fixed points were the melting point of gold and some higher temperature, *e.g.*, melting point of palladium. This has caused some difficulty, as has also the question of the dispersion curves of quartz. Their data are summarized in Table II. The present status is the adoption[95] of the value of C = 14300 and the melting point of palladium = 1557° C. Subsequent investigations[96, 97] appear to be made on this basis.

Observations of Coblentz:[30, 76, 4, 107] From the very beginning of this investigation on black-body radiation, it was found that the Wien equation did not fit the spectral energy curves. The calculations were therefore made on the assumption that the observed curves fit the Planck equation; and, at the completion of the investigation, this was found to be true for about 75 per cent. of all the observed curves. This conclusion was based upon the uniformity of the values of λ_m, which resulted from computation (by the method of equal ordinates) of values of λ_1 and λ_2, which were taken far apart, and also close together on the observed isothermal spectral energy curve.

The data are summarized in Table II. The main difficulty appears to have been the lack of exact knowledge and coördination of the indices of refraction of the fluorite prism used in obtaining the spectral energy curves. In the recent survey[107] of the whole subject it is shown that after making a small correction to the calibration curve, the value is $C = 14318$.

The various determinations of the constant of spectral radiation, *C*, are assembled in Table II. The sixth column gives the probable value, as determined from consideration of the data in the text. The mean value is C = 14320 micron degrees. The latest and most reliable determinations of the national laboratories are close to this value. Unfortunately perhaps this average value is too close to the theoretical value, arrived at from consideration of photoelectrical and similarly related phenomena, and experimenters may consider their task finished instead of just begun.

V. VERIFICATION OF THE LAWS OF RADIATION.

In the foregoing pages an inquiry was made into the instruments and methods used in, and the numerical values of, various determinations of the constants of radiation.

The various methods are classified, and a brief description is given of each research.[107] An attempt is given to indicate the good and the defective features in each research. This represents not only the writer's opinions but also those of other experimenters.

It is shown that the great variation in the various determinations of the numerical values of the constants, especially the constant of total radiation, is owing to the fact that in the original papers no corrections were made for atmospheric absorption of radiation in its passage from the radiator to the receiver.

In this paper conservative corrections for atmospheric absorption are made to the various determinations in which such corrections had not been made. As a result, instead of wide variations in the various determinations, which are free from other obvious defects, there is a remarkably close agreement among the various

TABLE II.

Observed Value and the Probable Value of the Constant C of Spectral Radiation.

Observer.	Date.	$\lambda_m T = A$ observed.	$\lambda_m T = A$ corrected.	$C = a\lambda_m T$ and from isochromatics.	C probable value.	Remarks.
Paschen.......	1899	2891	2891-	....	14360	Fluorite prism.
		2907?	2907?	....	.....	Temperature scale is questioned.
	1900	2921?	2894			
Lummer and Pringsheim...	1900	2879	2879	14290	.....	Fluorite prism; wave-length calibration of prism is questioned.
		2876	2876	....	.....	
		2940	2882	14310	14300	Also temperature scale.
Warburg and Collaborators	1911	....	....	14200 to 14600	.....	Fluorite prism.
	1912	....	....	14300 to 14400	.....	Fluorite prism.
	1912	....	....	14360	.....	Quartz prism.
	1913	2894	....	14370	.....	Quartz prism.
	1915	....	....	14250	.....	Temperature from Stefan-Boltzmann law.
	1915	....	....	14300 to 14400	14300	Refractive indices; calibration of prism questioned
Coblentz......	1913	2911	....	14456	.....	Fluorite prism.
	1916	2894	....	14369	.....	Revised calibration (refractive indices) of prism; 1912 data are recalculated.
	1920	....	....	14311 to 14326	14318	Correction for zero setting of bolometer.

Average value, C = 14320 micron degrees.

numerical values. While it cannot be said that the true numerical values are exactly as here recorded, obviously the time is past when the value of the constants of radiation are swayed by a single and perhaps novel method of research.

The best that an experimenter should expect is that his own little contribution to the subject may have sufficient merit to go into the melting pot with the other determinations.

The Formula and the Coefficient of Total Radiation.—In the foregoing review the data are assembled, and the evidence weighed pro and con. It is shown from various experiments that, beyond all reasonable doubt, the total radiation emitted from a uniformly heated enclosure is proportional to the fourth power of the absolute temperature—the so-called Stefan-Boltzmann law. Furthermore, the tabulated data show that the numerial values of the majority of the various determinations of the coefficient, σ, which enters into the mathematical formula of total radiation, range about the value, $\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} deg^{-4} sec^{-1}. The average of 12 of the most reliable determinations is $\sigma = 5.72$ to 5.73×10^{-5} erg cm^{-2} sec^{-1} deg^{-4}.

The Formula and the Constant of Spectral Radiation.—Experimental evidence is cited showing that throughout the spectrum from 0.5μ to 50μ Planck's formula fits the observed spectral energy distribution more closely than any other equation yet proposed. This formula is based upon theoretical principles and after two decades of discussion it remains unchanged.

The constant, C; which determines the slope of the spectral energy curve, has been the subject of numerous investigations. The numerical value of C has fluctuated greatly in the various determinations. In the foregoing pages it is shown that this is owing to experimental difficulties, such as, for example, lack of precise knowledge of the temperature scale, and of the refractive indices of the prisms used. The tabulated data show that the various determinations of the constant of spectral radiation are of the order of $C = 14300$ to 14350, with a mean value of $C = 14320$ micron degrees.

Confirmatory Evidence.—One of the most interesting phases of the inquiry into the laws and constants of radiation is the confirmatory data which one obtains from a consideration of the interrelated phenomena of atomic structures of X-rays, of ionization and resonance potential and of photoelectrical action. From these data, as well as from the foregoing data on the two constants of radiation, one can compute the value of Planck's element of action, *h*. This gives seven independent methods of determining the universal constant, *h*. Or, from any one of four of these methods one can calculate [103, 104, 105] the radiation constants, and it seems truly remarkable how close the agreement is with the observed values.

For example, Coblentz's[4] value of the coefficient of total radiation, discussed on a preceding page, is $\sigma = (5.72 \pm 0.012) \times 10^{-5}$ erg cm^{-2} sec^{-1} deg^{-4}. This indicates[105] a value of $C = 14320$ micron degrees and a value of $h = 6.55 \times 10^{-27}$ erg sec. The value of *h*, determined by Blake and Duane,[102] by X-rays, is $h = 6.555 \times 10^{-27}$ erg sec, or an indicated value of $C = 14330$ micron degrees, which is close to experimental determinations of this constant (see Table II).

Again, starting with Warburg's value of $C = 14300$, the corresponding value of the coefficient of total radiation is $\sigma = 5.74$, which is the higher estimated limit of the average of 12 different determinations of this constant. Hennig,[103] on the basis of Sommerfeld's theory, the measurement of spectral lines and the value of the electron, obtains $C = 14320$ and $\sigma = 5.717$.

In summing up the evidence, it is of interest to include Birge's[104] comprehensive and exact calculations of the constant *h*. In these calculations he, of course, assumes the truth of (1) Lewis' and Adam's[67] theory of ultimate rational units; (2) of the relation between C, σ, and *h*, as given by Planck's radiation formula; (3) of the quantum relation as applied to X-ray data;[102] (4) of Einstein's photoelectric equation; (5) of Bohr's theory of atomic structure; and (6) of the quantum relation as applied to ionization and resonance potentials.

TABLE III.

Birge's[104] Calculation of Planck's Universal Constant h by Various Methods.

Value of $h \times 10^{27}$	Method.	Remarks.
6.551 ± 0.009	$\sigma = 5.72$	Total radiation.
6.557 ± 0.013	$C = 14330$	Spectral radiation.
6.542 ± 0.011	Rydberg constant.	Bohr's theory of atomic structure.
6.578 ± 0.026	Photoelectric equations.	Einstein's equations.
6.555 ± 0.009	X-rays.	Quantum relation.
6.560 ± 0.014	Ultimate rational units.	Theory of Lewis and Adams.
6.579 ± 0.021	Ionization potential.	Quantum relation.

Mean value, $h = (6.5543 \pm 0.0025)\ 10^{-27}$ erg sec.

In this manner he obtains seven separate calculations of Planck's constant of action, h, the mean value of which, as shown in Table III, is $h = 6.5543 \times 10^{-27}$ erg sec.

This is close to the average of the value of h, which results from consideration of the two radiation constants.

From this calculation and intercomparison by Birge[104] of the data on C, σ, and h, as determined by thermal radiometric, photoelectric, X-rays, and ionization potential measurements, it appears that the value of h, computed from radiometric data, compares very favorably with that obtained by more direct measurement. In other words, it appears to prove the validity of laws of radiation and to establish the level of the numerical values of the constants entering therein.

The outstanding disagreement between all the observed and computed data appears to be of the order of 1 to 3 parts in 1000, whatever the method of experimentation. This is a very close agreement, considering the variety of the data and the difficulties involved in making the experiments, which seems to indicate something more than a fortuitous relation between properties of matter.

In conclusion it may be added that, to a close degree of approximation, we have the following constants:

$A = \lambda_m T = 2885$ micron degrees
$C = 14320$ micron degrees
$\sigma = 5.72 \times 10^{-5}$ erg cm^{-2} sec^{-1} deg^{-4}
$h = 6.55 \times 10^{-27}$ erg sec
Melting pt. of Pd $= 1556°$ C.

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SPECTRORADIOMETRIC STUDIES OF POLARIZATION PHENOMENA.*

BY

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1. *Magnetic Rotation*[1-7] *Method of Measurement*[6].—Light from a Nernst glower, or a strip filament tungsten lamp, is polarized by passage through a double-image prism (one beam only being used) and converged by a concave mirror to a focus between the poles of the electromagnet. After reflection from another concave mirror it passes a large Wollaston double-image prism whose principal planes are at 45° to the initial plane of polarization. Accordingly, two images of the source of equal intensity but polarized at right angles are formed, and these fall vertically above one another on the slit of the spectrobolometric part of the apparatus. This comprises a mirror spectrometer with large glass prism (45° angle), Wadsworth mirror and special bolometer, whose two strips (½ mm. wide) lie vertically above one another and receive the two spectra resulting from the two images on the slit. The galvanometer is of the Thomson type of 4 or 8 coils with a working sensibility of about 3×10^{-10} amp./mm. for 6 ohms resistance.

The material under examination is placed in the path of the beam between the poles of the magnet. When the latter is excited the rotation of the plane of polarization—which in practice is always small—causes a diminution in the intensity of the beam incident on the upper half, say, of the slit and a corresponding increase in the other. The result is a galvanometer deflection which is doubled by reversing the magnet current. To interpret this in degrees, the polarizing prism is given an accurately known rotation of one or two degrees and a comparison of the galvanometer deflection which this produces with that due to the magnetic rotation allows the calculation of the latter.

The possibilities of this method in measuring very minute magnetic rotations considerably exceed those of visual methods. A rotation of only .001° is readily detected, and, if desired, the sensibility can be pushed much farther than this. The accuracy of measurement for rotations of a few minutes of arc is of the order of 1 per cent. In spectral range the limits are about .5μ to 2.3μ, the lower limit being set by energy considerations and the upper by the absorption of the calcite and glass. A simplification of this method, following somewhat more closely the original scheme of Provostaye and Desains and using as polarizer and analyzer reflecting surfaces of glass—or better, as suggested by Pfund, selenium—would, at some expense in sensibility, enable one to proceed farther into the infra-red. The writer's first experiments were carried out along this line, using a rock salt prism and, as polarizer, piles of cover glass plates.

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* Section of 1920 Report of Standards Committee of Spectroradiometry, W. W. Coblentz, Chairman.

Results: (A) *Faraday Rotation.*—The magnetic rotatory dispersion of CS_2 has been measured[4] to 4.3μ and found to be well represented by a formula based on the Hall effect and taking account of an absorption band beyond 8μ. Water,[5] which may be tested about as far as 1.3μ, shows a similar curve, but one which does not fit the formula as well.

Films of iron, cathodically deposited, show,[6] on the other hand, an increase of rotation for longer wave-length, followed by a decrease. The curves are, in general, somewhat similar to those obtained on reflection (Kerr effect) but of opposite sign.

(B) *Kerr Effect.*—Iron, nickel, cobalt and magnetite show[4] on reflection a maximum of rotation in the neighborhood of 1μ and a decrease for longer wave-lengths. Nickel reverses its sign of rotation at about 1.5μ. These results support the view that the visible spectrum corresponds to a region of anomalous dispersion for such metals. This point of view is somewhat strengthened by other evidence. Heusler's magnetic alloy shows no Kerr effect.

The above measurements on the Kerr effect are for normal magnetization and normal incidence. When the magnetization is transverse and the incidence is varied,[8] the effect is more complicated, but may be shown to be in good agreement with Wind's[9] and also with Voigt's[10, 11] magneto-optic theory.

2. *Natural Rotatory Dispersion*[12-15].—A slight modification of the foregoing method enables the measurement of natural rotation of the plane of polarization over this same spectral region, *viz.*, from .5μ to 2.3μ, with an accuracy only slightly less than that obtained in the case of magnetic rotation. Quartz, widely

investigated by other experimenters,[12, 13, 14] shows a rotation dispersion over this spectrum range which is extremely well represented by a dispersion formula involving two free periods, one in the ultra-violet and the other in the infra-red. Two terpines and three solutions—tartaric acid, cane sugar and camphor—have also been tested in this way as far as their absorption permits. The results, as a whole, are in substantial agreement with the Wiedemann Law of the proportionality of magnetic and natural rotations, the best agreement being in the case of quartz and the least satisfactory for the solution of camphor in alcohol.

3. *Optical Constants of Metals* [16, 17].—The same spectrobolometric apparatus serves here as in the other work. The other arrangements, however, must be considerably modified. The polarized radiation is allowed to fall on the metal plate under investigation in various azimuths and at various angles of incidence. When both azimuth and incidence are so adjusted that the radiation coming from the plate is circularly polarized, the angles of incidence and azimuth are the so-called principal angles and the refractive and absorption indices can be readily computed. To enable one to determine when the radiation is circularly polarized, the metal surface, polarizer, source of light, etc., are all mounted in a massive framework, which can be rotated about the axis of the emergent beam. If the polarization, instead of being exactly circular, is elliptical, this rotation of the ellipse will produce a varying galvanometer deflection, which reduces to zero upon proper adjustment of the incidence and azimuth angles.

Results.—Steel, cobalt and nickel, all show [16] a considerable increase of the refractive index with longer wave-lengths and only relatively small changes in the absorption index. Silver and copper, whose indices of refraction are less than unity for visible wave-lengths, both show a decided increase in the infra-red, copper reaching the value unity at 2.25μ and silver approaching it. Silicon shows a slight decrease in refractive index with longer wave-lengths and a relative constancy of absorption index.

4. *Polarization by Gratings* [18, 19].—When light from a speculum grating is passed through the double image prism and the radiation examined for each state of polarization, certain phenomena are observed. For the radiation whose electric vector is perpendicular to the rulings there is found a sharp minimum of energy for such wave-lengths as are, in some order of spectrum,

passing tangentially from the grating. Such minima are observed either in the central image[19] or in spectra (work not yet published) of the first, second, etc., order, and are, as a rule, more easily studied in the early infra-red (λ about 1μ) than in the visible spectrum. At normal incidence the minimum in the directly reflected radiation for a wave-length equal to the grating space (first order spectra tangential) is very marked.[19]

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SYSTEMS OF COLOR STANDARDS.

BY

A. AMES, JR.

THE principal object of a color standard is to have some means by which the color of any object can be determined. This can be done either by an instrument such as a colorimeter or a standard composed of colored cards. As any instrument is too complicated for general use, only those standards composed of colored cards will be considered in this paper.

The purpose of a color standard and requisites to which it must conform will be first considered, after which the good points and defects of various standards will be taken up.

As was stated, the principal object of a color standard is to have some means by which the color of any object we may happen across can be determined. To do this it is necessary that the cards in the standard be properly arranged so that we can find as easily as possible the card similar to the color in question. As in most instances we wish to record what that particular color is we should have the simplest and best nomenclature. There should moreover be a sufficient number of cards, so that we can find any color. As such a number of cards will tend to make the standard bulky, it should contain the largest possible range of colors with the fewest cards. And last, though very important, every color in the standard should be based on exact measurements, so that people at other times and places will know exactly the color of any card we may designate.

This makes five requisites which a color standard of this kind must satisfy. First, *Arrangement*. It must be such that it is as easy as possible to find any desired color in the standard; that is, such that you know a color's particular position in the standard and its direction from any other color. Second, *Notation or Nomenclature*. The notation of the different cards must be as simple as possible. It should call to mind a particular color and only that color, and should designate its particular position in the standard. Third, *Number*. There should be a sufficient number of cards, so that the color of any object can be closely matched. Fourth, *Spacing*. The spacing between the cards in all parts of the standard should be the same. With even spacing between all the cards a standard with a given number of cards will have the greatest usefulness. Fifth, *Standardization.* The exact nature of the color in every card should be determined by accurate measurement, so that when a particular card is named its particular color and no other will be designated. Furthermore, such notation and measurements should be universally accepted, so that a particular color will mean the same thing to all peoples at all times.

These five requisites will now be considered in more detail.

ARRANGEMENT.

All colors lie in a natural arrangement to each other, due to the three constants which every color possesses. These are, first, Hue (or wave-length); second, Chroma (saturation); third, Value (brightness).

We are indebted to Munsell for the most complete presentation of their mutual arrangement due to these three constants. He shows their arrangement is in the form of a color solid, roughly spherical in shape. Steps around the horizontal equator give differences in hue or wave-length. Planes at different heights along the vertical axis give different steps in value or brightness, white at the top, black at the bottom. Radial distances from this vertical axis give differences in chroma or saturation.

For practical reasons the cards in a color standard must be arranged on a sheet, *i.e.*, on a two-dimensional surface. It therefore becomes necessary to decide what arrangement there exists in a two-dimensional surface of this color solid which is most suitable for our purpose. It would seem without question to be best to use that grouping which would leave one of the color constants unchanged and the other two variable. Three such groupings, however, can be obtained in a two-dimensional surface.

You can get a grouping in which the hue or wave-length remains constant and the value and chroma change, as by taking a vertical plane from the axis outward. Or you can keep the value constant with the color and saturation changing by taking horizontal planes. Or you can keep the chroma constant with varying color and value by taking cylindrical surfaces at different distances from the axis.

The first grouping seems without question the best. Because, when we see a color we always think first of its hue or wave-length. Value and chroma are qualifying factors.

The best arrangement therefore is to have all the colors of the same hue or wave-length on one chart or page. We would have then a series of such charts, as many as we wished different hues in our standard. On all of these charts the lines of cards which were on the same level from the top would have the same value. Those lines of cards which were the same distance from the sides would have the same chroma. With the three constants of a given color in mind such an arrangement enables one to find it very easily; in fact, leads one directly to it.

NOTATIONS OR NOMENCLATURE.

The color names in ordinary use are of practically no help, due to the fact that they are too general and in themselves designate no particular color, except to those who have arbitrarily learned to associate them with some particular color.

Every color has the three constants of hue, chroma and value. If any one of these constants is varied the color is varied. A color therefore can be absolutely specifically designated if its hue, chroma and value are given. Here again we are indebted to Munsell for giving us a most convenient way of doing this. The first constant, hue, he designates by a letter; for example, R for red or B for blue. The second constant, value, he designates by a number above a line to the right of the letter; the third constant, chroma, by a number below this line. For example, R 6/5 gives all three constants and determines the color. When you see such a notation you think first of red, then you notice the figure six and realize it is a little above middle value. The figure five tells you that its chroma is half-way between a neutral gray and a full chroma red. A very definite color is thus brought to your mind.

An improvement that could be made in this notation would be in the matter of the letters used to designate the hue. In a color standard there should be at least thirty or forty different hues. The words in common use to designate hue are yellow, orange, red, purple, blue and green. We can get definite hue notation if we qualify one of these words by another. That will give us all together twelve names: yellow, yellow orange, orange, etc. This, however, is not enough to cover the necessary hues. It is possible to still further increase the number in two ways. First, by considering the last part of the combined word as the noun and the first part as the qualifying adjective and adding between yellow and yellow orange, for instance, the name " orange yellow," which would be a yellower hue than " yellow orange." After having used this notation for many years in my own system I am convinced it is very poor. When I come across it I always have to stop and think which is yellowest in hue, " yellow orange " or " orange yellow." Moreover, this adds only six more names, which does not give us enough. Second, the name could be repeated as " yellow yellow orange." But such a notation is subject to the criticism made above.

It is believed the best notation would be to use the simple and simple compound names, yellow, yellow orange, orange, etc., followed by the wave-length of the hue. The name or initial would give the general hue, the number the exact hue. The decrease in size of the number would suggest shorter wave-length and a hue slightly towards the blue end of the spectrum.

As there are no wave-lengths covering the purple region, an arbitrary set of numbers covering the least perceptible differences could be used in this region.

There is some question as to whether there is any necessity of using the letters placed before the wave-length number to designate the general hue.

The objections to the use of such letters are: that anyone using a standard would learn to know the hues by the number of their wave-length and that the letters therefore are unnecessary. That as words designating hue designate no particular wave-length, and as there are not enough of them to designate all the necessary hues, it would be better to develop the practise of thinking of hue in terms of wave-length and abandon the old names for hue.

The arguments in favor of the use of letters to designate the general hue are that the majority of persons using a color standard would not learn to know the hues by the number of their wave-length and that general designations by a symbol would be helpful, for although words designating hue do not designate particular wave-lengths, yet they do universally designate regions in the spectrum. That it would be impossible to ever develop a universal practice of thinking of hues in terms of wave-lengths. That the old names for hue are so universally accepted that they will always be used and that it would be best to assign them to particular regions in the spectrum.

NUMBER OF CARDS.

A color standard to be of practical use must have a sufficient number of different cards, so that the color of any object can be closely matched. The degree of closeness of course varies according to what use the system is put. But for all scientific purposes, biological, for instance, and for use in mercantile business, matching textiles, arts, etc., it should be pretty close. The ideal standard set would be one in which the number of cards was such that the difference between any card and the card next to it was two least perceptible differences. The color of any object lying between cards with a separation equal to two least perceptible differences would appear like one or the other of the cards unless it lay half-way between them. Its color would therefore be definitely determined and it could be designated as lying half-way between such cards.

The least perceptible differences in the three constants under the best conditions are roughly such that we can distinguish one hundred and fifty steps in hue, two hundred steps in value, one hundred steps in chroma. The least perceptible difference between small cards such as must be used in a color standard is, however, much greater than those given above. For instance, in the set of grays which the writer made up containing sixty steps, the difference between the cards is just about perceptible. It is these larger, least perceptible, differences which should be used as the basis of the separation of a standard of cards. I should roughly estimate them to be between two and three times as great as that which you can get under the most favorable conditions. This would give from fifty to eighty steps in hue, seventy to one hundred steps in value and thirty-five to fifty steps in chroma. If the cards are separated by two such perceptible differences, we would have from twenty-five to forty steps in hue, thirty-five to fifty steps in value and sixteen to twenty-five steps in chroma. This would give a total number of about thirteen thousand different cards. It would be advisable to furnish sets with separations of three or four or more least perceptible differences between the cards, the different standards being supplied according to the use to which they would be put.

SPACING.

Whatever degree of separation is used, it should be the same throughout the entire system. That is, if the difference in value between a card and the next one above or below it was two least perceptible differences, the differences in chroma between that card and the next one to the left, or the right of it, should be the same. Also the difference in hue between the cards on one chart or page from those on the next chart or page should be the same.

The data on least perceptible differences in value or brightness have been worked out by many investigators, which makes this spacing easily determined. The least perceptible difference in hue in the spectrum have been very definitely determined for a limited number of observers by A. L. Jones.[1] It is believed, however, that it should be worked out for a large number of observers before it will be sufficiently definitely determined to use in making up a color standard.

In regard to spacing in chroma, although some data have been collected of the least perceptible differences in that constant, the best by Geissler,[2] it is not sufficient for the purpose. Further measurements would have to be made to determine the average least perceptible differences in this constant. This, however, would not be a difficult or long job, as the path has been blazed and methods established.

STANDARDIZATION.

The three constants of every card in the standard should be determined by the most accurate measurement. The best way to do this would be to first calculate in the three constants of hue, value and chroma; *i.e.*, wave-length, intensity and saturation or the amount of white light added, just what every card in the standard should be. Then these combinations should be obtained by means of a colorimeter with a standard light source in a standard illumination. Finally the cards should be made to match these predetermined combinations obtained with the colorimeter. The spectral curves of the color patches obtained by the colorimeter should be determined as should those of the various cards. If possible those of the cards should be made similar to those obtained by the colorimeter. The determination of the spectral curves of the cards would not be so difficult as might appear. For if, as suggested, all the cards on each page of the standard were various values and chromas of the same hue, they would be obtained by mixing white and black paint with the saturated pigment of that hue. If the spectral curve of the saturated pigment was determined, the curves of all the other cards on that page would be of the same type. This work should preferably be done by some government bureau, as the Bureau of Standards, and the colorimeter kept by them as the standard. A set of measurements of every card in the standard should accompany the standard.

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[1] JOURNAL OPT. SOC. OF AMER., Vol. 1, Nos. 2 and 3, March–May, 1917.

[2] *Amer. Jour. of Psych.*, 1913, 24, p. 171; see also Luckiesh: Color and Its Applications (Van Nostrand), p. 127.

Various color standards will now be considered, bearing in mind the above-mentioned requisites.

RIDGWAY'S.

Ridgway's system[3] is probably the most used to-day, and on the whole is believed to be the best system. It is defective, however, in that it satisfies but one of the aforementioned requisites of a perfect color standard.

First, *Arrangement*. It is defective in that he has grouped together in one page the fully saturated red, orange red and orange orange red, of all values from lightest to the darkest. On the next page he has red orange, orange red orange and orange, and so on around the spectrum. On later pages he has the same colors similarly grouped, but one step less in saturation. That is, on any one page he has different hues, chromas, and values; in other words, all three of the color constants are varying.

Further, different vertical lines of cards give differences in hue, but nothing in his arrangement gives differences in chroma or value. The chroma on a single sheet varies, because in mixing white or black with his pure color to get his differences in value he also gets differences in chroma in the same vertical line. And by putting all his fully saturated colors in the same horizontal line, *i.e.*, the fifth from the top, he varies his values because a full chroma yellow, for instance, is much lighter in value than a full chroma red.

Second, *Nomenclature*. The color names he uses are too general and do not materially help to designate the exact color. The letters and numbers he uses are subject to the following criticism: He has the hues both lettered and numbered, his lettering being complicated and his numbering arbitrary. He begins with number one for red and goes to seventy-one for violet red red. Accompanying these numbers he has letters. From orange to yellow, for instance, they run as follows: OY-O, Y-O, OY, YO-Y.OYY. This notation is clearly confusing. The different values he designates by small letters, beginning with (a) in middle value he designates the lighter values by (b), (d), (f), and the darker values by (i), (k), (m), differentiating between tints and shades. The different chromas he designates by dashes above the number for hue, one dash (′) designating one step from full saturation, two dashes (″) two steps, etc. It is evident that this is a very complicated notation.

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[3] "Color Standards and Color Nomenclature," Robert Ridgway, Washington, D. C., 1912.

Third, *Number of Cards.* In all he has one thousand one hundred and fifteen different cards. This number is too small. Defective arrangement and notation can be surmounted, but there is no way to make up for an insufficient number of cards.

Fourth, *Spacing.* His spacing is not even, primarily due to his faulty arrangement of putting all full chromored colors in the middle of his value scale. He gets, for instance, the same number of steps from yellow to white that he does from blue to white, and the yellow is much lighter than the blue. Even the spacing of his hues is not as even as can be determined by present scientific methods, Jones.[4]

Fifth, *Standardization.* We have Ridgway to thank for the best standardization as far as I know up to date. He based his colors on combinations from the Maxwell color disk. His data cover the spacing for all three constants and fulfill all the requirements of standardization. With the means he used it could not be improved upon. It is believed it would be much better, however, to obtain the various combinations of color constants by direct mixtures of light with a colorimeter. In the last analysis he has to base his six fundamental hues on wave-lengths. It would seem best therefore to start with actual wave-lengths, getting the various values and chromas by changing the intensity and adding white light.

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[4] *Loc. cit.*

MUNSELL.

Munsell's system,[5] which has been developed since Ridgway's, is in some of the requisites better, some worse, than Ridgway's.

First, *Arrangement*. Its arrangement, which has already been described, cannot be improved upon.

Second, *Nomenclature*. His nomenclature, as far as it goes, is also, it is believed, as good as can be devised. It is wanting only in that it does not give any suggestion as to how the intermediate hues between his eleven standard ones should be designated.

Third, *Number of Cards*. As a complete color system, for which I do not believe he intended it to be, it falls down in that there are not enough different colors in the system. He only has eleven different hues, as compared with Ridgway's thirty-six.

Fourth, *Spacing*. His spacing is not the same throughout the system, that between hues being greater than those between value and chroma. His spacing between values and chromas looks very good indeed. Priest, *et al.*,[6] have shown, however, that his series of values is not in accord with the natural system expressed by Weber's law.

Fifth, *Standardization*. Munsell gives no specific account as to what measurements his colors are based upon. His system, therefore, fails in this requisite, although the examination of his system by Priest *et al.*[7] goes a long way towards meeting this deficiency.

AMES.

This standard, which is in triplicate and has never been published, was made by the writer and his sister, Mrs. Oakes Ames, before Ridgway's last edition came out and before Munsell completed his work. It was made as an assistance in painting to make exact color reproductions of scenes. The method was to match the color seen by holding up a card, noting the color on the canvas, and after the whole scene had been matched putting in the paints from the color notations. It contains twenty-seven hues, fifteen steps in value and ten steps in chroma for each hue, making in all about thirty-three hundred different cards. It fulfilled every requirement expected of it. It has, however, marked defects and can be much improved upon. It will be considered in view of the five requisites of a perfect color standard.

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[5] "Atlas of the Munsell Color System," Wadsworth, Howland & Co., Boston, Mass.

[6] *Technol. Papers, Bureau of Standards, No. 167*, Sept., 1920.

[7] *Loc. cit.*

First, *Arrangement.* The arrangement, which was in general after Munsell, cannot be improved upon, except in so far as it does not follow Munsell. That is, in the following particulars: First, all the fully saturated colors are put ten steps out from gray. For example, there are ten steps from a fully saturated red to gray and the same number from a fully saturated green to gray. Now a fully saturated red is much stronger in chroma than a fully saturated green. The arrangement, therefore, is wrong. Second, vertical lines at an equal distance from the side of the page do not designate the same chroma. This is due to the fact that in making lighter values of a saturated color we mixed white with them and kept them in the same vertical line as the saturated color, although of course their saturation was reduced by the added mixture of white.

Second, *Notation.* The notation which followed Munsell is very good, except in the designation of the hues. Names as orange yellow and yellow orange were used which, as has been said before, are confusing. The use to which the standard has been put most thoroughly tested and proved the value of Munsell's method of notation, *i.e.,* a designation for each color constant.

Third, *Number of Cards.* The total number of cards in the standard is about thirty-three hundred. In this particular it is markedly superior to the other standards. From all my experience, however, it is believed the number should be increased.

Fourth, *Spacing.* The spacing between values and chromas is about right, but there should be more hues in certain sections of the spectrum. The spacing, however, is not the same throughout the system. This is due to the poor arrangement above spoken of. The spacing in the green chart is not as great as that in the red, and the spacing in the lighter values of the red chart is not as great as it is in the lower part. If the spacings were the same throughout the system would have its same usefulness with about one-third less cards.

The spacing in value was carefully worked out with Maxwell disks, taking into account Fechner's law, and though it has not been measured, gives, it is believed, substantially even steps in sensation. The spacing in hue and chroma was done only by eye.

Fifth, *Standardization.* In this requisite the system falls down entirely. Every card can be duplicated from a tube of color which matches it, but this in no way fulfills the demands of this requisite.

The arrangement of the colors on separate cards that can be taken out of the sheets has proved to be of the greatest convenience, as it makes it possible to match surfaces that cannot be brought into juxtaposition with the different parts of the sheet.

JORGENSEN.

Jorgensen's work[8] cannot really be considered a standard for the purposes above described. He gives, however, a suggestion that might be of value in making up a color standard. That is to have the different values and chromas of each hue gradually blend into one another, the value and chroma being designated by a scale at the top and side of each chart. I imagine this would have to be done by some printing process, and I am not sufficiently conversant with the technique of color printing to know whether or not it would be possible. The great advantages of such a method, in that it would give all of the colors and in the smallest possible space and at an enormously reduced cost, seem to make it at least worth considering.

CONCLUSION.

The demand for a perfect standard color system is very great and from innumerable branches of activity, art, sciences, manufacturers, retailers.

The lines of procedure and work necessary to produce a standard system have been pretty definitely laid out.

The only reason for delay is the lack of sufficient number of individual determinations of the least perceptible differences in hue and chroma. It is most earnestly advised that these determinations be made by various laboratories and as soon as possible, in order that the great need for a proper color standard can be met.

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[8] "The Mastery of Color," C. J. Jorgensen, Milwaukee, 1906.

THE ORIENTATION OF THE GRAINS IN A DRIED PHOTOGRAPHIC EMULSION.*

BY

L. SILBERSTEIN.

A SURFACE of film base is uniformly coated with a freshly made emulsion, forming a layer of thickness b, say. The orientation of the silver halide grains which, in accordance with all available experimental evidence, will be treated here as small plates, can be assumed to be completely haphazard, *i.e.*, such that in this, the initial state of the emulsion, no directions of the plane-normals of the plates are privileged. In symbols, if θ be the angle of inclination of a grain (plate) to the film base, ranging from 0 to 90°, the number of grains whose inclination is contained between θ and $\theta + d\theta$ is

$$N\,d\theta : \frac{\pi}{2} = \frac{2}{\pi} N\,d\theta$$

N being the total number of grains. Such is, by assumption, the initial state of things.

Let now the layer, left to dry, shrink down to the thickness b' and let it be assumed that this process consists, at least macroscopically, in a *uniform contraction* of the gelatine without lateral expansion, the axis of this simplest kind of deformation being the normal to the film base.

Such being the assumptions, it is required to find the amount of orientation of the grains in the final dry state of the emulsion. More definitely, a being the average area of the microscopic plates, it is required to find the average *efficient* area per grain, say κa, *i.e.*, the average value of the projection of a upon the horizontal plane, in the final state.

The efficiency coefficient κ thus defined amounts simply to the average of cos θ', if θ' be the final inclination of a grain-plate.

The problem is thus reduced to writing down the relation between θ' and θ and integrating over a quadrant in θ.

Now, with regard to the transformation of θ into θ', two equally simple possibilities (both compatible with the requirement of a macroscopically uniform contraction of the medium) present themselves.

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* Communication No. 112 from the Research Laboratory of the Eastman Kodak Company.

These are:

$$(1) \qquad \sin\theta' = k \,.\, \sin\theta,$$

$$(2) \qquad \tan\theta' = k \,.\, \tan\theta,$$

where $k = b'/b$ is the contraction ratio of the emulsion layer.

The first of these relations would correspond to the assumption that there is no slipping between the silver halide grains[1] and the adjacent gelatine, and that the distance x of every point of each plate from the base is being reduced in the same ratio, *i.e.*, to $x' = kx$. Let $AB = l$ (Fig. 1) be a diameter of the grain-plate, *viz.* the diameter contained in the vertical plane passing through the normal to the film base, and let x_1, x_2 be the ordinates of the end-points, A, B. Then

FIG. 1.

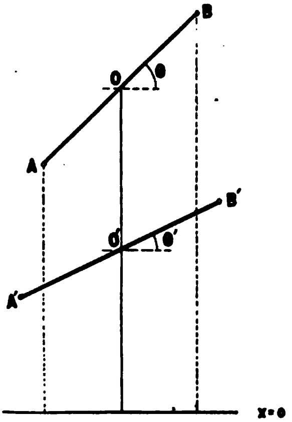
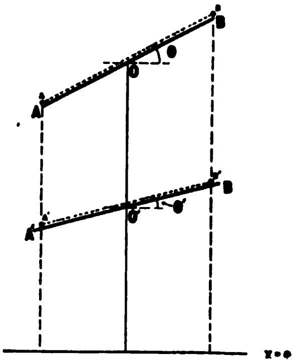


FIG. 2.



$$\sin\theta = (x_2 - x_1) : l, \text{ and } \sin\theta' = (x_2' - x_1') : l,$$

and since

$$x_1' = k\,x_1,\ x_2' = k\,x_2,$$

we have

$$\sin\theta' = k \sin\theta.$$

It will be noted that this kind of displacement of the (rigid) plates implies a torsional deformation of the gelatine in minute regions around the plates in addition to the general uniform contraction. For, if a point of the plate, say its centre, describes a straight, other points of the plate, together with the adjacent gelatine particles will, under the made assumptions, describe necessarily curved paths.

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[1] Whose thickness is being neglected for the present.

The alternative relation, (2), is obtained at once if it be assumed that each particle of the medium (gelatine), such as A or B in Fig. 2, describes strictly a straight path, its ordinate being reduced in the ratio k. In fact, in this case the horizontal projection of the distance (h, Fig. 2) of two gelatine particles (A, B) remains constant, and we have $\tan\theta = (x_2 - x_1) : h$ and $\tan\theta' = (x_2' - x_1') : h = k \tan\theta$. At the same time we see that, since the gelatine filament AB is contracted to A′B′, while the originally adjacent plate diameter AB retains its length, the plate must slip or slide, in its own plane, relatively to the medium.

Which of these two possibilities (or, perhaps, other intermediate ones) takes place in reality we do not know. And it seems, therefore, advisable to consider, separately, the consequences of the relations (1) and (2).

1. By formula (1), which will be referred to as the *sine-law,* we have

$$\cos\theta' = \sqrt{1 - k^2 \sin^2\theta},$$

and therefore,

$$\kappa = \frac{2}{\pi}\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta}\,.\,d\theta.$$

The integral on the right hand is the well-known (and tabulated) complete elliptic integral of the second kind, which is usually denoted by E (k); our contraction ratio k is the modulus of this integral. Thus we have the efficiency coefficient

$$\kappa = \frac{2}{\pi}\mathrm{E}\,(k). \qquad \text{(I)}$$

A short, and more than sufficiently accurate, table of E, with $\alpha = \text{arc}\sin k$ as entry, will be found, for instance, on page 68 of Jahnke and Emde's "Tafeln." We may as well use the rapidly converging power series for E(k) giving

$$\kappa = 1 - \left(\frac{1}{2}\right)^2 \frac{k^2}{1} - \left(\frac{1.3}{2.4}\right)^2 \frac{k^4}{3} - \left(\frac{1.3.5}{2.4.6}\right)^2 \frac{k^6}{5} - \ldots\ldots, \qquad \text{(I')}$$

of which, in practice, the first two or three terms will give all that is required. For $k = 0$ we would have $\kappa = 1$, an obvious result, and for $k=1$ (which is the initial state), $E(1)=1$ and $\kappa=\frac{2}{\pi}$, as it should be, this being the average of $\cos\theta$ in absence of any orientation. Thus the whole range of κ is contained between 0.6366 and unity. Thus far the thickness of the grain-plates or the breadth of their edges has been neglected. If it is taken into account and if, say, the thickness of an average grain is a fraction ϵ of the average diameter, the "efficient area" is increased by $a\,\epsilon\,k\sin\theta$, and we have the correction term $\frac{2}{\pi}\epsilon\,k$, to be added to the above value of κ. The whole reasoning is valid, of course, only when ϵ is a small fraction. In practice its value does not amount to more than 1/15.

2. Passing to the alternative assumption (2), which will be referred to as the *tangent-law,* we have

$$\cos^2\theta' = 1 : (1+\tan^2\theta') = 1 : (1+k^2\tan^2\theta),$$

so that

$$\kappa=\frac{2}{\pi}\int_0^{\pi/2}\frac{d\theta}{\sqrt{1+k^2\tan^2\theta}}=\frac{2}{\pi}\int_0^{\pi/2}\frac{\cos\theta\,d\theta}{\sqrt{1-(1-k^2)\sin^2\theta}},$$

or, substituting $u=\sin\theta$,

$$\kappa=\frac{2}{\pi}\int_0^{1}\frac{du}{\sqrt{1-(1-k^2)\,u^2}}.$$

The value of the integral is

$$\frac{\operatorname{arc\,sin}\sqrt{1-k^2}}{\sqrt{1-k^2}}=\frac{\operatorname{arc\,cos} k}{\sqrt{1-k^2}}.$$

Thus the efficiency coefficient corresponding to the tangent-law is

$$\kappa=\frac{2\operatorname{arc\,cos} k}{\pi\sqrt{1-k^2}}. \qquad \text{(II)}$$

This gives for $k=1$ and $k=0$ the values $\kappa=\frac{2}{\pi}$ and 1, respectively, coinciding, of course, with the previous ones, while for all intermediate values of k the values of κ are somewhat smaller than those given by formula (I). For contractions such as $k=1/10$ (as is usually the case in film manufacturing) the two κ – values differ only by about 6 per cent.

It is possible that in reality we have something intermediate between the conditions of (1) and (2), so that, in absence of better knowledge we can take some intermediate value between κ_{I} and κ_{II}, say their arithmetical mean. This becomes for such contractions as $k = 1/10$ a purely academic distinction, for then both κ – values differ but little from unity.

The following table gives, to four decimals, the values of κ on either assumption, for a few values of the contraction ratio k:

k	κ (I)	κ (II)
1.0	$\frac{2}{\pi} = 0.6366$	0.6366
0.5	0.9342	0.7698
0.3	0.9771	0.8442
0.2	0.9899	0.8899
0.1	0.9975	0.9410
0.0	1.0000	1.0000

If the average efficient area κa could be measured experimentally for various contractions k (preferably nearer to unity than to zero), we might readily decide whether in the actual drying process the behaviour of the emulsion answers better to the first or the second of our assumptions (*i.e.,* to the sine-law or the tangent-law). For the present we can say only that these theoretical results agree substantially with the observations made, for k equal to about 1/10 by Mr. Trivelli, who found that all or almost all plates (grains) lay "flat," *i.e.,* were parallel to the film base.

It may be well to make a few more remarks on the consequences of the sine-law (1) as compared with those following from the tangent-law (2).

Although, as we just saw, the difference in the κ– values, especially for low values of k, is rather small, the range of inclinations θ' in the dry plate and the distribution of the grains over this angle-range is entirely different for the two cases.

In fact, on the assumption (1), the upper limit of inclination, say α, is given by

$$\sin \alpha = k, \qquad (3)$$

so that no silver halide plate has an inclination exceeding the angle arc sin k, *i.e.,* about 5° 44′ for $k = 1/10$, for instance.[2]

On the other hand, under the assumption (2) in view of $\tan \frac{\pi}{2} = \infty$, we have for $\theta' = \text{arc tan } (k \tan \theta)$ the full range from 0 to 90°.

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[2] Of course, if a grain (plate) was initially rigorously perpendicular to the base ($\theta = \frac{\pi}{2}$) there would be no tendency to tilt it, thus leaving $\theta' = 90°$; but it is scarcely necessary to say that the probability of a grain having exactly $\theta = 90°$ is zero.

Thus far the total ranges of the angles. Now for the distribution of the grains through the angle domain.

Let us denote by $N'd\theta'$ the number of grains whose inclinations in the final state are contained between θ' and $\theta + d\theta'$. Then, since all these (and these only) were originally contained within the angle interval θ to $\theta + d\theta$ and were therefore $\frac{2}{\pi} N d\theta$ in number, we have, for the determination of the unknown function N',

$$N' \, d\,\theta' = \frac{2}{\pi} N \, d\,\theta.$$

Now $\frac{d\theta}{d\theta'}$ is $\frac{\cos\theta'}{k\cos\theta}$ under the first, and $\frac{\cos^2\theta}{k\cos^2\theta'}$ under the alternative (second) assumption. Thus the number of grains whose final inclinations are contained between θ' and $\theta' + d\theta'$ is, in the case of the *sine-law,*

$$N' \, d\,\theta' = \frac{2}{\pi} N \frac{\cos\theta' \, d\,\theta'}{\sqrt{k^2 - \sin^2\theta'}}, \quad 0 \leq \theta' \leq \text{arc sin } k, \tag{4s}$$

and in the case of the *tangent-law,* remembering that $\cos^2 = 1 : (1 + \tan^2)$,

$$N' \, d\,\theta' = \frac{2}{\pi} N \frac{k \, d\,\theta'}{k^2 \cos^2\theta' + \sin^2\theta'}, \quad 0 \leq \theta' \leq \frac{\pi}{2} \tag{4t}$$

The first formula gives, for the number $n'(\theta')$ of grains whose inclinations are contained between 0 and θ',

$$n'(\theta') = \int_0^{\theta'} N' \, d\,\theta' = \frac{2}{\pi} N \int \frac{d(\sin\theta'/k)}{\sqrt{1 - \left(\frac{\sin\theta'}{k}\right)^2}},$$

i.e.,

$$n'(\theta') = \frac{2}{\pi} N \text{ arc sin}\left(\frac{\sin\theta'}{k}\right), \quad 0 \leq \theta' \leq \text{arc sin } k, \tag{5s}$$

as it should be, for all these grains were originally contained between 0 and $\theta =$ arc sin $\left(\frac{\sin\theta'}{k}\right)$ of the angle domain. Similarly, the formula (4*t*) gives

$$n'(\theta') = \int_0^{\theta'} N' \, d\,\theta' = \frac{2}{\pi} N \text{ arc tan}\left(\frac{\tan\theta'}{k}\right), \tag{5t}$$

valid for any θ' between 0 and 90°.

By the sine-law formula (5^s), we have, for example, for

$$\theta' = \text{arc sin } k = \alpha\,,$$

$$n'(\alpha) = \frac{2}{\pi}\,\text{N arc sin } 1 = \frac{2}{\pi}\,\text{N} \,.\, \frac{\pi}{2} = \text{N},$$

that is to say, the total number of grains, as it should be, and for $\sin \theta' = \frac{1}{2} k$, for instance, we have

$$n' = \frac{2}{\pi}\,\text{N arc sin}\frac{1}{2} = \frac{2}{\pi}\text{N} \,.\, \frac{1}{3}\,\frac{\pi}{2} = \frac{1}{3}\text{N}\,,$$

i.e., just one-third of all grains (the remaining two-thirds being contained between arc sin $\frac{k}{2}$ and $\alpha = \text{arc sin } k$). For small k (such as 1/10), and therefore small θ', we can write formula (5^s), approximately,

$$n'(\theta') \doteq \frac{2}{\pi}\,\text{N . arc sin}\left(\frac{\theta'}{k}\right),\ \alpha \doteq k\,.$$

The tangent-law formula (5^t), which might also be obtained without integration by remembering that $\theta = \text{arc tan}\left(\frac{\tan \theta'}{k}\right)$, gives for

$$\theta' = \frac{\pi}{2}, n'\left(\frac{\pi}{2}\right) = \frac{2}{\pi}\,\text{N arc tan } \infty = \text{N}\,,$$

as it should be. For $\theta' = 45°$, for instance, we have $\tan \theta' = 1$, and

$$n'\left(\frac{\pi}{4}\right) = \frac{2}{\pi}\,\text{N . arc tan}\left(\frac{1}{k}\right).$$

Thus, if for example, $k = 1/10$,

$$n'(45°) = \text{N}\,\frac{84{\cdot}3}{90} \doteq 0.94\,\text{N}\,,$$

that is to say, 94 per cent. of the grains have their inclinations contained between 0° and 45°, and the remaining 6 per cent. between 45° and 90°.

EASTMAN KODAK CO.,
ROCHESTER, N. Y.

A METHOD OF OBTAINING RADIANT ENERGY HAVING THE VISIBLE SPECTRAL DISTRIBUTION OF A COMPLETE RADIATOR AT VERY HIGH TEMPERATURES.[1]

BY

IRWIN G. PRIEST.

THE author has previously published several detached notices and papers showing the utility of rotatory dispersion in producing radiant energy of some desired spectral distribution.[2] It is possible to obtain by this artifice spectral distributions corresponding to a complete radiator (black body) at temperatures far above anything possible in the actual operation of a furnace, or any artificial incandescent source. In certain studies in colorimetry and physiologic optics[3] it is desirable to be able to produce at will such high temperature energy distributions in the visible spectrum. In the previous papers this matter was not treated in a sufficiently unified manner, and data now at hand were not available when they were published. *The purpose of the present notice is to state the precise specifications for producing radiant energy of the visible spectral distribution of a theoretical complete radiator (by Planck's formula, $c_2 = 14350$) at any temperature between 3100° K and 7000° K, by means of an apparatus which permits of the convenient adjustment of the distribution to correspond to any temperature within this range.*[4] The detailed treatment of the theory of this method and the computations leading to these results are reserved for a subsequent paper.[5]

The source of radiant energy is a gas-filled tungsten lamp[6] at a color temperature of 2830° K (Cf. Fig. 1). The distribution is modified by the rotatory dispersion of quartz,[7] using a quartz

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[1] Published by permission of the Director, Bureau of Standards.

The author is indebted to K. S. Gibson, H. J. McNicholas and M. C. Malamphy for assistance in some of the computations leading to the results given in this paper. The figures have been drawn by Miss Gertrude Clemens.

[2] *Phys. Rev.* (2) **6**, p. 64, July, 1915; (2) **9**, p. 341, Apr., 1917; (2) **10**, p. 208, Aug., 1917; (2) 11, p. 502, June, 1918; (2) 14, p. 264, Sept., 1919; (2) 15, p. 538, June, 1920. J. OP. SOC. AM., Nov. 1920.

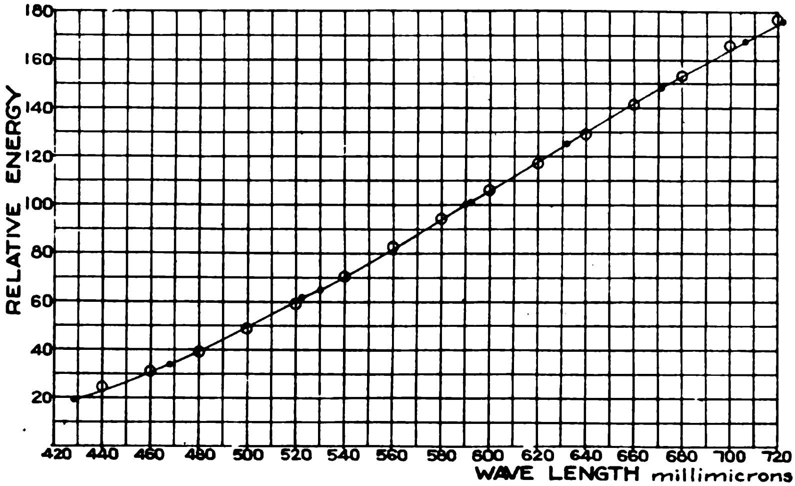
[3] *E.g.*, Troland: Trans. I. E. S., 13, p. 26, 1918. Priest: New Study of the Leucoscope, J. OP. SOC. AM., Nov., 1920.

[4] Temperatures up to about 3000° K can be attained in actual incandescent sources—gas-filled tungsten lamps.

[5] The Application of Rotatory Dispersion to Colorimetry, *B. S. Sci. Paper.*

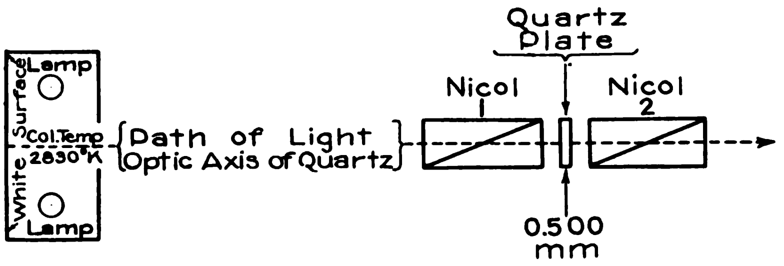
[6] B. S. Lamp 1717. The color temperature has been derived from the energy distribution curve determined radiometrically by Dr. W. W. Coblentz.

FIG. 1.



Spectral Distribution of Energy.
Solid curve: Complete radiator at temperature 2830° K, by Planck's formula with $c_2 =$ 14350. The small solid black circles indicate the points actually computed by Planck's formula.
Open circles: B. S. Lamp 1717 at 118.0 volts (about 15.6 l. p. w.) determined radiometrically by W. W. Coblentz.

FIG. 2.



Simple Arrangement of One Quartz Plate Between Nicols.
The spectral distribution of energy emerging from Nicol 2 in the direction of the dotted arrow may be adjusted by rotation of Nicol 2 relative to Nicol 1 (angle, ϕ).

plate 0.500 mm. thick. Two arrangements of apparatus are used, namely:

(1) The simple arrangement (Fig. 2);

(2) The duplex arrangement (Fig. 3).

The combinations of nicol prisms and quartz plates shown in these figures constitute, in effect, selective ray filters of adjustable spectral transmission. The relative transmissions as a function of wave-length are:

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[7] Cf. Priest: A New Study of the Leucoscope, Appendix I, J. OP. SOC. AM., Nov., 1920.

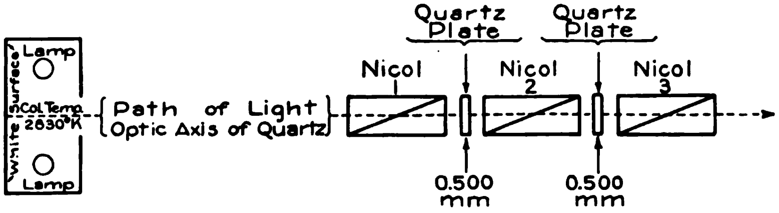
(1) For the simple arrangement,

$$\sin^2 (\phi - 0.5\,\alpha_\lambda); \qquad (1)$$

(2) For the duplex arrangement,

$$\sin^2 (\phi_1 - 0.5\,\alpha_\lambda) \sin^2 (\phi_2 - 0.5\,\alpha_\lambda); \qquad (2)$$

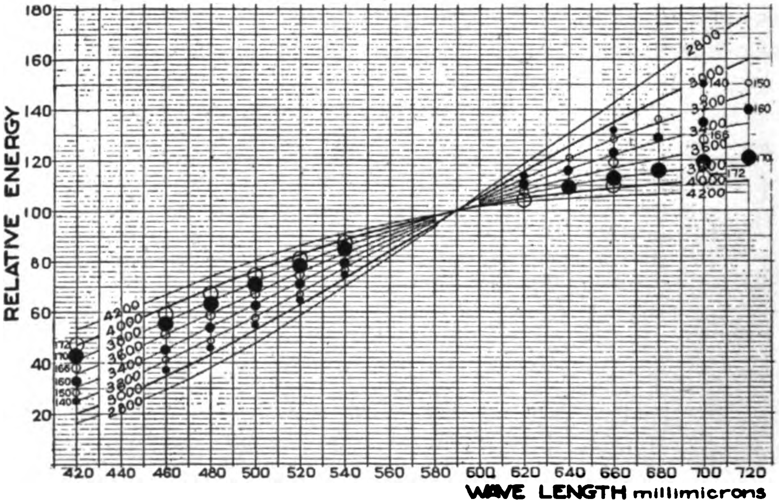
FIG. 3.



Duplex Arrangement of Two Quartz Plates and Three Nicols.

Nicols 1 and 2 are fixed. The spectral distribution of energy emerging from Nicol 3 in the direction of the dotted arrow may be adjusted by rotation of Nicol 3 relative to Nicol 2 (angle, ϕ_2).

FIG. 4.



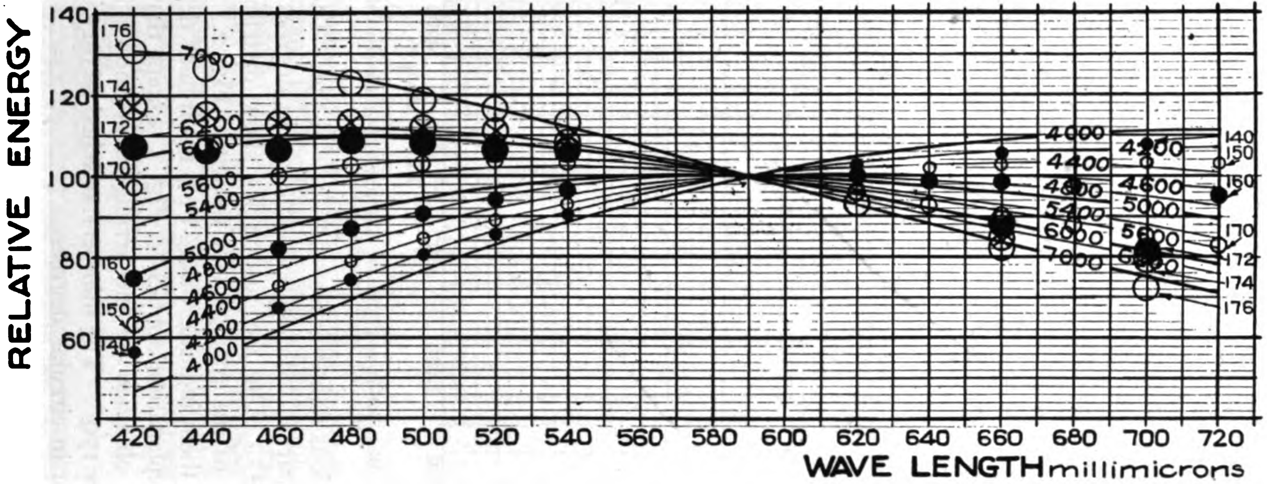
Illustrating the Production of Radiant Energy Having the Spectral Distribution of a Complete Radiator by Means of the Simple Apparatus (Fig. 2).

▶ The solid black curves represent Planck's formula with $c_2 = 14350$. The numbers attached to these curves indicate temperatures in degrees K.

The various circles represent distributions obtained by the arrangement shown in Fig. 2. Each different style and size of circle refers to a particular value of ϕ; and the numbers attached to the circles indicate values of ϕ in circular degrees.

In all cases, Energy = 100.0 at wave-length 590 (arbitrary convention).

FIG. 5.



Illustrating the Production of Radiant Energy Having the Spectral Distribution of a Complete Radiator by Means of the Duplex Apparatus (Fig. 3). The solid black curves represent Planck's formula with $c_2 = 14350$. The numbers attached to these curves indicate temperatures in degrees K. The various circles represent distributions obtained by the arrangement shown in Fig. 3. Each different style and size of circle refers to a particular value of ϕ_2; and the numbers attached to the circles indicate values of ϕ_2 in circular degrees. ϕ_1, constant = 170°.

In all cases, energy = 100.0 at wave-length 590 (arbitrary convention).

where:

$\alpha_\lambda \equiv$ rotation of the plane of polarization by 1.000 mm. of quartz for wave-length λ;

$\phi \equiv$ rotation of nicol, 2, relative to nicol, 1, in the same sense as α, from the extinction position with quartz removed (Fig. 2);

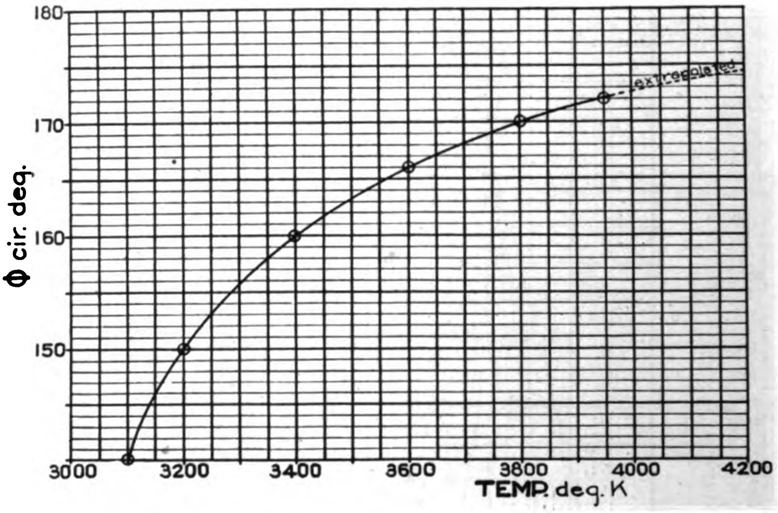
$\phi_1 \equiv$ rotation of nicol, 1, relative to nicol, 2, in the direction *contrary* to α, from extinction position;

$\phi_2 \equiv$ rotation of nicol, 3, relative to nicol, 2, with the same conventions as for ϕ above.

} (Fig. 3)

In Fig. 4 the solid curves show the spectral distributions of a theoretical complete radiator computed by Planck's distribution formula for various temperatures with $c_2 = 14350$. The numbers attached to the curves indicate temperatures in degrees K (absolute temperature). The various circles show distributions obtained by means of the arrangement shown in Fig. 2. The numbers attached to them indicate values of ϕ in circular degrees.

FIG. 6.



Temperature of Energy Distribution as a Function of ϕ with the Arrangement shown in Fig. 2. (Plotted from inspection of Fig. 4.)

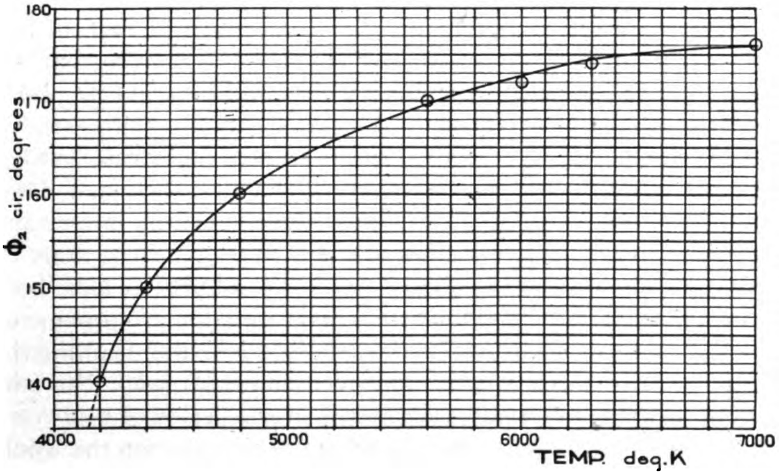
The same explanation applies to Fig. 5, except that it refers to distributions obtained by the arrangement shown in Fig. 3. ϕ_1 is a constant = 170°. The numbers attached to the circles indicate values of ϕ_2 in circular degrees.

In Fig. 6 the temperature corresponding to the distribution obtained by the arrangement in Fig. 2 is shown as a function of ϕ.

Likewise, in Fig. 7, the temperature corresponding to the distribution obtained by the arrangement shown in Fig. 3 is shown as a function of ϕ_2 (ϕ_1 being a constant = 170°).

It will be observed that very accurate representations of black-body distributions have been obtained by this method (Figs. 4 and 5). Curves drawn through each set of circles in Figs. 4 and 5 would be smooth curves very closely approximating the form of black-body curves, the only noticeable deviations occurring in the ends of the visible spectrum ($430m\mu > \lambda > 700m\mu$). Temperatures above 5000° K read as a function of ϕ_2 (Fig. 7) may be in error by about 100°; but lower temperatures read from either Fig. 6 or Fig. 7 are apparently reliable to about 10° to 20°.

FIG. 7.



Temperature of Energy Distribution as a Function of ϕ_2 (for $\phi_1 = 170°$) with the Arrangement shown in Fig. 3. (Plotted from inspection of Fig. 5.)

The Arons chromoscope [8] or similar apparatus provides a convenient means for the experimental realization of the principles outlined above. By means of such apparatus the conclusions drawn above have been experimentally verified in several cases. A determination of the spectral distribution of "normal gray light" by means of this apparatus is in progress.[9]

NATIONAL BUREAU OF STANDARDS,
    WASHINGTON,
        December 6, 1920.

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[8] *Ann. der Phys.* (4) 33, p. 799, year 1910; and (4) 39, p. 545, year 1912.

[9] See abstract, "The Spectral Distribution of Energy Required to Evoke the Gray Sensation," elsewhere in this number of this JOURNAL.

COURSES IN OPTICS AND OPTOMETRY IN COLUMBIA UNIVERSITY.

BY

JAMES P. C. SOUTHALL.

In 1909 a law was passed regulating the practice of optometry in the State of New York, and in the following year, partly in response to the provisions of this law and at the special solicitation of the Department of Education at Albany, courses in practical optics were offered for the first time in Columbia University. Inasmuch as this new profession was to be developed on a strictly scientific basis and as optometry was properly to be regarded as a branch of applied optics, it was expressly stipulated that the academic direction and supervision of the work should be placed in charge of the Department of Physics; but the administrative control was entrusted to Extension Teaching, the great development organization of the University, through which new ideas can be tried out without being hampered too much by usages and precedents. Year by year these courses have gained steadily in favor and importance, the qualifications for admission have been raised, the attendance has continually increased, the programme of studies has been intensified and extended, and the facilities of instruction have been greatly improved and enlarged. The curriculum, as at present arranged, extends over a period of two years and demands the student's single and entire time during the whole of that period. The principal subjects are mathematics, physics, theoretical optics, general anatomy and physiology, special anatomy of the eye, theoretical and practical optometry, physiological optics and pathological conditions of the eye. Not only is this course intended to provide such special training in optics as is needed for those who are, or expect to be, optometrists in the broader professional sense, but it can be pursued with the greatest benefit by any one who has refraction work to do and who, no matter what other useful training he may have had, wishes to obtain a real knowledge of the subject and a degree of practical skill at least equal to that required of the optometrist. The entrance requirements are precisely the same as those for admission to Columbia College.

Meantime, a longer course of study has also been planned, which will take four years to complete and which is designed to give a more extensive liberal education in addition to the professional training. This new programme, which has been presented to the authorities of the University for their approval, will enable a student to obtain not only a certificate of graduation in optometry, but at the same time an academic degree corresponding to that of bachelor of science. The completion of this course will likewise satisfy the requirements for admission to the medical school; and obviously this plan affords an unusually fine foundation for a person who looks forward to specializing in diseases of the eye after having had a medical education.

When one pauses to consider how nowadays nearly everybody suffers from eye-strain and the countless ills that follow in its train and reflects that it would be hard to overestimate the inefficiency, not to say incompetence, in all the affairs of life that may be traced to the lack of suitable eye-glasses, the far-reaching importance of this kind of special training begins to be duly appreciated. Until quite recently the intellectual and scientific level of optometrical practice outside of the work of those physicians who had made a special study of it was relatively very low. The truly remarkable elevation of standards through legislation and otherwise, which has come about within the last decade or more, is due in large measure to the single-hearted devotion and efforts of a few outstanding leaders among the optometrists. Often these have been very capable and gifted men with high ideals; and here and there some of these pioneers have made notable scientific contributions of recognized value and utility. The spiritual disciples of these men, fired with youthful enthusiasm for their new work, are eager at present to prove their skill and make further advances. According to this view optometry is a profession which, though still in process of development, and in spite of its comparatively humble origins and environment, is rapidly forging ahead and gaining recognition and prestige. Undoubtedly, it is a movement that both deserves encouragement and needs wise guidance. If optometry in the future is to take rank among the learned professions and is to enlist among its votaries the highest types of trained students, it must be taught systematically in the colleges. The fact that scientific training in refraction-work is now being given in at least two of the great universities of this country indicates that real progress is being made in this direction. It is in this field more than anywhere else that applied optics becomes a matter of vital interest in the large question of human welfare. It may be added also that although optics is one of the oldest of the sciences and its modern applications are very manifold, nevertheless spectacle glasses may be said to have been its only outcome of worldwide commercial importance. This statement needs to be emphasized, because in considering the larger aspect of the establishment in this country of a school of optical engineering and the scientific development of the optical industries in all their various branches, I think it is well perhaps to see clearly that the heart and nucleus of all projects of this kind are to be found in such beginnings as have been made in Columbia University in connection with the courses in Optics and Optometry.

In order to appreciate rightly the importance of this work, it should be viewed in its historical perspective; and for that purpose a brief survey of the development of modern ophthalmology will probably not be without interest here. Some of the statements in this sketch will be found to be quoted more or less freely from the current *Announcement of the Courses in Optics and Optometry*.

The oldest applications of optics were undoubtedly for the purpose of aiding vision, and the history of spectacle glasses dates back to about the time of Roger Bacon in the thirteenth century (or "Frier" Bacon, as he is called in Robert Smith's "Compleat System of Opticks"); although whether they were first invented in England or in Italy is one of those obscure questions which will probably never be answered. The late Professor Silvanus P. Thompson has called attention to a portrait of Pope Leo X, painted by Raphael in 1517, in which that pontiff is represented as wearing spectacles; but at the same time Professor Thompson reminds us that there is likewise a picture in one of the churches in Florence in which St. Jerome, who flourished as far back as the fourth century, is depicted in his cell with a pair of spectacles beside him; which is just one of those anachronisms which are so common in mediæval art.

Not until towards the latter part of the eighteenth century was there any really noteworthy progress in the art of making spectacles. Benjamin Franklin, as might almost be guessed, was the first to use bifocal glasses, and Sir David Brewster (whose remarkable and acute insight into all the problems of practical optics seems, so to speak, uncanny!) proposed and worked out multifocal lenses, which, however, have never had any wide application. "Periscopic" lenses, indeed quite similar to the glasses which are worn nowadays, were recommended by Wollaston in a notable paper which attracted little attention in his own day. Meanwhile, the foundations of physiological optics had been laid by Christopher Scheiner, whose celebrated treatise entitled "Das Auge oder die Grundlage der Optik" was published in 1619, and also by DesCartes. Many years later the versatile Thomas Young found himself irresistibly drawn to the same fertile but uncultivated fields of research; and incidentally it was he who in 1801 first recognized the existence, not to say prevalence, of astigmatism in the human eye. About a quarter of a century afterwards Sir George Airy corrected this error in his own eye by means of a cylindrical lens. From this time onwards progress in ophthalmology has been unretarded. Probably no scientific labors have ever been attended with greater blessings to mankind than the brilliant work in this vast new territory which is associated especially with the names of Donders and von Helmholtz along about the middle of the nineteenth century. To the student of science it is interesting to note how the training of both Young and von Helmholtz, each of whom, starting out at first on a medical career, afterwards gained his great renown as a physicist, peculiarly qualified them to investigate and chart the unexplored regions of physiological optics, which remains to this day very much as those great pioneers left it. Their horizons were not bounded by the narrow domains of a single science, as is too often the case with more recent investigators of these obscure phenomena.

Coming down nearer to our own time, we all know of the extraordinary modern developments in all directions in both ophthalmology and applied optics, particularly in Germany and other foreign countries. Of late years England is showing signs of resuming her rightful place in optical science which she had in the days of Newton and his successors. Few persons are perhaps aware of the rapid rate at which spectacle optics is developing into a severe scientific pursuit. It would be a very superficial view to suppose that modern improvements in the manufacture of spectacles consisted chiefly in the introduction of lighter and more adaptable frames and of new and ingenious processes of fusing bifocal lenses or of grinding toric surfaces; although these and other mechanical refinements are by no means insignificant achievements. This would be quite a mistake. It may be that the teaching in the schools has remained much as it was; but, as has been already intimated, the training of spectacle opticians to deal with the problems of astigmatism not only in the eye itself, but in the correction-glass, and with many similar problems, has made great strides and under the stimulus of the recent optometry legislation both here and abroad is assuming an important development. Improved clinical methods and apparatus are in evidence, new optical journals are being started, and the young practitioners are eager to be in the forefront. It would be difficult to overestimate the importance of the advances in ophthalmology which are to be traced to the work of Allvar Gullstrand, in Sweden, who, in collaboration with the disciples of Abbe, in Jena, has made contributions, both by his writings and by his optical appliances, which are quite worthy to be compared with those of von Helmholtz himself. As long ago as 1914 Carl Zeiss had made a new ophthalmoscope according to Gullstrand's optical specifications, which for the examination and diagnosis of diseased conditions of the eye is a marvel of instrumental perfection; although, alas! this instrument is almost unknown in this country. By means of telescopic spectacles designed by von Rohr thousands of old soldiers whose eyesight was almost ruined in the war are now being enabled to pursue useful employments.

The main purport of this argument is to set forth as simply as possible the ideals which we have had in mind in the development of the courses in Optics and Optometry in Columbia University; and while we do not pretend to have attained as yet anything beyond an imperfect and partial realization of some of our ideals, we believe profoundly that year by year we are making steady progress towards our goal. The two-year curriculum which we have at present is probably as nearly satisfactory as it can be, and it would be hardly practicable for a student who is not presumed to have had any previous college training whatever to acquire within the same length of time a deeper insight into the fundamental problems of spectacle optics and vision than is intended by this programme of studies. The fact is, it takes a boy or girl of rather exceptional ability to complete it in the minimum period of two years. The entire structure is reared on a basis of physics and elementary mathematics, which are two of the principal subjects in the first year; but at the same time from the very outset the pupil begins to take up other subjects which bear more directly on his vocation; for example, theoretical optics, laboratory practice, shop-work and general anatomy, with special reference to the anatomy and physiology of the eye. Naturally, when he comes to his second year, his work becomes more specialized and more practical; and although he still continues to devote a considerable portion of time to theoretical optics, and particularly to the theory of spectacle glasses, now he enters also on the study of physiological optics, and at the same time he begins to acquire the more distinctly professional part of his training; for example, in such courses as theoretical and practical optometry, where he is taught systematically and thoroughly all the various processes of optometry and the most approved modern methods of sight-testing and refraction. In the latter half of this year, beginning early in February and continuing until the end of May, an optometrical clinic is conducted every Monday and Thursday afternoon. This clinic is free to the public, and the students themselves act as assistants to the instructors in charge. A preliminary examination of each patient is made by one of the students in attendance, which is then checked and criticized by the members of the clinical staff. Two examination rooms have been furnished and equipped with every necessary appliance for making these tests. In addition to obtaining this clinical experience, the student has also laboratory courses in physics, lens-measurements and physiological optics; and during the whole of his second year he is required to spend four hours each week in shop-work, with a view to acquiring skill in all the ordinary operations of mechanical optics (grinding, surfacing, centering, edging, neutralizing, and transposing lenses, bridge-bending, frame-fitting, etc.). Thanks to the generous and helpful coöperation of the American Optical Company, the Bausch & Lomb Optical Company, the Standard Optical Company, the General Optical Company, the Globe Optical Company, and others, the laboratories and shops are equipped with all the most modern optical instruments, including clinical appliances of all kinds, and with the latest types of machines for grinding, surfacing, edging, drilling, cutting, etc.; so that the facilities for work in all these manifold directions are probably quite unsurpassed. Moreover, during the whole of the second year the pupil is given a special course in pathological conditions of the eye, which is intended chiefly to show him where the territory of the optometrist ends and that of the physician and surgeon begins, and to familiarize him to some extent with those numerous and complex cases of eye-trouble which, as a mere refractionist, it is beyond his power to do more than ameliorate, if indeed he can do even so much as that. Here he is constantly reminded of the limitations of his powers and taught that in such cases it is his duty to send the patient to some other specialist.

As might be supposed, already there is beginning to be a demand for postgraduate courses in optometry, and either something of that kind or perhaps the extension of the present curriculum so that it will require three years for completion instead of two is the next step to be considered. And here, it must be confessed, we are now more or less at sea, because beyond this point it is all uncharted.

After all, as was said in the beginning, optometry is but one branch of applied optics. Already in England a significant effect of the increased interest in the study of spectacle optics is to be seen in the establishment of one or more schools of optical engineering, where it is now possible for a student to pursue extensive studies in optics, both theoretical and applied, including courses not only in such subjects as physiological optics and the making, testing and fitting of spectacle lenses, but also in the principles, design and construction of optical instruments in the arts of both war and peace, and where presumably optical researches of all kinds are to be systematically fostered and encouraged. As I write, I have before me a prospectus of two courses of lectures on designing and computing of lens-systems which were given first in 1917 by Professor Conrady in the Department of Technical Optics in the Imperial College of Science and Technology at South Kensington. But it was the great war more than anything else that roused England at last to a realization of her plight, due to the long neglect of the optical sciences and industries. Professor Cheshire, writing, recently in *Nature* (civ., 1920, pages 530, 531) on "The Outlook of British Technical Optics," says:

> "We shiver yet when we remember the single thread upon which the production of optical munitions depended in this country. Our glass-makers, beaten by their foreign rivals, receiving neither help, encouragement, nor even recognition from the Government, had been content to continue their patriotic efforts to maintain the industry, on the urgent representations of a few far-seeing scientific men, until long after those efforts held out any promise of pecuniary reward.

That danger, happily, has passed, and the complete solution of the optical glass problem is now only a question of time. Many of the glasses now produced in this country compare favourably with the best of those of our foreign rivals. The varieties available are limited, but the leeway is being rapidly made up.

"It is often stated that the late supremacy of the Germans in optical production was the direct and necessary result of the glass-making labours of Abbe and Schott, completed in the year 1886. This is not a correct statement of the case. The fact is that, when Abbe and Schott broke down the barriers to optical progress imposed by the limited varieties of glass available, Germany had in reserve a small army of scientific workers, equipped with the necessary technical knowledge and skill, ready to fill the breach and carry on the work of utilising the new glasses in the invention of new optical systems and in the improvement of old. But the world moves quickly, and inventions and discoveries, however valuable intrinsically, are likely to remain barren unless a country has a sufficient number of men equipped with the necessary knowledge to exploit them instantly and to the full. Indeed, it is only such men that can appreciate the value of inventions and discoveries. The necessity for a broad and generous scheme of national education in optical matters thus becomes apparent. When the users of optical instruments are sufficiently educated to be able to distinguish and appraise good designs and work, makers will be encouraged to meet their demands. In the absence of such education the faddist has his day, and the maker concerns himself too often in meeting the demands of fashion.

"It is satisfactory to know, then, that, so far as this country is concerned, a great deal has already been done to foster optical education. The establishment of the Technical Optics Committee, which includes representatives of the British Optical Instrument Makers' Association, the War Office, the Admiralty, the National Physical Laboratory, the London County Council, the Royal Society, and the Imperial College of Science, is in itself sufficient evidence that the question has been taken up with great thoroughness. The establishment of a department of optical engineering and applied optics at the Imperial College will ensure a supply of capable and well-educated young men for the needs of the industry generally. Prof. Conrady is doing yeoman service in the establishment of an English school of optical designers and computers, the need for which was so acutely felt during the war. The outlook, then, so far as education is concerned, is decidedly promising. Indeed, in some important respects the scheme of education here is already in advance of that of any other country."

During the war also an Institute of Applied Optics was founded in France. The studies which should be included in a school of this kind are, for example, geometrical optics, physical optics, physiological optics, optometry, theory of modern optical instruments, lens design and lens testing, manufacture of optical glass, refractometry, photometry and illuminating engineering, spectrophotometry, colorimetry, etc. Courses corresponding to some of these mentioned above are already being given in Columbia University, and the writer himself is strongly of the opinion that this university with its commanding position and unequalled facilities offers by far the best place in the United States for the establishment of a school of optical engineering, especially in conjunction with the courses in optics and optometry which are now being carried on there; and with a modicum of encouragement from outside, he has reason to hope that the authorities of the University would view such a project with favor.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY,
November 15, 1920.

THE ANGULAR DEFLECTIONS PRODUCED ON TRANSMITTED LIGHT RAYS BY SLIGHTLY INCORRECT INTERFACIAL ANGLES OF REFLECTING PRISMS.

BY

F. E. WRIGHT.

IN the manufacture of precision optics a tolerance of ± 2 minutes of arc is considered a fair and reasonable specification for most prism angles. In certain prisms, such as the Dove erecting prism and the penta prism, the angle must be correct within ± 1′ of arc; in the roof-angle prism, the roof-angle must be correct within a few seconds of arc. These prism-angle tolerances are commonly measured optically and the specifications are stated in terms of permissible angular deflections of transmitted rays from their prescribed paths.

In conformity with the practice adopted at Frankford Arsenal on the recommendation of Col. G. F. Jenks, in 1910, for the inspection of the optical performance of prisms, the following conventions may well be established and followed: A reflecting prism in an optical instrument serves to change the direction of the optical axis of the instrument through a definite, prescribed angle. Let the plane in which this angle is measured be called the "axial plane." Then the "axial-angle error" of a prism placed with its reflecting surface in the correct position is the angular component, in the axial plane, of the deviation of the reflected ray from its prescribed path after emergence from the prism. The "side-angle error" is the angular component, in the plane normal to the axial plane, of the deviation of the reflected ray from its prescribed direction after emergence from the prism. The Dove erecting prism serves only to invert and not to change the direction of the axial rays after emergence; for this prism, which presents a limiting case, let the axial plane be perpendicular to the hypothenuse surface and to the first surface entered by the incident rays; it is the plane of incidence of the incident rays and contains the normals to the incident refracting surface and to the hypothenuse surface. These conventions establish the reflecting surface of a prism as the surface of reference. Any other surface might be used, but for the sake of uniformity in the testing and designation of the different types of prisms the reflecting surface appears to serve the purpose best. In the case of roof-edge prisms and penta prisms two reflecting surfaces replace the single reflecting surface and serve to invert the image; in these types of prisms the two reflecting prism-faces function as a unit and may be treated as such.

Experience has shown that different optical methods of different degrees of sensitiveness may be used to test the optical performance of any given type of reflecting prism. The deviations should be measured of those rays which traverse the prisms along the path which they follow in the actual instrument; in other words, a measure of the optical performance of the prism should be taken.

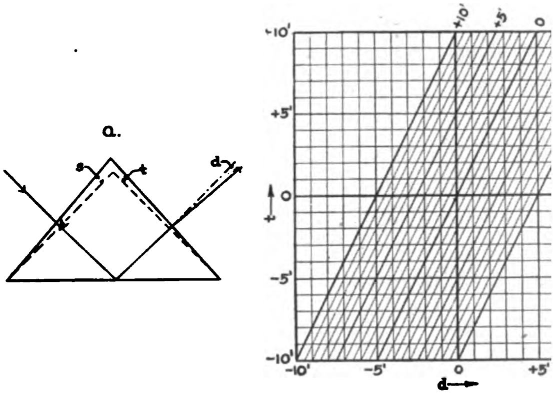
In view of the fact that the path of a ray through a prism depends on a number of different factors, such as defects in the glass, degree of flatness of prism surfaces in addition to different interfacial prism angles, the only feasible method for ascertaining the deviations of transmitted axial rays from the prescribed path, which result from slight departures of the several prism angles from the prescribed angles, is to consider the glass to be optically homogeneous and free from defects and the prism surfaces to be optically flat. Under these assumptions it is possible to ascertain the competency of the several different optical methods available for testing and to determine the significance, in terms of actual prism angles, of different optical tolerances which may be set. The relations between actual prism angles and the resulting ray deflections in the axial and side (normal to axial) planes are indicated for the different types of prisms in Figs. 1 to 7. In these figures a section of the correct prism is indicated by dotted lines; the incorrectly oriented surfaces are indicated by full lines; the paths of the rays are indicated by full lines. In each set of curves the ordinates are the angles of departure *t* of the prism surface from which the ray emerges; the abscissæ the deviations *d* of the emergent ray from the prescribed path; the sets of diagonal lines represent the departure angles *s* of the incident prism surface.

THE RIGHT-ANGLE REFLECTING PRISM.

Three optical methods may be used to test this type of prism; in two of the methods a telescope and a distant target or collimator are employed; in the third method an autocollimator serves the purpose. A variation of these methods is obtained b projection screen in place of the observing telescope.

FIRST METHOD. *Axial-angle Errors.*—Let it for the sake of simplicity, in this and other cases (error, that the faces of the prism are in the same practically the case in all prisms and the slight dep this condition introduce departures of the second ord axial-angle errors. Let s be the error of the angle (total angle exceeds 45°), which the first side incl hypothenuse (Fig. 1*a*) and t the error of the sec These errors are always small and do not exceed 10

FIG. 1.

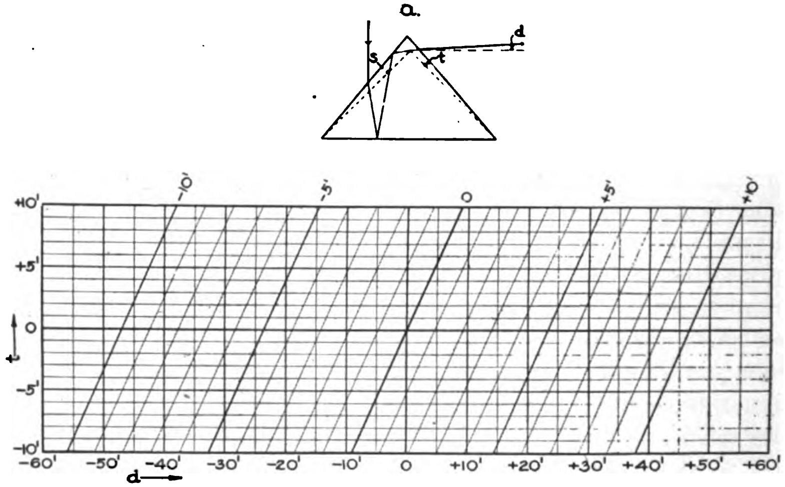


In this diagram are given the deviations of the emergent ray from its p transmission through a right angle prism of slightly incorrect interfacial in Fig. 1(*a*).

in very poor work. Let the direction of the incident angle of 45° with the reflecting face of the prism. of the emergent ray from its prescribed path in the then $d = (n-1)(t-s)$ wherein $n = 1.515$ is the re of the prism. In Fig. 1 the deviations of the rays a by the abscissæ, the angles t by the ordinates and by the series of oblique lines; thus a prism for w interfacial angle is 45° 02′ ($s = +2'$) and the seco angle is 45° 01′ ($t = +1'$) causes a deviation of the in the axial plane of $-\frac{1}{2}'$ from the prescribed path.

Side-angle Errors.—In this and other cases of si the angles which the side faces include with the face are considered to be exactly 45°. This assumption is made in order to facilitate the computation, and the error introduced by it is of the second order only because the departures in these angles from 45° are very slight. In this method the incident ray enters along the normal to the first side surface, is reflected at the hypothenuse face, and is then refracted at the second side surface whose trace on the hypothenuse face includes a small angle t with the trace of the first side face on the hypothenuse face. Under these conditions the deviation of the emergent ray for a prism of refractive index, $n = 1.515$, is $d = 1.07 \cdot t$; in other words, the deviation is practically equal to the departure angle t.

FIG. 2.



Constant deviation method. In this figure are shown the deflections of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 2(*a*).

With this method, therefore, the angular deviation of the emergent ray from the prescribed path, both in the axial plane and the side plane, is of the same order of magnitude as that of the errors in the interfacial angles, and the method is not especially sensitive. The method has the advantage, however, that the prism is tested under conditions resembling those of actual use.

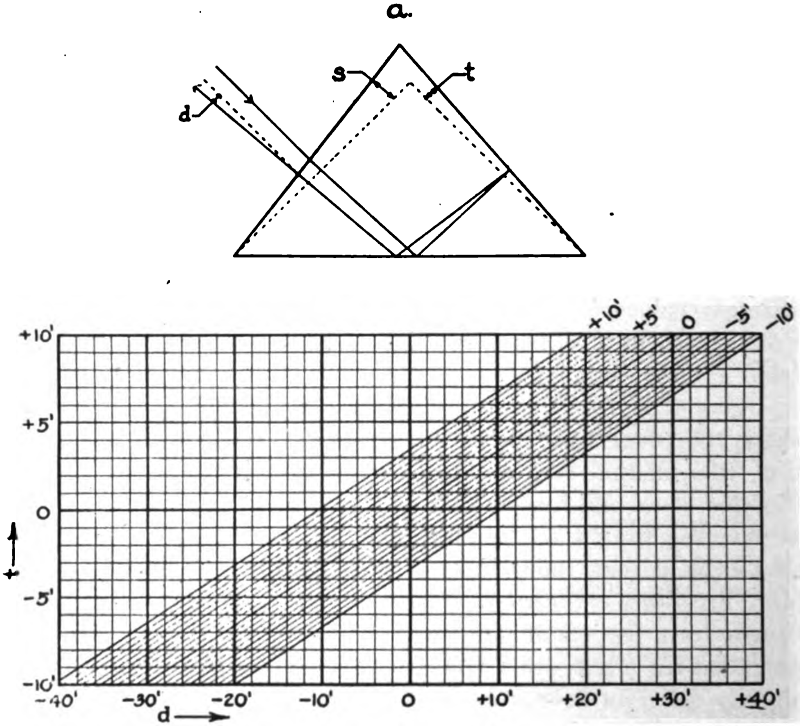
For routine testing by this method, either a spectrometer or a combination of a telescope of adequate resolving power and a collimator or telescope and distant target is used. The prism is placed with the hypothenuse surface against a fixture having rounded bearing points for both the hypothenuse and first side surface. The fixture is accurately adjusted so that the plane of incidence contains the normals to the hypothenuse face and to the first side face. A coördinate scale in the image plane of the telescope indicates the axial-angle and the side-angle departures from the prescribed path. In place of the observing telescope a projection screen with properly arranged coördinate lines to mark the tolerance limits may also be used.

SECOND METHOD. THE CONSTANT DEVIATION METHOD OF FRANKFORD ARSENAL. *Axial-angle Error.*—The paths of the rays through the prism when used as a constant deviation prism are shown in Fig. 2*a*. The incident ray is considered to be perpendicular to the hypothenuse face. The deviation d, of the emergent ray from 90°, is then $d = 2s + (i - r) - (i' - r')$, wherein i is the angle of incidence, r the angle of refraction, and $\sin i = n \cdot \sin r$ $(n = 1.515)$; r' is the angle of incidence of the emergent ray and equal to $(r - 3s - t)$, and $\sin i' = n \cdot \sin r'$. The deviations d for different values of t and s are shown by the curves of Fig. 2. From these curves it is evident that in a prism whose angles differ from the correct angles by $\pm 2'$, the maximal deviation of the emergent ray from the prescribed path may be $11'$. The curves show, moreover, that if the prism be observed only in one position, this method may not be adequate to determine its quality; thus for $s = 0'$ and $t = +10'$ the deviation is $+9'$; if now the prism be reversed so that $s = 10'$ and $t = 0'$ the deviation $d = 46.5'$. It is thus evident that, unless care is taken to measure a prism from both directions, erroneous conclusions regarding its quality may be drawn from the result obtained. If, moreover, the incident ray enters at an angle other than normal to the hypothenuse face, slightly different deviations result. The deviation depends therefore not only on the prism angle errors, but also on the position of the prism during the observation. The last uncertainty is eliminated by prescribing that the incident ray shall coincide in direction with the normal to the hypothenuse face.

Side-angle Error.—Let the prism side-angle be 45°; let the direction of the incident ray coincide with the normal to the hypothenuse face; let the plane of incidence include the normals to the first side face and to the hypothenuse face; let t be the angle between the traces of the first side face and of the second side face, respectively, on the hypothenuse face. The deviation d of the emergent ray from the prescribed path is then approximately $t/2.1$ for $n=1.515$; in other words, the deviation is about half that of the angle t and the method is not especially sensitive in this respect.

THIRD METHOD. AUTOCOLLIMATION METHODS. *Axial-angle Error.*—In these methods a well corrected autocollimator of

FIG. 3.

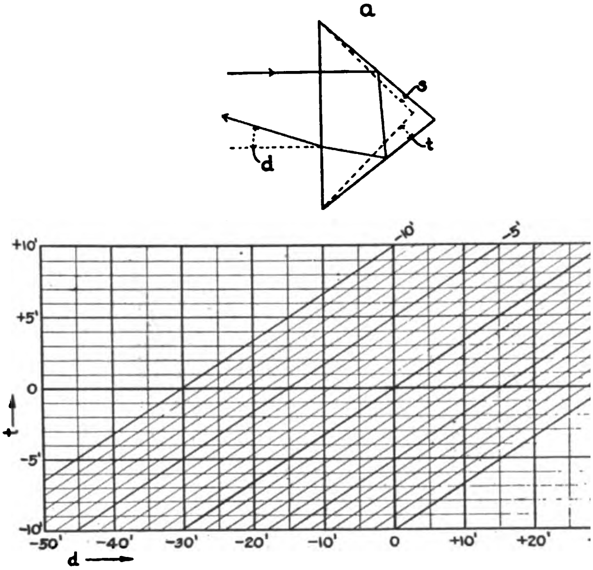


Autocollimation method. **In this diagram are shown the deviations of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 3(*a*).**

adequate resolving power is used; the hypothenuse surface is placed at an angle of 45° with the axis of the collimator (Fig. 3*a*). The deviation of the emergent ray from the original line of sight is then $d=2nt-2(n-1)s=3.03\,t-1.03\,s$ for a prism of refractive index $n=1.515$. The curves of Fig. 3 show that for errors in the prism interfacial angles of $s=+3'$ and $t=+1'$ the deviation

$d=0$; for $s=-3'$ and $t=+1'$ the deviation i
method alone is therefore not adequate to test t
If, however, the prism be rotated and the hyp
placed normal to and facing the collimator (Fig.
of Fig. 4 show that the deviation, as computed f
$d=2n(t+s)$, for $s=3'$ and $t=+1'$ is $d=+12'$; s
the deviation $d=-6'$. The measurement of the
prism in these two positions suffices therefore
angles satisfactorily. The autocollimation meth

FIG. 4.



Autocollimation method. A graphical representation is given in th
tions of the emergent ray from its prescribed path after transmission th
incorrect interfacial prism angles, as indicated in Fig. 4 (*a*).

sensitive than the first method and is less encumb
than is the second method.

Side-angle Error.—With the prism placed
of Fig. 4*a* the side-angle deviation is practica
that obtained in the first method, namely, $d=$
approximately.

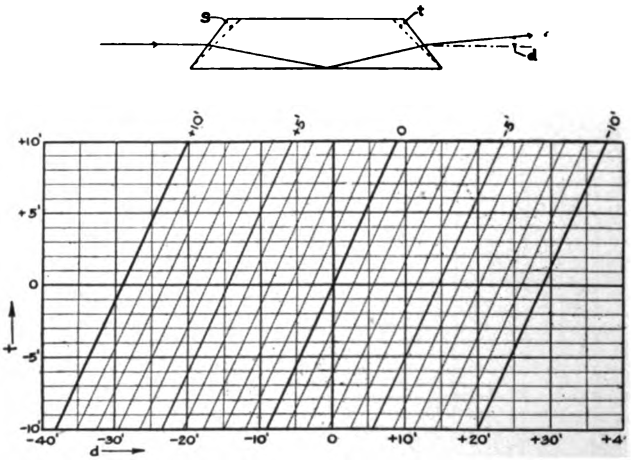
A projection method may also be used with
but this method is simply a modification of the t
and involves nothing new in principle.

THE DOVE ERECTING PRISM.

Axial-angle Error.—This prism is used to invert the image and not to change the direction of an axial ray in the telescope. In the method commonly used for testing this prism a collimator or a distant target is employed, together with an observing telescope with coördinate scale in its image plane. The prism is placed in front of the telescope with the hypothenuse face parallel with the line of sight. The path of the axial ray through the prism is shown in Fig. 5*a*. The deviation d for prism interfacial angle errors s and t is $d = 45° - t - i'$ wherein $\sin i' = n \cdot \sin r'$; $r' = r + s - t$; $\sin (45° - s) = n \cdot \sin r$ and $n = 1.515$. The curves of Fig. 5 show that for $s = +1'$ and $t = -1'$, $d = -3.8'$; for $s = +3'$ and $t = +10'$; $d = 0$; but if the prism be reversed so that $s = +10'$ and $t = +3'$, the deviation $d = -26'$. In testing a prism by this method, therefore, it should be examined in both positions, otherwise serious errors may arise. This fact should also be noted in the assembly of the prism.

FIG. 5.

a.



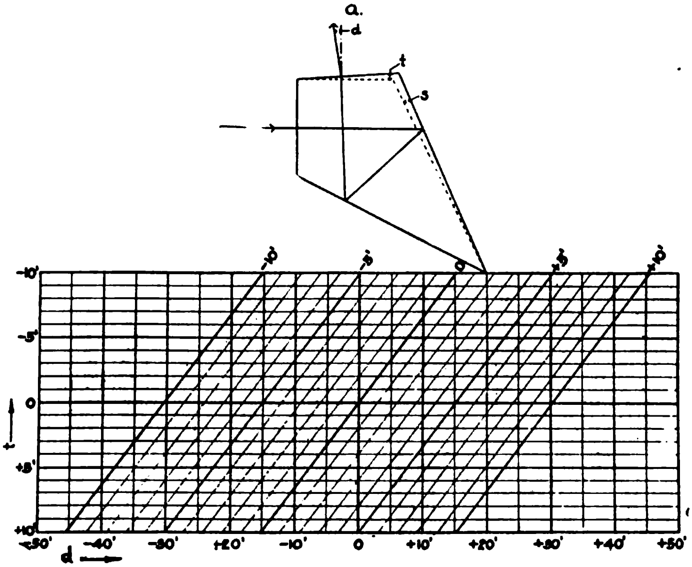
In this figure are shown the deviations of the emergent ray from its prescribed path after transmission through a prism of slightly incorrect interfacial prism angles, as indicated in Fig. 5(*a*).

Side-angle Error.—On the assumption that the side-angles are correct and equal to 45° and that the axial plane is normal to both the hypothenuse and first side faces, the trace of the second side surface on the hypothenuse face includes a small angle t with the trace of the first side face on the hypothenuse face, the resulting side-angle deviation of a transmitted axial ray is approximately $d=-t/2.1$ for $n=1.515$.

THE ROOF-EDGE PRISM.

This prism may be treated as a simple reflecting prism in which the hypothenuse reflecting surface has been replaced by two reflecting surfaces mutually at right angles and truncating, like a roof, the elongated edges of the hypothenuse face. The roof-angle must be exactly 90° within a few seconds of arc, otherwise a double image results. The incident light strikes the first reflecting surface at an angle of 120°, is reflected across to the second face, and is reflected by it at an angle of 120°, thus inverting the image completely. The tests for axial-angle and side-angle errors may be made by the methods used for ordinary reflecting prisms; of these the autocollimating methods appear to be the most satisfactory. The correctness of the roof-angle is determined by the absence of a double image of the lines on the horizontal line of the

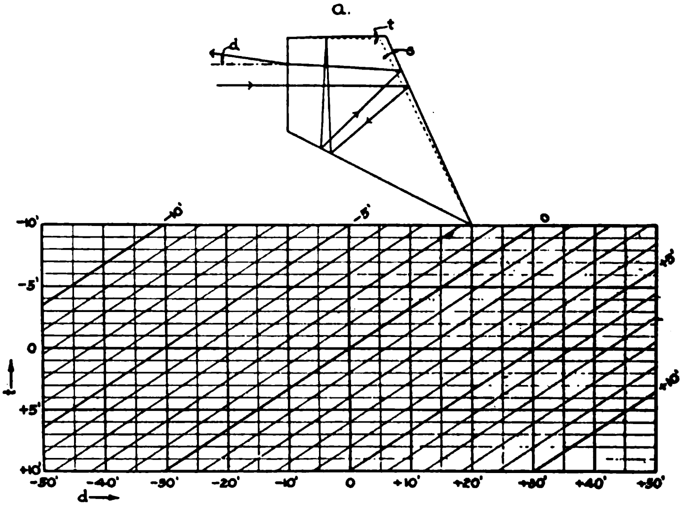
FIG. 6.



In this diagram are represented the deviations of the emergent ray from its prescribed path after transmission through a penta prism of slightly incorrect interfacial prism angles, as indicated in Fig. 6(*a*).

target. The following rule is useful in determining, in the case of a double image, whether the roof-angle is greater or less than 90°. Rack the eyepiece of the observing telescope inward toward the prism; if the two images approach each other the roof angle is less than 90°; if they approach when the eyepiece is racked outward the roof angle is greater than 90°. Another method for testing the roof-angle is to grind and polish a small face truncating the edge between the two side faces of the prism; a ray of incident light normal to this face should be reflected by the roof faces as a single ray parallel with the original direction. This test is made with an autocollimator; the method is, however, hardly to be recommended, as it offers no advantage over the foregoing method and requires the preparation of an additional face on the prism.

FIG. 7.



Autocollimation method. In this diagram are shown the deviations of the emergent ray from its prescribed path after transmission through a penta prism of slightly incorrect interfacial angles, as indicated in Fig. 7(*a*).

THE PENTA PRISM.

Methods similar to those described in the foregoing paragraphs are used for testing penta prisms optically. In all cases the prism should be held against a proper fixture so that the incident ray enters normally to the first surface of the prism as indicated in Figs. 6 and 7.

FIRST METHOD.—A collimator or distant target with horizontal and vertical lines is used in conjunction with an observing telescope or projection screen.

Axial-angle Errors.—For a ray entering the prism normal to the first face as shown in Fig. 6*a*, the deviation of the emergent ray from the prescribed path is $d = n(2s - t)$ wherein t and s are the small angular errors in the interfacial angles of the penta prism, and $n = 1.515$ is its refractive index. The series of curves in Fig. 6 are a graphical solution of the foregoing equation; thus for $t = +2'$ and $s = -1'$, $d = -6'$; for $t = +2'$, $s = +1'$, $d = 0$. The equation shows, moreover, that a slight change (s) in the angle between the two reflecting faces has an effect in deflecting a transmitted ray twice as great as that produced by the same change (t) in the angle between the other two faces. The reflecting faces here function as a unit just as do the reflecting faces of a roof-edge prism.

Side-angle Error.—Let the line of intersection of the two reflecting faces include an angle t with the line of intersection of the other two faces. This gives a twist to the prism. Let the angle between reflecting faces be 45° and that between the other two faces 90°. Then the angle of deflection is approximately equal to $d = n \cdot t$ wherein $n = 1.515$, the refractive index of the glass.

SECOND METHOD. AUTOCOLLIMATION METHOD. *Axial-angle Error.*—The deflection in this case (Fig. 7) is $d = 2n\ (2s - t)$ or twice that obtained by the first method; thus for $t = 2'$ and $s = -1'$, $d = -12'$; for $t = 2'$ and $s = 1'$, $d = 0$. Similarly the side-angle error is approximately twice that found by the first method. The results prove, however, that in these methods a compensation may take place whereby zero deflection may be obtained from a prism whose angles are incorrect.

SUMMARY.

Different methods are available for the testing of the interfacial angles of a reflecting prism of any given type. The angles may be measured directly on a goniometer or a spectrometer or by means of delicate gauges or of specially mounted test plates, such as the optical square. They may also be measured by the deflections produced by the prism on transmitted rays of light. A prism of the prescribed shape deflects the transmitted rays along a prescribed path. A prism of slightly incorrect interfacial prism angles deflects transmitted light rays so that these emerge along a direction slightly different from that prescribed; from this deviation of the emergent rays of light from the prescribed path it is possible to determine the degree of exactness of the interfacial prism angles. Methods of test based on this principle enable the observer at the same time to draw conclusions regarding the quality of glass in the prism and the degree of flatness of the prism faces; in short, to judge of the fitness of the prism as a component of any given optical system. In the present paper prisms of different types are considered and different sets of curves are shown in diagrams illustrating the deviations of transmitted light rays from the prescribed paths caused by slightly incorrect interfacial prism angles. From these curves the sensitiveness of the several optical methods which have been proposed for testing the accuracy of prism angles by the use of transmitted light rays can be determined directly and the suitability of each method thereby ascertained.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
January, 1921.

THE SPECTRAL DISTRIBUTION OF ENERGY REQUIRED TO EVOKE THE GRAY SENSATION.*

[ABSTRACT.]

BY

IRWIN G. PRIEST.

General Nature of Paper.—The chief significance of this paper lies in the development and testing of an experimental method for determining an objective physical standard of white light.

Common experience teaches that the color evoked by the light from an incandescent solid (oil and gas flames, electric incandescent lamps, etc.,) depends upon the temperature of the source. Sources at comparatively low temperatures appear red; as the temperature rises the color becomes yellow of paler and paler tint until, for the highest temperatures at which artificial sources can be operated, it approximates to white. Still, all artificial incandescent sources appear yellow relative to the sun. On the other hand, some stars appear blue relative to the sun. Following Ives, Troland and others, the basic concept of this paper is this: Light conforming to the Planckian distribution formula for some particular value of temperature should evoke the hueless sensation of brilliance commonly called "white" or "gray," and recognizable neither as bluish nor yellowish.

Specific Purposes of Investigation.—The purposes of this investigation have been:

(1) To develop and test an experimental method of determining this hypothetical transition temperature, below which a Planckian radiator would appear yellow, at which it would appear white, and above which it would appear blue.

(2) To determine this temperature for a few individuals, under the following conditions:

(*a*) Field size, about 3.5°.
(*b*) Surrounding field dark. (No comparison field.)
(*c*) Intensity, medium; that is, low enough to avoid after images, and high enough to avoid the Purkinje effect.
(*d*) The effect of previous selective fatigue is eliminated.

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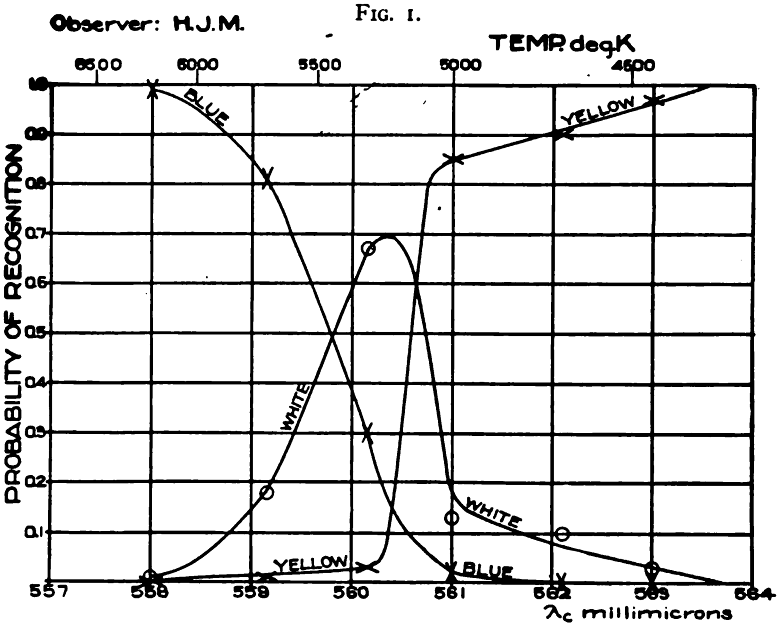
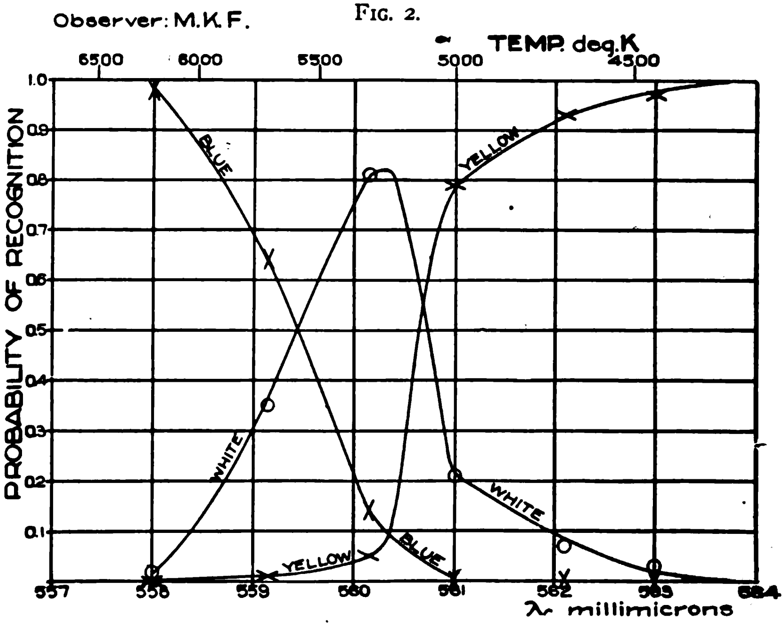
* Abstract of a paper presented at the joint meeting of the American Physical Society and the Optical Society of America, Chicago, Dec. 29, 1920. The complete paper will be published as a Bur. of Stands. Sci. Pap.

FIG. 1.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

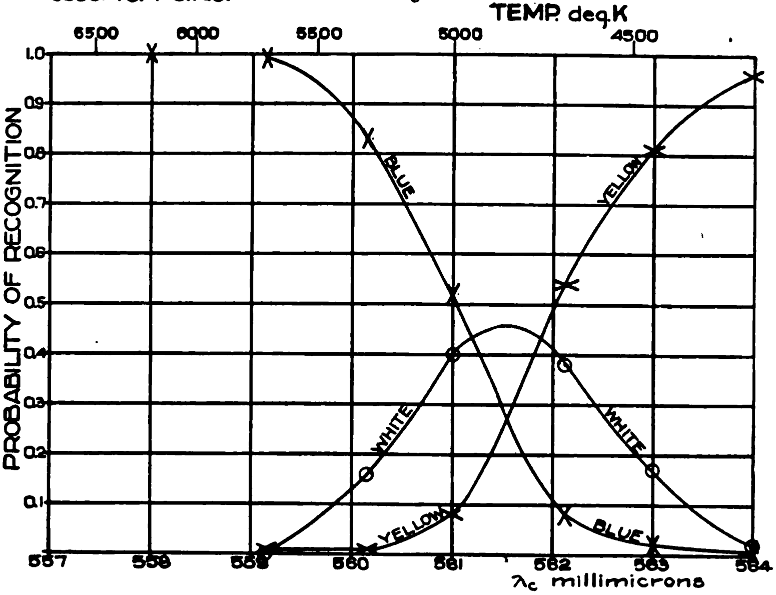
FIG. 2.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

Observer: C.A.S.

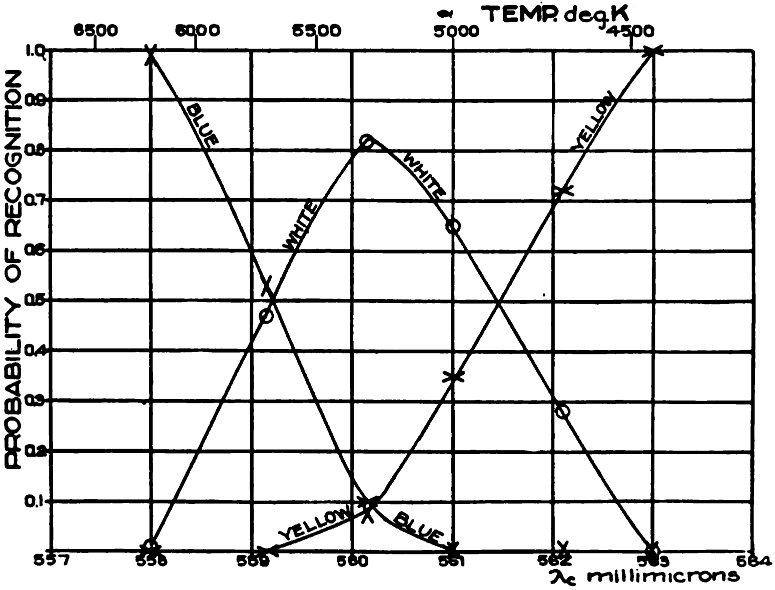
FIG. 3.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

Observer: I.G.D.

FIG. 4.



Probability of recognizing the Planckian stimulus as yellow, white or blue.

Experimental Methods.—The method of producing and adjusting the spectral distribution of the stimulus is this: Light from a lamp of known spectral distribution is modified by rotatory dispersion in a system of quartz plates and nicol prisms in such a way that by rotating one of the nicols the light emerging from it can be made to assume the spectral distribution of a Planckian radiator at any desired temperature between 4000° and 7000° K. Such a system is in effect a selective light filter of adjustable spectral transmission.[1]

Two methods of observation were used:

(1) The method of adjustment by trial, in which the observer, himself, adjusts the stimulus until he calls the sensation, "white."

(2) The method of answers, in which the operator conducting the experiment adjusts the stimulus to correspond to certain fixed temperatures of the hypothetical Planckian radiator; and then records the observer's reactions as "blue," "white," or "yellow," as the case may be. The latter method proved to be the more satisfactory.

Results and Conclusions.—Experimental results are given from four observers.

The results obtained by the method of answers are shown graphically in Figs. 1 to 4, inclusive. The stimuli are specified by the abscissæ, on which the scale of equal parts is in terms of λ_c, the wave-length of the centre of gravity of the spectral distribution of light.[2] Corresponding source temperatures are given in the upper margin. The ordinates show the probability of the observer's recognizing the stimulus as blue, white or yellow.

A graphic synopsis of the results is shown in Fig. 5, which is self-explanatory.

Previous selective fatigue has a notable effect on the temperature found, but recovery from such fatigue is rapid and complete in a few minutes.

The average results of these observers indicate that "white light" may be represented: (1) *theoretically,* by the light from a Planckian radiator at a temperature of 5200° absolute; (2) *practically,* to a fair approximation, by average noon sunlight at Washington. It is, however, emphasized that the final establishment of such a standard should be based on a more extensive statistical investigation.

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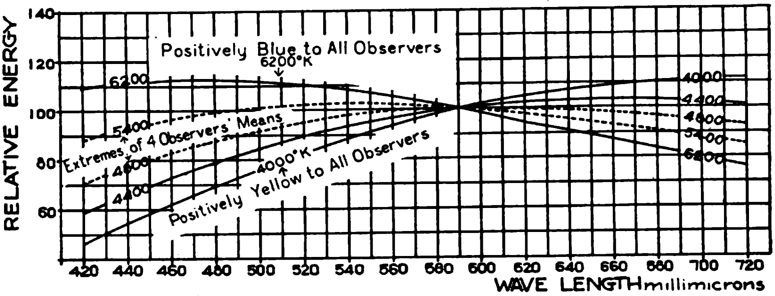
[1] Cf. Priest: "Radiant Energy at High Temperatures," this JOURNAL, p. 178.

The results are compared with previously proposed standards for "average daylight." Accepting the results of this investigation tentatively as a standard for the stimulus of white, it is observed that Ives' proposed "average daylight" (Planckian radiator at 5000° K) is a comparatively close approximation to this standard, while the sensation evoked by Gage's proposed "average daylight" (Planckian radiator at 7000° K) is undoubtedly blue.

FIG. 5.



Spectral Distributions of Radiant Energy Evoking the Sensations, White, Yellow and Blue.

Dotted curves, 5400° and 4800°K delineate the extreme range of four observers' final averages in determining the stimulus for white.

The solid curve, 6200°K shows the spectral distribution always recognized as blue by all four observers.

The solid curve, 4000°K shows the spectral distribution always recognized as yellow by all four observers.

The solid curve, 4400°K shows the spectral distribution always recognized as yellow by the three observers of ages less than 35 years.

An appendix to the paper sets forth the desirability of an extensive statistical determination and correlation of all the various relations between visual sensation and stimulus.

BUREAU OF STANDARDS,
January 29, 1921.

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[3] See JOUR. OP. SOC. AM., 4, pp. 389–402; 1920.

NOTICES.

OPTICAL SOCIETY OF AMERICA, FIFTH ANNUAL MEETING.

The fifth annual meeting of the Optical Society of America was held in Chicago, at the University of Chicago, December 27, 28 and 29, 1920. On December 27th the dinner of the Society, attended by about forty members, was held at the Del Prado Hotel. The next meeting of the Optical Society will be in Rochester, N. Y., October, 1921. The following papers were presented before the Optical Society at the Fifth Meeting:

PROGRAM.

MONDAY MORNING AT 11 O'CLOCK.

(1) Courses in Optics and Optometry in Columbia University. **JAMES P. C. SOUTHALL**, Columbia University.

(2) Thermal Expansion of Wires Used in Glass Seals. **C. G. PETERS** and **C. H. CRAGOE**, Bureau of Standards.

(3) Refractive Index of Glass Through the Annealing Range. **C. G. PETERS** and **C. H. CRAGOE**, Bureau of Standards.

(4) Notes on the Theory of Photographic Spectrophotometers. **E. D. TILLYER**, American Optical Company.

(5) A New Ocular Micrometer. **HERMANN KELLNER**, Bausch & Lomb Optical Co.

(6) Presentation and Discussion of the Reports of the Committees on Nomenclature and Standards. **P. G. NUTTING**, General Chairman.

1. Colorimetry. L. T. Troland.
2. Lenses and Optical Instruments. J. P. C. Southall.
3. Optical Glasses. George W. Morey.
4. Photographic Materials. W. F. Meggers.
5. Photometry and Illumination. E. C. Crittenden.
6. Polarimetry.. F. E. Wright.
7. Projection. L. A. Jones.
8. Pyrometry. W. E. Forsythe.
9. Reflectometry. A. H. Taylor.
10. Refractometry. C. A. Skinner.
11. Spectacle Lenses. E. D. Tillyer.
12. Spectrophotometry. A. H. Pfund.
13. Spectroradiometry. W. W. Coblentz.
14. Visual Sensitometry. Prentice Reeves.
15. Wave Lengths. W. F. Meggers.

About half of the above reports were presented before the general meeting by title only.

MONDAY AFTERNOON AT 2.30 O'CLOCK.

(7) A Comparison of Monochromatic Screens for Optical Pyrometry. **W. E. FORSYTHE**, Nela Research Laboratories.

(8) An Improved Form of Pickering Polarimeter for Gloss Measurements (by the Polarization Method). L. R. INGERSOLL, University of Wisconsin.

(9) An Unfamiliar Anomaly of Vision and Its Relation to Certain Optical Instruments. W. B. RAYTON, Bausch & Lomb Optical Co.

(10) Double Refraction of Glass Tubing as Indicating the Strains Present. A. Q. TOOL and C. G. EICHLIN, Bureau of Standards.

(11) Monocular and Binocular Perception of Contrast and Brightness. PRENTICE REEVES, Eastman Kodak Company.

(12) Systems of Color Standards. A. AMES, JR., Dartmouth College.

(13) A New Study of the Leucoscope and Its Application to Pyrometry. (Extension of work reported at N. Y., February, 1920.) IRWIN G. PRIEST, Bureau of Standards.

(14) Address of the Retiring President of the Optical Society of America. "Some Outstanding Problems of Physiological Optics." F. K. RICHTMYER, Cornell University.

(15) Atmospheric Corrections for the Harcourt Standard Pentane Lamp. E. B. ROSA, E. C. CRITTENDEN, A. H. TAYLOR, Bureau of Standards.

(16) Some Major Problems in Photometry. E. C. CRITTENDEN and J. F. SKOGLAND, Bureau of Standards.

(17) Comparative Tests as to the Accuracy of Various Methods for Precision Measurements of Focal Lengths (by title). W. O. LYTLE and A. K. BENNETT, Bureau of Standards.

TUESDAY MORNING AT 10 O'CLOCK.

(18) The Diffusion of Light in a Searchlight Beam (by title). ENOCH KARRER and U. M. SMITH, Bureau of Standards.

(19) Further Results on the Heat of Absorption of Glass. A. Q. TOOL and C. G. EICHLIN, Bureau of Standards.

(20) A Recent New System of Formulæ for Tracing Rays Through a Combination of Lenses. JAMES P. C. SOUTHALL, Columbia University.

(21) Notes on Lens Computation. HERMANN KELLNER, Bausch & Lomb Optical Co.

(22) A New Astronomical Lens. FRANK E. ROSS, Eastman Kodak Company.

(23) Note on the Extended Theory of the Sectored Disk Used in Photometry (by title). ENOCK KARRER, Bureau of Standards.

(24) Measurements of Aberrations of the Eye. C. A. PROCTOR and A. AMES, JR., Dartmouth College.

(25) Characteristics of Retinal Image. A. AMES, JR., and C. A. PROCTOR, Dartmouth College.

(26) Some Notes on Condenser Correction in Optical Projection (by title). G. W. MOFFIT, Eastman Kodak Company.

(27) The Use of the Ulbricht Sphere in Measuring Reflection and Transmission Factors (by title). ENOCH KARRER, Bureau of Standards.

(28) A Comparison of Retinoscopic, Subjective and Finally Acceptable Ocular Corrections. CHARLES SHEARD, American Optical Company.

(29) A New Method of Joining Glass. C. O. FAIRCHILD, Bureau of Standards.

(30) The Effect of Variations in Intensity of Illumination of Functions of Importance to the Working Eye (by title). C. E. FERREE and G. RAND, Bryn Mawr College.

(31) Optical Determination of Stress in Transparent Materials. A. L. KIMBALL, General Electric Co.

WEDNESDAY MORNING AT 9.30 O'CLOCK.

This was a joint session with the American Physical Society. The following papers were contributed by the Optical Society:

(32) Photographic Reproduction of Tone. L. A. JONES, Eastman Kodak Company.

(33) The Spectral Distribution of Energy Required to Evoke the Gray Sensation. IRWIN G. PRIEST, Bureau of Standards.

(34) The Propagation of Light in Rotating Systems. L. SILBERSTEIN, Eastman Kodak Company.

(Signed) PAUL D. FOOTE, *Sec.*, 1920.

PRESS OF
J. B. LIPPINCOTT COMPANY
PHILADELPHIA

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Journal
of the
Optical Society of America
Devoted to all Branches of Optics

Vol. V MAY, 1921 Number 3

THE GLARIMETER

AN INSTRUMENT FOR MEASURING THE GLOSS OF PAPER

BY
L. R. INGERSOLL

Some years ago the writer developed a method[1] and instrument for measurement of the gloss or degree of finish of a sheet of paper by means of its polarizing action on obliquely reflected light. The instrument—called a "glarimeter" since it measured the gloss in terms of the "glare" or specularly reflected light—was not placed at that time on the market, in spite of the fact that it seemed to supply a long-felt want for an exact means of gloss measurement. This was partly because of the difficulty of securing the necessary optical parts during the war time and partly because of some inherent defects.

Not long ago, however, it was found that if the optical system of the polarimetric part was replaced by a modified form of that used in the Pickering[2] polarimeter, these defects would be removed and at the same time the sensibility and accuracy greatly increased. Furthermore the required optical parts—Wollaston double-image prism, nicol and lens—could be fairly readily obtained. A number of these glarimeters have been made up and have already proved their usefulness in various paper mills, and accordingly it seems worth while to describe the new form of the instrument.

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[1] Electrical World, **63,** p. 645 (Mar. 21, 1914). It is of interest to note that a German scientist, Dr. Karl Kieser, has recently (apparently independently) developed a similar instrument making use of the same principle. See Zeit, für Angew. Chemie, **32,** 1, p. 357 (Nov. 11, 1919).

[2] E. C. Pickering, Proc. Am. Acad. Arts and Sci. **9,** 1 (1873) :**21,** 294 (1885).

The general principle of the instrument, i.e. the polarization method of gloss measurement, has been already described,[3] but may be briefly reviewed. Light from a source subtending a certain solid angle—in this case .038, i.e. a 44.5mm diameter circular aperture at 203mm distance—falls on the surface of paper at an angle of approximately 57.5 degrees. Part of this light is diffusely reflected and part specularly, the latter being shown by experiment to be (almost) completely plane polarized. The instrument accordingly measures gloss in terms of the fraction, which is polarized, of the light entering the eyepiece from the paper, that is, in terms of the fraction of its brightness which is due to polarized light.

In the new optical system of the polarimeter the light passes in turn an adjustable slit, Wollaston prism, convex lens, and nicol rotating in graduated circle. The object of the lens is to throw the slit sharply in focus and to throw out of focus the paper surface so that its minute irregularities will not appear. It also allows the instrument to be shortened somewhat.

On looking in the eyepiece one sees the two images of the slit formed by the double-image prism. These are brought into focus by pulling in or out the tube carrying the slit, and the slit width is adjusted until they form a clear bi-field with sharp dividing line. The nicol is then rotated until they are of equal brightness. Then if D^2 and S^2 represent the intensities of the diffusely and specularly reflected light (so directed as to enter the eye) before entering the Wollaston, and if the latter is set so that the specularly reflected light is completely extinguished in the lower image, the brightness of the upper and lower images—before nicol is interposed—will be proportional to $D^2/2 + S^2$ and $D^2/2$, respectively. When they are viewed through the nicol and balanced by turning its principal plane through an angle A from parallelism with that of the lower slit image, we have

$$(D^2/2 + S^2) \sin^2 A = D^2/2 \cos^2 A$$

from which we readily obtain

$$\frac{S^2}{D^2 + S^2} = \cos 2A$$

---

[3] Electrical World, *loc. cit.*

or the percentage polarization is given at once in terms of the cosine of a double angle.

The gloss may be expressed directly in terms of this per cent of polarization, if desired, and indeed it was formerly recommended that this be done. It is a mistake, however, to imagine that this furnishes a scale of measurement which is absolute,[4] since it will vary with the angle subtended by the source, and the last is essentially arbitrary. If one is to be arbitrary then, it is believed better to be frankly so in this case and express the gloss in the most convenient fashion, i.e. *directly in terms of the reading of the circle*, the latter being divided in ordinary degrees.

The worst that can be said of such a scale is that it is at least as rational as the Fahrenheit thermometric scale, and the best that it is practical, convenient, direct-reading and involves no manufacturing difficulties. On the scale chosen a very low gloss, such as that of blotting paper, reads about "20 degrees," while a high finish—glossy Azo or Solio—comes in the neighborhood of 50. As determinations may be made to the tenth of a degree we may distinguish between these limits a possible 300 gradations of gloss. Ordinary magazine paper runs between 25 and 40 degrees and the stock on which the OPTICAL SOCIETY JOURNAL is published averages about 30. It is hardly necessary to add that all the instruments will be set so that they read the same, with a small calibration correction to be used for accurate work. If one wishes to express gloss in per cent as explained above it is only necessary to make settings at each of the two possible points on the scale, e.g. at 34 degrees, say, and 86 degrees, and take the cosine of the difference.

The external features of the new Glarimeter are shown in the figure although certain improvements which add to the convenience of operation have been incorporated since this photograph was taken. It makes a fairly rugged, compact, portable instrument with which gloss determinations may be quickly made (15 to 20 seconds) without any darkening of the room. It is accordingly an essentially practical instrument equally suitable for mill or laboratory use. Nor is its application limited to papers, for it may be used with any surface which does not show metallic reflection. Indeed, in somewhat different form it might prove useful in connection with the finishing of fine furniture or automobiles or in determining the gloss of paints, textiles, walls, etc.

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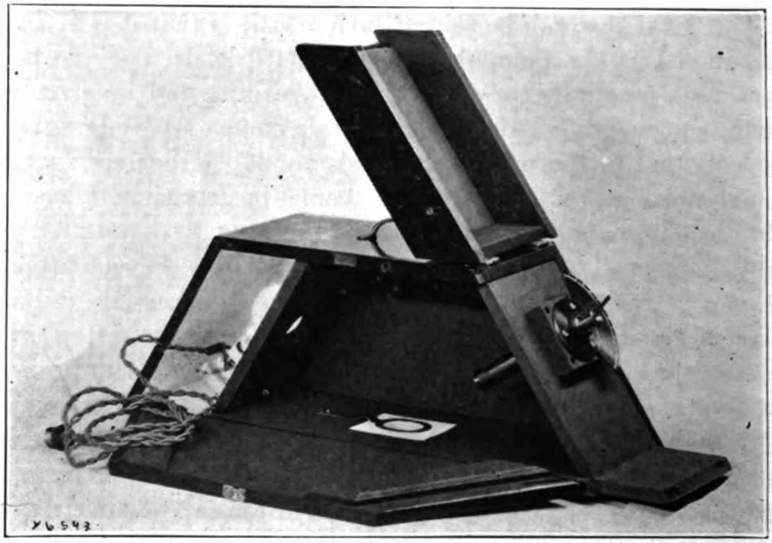
[4] Dr. Nutting in a criticism, Electrical World, **63,** 1156 (1914), of my original paper suggests a scale which may be considered absolute but it is doubtful wisdom to apply it here. See also Electrical World, **64,** 35 and 717 (1914).

The limitations of the polarization method of paper gloss measurement may be considered for a moment in closing. While it gives a direct measure of the "optical smoothness" or glare-producing property it does not primarily measure the "mechanical smoothness" or quality which determines the reproduction of half-tones, and it might at first sight be thought that this would seriously limit its use. Such, however, is not found to be the case. The calendering process which gives the finish to the surface increases at the same time the optical and mechanical smoothness and in sensibly the same proportion; the measure of one then serves to determine the other as well. It is true that one might imagine exceptions to this—e.g. a smooth-finished paper which has been



One Form of Glarimeter

punctured with fine pin-holes: this would still show a high gloss but would be, on the burr side at least, mechanically rough—but such as a rule are highly artificial and never met with in practice. Experience shows that, for any given type of paper, the glarimeter serves every purpose for testing and controlling the uniformity of the calendering process.

When we have under consideration at one time, however, radically *different sorts* of paper surfaces we cannot generalize so easily. To assume that glarimeter readings alone would serve as an exact measure of the half-tone possibilities of any paper, without reference to the character of its finish, coated or uncoated, etc. would be obviously ridiculous. This consideration applies particularly to colored papers, where a darkening of shade increases the gloss reading. That this must be so is readily seen by remembering that darkening reduces the amount of diffusely reflected light while it alters only in a minor degree the specularly reflected portion: experiments with solio paper of various shades show this effect very well. It may be said, however, that only in special cases does this affect the usefulness of the instrument. It is found in practice that good settings on colored papers are obtainable when a piece of colored glass is inserted in the eyepiece, and while this should, theoretically, be of the color of the paper, a light red stained glass works very well in practice with a variety of colors.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN.
Feb. 7, 1921.

THE INTRINSIC INTENSITY OF LIGHT TRANSMITTED THROUGH A SLIT AS A FUNCTION OF THE WIDTH AND DEPTH OF THE SLIT AND OF THE WAVE LENGTH OF THE LIGHT

BY

L. P. SIEG AND A. T. FANT

INTRODUCTION

It appears that very little work has been done on the absorption of light by a narrow, deep slit. This offers a very interesting problem, for even a simple observation indicates that there is apparently a much stronger absorption than one might anticipate. In addition there is a strong polarization of the light. This latter phenomenon is very striking, but it has not been studied quantitatively in this first work. Rayleigh[1] has treated theoretically the polarization of light transmitted by a narrow slit in an infinitely thin, opaque screen. That work serves in this present instance principally to enable us to define a "deep" slit. A "deep" slit shall here be considered as one which causes polarization of incident natural light such that the electric vector parallel to the length of the slit is greater than that perpendicular to the slit. In this sense it is well nigh impossible to attain to a thin slit. Even a thin scratch in a very thin silver film on glass constitutes a "deep" slit. Stewart[2] noticed strong polarization and absorption by a narrow slit in a thick screen in some experiments performed in another connection. It was thought a matter of interest, and of some importance to investigate the intensity of light transmitted by a deep slit as a function of the width and depth of the slit, and of the wave length of the light. Further work should be done on the variation of the material constituting the jaws of the slit.

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[1] Rayleigh. Roy. Soc. Lond. Proc. A, 89, p. 194, 1913–14.

[2] G. W. Stewart. Abstract. Phys. Rev. 5, p. 73, 1915.

APPARATUS AND METHOD OF PROCEDURE

A sketch of the arrangement of the apparatus used is shown in Fig. 1. Light from a tungsten strip lamp was focused on S_1 of the monochromatic illuminator, *A*. By means of prism P_1 any desired wave-length could be obtained at S_2. A lens L_2, placed so that S_2 was at its principal focus, secured parallel light at S_3. A mirror at M_1 divided the pencil into two parts, one reflected at right angles, the other passing straight through. S_3 was the slit under investigation. It was formed by two plane steel surfaces of a pair of Hoke Precision Gauges. One of the gauges was clamped securely to the bed of the interferometer, *K;* the other was set in wax in a frame attached to the carriage of the interferometer, brought into contact with the first gauge, and allowed to remain in that position until the wax had hardened. The slit was then placed parallel to the direction of the light. A template, *D*, prevented any light from passing below or above S_3. A short focus lens was placed at L_3, the center of the edge of

FIG. 1

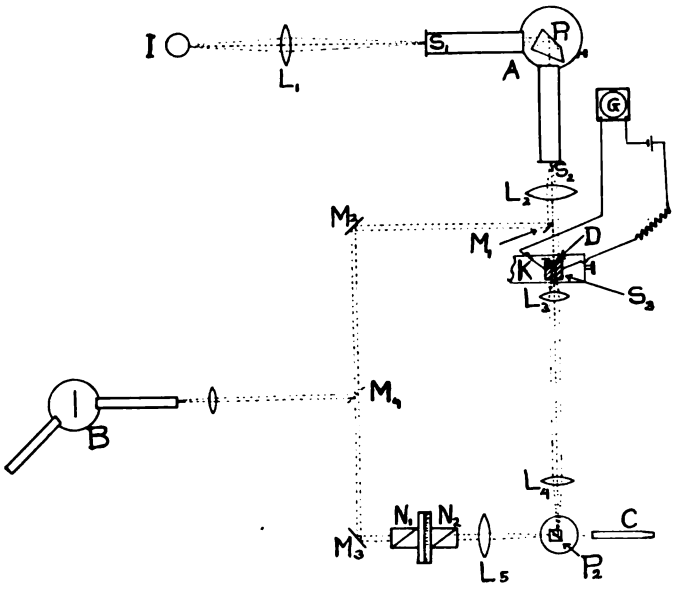


Diagram of Apparatus

S_3 nearest it being at its principal focus. This pencil was then brought to a focus by L_4 on a series of parallel silver bands between two right-angled prisms, P_2. The image of the near edge of S_3, thus formed, was viewed through a low power microscope at C.

That portion of the light which was reflected by M_1 was, by means of the mirrors M_2 and M_3, made incident normally on the nicols N_1N_2, the rotation of one of which enabled one to match the intensities of the two images in P_2. By means of L_5 the light coming through N_1N_2 was focused on the silver bands in P_2. Thus there was formed at P_2 a continuous band of light; the portion *reflected from* the silver being the image of the near edge of S_3, and the portion seen *between* the silver bands being the image of S_2, formed by the light coming by way of $M_1M_2M_3$. It was found by careful trial that the results were not appreciably altered by placing the microscope, C, in line with S_2S_3; and this position being more convenient, the final measurements were taken with that arrangement. By means of an auxiliary mirror at M_4 and the spectrometer, B, the mean wave-length of the light used was determined.

Denoting by I_0 the intensity transmitted through the nicols when the angular separation of their transmission planes is zero, the intensity, I, for any other angle of separation, θ, is

$$I=I_0\cos^2\theta \tag{1}$$

For a wide opening of S_3 the intensities of the images at P_2 were made equal by the use of a neutral tint glass in conjunction with the nicols. Letting $I_0 = 1$, since only comparative values are wanted, (1) becomes,

$$I=\cos^2\theta \tag{2}$$

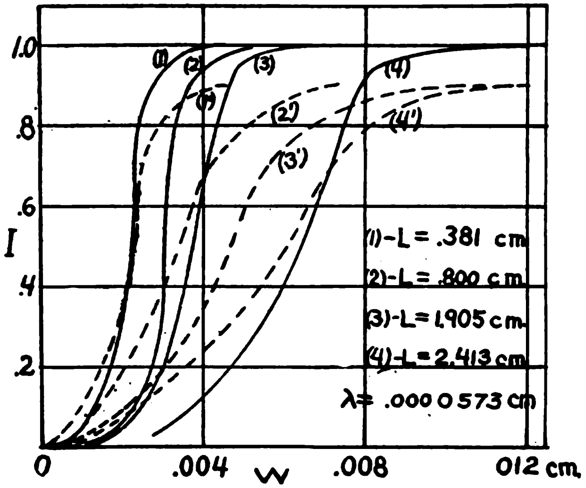
After S_3 is opened to a certain width, the image at P_2 becomes constant in intensity, any further opening of S_3 simply serving to make the image wider. Hence intrinsic intensities are obtained, not total transmitted light.

Contact of the surfaces of S_3 was determined, after thoroughly cleaning the surfaces, by the deflection of a galvanometer, G, placed in series with a dry cell, and the surfaces of S_3. Then by means of the interferometer screw the separation of the surfaces was ascertained. For each width of S_3, three independent settings of N_2 were made, and the mean relative intensities were thus obtained.

RESULTS

The results, reduced to tabular form would occupy more space than is desirable. However, their essential character can readily be represented by a few typical curves. These are drawn to illustrate three separate relations; I as a function of w, λ and L constant (Fig. 2); I as a function of L, λ and w constant (Fig. 3); and I as a function of λ, w and L constant (Fig. 4). In the above λ represents the wave-length, w the width of slit, and L the depth of slit, all expressed in cms. The full line curves represent the experimental results: the broken line curves represent certain calculated results based on two assumptions to be discussed in the next section.

FIG. 2

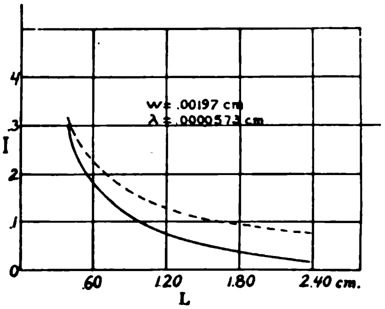


The intensity as a function of the width of slit, for various depths

THEORETICAL CONSIDERATIONS

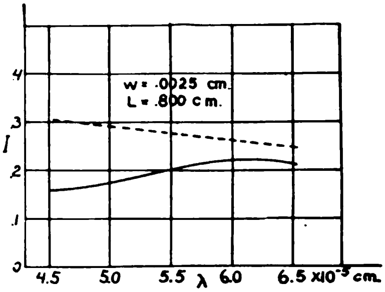
Undoubtedly the passage of a wave through such a deep channel is a complicated matter, in which diffraction, selective reflection, and possibly interference are the chief factors. An accurate test of any theory would necessitate a knowledge of the reflection

FIG. 3



The intensity as a function of the depth of slit, the width and wave length constant

FIG. 4

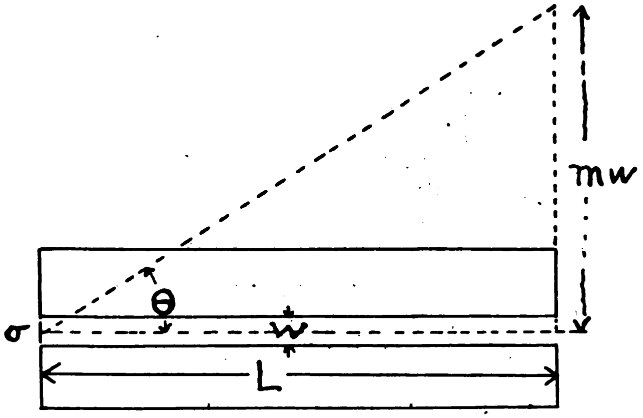


The intensity as a function of the wave-length, the width and depth constant

constants of steel (in this case) for both electric vectors, for many angles of incidence, and for all wave-lengths employed in the experimental work. Only one set of such constants, that recorded by Drude[3] for sodium light, has been found. Steel is such a variable substance, that one is in some doubt as to the validity of applying his results to the particular samples chosen for this work.

Let us assume a wave front, σ incident normally on a slit of depth L, and width w (Fig. 5). Consider a point on σ from which

FIG. 5



Section of jaws of the slit

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[3] Drude. Winckelmann's Handb. der Phys., Vol. III, p. 823, 1st Ed.

a pencil is diffracted through an angle θ. The intensity of this diffracted pencil is proportional to $\sin^2 \alpha / \alpha^2$, where α is an auxiliary angle defined by the relation

$$\alpha = \frac{\pi w}{\lambda} \sin \theta \qquad (3)$$

From Fig. 5 we have

$$\theta = \tan^{-1} mw/L \qquad (4)$$

where we assume m to be some integral number. It can readily be shown that the number of reflections of a diffracted pencil before emergence from the slit is m. Substituting for θ in (3) its value given by (4), we obtain

$$\alpha = \frac{\pi w^2 m}{\lambda \sqrt{m^2 w^2 + L^2}} \qquad (5)$$

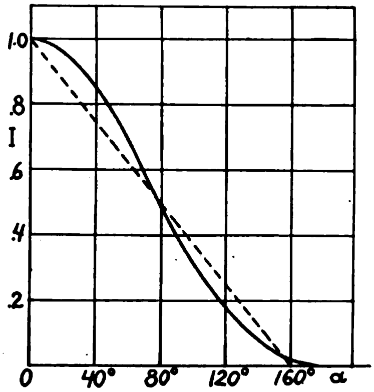
Since $m^2 w^2$ is small as compared with L^2, we may write

$$\alpha = \frac{\pi w^2 m}{\lambda L} \qquad (6)$$

Negative values of θ can be neglected on account of symmetry.

Plotting the relation $I = \sin^2 \alpha / \alpha^2$, we obtain the well-known (full line) curve in Fig. 6. Beyond $\alpha = \pi$ there is a series of rapidly decreasing maxima, which we shall here neglect. The entire area under this curve is π, and the area under the curve from $\alpha = 0$ to $\alpha = \pi$ was calculated to be approximately 0.45 π.

FIG. 6



Diffracted intensity as a function of the auxiliary angle α

If we assume the intensity of the light passing through a given slit, which is infinitely thin to be unity, the resultant intensity of light passing through a given deep slit will be given by the following expression,

$$I=\frac{1}{0.45\pi}\cdot\frac{\pi}{m}\cdot\frac{1}{2}\left[\sum_{i=0}^{i=m}\frac{\sin^2\alpha_i}{\alpha_i^2}\overset{(i)}{R_s}+\sum_{i=0}^{i=m}\frac{\sin^2\alpha_i}{\alpha_i^2}\overset{(i)}{R_p}\right]\text{............(7)}$$

where α_i represents the value of α for i reflections, R_s and R_p the reflection constants for the electric vector parallel and perpendicular respectively to the plane of incidence, and m the number of reflections when $\alpha = \pi$. Strictly the expression (7) sums up, not a smooth curve, but a series of rectangles, which more and more closely approximate the curve as m increases. A more rigorous expression than (7) could be formulated as a definite integral, but when one realizes that the R's are not constants, but are complicated functions of the angles α, the wellnigh insuperable difficulty of evaluating the series becomes apparent. A substitution of Drude's[4] values in (7) for a typical case, yielded results that conformed in a general way with the curves of Fig. 2, but it was quite apparent that the reflection constants given by him were larger than those in the present work.

In view of the difficulty of using even (7), the following, admittedly somewhat crude method was employed. Its chief merit consists in the fact that the results obtained by it agree fairly well with the experimental results, and that the labor of computation was much abbreviated.

Then for the sake of simplicity, as stated above, let the intensity curve (Fig. 6) be replaced by a straight line running from $I = 1$ at $\alpha = 0$, to $I = 0$ at $\alpha = 160°$. From (6) we see that α is directly proportional to m. Hence an m can be found that will make α any desired value. Thus with a given depth of slit, L, and a given λ, we can assume various w's, and for each w we can determine α for one reflection. To obtain the number of reflections for $\alpha = 160°$ we have merely to divide 160° by the value just determined for $m = 1$.

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[4] Drude, loc. cit.

Let a represent the ordinate (Fig. 6, dotted curve) at $\alpha=0$, and m the number of reflections. The second ordinate is then, $a-a/m$; the third, $a-2a/m$, etc. The intensity is given by the areas of these strips, each multiplied by the reflecting power, R (merging R_s and R_p), raised to the power $(m-1)$. Since we wish to call the length of the base unity, the base of each strip is $1/m$. Hence the expression for the intensity is

$$I=\frac{1}{m}\left\{\frac{a+(a-a/m)}{2}+\frac{(a-a/m)+(a-2a/m)}{2}R+\ldots\ldots\right.$$
$$\left.\ldots .+\frac{[a-(m-1)\,a/m]+[a-ma/m]}{2}R^{m-1}\right\}$$

which reduces to

$$I=\frac{a}{2m^2}\left\{\frac{R^{m+1}+R^m-(2m+1)R+(2m-1)}{(1-R)^2}\right\} \tag{8}$$

Applying this to the present problem we have

$$m=\frac{160\,\lambda\,L}{180\,w^2}=\frac{8\lambda L}{9\,w^2}, \qquad a=1, R=0.6$$

For m we must employ the nearest integer, and m should not be smaller than about 5. Substituting in (8) the values for α and R, we obtain

$$I=\frac{1}{0.32m^2}\left\{0.6^{m+1}+0.6^m+0.8\,m-1.6\right\} \tag{9}$$

For widths of slit for which m was smaller than 5, the full line intensity curve (Fig. 6) was employed, and areas were directly counted.

The broken-line curves of Figs. 2, 3, and 4 represent the results obtained as described above. In Fig. 2, (1) is the experimental curve, (1′) the corresponding theoretical curve, etc.

DISCUSSION OF RESULTS

In attempting a possible explanation of the experimental curves (Figs. 2, 3, and 4), two simple assumptions have been made; first, that the light is diffracted at the entrance to the deep slit; and second, that this diffracted light is weakened through successive reflections. These lead to results that fit the experimental curves approximately, with the exception of the case represented in Fig. 4. The differences undoubtedly arise from the rather rough assumptions, made for the sake of simplicity. The essential character of the curves is, however, with one exception, true to experiment. Those results in which I is plotted as a function of λ cover a much smaller range of values than do those others where I is a function of w, and of L. The change of I with the wave-length is small, and the discrepancy may not be so serious as it at first appears.

In practice then, even with very thin slits, a certain width of slit must be attained before the intrinsic transmitted intensity is approximately constant. Indeed, the strong polarization of light transmitted through narrow slits is of itself sufficient evidence that at least all of one vector is completely suppressed. This result in practice probably applies only to an instrument wherein intensities are matched quantitatively by permitting light to pass through a slit of variable width, particularly when the widths are small. In such cases serious error would result.

STATE UNIVERSITY OF IOWA,
Jan. 1921.

THE PHOTOCHEMISTRY OF THE SENSITIVITY OF ANIMALS TO LIGHT[1]

BY

SELIG HECHT

The stimulation of certain animals by light furnishes the opportunity for an objective and quantitative analysis of the underlying mechanism of photoreception. The reason for this is that both the stimulus,—light,— and the response of the animals, —a qualitatively invariable retraction reflex,—are capable of precise and easy control. Fortunately the characteristics of this sensitivity are not peculiar to these particular animals (a clam and an ascidian), but are to be found in one form or another generally distributed among animals possessing a light sense. The present analysis is therefore of broad application in its fundamental ideas.

The characteristics referred to are as follows. (1) A measurable interval, the reaction time, intervenes between the beginning of the application of the light and the beginning of the response. (2) A response is elicited only when the intensity of illumination has been increased. (3) Once a response has been given to a light, the continued application of the same intensity fails to produce any additional effect. (4) If, following this, the animal is placed in the dark, it soon recovers its sensitivity to the intensity which had become ineffective. The investigation of each of these characteristics has resulted in the formulation of certain principles and in the postulation of an hypothesis for their explanation.

A study of the reaction time shows first that it represents practically the time taken up in the sensory process alone, the conduction of the impulse and the early part of the response taking almost no measurable time in comparison. Second, it shows that the reaction time represents two definite intervals.

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[1] Abstract of a paper delivered at the Symposium on General Physiology, held by the American Society of Naturalists on Dec. 30, 1920, at its Chicago meetings. The data on which this summary is based may be found in a series of articles in the *Journal of General Physiology* from 1918 on.

The first is the sensitization period, or the time during which illumination is actually necessary. The second is the latent period, or the time during which the animals may be in the dark, and at the end of which the animal gives its response as if it had been illuminated all the time. In the clam, *Mya arenaria*, the sensitization period may be 1/100 second or even less, depending on the intensity. The latent period, however, is constant, and lasts about 1.5 seconds. The discovery of the complexity of the reaction time opened up the field for quantitative investigation.

The action of light in the photosensory process is probably photochemical. Experiments show that this action possesses the ordinarily well-demonstrated properties of photochemical reactions. First, the relation between the intensity of illumination and the sensitization period, or time of necessary exposure, follows the Reciprocity Law of Bunsen and Roscoe. The product of the two variables is equal to a constant. Second, the temperature coefficient for the action of light on the sensory activity of these animals is very low, near 1 for a rise of 10° C. This is in contrast to the temperature coefficient of ordinary chemical reactions which is usually between 2 and 3. Therefore, no matter what the temperature, a definite quantity of photochemical transformation is required to produce a stimulating effect.

The light adaptation of these organisms is well marked. They fail to give more than a single response even to intense sunlight. Dark adaptation is equally prominent. Studies of the rate and course of dark adaptation furnish the clue to the photosensory mechanism. In the dark the quantity of light required to elicit a response is at first very large. Gradually the amount of light necessary for a response decreases, at first rapidly, then more slowly, until after a definite interval it becomes a constant minimum. This means that less and less photochemical decomposition is required during dark adaptation to produce the same response.

During light adaptation the light decomposes a sensitive substance, and at the same time causes a decrease in sensitivity. During dark adaptation it is possible that the return of sensitivity is the result of the formation of fresh sensitive material. If we assume that the action of light is to break up a sensitive substance into its precursors, and that in the dark these precursors reunite to form the sensitive material, all the data acquired in these experiments may be explained in terms of the kinetics and dynamics of chemical and photochemical reactions whose general properties are well known and mathematically predictable.

The course of dark adaptation is found to follow with great accuracy the kinetics of a bimolecular chemical reaction. There are therefore *two* precursors (P and A,—precursor and accessory) uniting to form the sensitive material (S). The reversible photochemical reaction is then

$$S \underset{\text{"dark"}}{\overset{\text{light}}{\rightleftarrows}} P + A$$

and all the results fit this stoichiometric equation remarkably well. Moreover, certain predictions which can be made from it, have been tested, and have been verified on a number of occasions. For example, the "dark" reaction, $P+A \longrightarrow S$, is an ordinary chemical reaction. Its temperature coefficient should therefore be between 2 and 3 for 10° C. This reaction represents the process of dark adaptation, and experiments show that dark adaptation in these animals has a temperature coefficient of 2.4.

The phenomena so far considered are those concerned with the sensitization or exposure period. But these animals possess a very definite latent period which very frequently is of longer duration than the sensitization period. A study of the latent period shows that it represents the duration of a process closely connected with the products formed by the primary photochemical reaction. The velocity of this process turns out to be a linear function of the concentration of precursors, P and A, formed by the light.

From this and other evidence the following explanation of the latent period has been proposed. During the latent period an inert material, L, is changed into a chemically active substance, T, which then acts on the nerve to produce the outgoing sensory stimulus. This reaction, $L \longrightarrow T$, is catalyzed by the presence of the freshly formed photochemical decomposition products P and A.

Thus the latent period reaction becomes really the principal reaction in the sensory process. It is all set and ready to go, and requires only that the light change S into P and A so that the latter can catalyze the transformation of L into T. The whole photosensory mechanism may then be summed up in the two reactions

$$S \rightarrow P + A; \qquad L \,||\, P + A \,||\rightarrow T$$

in which $||P+A||$ means catalysis by one or both of the precursors. The first reaction occurs during the sensitization period; the second during the latent period.

This rather concrete hypothesis has proved to be a useful tool in getting further knowledge, because of experiments designed to test it in various ways. A few of these tests may be briefly mentioned. The latent period is supposed to be a simple reaction, perhaps an oxidation. Its behavior with the temperature should therefore follow the equation deduced by Arrhenius for the relation between the velocity constant and the absolute temperature. Experiments show that the reaction $L \rightarrow T$ does follow this prediction accurately. Moreover, the value of the constant $\mu =$ 19,680, found for the reaction is in accord with those usually found for hydrolyses, oxidations, etc. in pure chemistry.

Another test concerns the interrelations between the exposure and the latent period. If the intensity of the stimulating light is constant, the velocity of the latent period reaction is found to be directly proportional to the time of exposure. On the other hand, if the time of exposure is kept constant, and the intensity varied, the velocity of the latent period reaction is found to be proportional to the logarithm of the intensity. If now both time and intensity are varied, the velocity should be proportional to the product of the time and the logarithm of the intensity. Experiments prove this to be true.

A perhaps more significant application of the hypothesis has been made to the dark adaptation of the human eye. A proper analysis of the dark adaptation data has shown that the process in the human eye is fundamentally similar in principle to the process occurring in the clam and the ascidian. As a result there has been opened up a new field of investigation in retinal photochemistry which may some day enable us to possess a reasonable theory of vision.

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PHOTOGRAPHIC REPRODUCTION OF TONE

BY

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The ideal pictorial representation of any object or group of objects is one that produces when viewed by the observer exactly the same visual sensations as are produced when the objects themselves act as the source of visual excitation. The problem of finding how closely this ideal may be approached in practice has engaged the attention of workers in the field of photography for many years, and is commonly referred to as the problem of "tone reproduction." There are many phases of the subject to be considered, and a complete treatment requires a careful analysis of the factors upon which depends the operation of our visual perception of space and spatial relations. A complete analysis, therefore, leads not only into the realm of physical science, but also into those of psychology and philosophy. A careful consideration of these factors leads to the conclusion that brightness and brightness difference, that is, contrast, are by far the most important of those which are reproducible by the photographic process.

It is found in nature that different degrees of brightness or tones, as they are frequently called, are produced either by variation in the reflecting power of surfaces subject to the same illumination or by variation in the illumination incident upon various parts of the surface having uniform reflecting power. Visibility, normally, is due to differences either in the color or brightness of the various objects occupying the field of vision. Since, however, a photographic process does not in general reproduce color, it may be said that photographic visibility is due entirely to brightness differences. The proper reproduction of these differences, therefore, is of pre-eminent importance and it is with this brightness factor and the possibility of its correct reproduction by the photographic process that this paper deals.

The problem of tone reproduction has frequently been regarded as requiring only the correct reproduction of the actual physical brightness and brightness difference. When it is considered, however, that a given brightness or contrast may produce an entirely different subjective impression, depending upon the state of adaptation of the observer's eye, it is obvious that such a treatment of the subject cannot afford a complete solution of the problem. The problem may, in fact, be divided into two general parts, the one which we term the objective phase dealing with the reproduction of the actual physical brightnesses and contrasts, capable of being completely solved by purely physical methods. The other, which may be termed the subjective phase, includes a consideration of those factors which determine the nature of the subjective impressions produced by the action of given physical stimuli under various conditions and requiring the use of psychophysical methods and data for its adequate treatment.

Hurter and Driffield,[1] who may be considered as the founders of the science of photographic sensitometry, treated this subject to some extent, but confined themselves to a consideration of the objective phase. Lord Rayleigh[2] also has treated, although rather briefly, the objective phase of the subject. F. F. Renwick has made valuable contributions to the solution of this problem at various times, dealing in particular with the brightness reproduction possible when using the region of underexposure,[3] and later presenting a more complete treatment of the entire subject both from the objective and subjective viewpoints.[4]

Recently A. W. Porter and R. E. Slade[5] have published a paper on the subject in which they take exception to some of the conclusions reached by Hurter and Driffield and outline the relations which they consider essential for the solution of the problem. They also deal only with the objective phase, the reproduction of the actual physical factors. Following this, two short papers of a somewhat controversial nature dealing with special cases of the tone reproduction problems have appeared, one by

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[1] Jour. Soc. Chem. Ind., May, 1890, and Feb. 1891, and B. J. 1894, p. 417.

[2] Phil. Mag. Vol. 22 (1911) p. 734.

[3] Phot. Jour. Vol. 36 (1913), p. 127.

[4] Phot. Jour. Vol. 40 (1916), p. 222.

[5] Phil. Mag. Vol. 38 (1919), p. 187.

F. F. Renwick[6] and the other by Messrs. Porter and Slade.[7] Finally Mr. Renwick[8] has published a paper in which he gives in detail the method by which he obtained the results previously published (loc. cit.) relative to the objective phase of the subject. His conclusions are in substantial agreement with those arrived at in the following treatment of the subject although a quite different graphic construction was used.

During the past five years a large amount of work has been done in this laboratory on the theory of tone reproduction, and a satisfactory solution for the objective phase was obtained some two or three years ago. Simultaneously with the work on tone reproduction, research in the field of visual sensitometry has been in progress, which has resulted in data applicable to the solution of the subjective phase of the more general tone reproduction problem.

While some particular details of the subjective data are still lacking, it is felt that sufficient information is now available to establish the general nature of the function necessary for the conversion of objective reproduction into its subjective equivalent. In the meantime further work is being carried out in order to make possible a more precise quantitative evaluation of this function.

SIMPLIFYING ASSUMPTION

In order to simplify the subject and bring it within the limits of a single paper, it will be necessary to make certain simplifying assumptions. Since the sensitivity of the photographic plate to radiation of different wave-lengths is in general radically different from that of the eye, objects in which color is present have not in general the same brightness when considered from the visual standpoint as when the evaluation is in terms of the photographic plate.

Let us, therefore, assume that all areas of the object considered are both visually and photographically non-selective (i.e., colorless), and further that the optical system forming the image of the object on the negative material is likewise non-selective. Under such conditions values of visual brightness are directly proportional to those of photographic brightness. Thus this treatment will deal with brightness relations uncomplicated by any consideration of selectivity. Let us further assume that all photographic deposits considered are also non-selective in order that visual and photographic density values may also be considered as identical.

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[6] Phil. Mag. 38: 637, 1919.

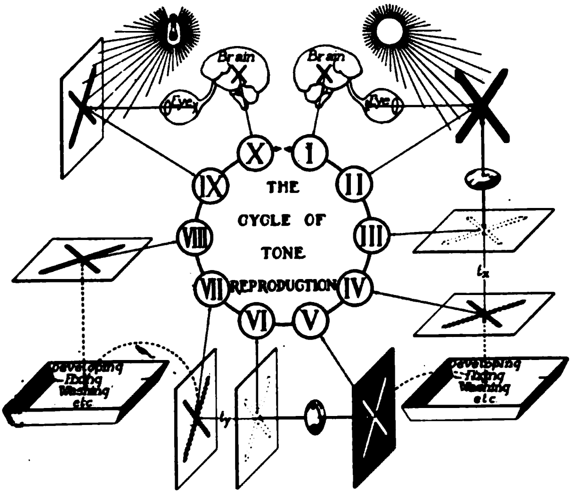
[7] Phil. Mag. 38: 637, 1919.

[8] Phil. Mag. 38: 151, 1920.

THE CYCLE OF TONE REPRODUCTION

In Figure 1 is given a graphic diagram, which although it may be somewhat fanciful will convey a more concrete idea of the problem than a mere verbal statement of the case.

FIG. 1



Graphic Diagram of the Cycle of Tone Reproduction

For the purpose of discussion it is found convenient to divide the cycle into ten distinct steps and these are designated by the Roman numerals enclosed in the small circles.

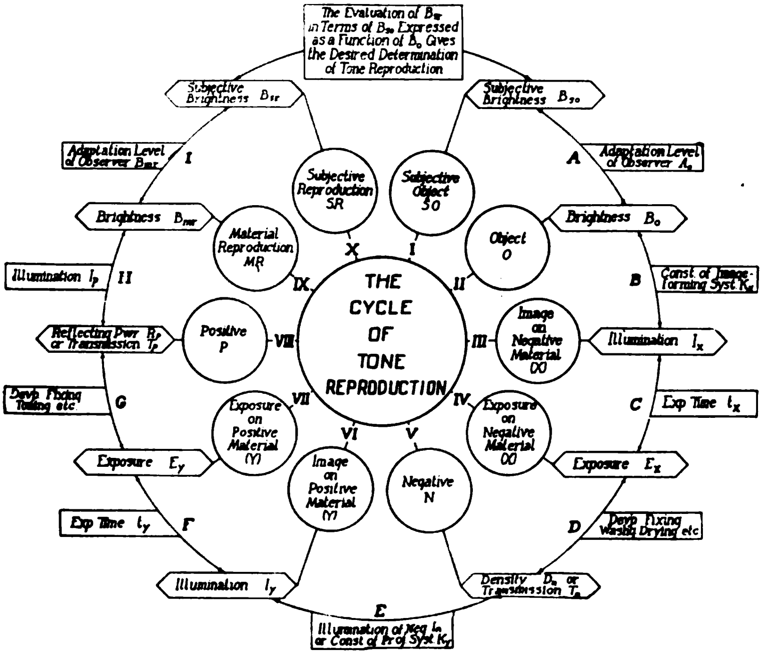
The first phase, indicated by I, is the subjective object which is defined as the subjective impression existing in the brain of the observer and resulting from the action of the retinal image of the object (II) considered. This subjective impression may be considered as the fundamental starting point in all tone reproduction problems, and is the thing which is to be reproduced.

The second phase is the object itself represented by the black cross,(II). While it may be convenient in some cases to interpolate one or more distinct steps between this actual physical object and the subjective impression due to this observation, such intermediate steps are unnecessary in the present discussion.

The third phase (III) is the image of the object formed on the negative sensitive material by some type of image forming system. This image when allowed to act through some finite period of time, t_x, produces upon the negative sensitive materials an exposure (IV) indicated in the diagram by the shaded cross. The exposed material is then subjected to various chemical operations such as development, fixing, washing, etc., the result being the fifth step in our cycle, the negative (V). The image (VI) of this negative is then formed either by contact or projection upon the positive sensitive material and forms the sixth step in the series. This image acting during a period of time, t_y, results in an exposure (VII) on the positive material which after development, fixing, or similar chemical processes is converted into a positive (VIII) the eighth step in the cycle. When the positive thus produced is illuminated it becomes the material reproduction (IX), and a comparison of the characteristics of this material reproduction with those of the object constitutes a complete solution of the tone reproduction problem from the objective standpoint. An image of this object is formed on the retina of the observer and creates in his mind an impression which we shall designate as the subjective reproduction (X) the last step in the cycle of tone reproduction. The evaluation of this subjective reproduction in terms of the subjective object constitutes a complete solution on the tone reproduction problem from both the objective and subjective viewpoints. In practice it may be more convenient to consider the object (II) as the starting point of the cycle and to express finally the characteristics of the subjective reproduction evaluated in terms of the subjective object as a function of the characteristics of the object itself. This follows largely from the fact that it is possible to measure directly by ordinary physical means the essential characteristics of the object while the characteristics of the subjective object must be deduced in an indirect manner from the knowledge of the characteristics of the object itself and of the condition of the observer's eye at the time considered.

In Fig. 2 is given a schematic diagram somewhat analogous in structure to the graphic diagram in Fig. 1, but expanded so as to include not only a statement of the nature of each phase of the problem but also the particular data necessary for the specification of each phase and the factors required for the transformation of the data relative to one phase into that relating to each of the contiguous phases.

FIG. 2


The Evaluation of B_{sr} in Terms of B_{so} Expressed as a Function of B_o Gives the Desired Determination of Tone Reproduction
Subjective Brightness B_{sr}
Subjective Brightness B_{so}
Adaptation Level of Observer B_{mr}
Adaptation Level of Observer A_o
Brightness B_{mr}
Brightness B_o
Illumination I_p
Const. of Image-forming Syst. K_x
Reflecting Pwr R_p or Transmission T_p
Illumination I_x
Devp Fixing Toning etc
Exp Time t_x
Exposure E_y
Exposure E_x
Exp Time t_y
Devp Fixing Washg Drying etc
Illumination I_y
Density D_n or Transmission T_n
Illumination of Neg I_n or Const of Pr of Syst K_y
Subjective Reproduction SR
Subjective Object SO
Material Reproduction MR
Object O
Positive P
Image on Negative Material (X)
Exposure on Positive Material (Y)
Exposure on Negative Material (X)
Image on Positive Material (Y)
Negative N
THE CYCLE OF TONE REPRODUCTION
I
II
III
IV
V
VI
VII
VIII
IX
X
A
B
C
D
E
F
G
H
I

Schematic Diagram of the Cycle of Tone Reproduction

In the small circles designated by the Roman numerals from I to X are the designations of the ten steps in the process corresponding to the graphic diagram of Fig. 1. The radial line extending outward from each of these circles connects with the enclosure in which is a statement of the essential characteristic or attribute which from the standpoint of this problem completely specifies the particular phase considered. For instance, in the case of the object (II) the essential data relative to this phase are the values of the brightness, B_o, of the various areas, while in the case of the negative, complete specification of its characteristics is given by values of density, D_n or transmission, T_n. In the approximately rectangular enclosures marked, A, B, C, etc., are given the transformation factors required for converting the data relative to one phase into that of the next. For instance, if the brightness values, B_o, of the various areas of the object be known and it is desired to compute the illumination of the image formed on the negative sensitive material, such computation can be carried out if the value of K_x the constant of the image forming system is known. Likewise illumination may be converted into exposure by the transformation factor t_x, the exposure time. A careful study of Fig. 2 will convey an excellent idea as to the various factors which must be considered in the treatment of this problem. The final comparison between I and X as indicated

TABLE 1

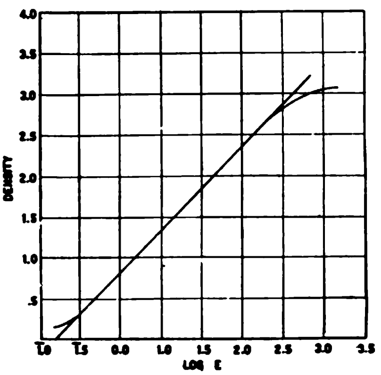
	Part I. "Things"	Symbol
1	Subjective Object	S O
2	Object	O
3	Negative Sensitive Material	X
4	Negative	N
5	Positive Sensitive Material	Y
6	Positive	P
7	Material Reproduction	M R
8	Subjective Reproduction	S R
	Part II. "Characteristics"	
1	Radient Flux	F
2	Illumination	I
3	Brightness	B
4	Reflection Coefficient	R
5	Transmission Coefficient	T
6	Exposure	E
7	Time	t
8	Density (Optical)	D
9	Adaptation Level	A

at the top of the diagram gives the desired information relative to the quality of tone reproduction obtained in any specific case.

TERMINOLOGY AND SYMBOLS

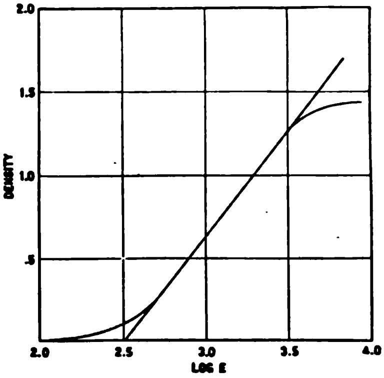
Since the problem to be treated involves so many operations, materials and inter-relations, it is very necessary to adopt at the outset a logical system of terminology and symbols. The essential factors belonging to what we may term the *thing* category are as shown in Part I, Table 1, the symbols adopted being as indicated in the symbol column. The essential *characteristics* or *attributes* descriptive of or applying to these things are given in Part II to Table 1 with the symbols adopted. When any characteristic applies to some particular factor in the thing category this is indicated by attaching the symbol of that thing as a subscript letter to the symbol of the characteristic in question. For instance, B_{mr} indicates the brightness of the material reproduction, while E_x should be read as the exposure on the negative sensitive material. This system of nomenclature, it will be noted, is applied in Fig. 2.

FIG. 3



Characteristic Curve of Negative Material

FIG. 4



Characteristic Curve of Positive Material

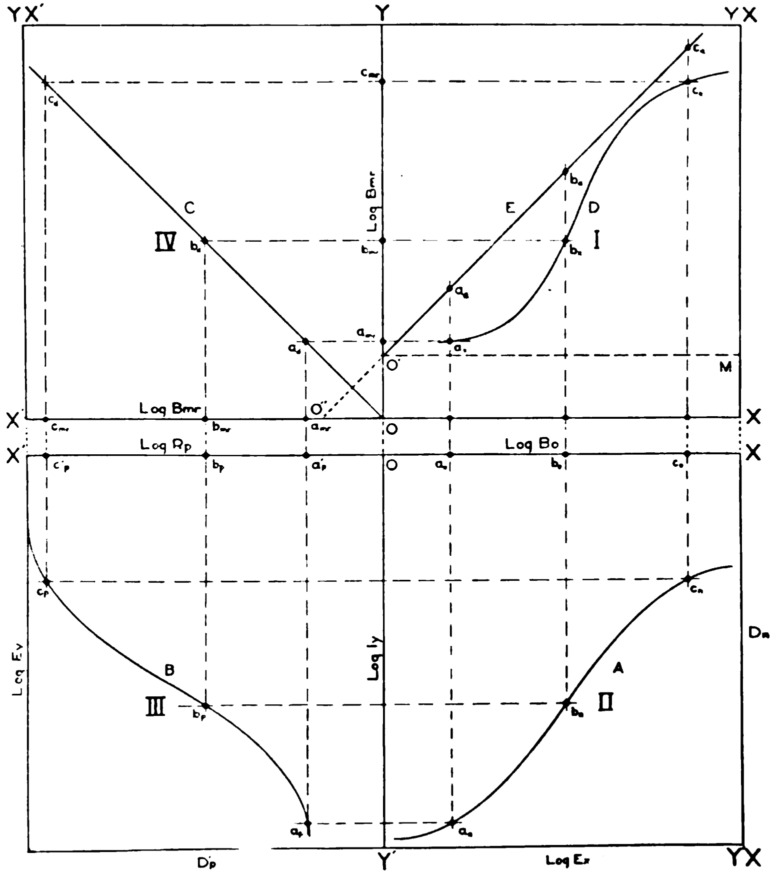
CHARACTERISTIC DATA

Some of the data necessary for the solution of this problem are contained in the characteristic curves of the photographic materials and of the retina. As illustrative of these important groups of data, the characteristic curve of a typical negative material is given in Fig. 3. This contains the necessary information for the transformation of exposure into the resulting negative density. In Fig. 4 is given a typical example of the characteristic curve of the positive material, while in Fig. 9, which will be described in detail later, are given the curves of retinal sensitivity which enables an evaluation of the sensation resulting from the action of the physical stimulus to be made.

GRAPHIC SOLUTION

Having considered thus briefly the problem as a whole, the data necessary for the solution, and the separate steps in which

FIG. 5



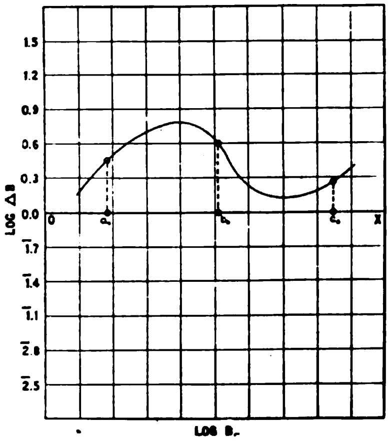
Graphic Solution for the Objective Phase of Tone Reproduction it can be conveniently divided, we will proceed to a consideration of how all the various factors may be combined in order to obtain the desired solution. In Fig. 5 is given a graphic solution of the objective phase of the problem.

This consists of a four quadrant diagram in which are plotted the characteristic curves of negative and positive materials and the curve resulting from a combination of these two which represents the quality of reproduction obtainable by the use of these materials and which we shall refer to as the reproduction curve. Along the line OX of the second quadrant is laid out a logarithmic scale on which may be plotted the brightness values of the various areas of the object considered. Below this scale is plotted the characteristic curve of the negative material, the ordinates being values of density and the abscissae values of log exposure. (log E_x). This curve must be determined by sensitometric methods such that it is a true representation of the characteristics of the negative material when used under the conditions existing in the making of the negative. These conditions include those factors operative in transforming the latent into a real image, as well as the necessary physical conditions relative to quality and intensity of exposing radiation.

The curve A must be so placed in the horizontal direction as to include properly the various points on the log E_o scale which represent the brightness values of the various object areas. For instance, if the point a_o represent the brightness of the deepest shadow in the object considered, and it is desired to render this brightness by a just perceptible density in the negative, or by any particular density value the curve A is plotted in such position that a perpendicular dropped from the point a_o will cut the characteristic curve at the desired value of density (D_n). Now the characteristic curve B, of the positive material is plotted in quadrant III. The positions of the necessary co-ordinate scales are as indicated in the figure, that is, the scale of exposure, log E_y, is established along the line X′, X′Y, while the density scale is established on the line X′Y′, Y′. This is equivalent to the characteristic curve as usually plotted but rotated through 90°. The absolute values on the log E_y scale must be so chosen that when the characteristic curve is plotted it will occupy the proper position relative to the curve A in the first quadrant.

This position of curve B is determined by deciding what value of positive density is desired for the rendering of some chosen value of negative density. For instance, let it be required to render the highest brightness of the object by a just perceptible density on the positive. Curve B must then be so placed that a horizontal line through the point c_n (which we assume to be the negative density representing the highest light of the object) will cut the curve B at a just perceptible value of positive density (D′p). Now if we take for example three points, a_o, b_o, c_o representing various brightnesses of the object and project them around as indicated by the dotted lines passing through these points on the log B_o scale, three corresponding points will be located on the line O, X′ designated as a'_p, b'_p, c'_p. Now if on the line OX′ a suitable logarithmic scale, log R_p, be established the values read at the points mentioned will give the relative reflecting power values of the areas in the positive by which the originally assumed object areas are rendered. Values read from the log R_p scale therefore determine the characteristics of the positive. Now this positive becomes a reproduction of the object only when illuminated. Along the line OX′ (quadrant IV) is now

FIG. 6



Curve of Brightness Deviations

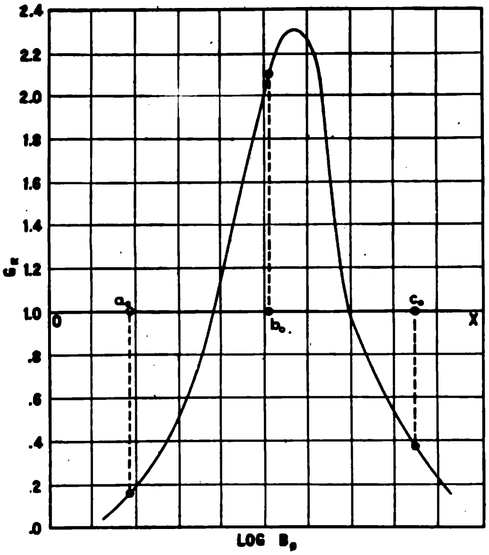
established another logarithmic scale (log B_{mr}) related to the scale of R_p by a constant the value of which depends upon the illumination incident upon the positive. This scale having been properly located and the points projected across as indicated, the values of brightness in the reproduction may be directly determined. The relation between the values of brightness in the reproduction to those of brightness in the object determines the quality of objective reproduction obtained. In order to obtain a graphic representation of such reproduction quality it is only necessary to transfer the points located along the line OX′ to the line OY. This is most conveniently accomplished by use of a dummy curve, C, in quadrant IV, which is a line passing through the origin at an angle of 45° to the X axis. The points on the line OX′ are projected onto this dummy curve as indicated and thence onto the line OY falling at a_{mr}, b_{mr}, c_{mr}.

Now, the points a_x, b_x, c_x in quadrant I are located at the intersections of the dotted lines through the points a, b, and c, as indicated, and the curve drawn through these points constitutes the reproduction curve. The curve of exact reproduction of both brightness and contrast is a straight line, E, at 45° to the axis and passing through the point O′. The position of O′ is again dependent upon the relation between the absolute values of the brightness of corresponding points of the object and material reproduction.

The distance between corresponding points, such as, for instance, a_a and a_x, on the curves E and D is a measure of the magnitude of the departure from exact reproduction of brightness of the object area considered, in this case, a_o. If, therefore, these values be determined for the various object areas such as a_o, b_o, and c_o and plotted as ordinates at the proper point on the log B_o scale a curve will be obtained which is a graphic representation of the departure from exact reproduction of brightness throughout the range of object brightnesses considered. Such a curve is shown in Fig. 6.

A comparison of the slopes or gradients of curves D and E at corresponding points gives an indication of the departure from exact reproduction of contrast. This departure is most conveniently expressed quantitatively by taking the ratio of the gradient, G_{mr}, of curve D to G_e the gradient of curve E, but G_e is unity by construction, hence the values of G_{mr} are quantitative expressions of the departure from exact reproduction of contrast. Since the gradient of any curve is expressed by the first derivative of the function a graphic expression of the departure from exact contrast reproduction will be obtained by plotting the first derivative of curve D. This is shown in Fig. 7.

FIG. 7



Curve of Contrast Deviations

Now referring again to Fig. 5 let B_0 be taken as the value of object brightness at some point on the log B_0 scale and E_x as the value of exposure at the corresponding point on the log E_x scale. Let K_x, the constant of the image forming system, be defined by the equation,

$$B_0 \cdot K_x = I_x$$

where I_x is the illumination at some point on the image formed on the negative material and B_0 is the value of object brightness at the corresponding point on the object.

If t_x denotes the required exposure time the following relations are found

$$t_x = \frac{E_x}{B_o \cdot K_x}$$

$$\log t_x = \log E_x - (\log B_o + \log K_x)$$

The required value of t_x can therefore be computed.

Now if the negative is to be printed by contact and I_n is the illumination on the negative during printing, the necessary exposure time can be determined by the relation,

$$\log t_y = \log E_y - \log I_n + D_n.$$

where log E_y and D_n are the values read at corresponding points on the log E_y and D_n scales respectively.

If I_p be the value of the illumination under which the positive is observed, the brightness of any given area can be computed from the relation,

$$\log B_{mr} = \log R_p + \log I_p,$$

and the magnitude of I_p will determine the relation between the numerical values of corresponding points on the log B_{mr} on log R_p scales thus establishing the scale of log B_{mr}.

The scale established on line OY is identical with that on OX′. The point O′ fixed at the point on the line OY where log B_{mr} is equal numerically to the value of log B_o read at the point O on the line OX.

Many interesting relations between the various factors involved can be derived from an analytical treatment of the diagram in Fig. 5, and while space does not permit of an extensive treatment of this phase one or two interesting results will be mentioned. Let the first derivative or gradient of the various curves be designated by the general symbol G, and in particular:

G_n = Gradient of negative characteristic (Curve A)
G_p = Gradient of positive characteristic (Curve B)
B_{mr} = Gradient of material reproduction (Curve D)

It can be shown that

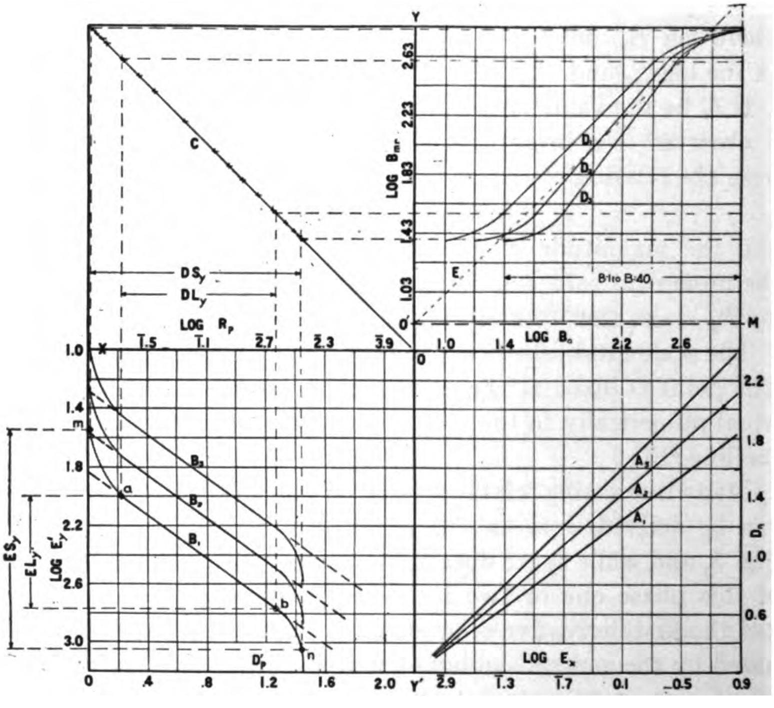
$$G_n \cdot G_p = G_{mr}$$

Since for exact reproduction of contrast it is necessary that $G_{mr} = 1.0$ it follows that the equation expressing this condition becomes

$$G_n \cdot G_p = 1.0$$

This is a general statement of gradient relations and assumes nothing as to the shape of the characteristic curves of the material used. In the special case involving the use of the straight line portions only values of gradient become equivalent to the well-known constant of sensitometry Υ (gamma), and may be written $\Upsilon_n \cdot \Upsilon_p = 1.0$. It should be pointed out, however, in the use of positive materials such as developing-out papers that but a relatively small portion of the curve is straight. Since the entire range of density must in general be utilized for rendition of the density scale of the negative the special case $\Upsilon_n \cdot \Upsilon_p = 1.0$ is of little value in the evaluation of the quality of contrast reproduction obtainable.

FIG. 8



Objective Reproduction Obtainable with Specified Materials and Conditions

It is evident from inspection of Fig. 5 that if any two of the curves A, B, and D be known or assumed, the third can be graphically constructed by methods analogous to those used in construction D from known values of A and B.

The applications of the methods outlined to practical problems are too numerous to mention in detail at this time. However, a specific example, Fig. 8, is shown. This illustrates the reproduction resulting from the use of obtainable negative and positive materials, the characteristic curves of which have been carefully determined by sensitometric measurements in this laboratory.

The object is assumed to have a contrast of 40 with B_o max = 1000 (apparent meter candles). The negative material is assumed to have a latitude greater than 40 (in exposure units), $\Upsilon\infty$ greater than 1.00, and log inertia equal to 2.5. The negative characteristic may therefore for the purpose of this discussion be represented as straight lines.

In order to show the effect upon reproduction of the extent to which the negative is developed, three times of development are assumed and the construction carried through for each.

The positive material used is specified by the curves B_1, B_2, and B_3 and has a standard exposure of 100,000 (m.c.s.)

This construction is so adjusted that in each case the highest light in the object is reproduced by a just perceptible deposit in the positive.

Values for various factors are assumed as follows:

$K_x = .01$
$I_n = 1000$ (m.c.)
$I_p = 851$ (m.c.)

By use of the relations previously derived the value of t_x, and t_y were computed and found to be

t_x = .0126 sec.
t_y (case 1) = 70.8 sec.
t_y (case 2) = 135.6 sec.
t_y (case 3) = 250.0 sec.

and from the relation

$\gamma_m \,.\, \gamma_p = \gamma_{mr}$
γ_{mr} (curve D_1) = 1.01
γ_{mr} (curve D_2) = 1.17
γ_{mr} (curve D_3) = 1.34

The reproduction curves obtained by the graphic construction are as indicated at D_1, D_2, D_3.

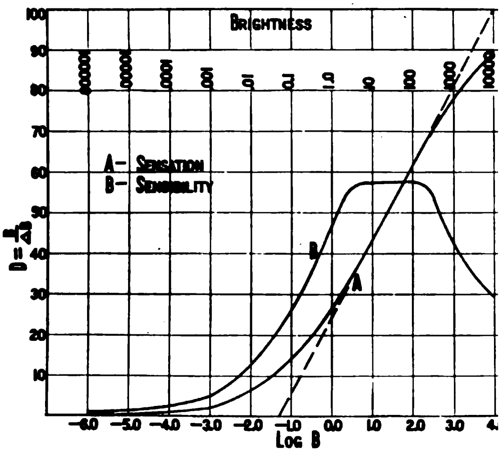
From a consideration of Fig. 8 a conclusion of considerable importance in practical work may be drawn. This relates to the problem of choosing the printing paper best adapted to a given negative or the inverse problem of making a negative of the proper quality for printing on a given printing paper. Considering Curve B, it will be seen that the useful portion is limited by the points *m* and *n*. On portions of the curve lying outside of these limits the gradient $\frac{d\,D'_p}{d \log E_y}$ is so small as to be practically useless in the reproduction of difference in E_y. Now in practice it is, in the great majority of cases, desirable to render the deepest shadow (lowest brightness) by the maximum useful density of the printing material, and at the same time to render all of brightness differences of the object as density differences in the finished print. If the density in the negative by which the highest light of the object is rendered be designated by the symbol $D_n max$, and that negative density which renders the deepest shadow be designated by $D_n min$, it is evident in order to fulfill the condition stated in the previous sentence that $D_n max. - D_n min. = ES_y$. The quantity $(D_n max. - D_n min.) = DS_n$ is termed the "density scale of the negative" and applies to a particular negative. This term (DS_n) should not be confused with the similar term DS_x which applies to the negative material itself with a meaning exactly parallel to that of DS_y as applied to the positive material.

From a consideration of Fig. 8, it will be seen if the negative and printing paper be such that $DS_n = ES_y$, that all of the density differences in the negative will be rendered as density differences in the print and further that the maximum negative density will be rendered as the minimum useful density of the printing material, while the minimum negative density will result in the maximum useful density in the print. Such procedure of course results in a diminished contrast in the extreme shadows and highlights due to the shape of the characteristic curve of the printing paper, even though (as is usually possible) all of the object brightnesses are rendered on the straight line portion of the negative curve. Of two printing papers having identical

values of ES_y (and equal to the value of DS_n fc
considered) the better result will in general be c
of the one of which the characteristic curve most
mates to a straight line between the points *m* ai
some special cases better results may be obtainec
procedure, it is undoubtedly true that in general
able relation between negative and printing pa
"density scale" DS_n of the former shall be as ne
equal to the "exposure scale" DS_y of the latter.

Since it is not the primary object of this pap
particular cases but rather to outline the genera
subject, the discussion of the objective phase w
this point leaving the application to practica
treatment in subsequent papers. Moreover, a fin
the quality of a reproduction cannot be made
sideration of the subjective phase of the proble
now be treated.

FIG. 9



Retinal Sensitivity Curves of the Retina

THE SUBJECTIVE PHASE

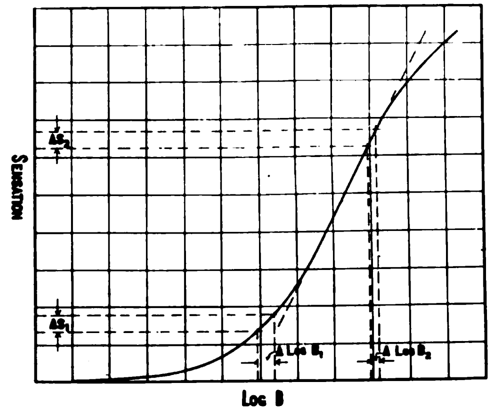
Turning now to a consideration of the subjecti
be well to discuss more in detail the significanc
showing Fig. 9. These contain the data relati

sensitivities necessary for the evaluation of the subjective relative-contrast function. Curve A gives the relation of the sensation to the stimulating brightness. Equal increments on the Y axis correspond to equal increments in the sensation, and the brightness intervals which give rise to these sensation steps can be determined from this curve. The shape of this function could be established by direct experiments, but since such a process is extremely laborious it is customary to arrive at the same end by other means. The curve B, the contrast sensibility, is the first derivative of the sensation curve, hence if the sensibility be measured and the curve B plotted the sensation curve may be obtained by integration. The sensibility at a given adaptation level is determined by measuring the least difference in brightness perceptible under the specified conditions. The least perceptible difference, ΔB, expressed as a fraction of the brightness at which it is determined is a direct measure of the sensibility of the eye to contrast or brightness difference. The data from which the curve B is plotted are from the work of Nutting and Blanchard (loc. cit.). The ordinate of curve B at any point is therefore equal to the slope of the curve A at the same value of B. Thus, if S be the sensation, $\frac{d\,S}{d\log B} = \frac{B.}{\Delta B}$ The values of the ordinates of the sensation curve are in terms of sensation units, and the construction of such a sensation scale is based on the assumption that a just perceptible difference at any point on the sensation scale is equal to a just perceptible difference at any other point. The basic unit upon which this scale is established is therefore the least perceptible brightness difference.

It should be pointed out that the curves in Fig. 9 are similar in shape to those given by Renwick (loc. cit.) but that the absolute values of log brightness are very different. This is due undoubtedly to the uncertainty in the value of the unit used by König from whose data the curves in question were plotted. Blanchard (loc. cit.) has discussed the question and computed the probable value of König's unit as being .044 ml. (approximately). The millilambert is the c.g.s. unit of brightness and is equivalent to a brightness of ten apparent meter candles or .93 apparent foot candle. The region of maximum sensibility is therefore from 3 to 250 foot candles (approximately) instead of from 20,000 to 400,000 as indicated by the curves published by Renwick. In order to determine the sensibility for a given adaptation level it is important that the correct abscissae values be used.

Now, if we consider a specified brightness difference in the object viewed with the eye adapted to some brightness level A_o, it is possible from curve A (Fig. 10) to find the corresponding sensation difference; and if the brightness difference between the corresponding areas of the material reproduction be known together with the adaptation level (A_{mr}) of the observer viewing this material reproduction, it is possible to determine also the

FIG. 10



The Relation between Sensation and Stimulus

magnitude of the resulting subjective contrast. A comparison of the magnitudes of these two contrast values namely: (a) the subjective contrast resulting from observation of the object at a given adaptation level, which subjective evaluation it is convenient to refer to as the subjective object; and (b) the subjective contrast due to the observation of the material reproduction at a specified adaptation level, which subjective evaluation will be termed the subjective reproduction; determines the exactness with which the subjective sensation resulting from the observation of the object is reproduced by observation of the material reproduction (the illuminated positive) under the conditions specified. It is also possible by similar methods to determine the magnitude of the contrast necessary, at any point in the tonal scale of the material reproduction, for the exact reproduction of the subjective contrast of the object, assuming of course, that the values of A_o and A_{mr} are known

A_o = Adaptation level while observing object
A_{mr} = Adaptation level while observing material reproduction

For instance, in Fig. 10, consider an increment of brightness, $\Delta \log B_1$ for which the corresponding increment in the sensation is ΔS_1. At any other point take a second increment in sensation ΔS_2 such that $\Delta S_1 = \Delta S_2$. Let the increment in log B corresponding to ΔS_2 be represented by $\Delta \log B_2$. Assume that the increment $\Delta \log B_2$ applies to some point in the object and that the value of B_2 fixes the adaptation level of the observer looking at that object, while for the material reproduction the adaptation is conditioned by the value of B_1

$$A^o = B_2$$
$$A_{mr} = B_1$$

Now the average slope of the curve (Fig. 10) over the range $\Delta \log B_2$ is given by,

$$\text{Slope (2)} = \frac{\Delta S_2}{\Delta \log B_2}$$

And the average slope over the interval $\Delta \log B_1$ is likewise,

$$\text{Slope (1)} = \frac{\Delta S_1}{\Delta \log B_1}$$

Since $\Delta S_1 = \Delta S_2$ it follows that

$$\frac{\text{Slope (2)}}{\text{Slope (1)}} = \frac{\Delta \log B_1}{\Delta \log B_2}$$

But the average slope of the sensation curve over any given interval is given by the average value of the ordinate, D, of the contrast sensibility curve (Curve B, Fig. 9) over the same interval. Hence, we may express the values of the slope by the corresponding values of D and the equation becomes

$$\frac{\textit{Average } D_2}{\textit{Average } D_1} = \frac{\Delta \log B_1}{\Delta \log B_2}$$

It is evident, therefore, if Δ log B_2 be the ol difference at some point in the object that the the value of the Δ log B_1, the objective brightne: sary in the photographic reproduction for the e of subjective contrast. In the case illustrated, th is less than the interval Δ log B_1 thus indicating th tive contrast in the reproduction will be requ the subjective contrast due to the smaller obj the object. Changing the numerical subscripts indicate that the various factors apply to obj reproduction the exposure becomes,

$$\frac{Average\ D_o}{Average\ D_{mr}} = \frac{\Delta \log B_{mr}}{\Delta \log B_o}$$

By allowing these finite increments to approac we may replace the average values of D by valu the contrast sensibility curve at a point and obt

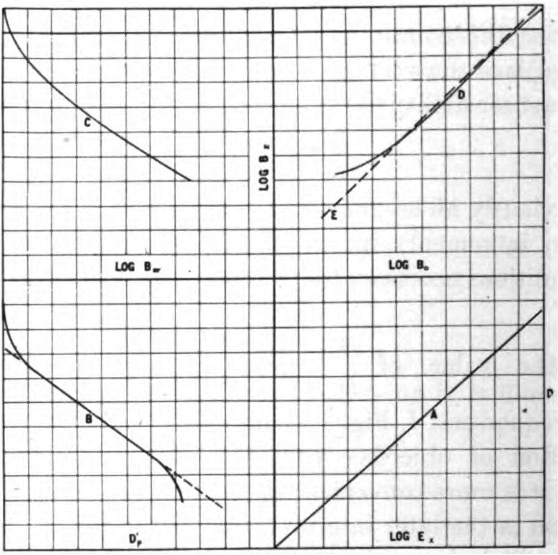
$$\frac{D_o}{D_{mr}} = \frac{d \log B_{mr}}{d \log B_o}$$

Since previously an assumption of equality of su (sensation increment) was made this equatior of the conditions necessary for the exact reproduc contrast.

Now the value of $\dfrac{D_o}{D_{mr}}$ gives the slope of plotted in quadrant I, Fig. 5 would be the line of reproduction of objective contrast. In the g however, it is more convenient in plotting the re D and also in the later interpretation of the res line E (of which the gradient is unity) as the line reproduction. This can be done by plotting in curve of which the gradient is equal to the r value $\dfrac{D_o}{D_{mr}}$, that is $\dfrac{D_{mr}}{D_o}$. This new function the dummy curve previously plotted in quadrar in the graphic solution for the objective phase.

It should be pointed out at this place that thi to a final expression of the exactness of reproduct of B_o, the brightness of the object. It is quite possible to obtain the final evaluation of the exactness of reproduction as a comparison between the contrast values of the subjective object and the subjective reproduction, but in view of the fact that the starting point in any tone reproduction problem must be the objective brightness values of the object it seems more logical to make these values the primary base of the computation and to express finally the departures from exact reproduction in terms of these physically measured values of object brightness. This necessitates the evaluation of the subjective contrast of the reproduction in terms of the subjective contrast of the object and a final expression of this complex relation as a function of the object brightness B_o. The final comparison must therefore be made between the actual measured (objective) contrast values of the object and the corresponding contrast values of the subjective reproduction evaluated in terms of the subjective object. The scale on which the points representing the various areas of the reproduction as evaluated in terms of this relative subjective contrast function

FIG. 11



A General Case of the Complete Solution Including the Subjective Phase must occupy the line OY in order that the curve representing the final reproduction may be graphically constructed with values of B_o as abscissae, Fig. 11. The required scale constructed on line OY will be termed the log B_z scale and the relative subjective contrast function which is used to determine the distribution of the points on the log B_z scale will be referred to as the Z function and its gradient will be designated by G_z.

The slope of this function (Curve C Fig. 11) at any point is given therefore by $G_z = \frac{D_{mr}}{D_o}$.

Now it can be shown that the relation between the gradients of the several curves (Fig. 12) is expressed by the relation,

$$G_n \,.\, G_p \,.\, G_z = G_r$$

Where G_n = Gradient of negative (Curve A)
G_p = Gradient of positive (Curve B)
G_z = Gradient of Curve C.
G_r = Gradient of reproduction (Curve D)

If any three of the four functions be known or are assumed the fourth can be obtained either by the graphic method or by computation. In the most general case such as is represented in Fig. 11 no assumption is made as to the constancy of the adaptation level of the observer as attention shifts from regions of high to those of low brightness in the object and reproduction, and hence the value of G_z may not be constant. However, it is probable that the value of G_z is in the great majority of cases practically constant in which case the curve C becomes a straight line of which the slope is specified by the value of $\frac{D_{mr}}{D_o}$.

It is well-known that other factors than the brightness of the area upon which the attention is fixed (foveal image) influence to some extent the adaptation level of the eye at any instant. Such factors include the brightness of the surrounding objects (i.e., the peripheral images) and the length of time during which the attention has been fixed on the area considered. In practice, however, the picture being observed occupies a very considerable portion of the field of vision, and, further, in the great majority of such pictures the actual range of brightness is relatively limited.

This is especially true of surfaces viewed by reflected light. Further, under the majority of conditions the field of vision not filled by the reproduction being considered does not contain any area contrasting extremely with brightness of the reproduction. It seems reasonable, therefore, to assume that the adaptation level will be fixed in practice by the brightness of the reproduction itself and likewise that the level when viewing the object will be conditioned by the object considered.

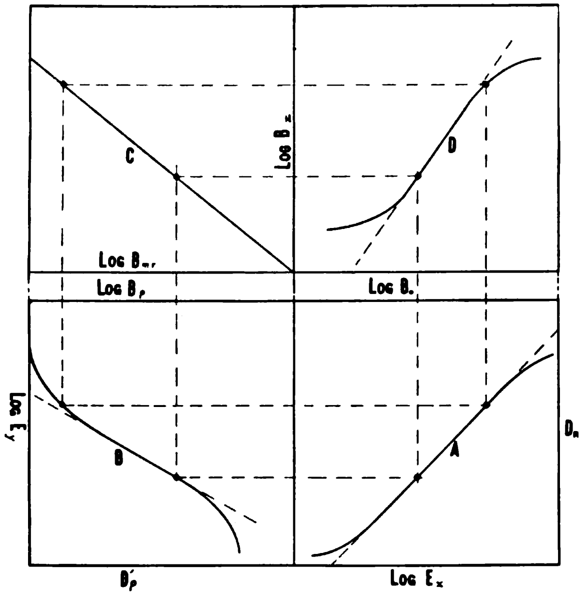
The question then arises as to what particular brightness in object and reproduction will determine the adaptation level in each case. Considering the reproduction as a reflection surface the average range of brightness may be taken as 1 to 40 (1.6 in log B_2) which even at the steepest part of the sensibility curve (Fig. 9) does not correspond to an excessive change in value of D. The change in adaptation as attention travels from highlight to shadow probably is not so great as that indicated by the brightness range, being limited to some extent by the stabilizing effect of the peripheral images as well as to the time lag of adaptation behind the changing foveal stimulus. It seems logical, therefore, to assume that A_o and A_{mr} are conditioned by the average brightness of object and material reproduction and this assumption will undoubtedly hold for a large percentage of the normal cases, although it may be necessary under extreme conditions to take into account the change in adaptation (and consequently in the value of D) which occurs as attention shifts from an area of one brightness to that of another.

A special case, therefore, which will probably fit most practical conditions depends upon the assumption that the adaptation level is conditioned by the brightness of object and reproduction half way between (on the log B scale) the highest and lowest brightness existing in these things. Such a case is shown in Fig. 12 where curve C, the relative subjective-contrast function is a straight line and D is the curve expressing the quality of final subjective reproduction as a function of B_o.

Now if

$$D_o = D_{mr}$$
$$G_s = 1.0 = G_d$$

FIG. 12



Special Case of the Complete Solution Including Subjective Phase

This is the gradient thus far used in the construction of the curve C. It is evident, therefore, if average B_o and average B_{mr} are such that they fall between the limits (Fig. 5) log $B = 0.5$ ($B = 3.2$ *ml*) and log $B = 2.2$ ($B = 166$ *ml*) that the subjective reproduction of contrast will be identical with the material reproduction curves already discussed.

If however, the brightness, B_o, of the object and the consequent adaptation level, A_o, of the observer viewing that object is relatively high, while the brightness of the material reproduction, B_{mr}, (the illuminated positive) and the corresponding adaptation level, A_{mr}, is low, the subjective reproduction will be different from the objective reproduction. In this case the contrast sensibility D_o is greater than D_{mr} and hence the gradient, G_s of the relative subjective contrast curve is less than unity.

$$G_s = \frac{D_{mr}}{D_o}$$

The reproduction curve, D, constructed in the usual way will also have a decreased gradient. This indicates that under such conditions a loss of contrast occurs due to the subjective factors and in order to obtain exact reproduction it will be necessary to enhance the objective contrast in material reproduction in order to compensate for the loss due to the subjective effect. It is evident from these considerations that a picture of a brilliantly lighted scene which is to be viewed under relatively low illuminations such as exist in interiors at night should be somewhat more contrasty than the scene itself. That is, best subjective reproduction will be obtained by a positive in which the actual objective contrast is somewhat enhanced. This condition may be stated symbolically thus:

If,

D_o is greater than D_{mr}
G_s is less than 1.0

When G_s is less than unity there will be a loss of contrast due to the subjective factors.

On the other hand if,

D_o is less than D_{mr}
G_s is greater than 1.0

and there will be an enhancement of contrast due to the subjective factors.

Many interesting relations between the various factors may be derived and applied to practical problems but the discussion of such details will not be taken up at this time. One point deserving mention at this time, however, is relative to the values of adaptation of the eye upon which depend the sensibility values from which the slope of the subjective relative-contrast function is computed. Few data are at present available as to adaptation levels when the visual field is filled by areas of different brightnesses such as exist under practical conditions. Experimental work is at the present time in progress in this laboratory from the results of which it is hoped more reliable evaluations of A_o and A_{mr} may be obtained for certain specified practical conditions. When these data are available it will be possible by the application of the general principles outlined in this paper to arrive at more certain conclusion relative to the exactness of reproduction in any particular case.

REPORT ON INSTRUMENTS AND METHODS OF RADIOMETRY*

BY

W. W. COBLENTZ

I. INTRODUCTORY STATEMENT

Under this caption a brief outline is given of the most important radiometric instruments for present day laboratory work in thermal radiation. The measurement of (1) solar, sky, nocturnal and stellar radiation, and (2) the study of polarization phenomena by means of spectroradiometry, require special apparatus which is described in separate reports on this subject.

Radiometric instruments are used for two classes of work: (1) the measurement of total radiation and (2) the measurement of thermal radiation intensities in the spectrum. The latter requires auxiliary apparatus including a spectrometer and prisms of quartz, rock salt, etc. The calibration of the spectroradiometer, the precautions necessary for eliminating the effect of stray radiation, etc., are described in a separate paper on transmission and refraction data of prism and lens material with special reference to spectroradiometry.[8] More specific information on various types of spectroradiometers for investigations in absorption, emission and reflection spectra, in the ultra-violet, visible, and infrared is given in the classified bibliography at the end of this report. Various details on the construction and operation of bolometers, thermopiles, etc., are given in papers by Coblentz[1] on "Instruments and Methods of Radiometry." A concise exposition of the whole subject of radiometry and spectroradiometry is given in Nutting's "Outlines of Applied Optics."[2]

One of the chief complaints of workers in radiometry is the uncertainty of the observations caused by unsteadiness of atmospheric conditions. Radiometers are essentially of small heat capacity and as a consequence are greatly affected by air currents. In planning a research, it is therefore desirable to place the radiometer in a container that can be evacuated, or to make the observations on dull, cloudy, windstill days, or in the summer when the air is more uniformly heated within and without the laboratory.

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*Section of 1920 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

The loss of heat by convection is very great. The sensitivity of a modern thermopile is doubled and that of a thermocouple of fine (0.01 mm) wire is quadrupled, by placing it in an evacuated enclosure.

Most of the radiometers to be discussed are non-selective in their response to thermal radiation of all wave lengths, and this sometimes causes difficulties. Hence in investigations employing ultra-violet radiations, (for example, ultra-violet transmission and reflection measurements) it is sometimes advisable to use a selective radiometer such as, for example, a potassium-hydride photoelectric cell.

II. THERMOPILES

Thermopiles are constructed of fine wires of different kinds of metal as for example copper and constantan. They function by generating an electric current when the alternate junctures of these two metals are heated by absorption of thermal radiation. The thermopile is therefore less affected by air currents than a bolometer which is slightly warmer than the air as a result of the electric current constantly flowing through the circuit. Improvements introduced during the past few years have made the thermopile a close competitor with the bolometer in sensitivity. It is not so quick in response as is a bolometer, requiring three to five seconds to attain a maximum. However, there are but few everyday problems requiring instantaneous action.

The theory of thermopile design has been worked out by Altenkirch[14] and by Johansen.[16] The latter finds that (1) the resistance of the thermopile should equal that of the galvanometer, (2) the radii of the two wires of the element should be so chosen that the ratio between the heat conductivity and the electrical resistance is the same in both, (3) the heat loss by conduction through the wires should equal the heat loss by radiation from the junctions, (4) the radiation sensitivity is proportional to the square root of the exposed surface. In an iron-constantan couple the theory required the use of iron wire 0.023 mm and constantan wire 0.045 mm diameter; and for a Bi-Fe couple, iron wire 0.024 mm and bismuth wire 0.075 mm diameter. If the diameter of both wires were twice as great, theory indicates that the sensitivity would be only 0.8 the maximum sensitivity. He used a single long junction consisting of a constantan wire 0.03 mm and an iron wire 0.015 mm diameter soldered to a strip of silver 10 by 0.2 by 0.015 mm.

The earlier form of Ruben's [13] thermopile has been modified by Paschen,[15] who constructed a thermopile of iron 0.1 mm and of constantan 0.08 mm in diameter. The constantan wire was 4 mm long. The wires were mounted end to end and fused with a little borax and silver solder. The junctions and wires were then rolled to 0.002 mm thickness. The maximum temperature was attained in four seconds and 98 per cent of the increase in two seconds. The sensitivity was about the same as that of a bolometer strip 0.001 mm thickness but was only about one-half that of his best bolometers on high battery current; its manipulation, however, was easier.

Reinkober[17] constructed a vacuum thermopile of 14 junctions of iron-constantan, of wires 0.05 mm diameter and having a resistance of 14 ohms. It was 3 times as sensitive as the old form of Rubens pile with wires 0.15 mm in diameter. In vacuo it was 1.6 times as sensitive as in air. He made also a thermopile (of wires 0.021 mm thickness, hammered thin) of only four elements, since the resistance was 18 ohms. In air, this was twice as sensitive as the old form, and in vacuo 10 times as sensitive as the commercial Rubens instrument. He found that its sensitivity was only one-half that of a vacuum bolometer.

The technique of rapid construction of thermopiles of bismuth and silver is given in papers by Coblentz.[18] In a separate paper[19] various modifications of the thermopiles having a continuous absorbing surface are described, suitable for physical photometers, for physiological investigations, for the measurement of nocturnal radiation, etc. The construction of thermo-couples for measuring heat from stars[20] requires considerable skill and special technique.

For ordinary laboratory work *Bi* wire 0.1 mm in thickness and *Ag* wire 0.035 to 0.04 mm, with receivers of tin (2 by 1 by 0.02 mm) are used. After completion the silver wire is given a thin coating of shellac. Such thermopiles are strong and can be shipped without breakage.

Symmetry of construction is important in a thermopile. To attain this end all the junctures should be suspended freely in the air which becomes warmed by the incident radiation. Thermopiles in which the "cold" (unexposed) junctions are soldered to metal supports invariably cause a drifting of the galvanometer as a result of warming of the air surrounding the "hot" junction. This can be neutralized by having a thermocouple of low e.m.f. (e.g. iron-copper) in the circuit. By slightly heating one of the junctures of this couple, the zero position of the galvanometer reading is brought back on the scale.

The radiation sensitivity of a thermopile is doubled by placing it in a vessel evacuated to 0.02 mm pressure. The radiation sensitivity of thermopiles of fine (0.02 mm diam.) wire is increased 4 to 5 times in a vacuum.

III. BOLOMETERS

The electrical connections of a bolometer and its auxiliary galvanometer are the same as that of a Wheatstone bridge. The latter is constructed of resistances having a negligible temperature coefficient. It is therefore misleading to speak of the bolometer as being "simply a Wheatstone bridge." In a bolometer two branches of the bridge are made of thin strips of metal having a high temperature coefficient of resistance. These strips, called receivers, are made as thin as possible in order to reduce the heat capacity. They are usually made of thin (0.001 mm) platinum, though iron[23] and nickel have also been used. They are blackened to increase the absorption of radiation. Two strips are used for symmetry. The remaining two branches of the bridge are usually made of manganin wire having 3 to 4 times the resistance[22] of the receivers.

It will be understood that the bolometer functions as a result of a change in resistance of one of these thin blackened strips when it is exposed to radiation. The resultin deflection is proportional to the temperature ris receiver and hence of the intensity of the th stimulus.

Convection currents of air cause great un bolometer. It is therefore necessary in practic place the receivers in a vessel from which the air c to a pressure of less than 0.01 mm.[27] The vacuu is much more sensitive than a bolometer in air. are many complications and disturbances, and ments in the sensitivity of thermopiles have ma substitute the latter for the bolometer in most ra

IV. RADIOMICROMETERS

The radiomicrometer is essentially a moving co having a single loop of wire with a thermojunct This instrument was invented independently b and by Boys.[31] Coblentz[34] has devised a vacuum ra combined the principle of the radiomicrometer a radiometer. One of the most noteworthy radiom constructed by Schmidt[35] in the laboratory of The instrument has undergone modifications and, judging from published work, is practically use. The commendable feature of the radiomi freedom from magnetic disturbances. However, it and to attain a high sensitivity requires a long pe

V. NICHOLS RADIOMETERS

The torsion radiometer by Crookes[37] as improv consists essentially of two similar, thin vanes of or platinum, attached to a horizontal arm and means of a fine quartz fiber, in a vessel which is e vanes are about 3 mm from a window. The measured falls upon one of the vanes which becom repelled from the window. A small mirror is att staff which supports the vanes. The deflectio by means of a telescope and scale. The devi magnetic disturbances but to attain high sensitivity it requires a long period, say 30 to 60 seconds as compared with a thermopile and Thomson galvanometer which requires 5 seconds. In its time it was very useful, but at present it is being replaced by vacuum thermopiles and iron-clad galvanometers.

VI. Galvanometers

One of the most important adjuncts to a radiometric laboratory is a magnetically well-shielded astatic galvanometer. The bolometer and the thermopile require an auxiliary galvanometer. Much radiometric work can be done with a thermopile and a sensitive d'Arsonval galvanometer. Such galvanometers, now constructed by Leeds and Northrup and by Weston, have a sensitivity of $i = 1 \times 10^{-9}$ ampere. They are designed to fit a bismuth—silver thermopile[19] having a resistance of 10 to 12 ohms. In other words they are designed to be critically damped on an external resistance of 12 ohms and a single swing of 8 to 10 seconds.

Recent great advances in radiometry have resulted from imbedding the coils of a Thomson galvanometer in soft iron[52,55] and providing laminated shields of transformer iron. Tests by Coblentz[55] showed that while the inner shields of laminated iron greatly reduced the magnetic disturbances, imbedding the coils in blocks of soft iron, reduced still further the effects of air currents and magnetic perturbations. Whether there is a further gain by making the inner shields of transformer iron regularly separated by thin paper, as described by Wente,[56] or by wrapping iron wire upon cardboard, as described by Esmarch[53] remains undecided. These like other refinements (as for example the older method of not having slots in the shields, for viewing the mirror) are usually negligible by an experimenter when he is anxious to get things done. For example, judging from published work, not until within recent years did Paschen[54] devise and use a special iron-clad mounting.

The galvanometer should be permanently mounted upon a substantial pier. Local tremors are sometimes avoided by shifting the position of the galvanometer. Serious earth tremors may be avoided by suspending the galvanometer as described by Julius.[57]

A good shield from the magnetic disturbances of the average laboratory is obtained by imbedding the coils in blocks of soft (Norway) iron, over which is placed a laminated shield of soft iron. The latter is made of 20 or more pieces of transformer iron (which is in strips about 75 cm (2.5 ft.) in length and 18 cm wide) and weighing 9 to 10 lbs. It is well to provide additional shields of black wrought iron welded pipe of "extra strong" quality in sizes 5, 7, 9, and 11, (See Kent's Mech. Eng. Pocketbook) and cut in lengths of 25 or to 30 cm. Researches [61] not requiring the highest radiometric sensitivity have been made successfully by using simply the iron-clad mounting and laminated cylinder just described, and one additional shield which consisted of an 8-inch black wrought-iron pipe inside of which was placed a series of laminations of transformer iron which practically doubled the thickness. A cover of pieces of transformer iron adds to the shielding effect[55] and keeps out dust.

VII. VACUUM MAINTENANCE

The most efficient device for easily maintaining a vacuum is metallic calcium[19,20] in a quartz or porcelain tube which is cemented to the thermopile container. Heavy pyrex tubing is useful but is likely to collapse on heating. At the beginning, the thermopile container is thoroughly evacuated by means of an oil pump and the stop cock is closed.

The metallic calcium should be free from carbide, and should be heated to a dull red before closing the stop cock. After this the vacuum is maintained by heating the calcium to a dull red by means of an alcohol lamp, a gas flame, or by electric means. The vacuum stellar thermocouples[20] have been heated but twice in 6 years to remove the gases liberated from the cell walls. On the other hand a laboratory thermopile which has always leaked badly has had its vacuum maintained for the past 4 years by frequently heating the calcium.

VIII. SOURCES OF RADIATION FOR SPECTRORADIOMETRY

The difficulties experienced by earlier workers in spectroradiometry are gradually being overcome by the development and application of apparatus suited for the purpose. It is essential to use spectrometers of high light-gathering power, as described in a separate report published elsewhere in this journal.[8]

1. ULTRA-VIOLET

For transmission and reflection measurements in the ultra-violet between 0.18μ and 0.28μ we have the strong emission lines [58] from the spark spectra of Al, Cd, Mg, and Zn.[59]

The emission spectrum of the mercury in quartz arc lamps,[58,61] is a source of powerful emission lines at 0.25μ to 1μ. Equipped with a Cooper-Hewitt 600-watt quartz-glass, mercury-vapor lamp the experimenter is now able to make measurements, at certain wave lengths in the ultra-violet, with as great ease and accuracy as in the infra-red.

Owing to the generation of ozone and nitrous oxide fumes this lamp should be operated in a closed metal box provided with a ventilator. Deep brown-colored glasses should be worn when exposed to the lamp, otherwise painful and perhaps permanent injurious effects will occur to the eyes.

2. VISIBLE AND INFRA-RED

Transmission and reflection measurements in the visible and in the infra-red to 4μ are now easily made by using a gas-filled tungsten lamp[60] (500 watt, "Mazda C," stereoptican type is recommended). This type of lamp is easily calibrated for an equal energy spectrum such as is used in studying the photoelectric sensitivity of substances.[62] For investigations in the infra-red of long wave lengths the ordinary gas-mantle[63] is a useful source of radiation. The Nernst glower is also useful, but it is easily affected by air currents.

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THE MEASUREMENT OF SOLAR, SKY, NOCTURNAL AND STELLAR RADIATION*

BY

W. W. COBLENTZ

I. THE MEASUREMENT OF SOLAR RADIATION

1. INTRODUCTORY STATEMENT

The measurement of thermal radiation from the sun, and from celestial bodies in general, is a field which is quite undeveloped, and the importance of which cannot be foretold. Animal and vegetable life exists primarily because of thermal and photochemical activity induced by the radiation emanating from a neighboring sun. We are therefore vitally concerned with the question of the constancy of the radiation from our sun. A continual fluctuation in the emissivity of the sun is to be expected and, if the terrestrial atmospheric transmission were more constant, this fluctuation in solar radiation could, no doubt, be recorded to the minute. But, limited as we are in our ability to eliminate the disturbing causes in making the measurements, perhaps the best that we can expect, for the present at least, is to record great fluctuations in the value of the solar constant of radiation and determine whether or not there is any regularity or periodicity in these fluctuations. This will establish a level of values and our successors of one hundred or more years hence, (if not at an earlier date and if interested in the subject) will be able to determine to what extent, and in what spectral radiation qualities, solar radiation differs from that of the present. Measurements of stellar radiation show that red stars have a much greater emissivity (are cooling off faster) than stars of the solar type. It is therefore interesting to speculate on the terrestrial conditions that will ensue when (and if) the sun emits, relatively at least, more infra red and less visible radiation than obtains at present.

In the meantime the observational data accumulated may perhaps be correlated and utilized (as, indeed, is already being attempted) in predicting conditions of the weather.

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*Section of 1920 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

2. THE INTENSITY OF SOLAR RADIATION AT THE EARTH'S SURFACE

The solar radiation intensities most commonly recorded in meteorological journals generally pertain to a given station, for a given date. They are useful for purposes of comparison, and may be obtained by means of simple apparatus.

One instrument frequently used for this purpose is the Ångström,[4]† compensated pyrheliometer. This device consists of two thin blackened manganin strips, the one of which is exposed to (and hence warmed by) solar radiation. The other strip is shielded from solar radiation and is heated electrically to the temperature of the exposed strip. Equality of temperature is determined by thermojunctions attached to the manganin strips and connected with a galvanometer. From a knowledge of the area exposed to radiation and the electrical data, the value of the solar radiation intensity is obtained in absolute value, which is usually expressed in gr. cal. per cm^2 per min. The Marvin pyrheliometer,[9] which consists of a resistance thermometer and a heating coil imbedded in a disk of silver, is used in a similar manner to measure solar radiation.

A dynamic method of observing the intensity of solar radiation consists in noting the rate of rise in temperature of a mercury-in-glass thermometer the bulb of which is imbedded in a disk of copper or silver, such as for example the silver disk pyrheliometer described by Abbot.[3,5] A paper by Whipple[10] describes and gives illustrations, of various types of instruments for measuring solar radiation. Instructive and important data on solar radiation intensities are being obtained, and in concluding this part of the discussion it is of interest to note the great seasonal variability of the intensity of solar radiation. For the central latitude of the U. S. A., Kimball[28] shows that the maximum solar radiation at perpendicular incidence varies from 1.37 gram calories per minute per square centimeter in January to 1.5 gram calories in May and September. The total radiation on a horizontal surface, with a clear sky, varies from 0.77 gram calories per minute in December to 1.55 gram calories in June. Clouds nearly in line with the sun, but not obscuring it, increase the radiation by about 0.15 gram calories.

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†References are given in the bibliography at end of text.

The use of daylight and artificial light is increasing in dye fading and other photochemical tests. In a recent investigation by Coblentz and Kahler[13] on the component radiations from the sun and from a quartz mercury vapor lamp, it was shown that there is no marked difference in the total ultra violet radiation, of wave lengths less than about 0.4μ, from these two sources. However, the spectral quality of the ultra violet (the energy distribution) is entirely different. The ultra violet of the solar spectrum terminates at about 0.3μ. On the other hand, in the quartz-mercury vapor lamp the ultra violet component of wave lengths less than 0.3μ is about 20 per cent of the total ultra violet component radiation from this lamp.

3. THE SOLAR CONSTANT OF RADIATION

By solar constant is meant the intensity of solar radiation (usually expressed in gr. cal. cm^{-2}, min^{-1}) in free space, at the earth's mean solar distance. The determination of this constant involves; (1) an accurate measurement of the solar radiation intensity at the earth's surface, and (2) an accurate estimation of the losses in intensity suffered by the solar rays, in passing through the earth's atmosphere.

The first step in the determination of this constant is to measure the solar radiation intensity by means of the silver disk[3,5] pyrheliometer. This is a secondary instrument which was calibrated against a primary standard water flow instrument (viz., a water flow calorimeter of special design).

Simultaneously with the pyrheliometric measurements, spectrobolometric energy curves are obtained, of the sun. From these spectral energy curves of the sun at different altitudes, the atmospheric transmission curve, for all wave lengths, is determined; and from these two sets of observations it is possible to compute the solar radiation as it would be outside the terrestrial atmosphere.

This method of determining the solar constant of radiation was introduced by Langley, and the numerical values obtained therewith are perhaps the first to command the confidence of experimenters. The fault to be found with other determinations which differ from each other by almost 100 per cent,[16] no doubt, lies in an inaccurate estimate of the amount of solar radiation lost in passing through the terrestrial atmosphere.

Recently Abbot[29] and his collaborators have worked out a new method of determining the solar constant, based upon a relation between measurements of sky brightness and the above mentioned spectral transmission coefficients. The sky owes its brightness to scattering of the solar rays by water vapor, etc. The more hazy the sky, the greater is its brightness and the less is its atmospheric transmission.

The measurement of the atmospheric transmission coefficients is a long tedious process. The measurements of sky brightness are very quickly made with a pyranometer.[6] They were able to work out graphically a relation between the atmospheric transmission coefficients and sky brightness which appears to be valid at least for the Calama, Peru, Smithsonian Station thereby greatly shortening the time of observing and calculating the results. All that appears necessary is to make simultaneous measurements of sky brightness and solar radiation intensities, and determine from the graph, the corresponding transmission coefficients.

The mean value of the solar constant of radiation for the epoch 1902–1912 (696 observations) is 1.93 gr. cal. (15° C) per cm^2 per min. (loc. cit.,[3] p. 134). The value of the so-called "solar constant" is in reality not a constant but is subject to variation with sunspot activity, etc. These fluctuations occur at irregular intervals and range over perhaps 8 per cent. They are thought to indicate a true variability of the sun[16] (loc. cit.,[3] p. 117). However, no definite periodicity in variation in solar radiation intensity has yet been observed.[30,23]

4. THE TEMPERATURE OF THE SUN

Various attempts have been made to obtain an estimate of the temperature of the effective radiating layer of the surface of the sun. The usual procedure is to apply the radiation laws of a perfect radiator, which of course cannot be exactly true for the sun.

Wilson[20] compared directly the radiation from the sun with that of a black body at a known temperature, using Kurlbaum's value of $\sigma = 5.3$. Assuming a zenith transmission at 71 per cent he obtained a value of 5500°C for the temperature of the sun. Poynting[26] concluded that either Wilson's estimate of zenith transmission is too high or Kurlbaum's value of the coefficient of total radiation is too small. This is very interesting; for recent work shows that Kurlbaum's original value of $\sigma = 5.3$ is about 7 per cent too low.

The most recent value of the effective temperature of the sun's radiating layer is by Abbot.[23] It is based upon the distribution of energy in the spectrum of the sun, as it would be outside of the terrestrial atmosphere. After correcting the bolographs for absorption by the spectrometer mirrors and the atmosphere, the wave length of maximum spectral energy is at $\lambda_m = 0.470\mu$. On the basis of the Wien displacement law. Using the most recent value of $A = \lambda_m T = 2885$ the effective solar temperature, T, is 6140°K or about 5870°C. (Abbot used the old value $A = 2930$ and obtained 6230°K).

On the basis of the fourth power law of total radiation using the coefficient $\sigma = 5.72 \times 10^{-5}$ erg and the above mentioned value of the solar constant(1.922) the computed effective solar temperature is about 5740°K or about 5470°C. Kurlbaum[24] has made measurements with an optical pyrometer which yielded values of the order of 5500°C on the basis of $C = 14600$ (for a black body $C = 14300$ but is higher for a selective radiator like platinum).

In view of the fact that the sun is highly selective in its spectral emission, these two methods of calculation must indicate temperatures which are lower than the true temperatures. The conclusion to be drawn is that the sun's effective radiating layer is roughly comparable with that of a black body at about 6000°K or about 5700°C.

II. THE MEASUREMENT OF SKY AND NOCTURNAL RADIATION

Noctural radiation is the expression commonly used for describing the loss of thermal radiation from terrestrial objects to space. The name no doubt had its origin from the early experiments on the cooling of objects, which is best observed at night. Being a low temperature radiation, the wave lengths most strongly emitted are in the region of the spectrum greater than 8μ. In connection with the question of the incoming solar radiation, it is interesting to note that the outgoing loss in nocturnal radiation may amount to 10 per cent of the solar constant. The study of the heat interchanges between the earth and the sky requires radiometers of novel construction and operation.

1. THE PYRGEOMETER

Ångström[7] has modified the original compensated pyrheliometer[4] by using polished and blackened strips of manganin. On exposure to space, the dark strip cools more rapidly than the light strip, and by heating the dark strip electrically to the temperature of the bright strip a measure is obtained of the outgoing, or nocturnal radiation.

In a recent paper, Ångström[8] describes a new modification of the pyrgeometer for measuring sky radiation. In the new device, one strip is covered with platinum black, and the other is covered with magnesium oxide which has a high diffuse reflecting power[31] for luminous rays and a low reflecting for radiation of wave lengths at 8 to 10μ. The two strips will therefore have practically the same emissivity when exposed to nocturnal radiation. However, as a mechanical protection, he covers the device with a hemispherical glass vessel. When exposed for measuring sky radiation (or solar radiation) the difference in emissivity for long wave length radiation does not enter the measurements as it does in the Callendar[12] sunshine recorder and in the pyranometer,[6] in which the heating or cooling of the glass cover may introduce errors from long-wave radiation.

2. THE PYRANOMETER

This instrument was devised by Abbot and Aldrich[6] for measuring sky radiation. The device consists essentially of two short strips of blackened manganin suitably mounted in the center of a circular nickel-plated block of copper. One strip is ten times as thick as the other, and hence, because of its greater thermal conductivity, on exposure to radiation the temperature rise in the thick strip is less than in the thin strip. This is indicated by thermojunctions attached to the back of the strips and connected with a galvanometer. After again shading the strips, the deflection, which was caused by the absorption of radiation, is reproduced by heating by means of an electric current which is divided between these two strips so as to produce equal heating effects in each. These receiving strips are covered with an optically figured, hollow, hemispherical screen of ultra-violet crown glass which admits direct or scattered solar radiation but which prevents the exchange of long wave length radiation between the manganin strips and the sky. On removal of the glass cover the instrument is useful at night for measuring the outgoing or so-called nocturnal radiation.

Interesting and important data have been obtained with these instruments, although the subject is comparatively new. In the discussion of the solar constant of radiation attention was called to use of measurements of sky radiation[29] in determining the transmission coefficient of the atmosphere.

The blanket effect of clouds in preventing nocturnal radiation from the earth is a common observance. Because of this effect, Humphreys[38] arrives at the conclusion that the intensity of the earth's outgoing radiation is much greater in middle latitudes than it is in equatorial regions. The extraordinary effects that result from nocturnal radiation are well illustrated in the book of investigations by Barnes[36] on ice formation, in which it is shown that the "anchor ice" formed at the bottom of the St. Lawrence river is owing to cooling by radiation.

Interesting data on nocturnal radiation as affected by atmospheric humidity are given by Ångström.[32] In a recent paper by Boutaric[37] it is shown that the intensity of nocturnal radiation may be expressed as a function of the absolute temperature of a black radiating surface and the vapor pressure in its immediate vicinity. The application of such data to horticultural problems is well illustrated in an interesting paper by Kimball[39] in which it is shown that the nocturnal, or outgoing radiation from the pyrgeometer at 20 C increased from about 0.13 gr. cal. for a vapor pressure of 12 mm, to 0.23 gr. cal. per cm^2 per min. for a 2 mm vapor pressure.

III. THE MEASUREMENT OF STELLAR RADIATION

Numerous attempts have been made to measure the radiation from stars. One method of attacking the problem is by means of photoelectric photometry. The instruments and methods for this type of (selective) radiometry have been developed principally by Stebbins,[40] who has been very successful in obtaining important data on the change in brightness of variable stars By this means, he has added important data to the subject of variable stars. In his earliest work, he used a selenium photometer, and in his most recent work he has used a potassium hydride photoelectric cell.

The second method of attack is by means of thermal radiometry with non-selective receivers. The most recent attempt, by Coblentz,[43] using thermocouples has yielded some interesting results. Measurements were made on 112 celestial objects. It was found that red stars emit from two to three times as much *total* radiation as blue stars of the same (visual) photometric magnitude. Measurements which were made through a cell of water showed that, of the total radiation emitted, the blue stars have about two times as much *visible* radiation as the yellow stars and about three times as much *visible* radiation as the red stars. The absolute value of the total radiation received from all the stars is estimated at less than 2 x 10^{-8} gr. cal. per cm^2 per minute. From this it appears that if the total radiation from all the stars, incident upon 1 cm^2, of the earth's surface were collected and conserved, it would require from 100 to 200 years to raise the temperature of 1 gr. of water 1°C, whereas the solar rays, which reach the earth's surface, can produce the same effect in about 1 minute.

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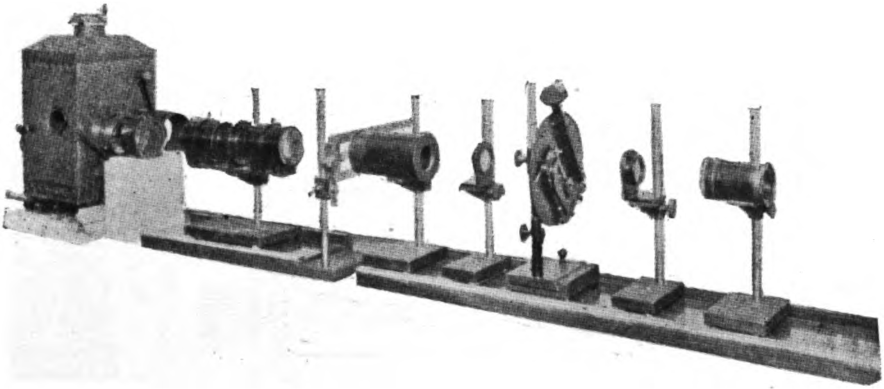
OPTICAL DETERMINATION OF STRESS IN TRANSPARENT MATERIALS

BY

A. L. KIMBALL, JR.

The purpose of this paper is to give a brief presentation of the Coker method for the determination of stress in transparent materials as now in use at the General Electric Research Laboratory at Schenectady, particularly with reference to the optical theory involved. This method was first developed and put on a practical basis by Dr. E. G. Coker, F. R. S., of the University College of London, and has been described by him at different times.[1] Its operation depends upon the well known law, discovered by Sir David Brewster in 1816, that transparent materials under stress become doubly refracting.

FIG. 1



Optical Apparatus

Fig. 1 shows the general set up, and Fig. 2 is a schematic diagram of the principal parts of the optical system. Light from a source, A, passes through a condensing lens, B, and water screen, C, and then through the polarizer, D. The latter is a special arrangement designed by the writer for making a Nicol polarizing prism of one inch circular aperture produce a beam of plane polarized light of much larger diameter, see Fig. 3. In this case the polarized beam produced has a diameter of four inches and the polarization is perfect over the entire field. The water screen, C, has a thickness of six inches, necessary to protect the polarizing prism which transmits a very concentrated beam. The source, A, may be an arc or a powerful incandescent lamp. In this set a 30 volt, 30 ampere tungsten filament moving picture lamp is used with good results, and is more convenient than an arc, as it requires no attention.

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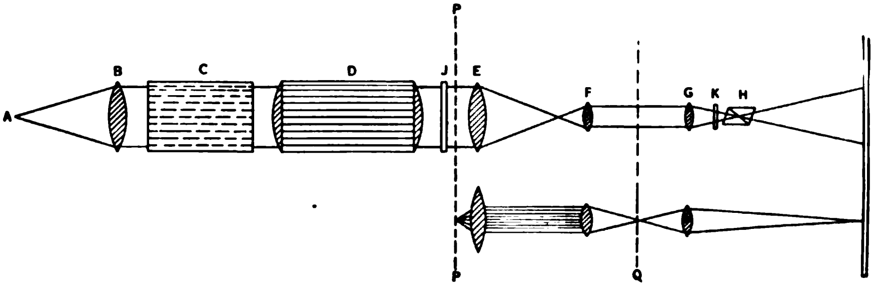
[1] E. G. Coker, Instit. of Mechanical Engineers, London, Feb., 1913.
Engineering, Sept. 4, 1914.
Institution of Automobile Engineers, Nov. 1917.

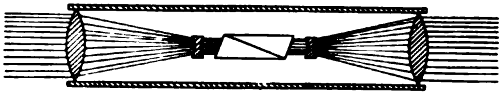
The plane polarized beam from D is then transmitted through a quarter wave plate of mica, J, so turned as to produce circularly

FIG. 2



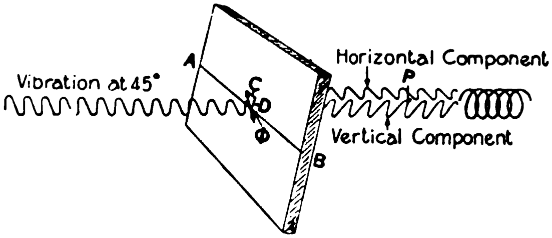
Schematic Diagram of Optical Set

FIG. 3



Polarizer

FIG. 4



Action of Quarter Wave Plate on Light Vibration

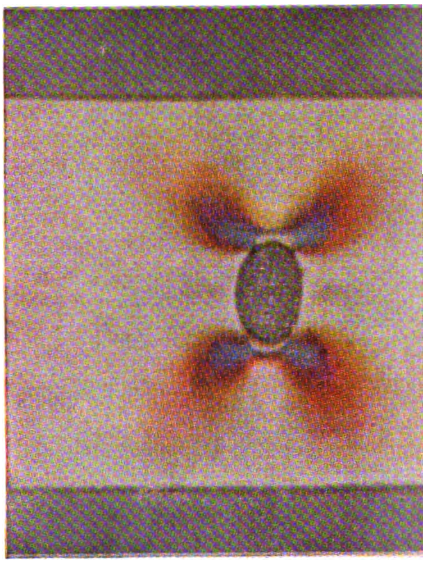


Plate I—Elliptical Hole in Tension Member, with Major Axis Tr

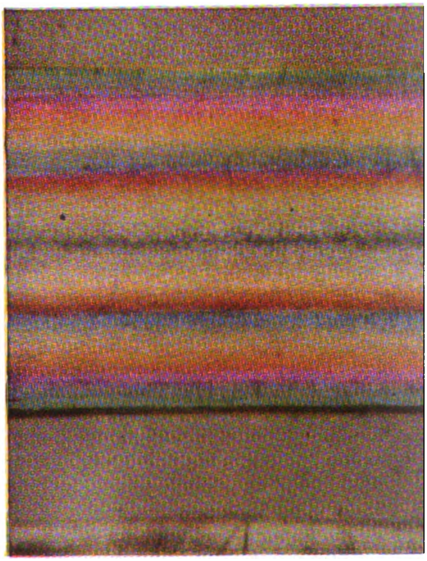


Plate II—Beam Under Uniform Bending, and Two Orders of Color D
Above and Below the Neutral Axis

polarized light, Fig. 4, and it is the circularly polarized light which is transmitted through the transparent model under stress. The model may be of glass, celluloid, bakelite or any other material with a good optical constant, but celluloid has been found best suited for the work because it may be easily worked, it has a large elastic extension, it has a high optical constant, and may usually be obtained free from initial stresses. A sample of celluloid is cut to the desired shape from a sheet ¼ inch thick or so, and the required tension or compression applied by means of a suitably devised frame with a thumb screw. The light, altered by transmission through this specimen, passes through another quarter wave plate, K, Fig. 2, with its axis at 90 degrees to the first and finally through an analyzing Nicol H whose polarizing plane is at 90 degrees to that of the polarizer.

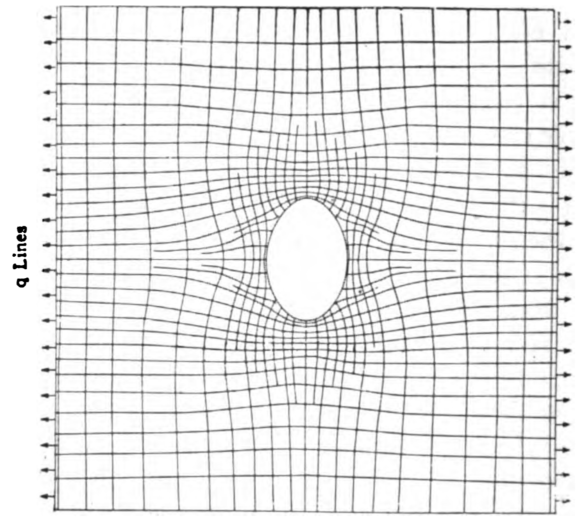
In the optical scheme shown in the figure, the specimen under stress may be placed in either the plane P or Q, extra lenses being necessary to accomplish this. Thus, the effect of two stress systems may be superimposed.

With this combination the transmitted light, which is projected upon a suitable screen, is all intercepted when the specimen is under zero stress and as the *difference* of the *principal stresses* p and q increases at a point in the specimen, the image of that point on the screen passes through a series of color changes as follows: black for (p = q), then straw, orange, red, blue green, straw, orange, red, blue green, etc., through several orders which begin to fade out after the fourth or fifth order.

Plate I shows the color effects from a tension member of celluloid with an elliptical hole in it, and plate II part of a beam, under uniform bending moment. These plates were made from Lumière color photographs.

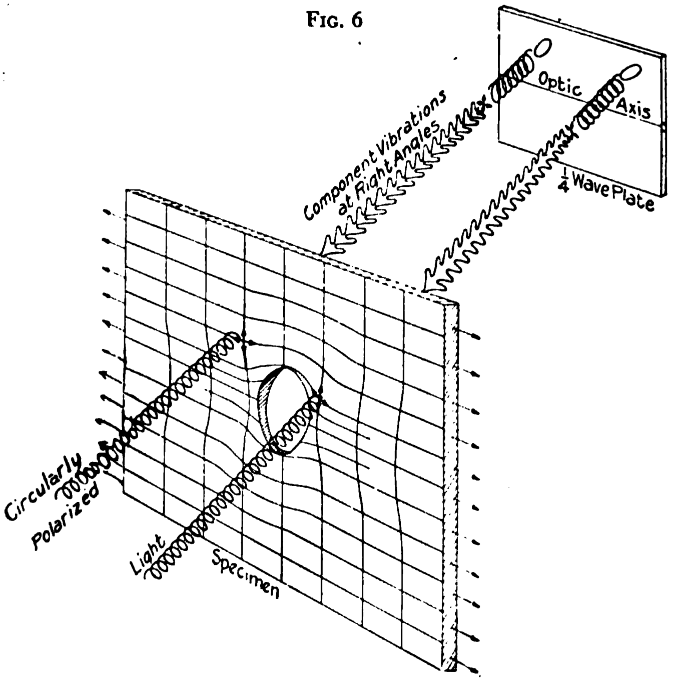
The color effects are produced by interference of the light components transmitted by the analyzer. To better understand this, a word or two will be said about stress. The stress distributions, which can be analyzed by this method are two-dimensional, or stress in a plane; light being transmitted normally to the stress directions. Any plane stress distribution consists of two systems, called, for convenience, the p and the q systems, see Fig. 5. The

FIG. 5



Principal Stress Lines About an Elliptical Hole in Tension Member

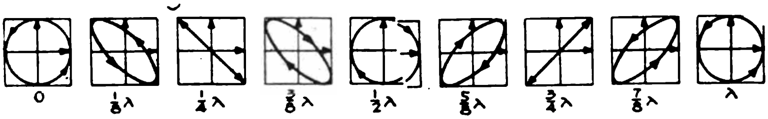
FIG. 6


Component Vibrations at Right Angles
Optic Axis
¼ Wave Plate
Circularly Polarized Light
Specimen

Light Rays Passing Through Stressed Specimen p and q systems are at 90° to each other at all points. The color figure produced on the screen, by means of its varying orders, gives a reproduction of p − q for every point of this system. On a specimen under tension, as shown in Fig. 6, at practically all points p is much greater than q. For a plane tension member q vanishes, and the order of color is a direct measure of p. Further, the p and the q stresses are always parallel and normal respectively to boundaries. Obviously, the normal stress must vanish at a boundary, so we conclude that at all boundaries the color order gives a direct measure of the tangential stress. This is a valuable point because boundaries are often points of maximum stress.

Referring to Fig. 6, two circularly polarized rays are shown from the beam of transmitted light incident on the specimen at two different points. All parts of the specimens become doubly refracting due to the stress, and the circularly polarized light is separated into two vibrations at right angles whose directions coincide with the principal stress directions, as shown, which are retarded relatively to each other according to the law

FIG. 7



Alterations of Circularly Polarized Light with Increasing Relative Retardation of Components

Relative retardation $= C(p-q)t$
where $C =$ optical constant
$p-q =$ principal stress difference
$t =$ thickness of sample

The transmitted light is the resultant of these vibrations at right angles, and is some form of elliptical vibration, depending upon the amount of relative retardation of these components. Fig. 7 shows the various forms of elliptical vibrations produced from different relative retardations of rectangular components of circularly polarized light measured in terms of wave lengths. When these forms of elliptical vibration are transmitted through the final combination of quarter wave plate and analyzer, the light which is unaltered by the model is entirely intercepted as is also that for all multiples of full wave length relative retardation of the components, and all odd half wave length relative retardations, 1/2 λ, 3/2 λ, 5/2 λ etc., are completely transmitted. The variation between these maxima and minima of transmitted light obeys the sine law, or, expressing mathematically,

$$A = A_m \sin \pi\lambda$$

where A = Amplitude of vibration
A_m = Maximum aplitude.

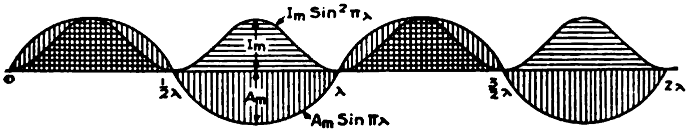
Since intensity varies as the square of the amplitude

$$I = I_m \sin^2 \pi\lambda$$

where I = Intensity of illumination
I_m = Maximum intensity

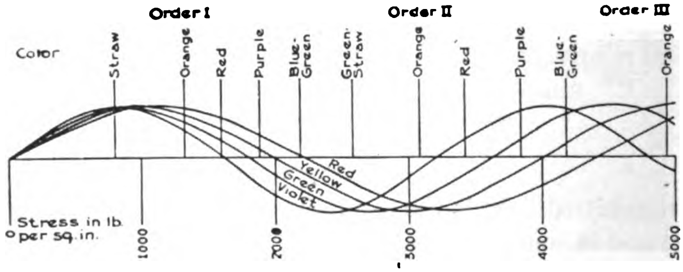
These results are shown graphically by Fig. 8 for a single wave length or color. It is interesting to note how the phase of vibration of the transmitted polarized light reverses for every wave length relative retardation.

FIG. 8



Amplitude and Intensity of Transmitted Light Corresponding to Increasing Stress for One Color

FIG. 9



Amplitude of Transmitted Light Corresponding to Increasing Stress for Several Colors Superposed

The above analysis is for a single wave length or color. For white light the same explanation applies, but to each color separately, and when all colors are superposed, Fig. 9, there results a repeating series of colors as the stress increases, since the relative retardation for a given stress is different for different colors. Fig. 9 is drawn for amplitudes.

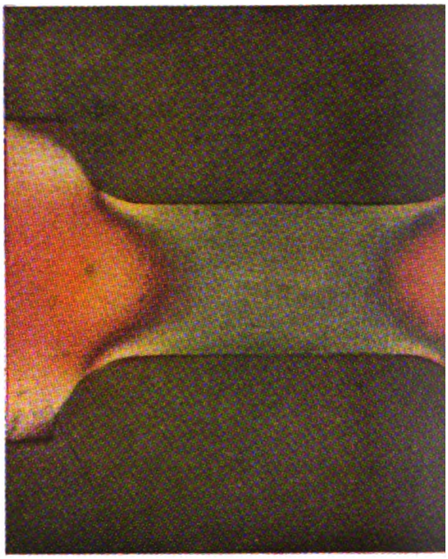


Plate III—Tension Member Showing Points of Maximum Stress at Ar

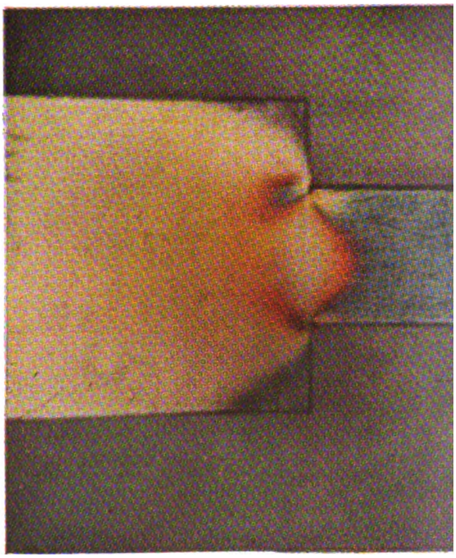


Plate IV—Tension Member Showing Sharp Local Stress at

The reason for a sine variation of light intensity for varying relative retardation, or varying principal stress difference is not self evident and the writer has worked out what appeals to him as rather an interesting proof, using the vector method, which is given in the appendix. This proof, together with the figures, will appear in the General Electric Review at an early date. There are other ways of proving this, such as the method outlined in Houstoun's Treatise on Light[2] which gives a brief discussion of stress analysis by polarized light as developed by Dr. Coker.

The question naturally arises, why is circularly polarized light used instead of plane polarized light. The answer is that different results are obtained for different positions of the specimen with plane polarized light. If no quarter wave plates are used, the color effects are exactly the same as with the quarter wave plates if the planes of polarization of the Nicols happen to be 45 degrees to the principal stress directions; otherwise less light is transmitted, and when the planes of polarization and principal stress directions coincide, all light is cut out. A black zone appears which is the locus of points on the model where the principal stress directions coincide with the polarizing planes of the Nicols, and gives a simple way of mapping principal stress directions. Those of Fig. 5 were determined in this way. Furthermore, with quarter wave plates a specimen may be rotated through any equatorial angle with respect to the light direction, and the color distribution of the image does not change. The use of quarter wave plates was suggested by Silvanus Thompson.[3] Plate III and IV show the color effect produced by tension members of two different shapes. These four photographs were taken in Dr. Coker's laboratory in London.

The optical measurement, therefore, gives a quantitative method of determining the stress at free boundaries of such a transparent model, or of the stress differences at internal points. A calibration member is used of the same material, in which the relation between stress and color is known, or the color effect may be directly balanced out for any point by superimposition of the images of the calibration member on that of the specimen, the stress on the calibration member being known. When a balance is accomplished, the point of the specimen under investigation appears black.

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[2] R. S. Houstoun, Treatise on Light, P. 228.

[3] S. P. Thomson and E. G. Coker, *Chemical News*, v. 100, pp. 161–62, Oct. 1, 1909.

For a complete determination of p and q at all points, measurements are made of changes of thickness of the model between the stressed and unstressed state by a delicate differential extensometer. This gives p + q, which in combination with the results of the optical measurements gives a complete determination of the stress distribution for the case under investigation.

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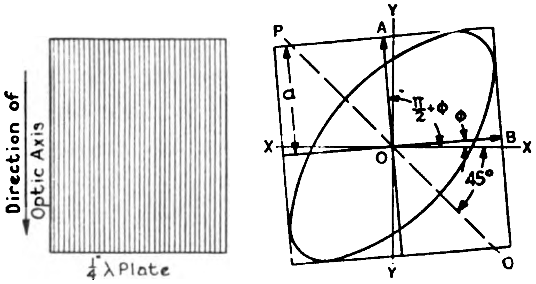
APPENDIX

Given a general case of elliptically polarized light. To find the laws according to which the intensity of monochromatic light varies when transmitted through a quarter wave plate and a polarizing prism with its axis at 45° to that of the quarter wave plate.

FIG. 10



In Fig. 10, PQ represents the line of vibrations transmitted by the polarizer which is at 45° to the axis of the quarter wave plate shown to one side. An elliptical vibration path is shown and the component vibrations OA and OB out of which it is composed. We will regard OA as retarded with respect to OB, the amount of this retardation determining the form of the elliptical motion of which the circle and straight line are limiting cases.

Expressing the amplitudes of these vibrations OA and OB in terms of components perpendicular and parallel to the optic axis of the quarter wave plate, or along the X and Y axes as shown in the figure, we obtain

$$\begin{aligned} \mathrm{OA} &= i\,a\cos\left(\frac{\pi}{2}+\phi\right)+j\,a\sin\left(\frac{\pi}{2}+\phi\right) \\ &= i\cos\left(\frac{\pi}{2}+\phi\right)+j\sin\left(\frac{\pi}{2}+\phi\right) \qquad (1) \end{aligned}$$

when $a=1$

This is a vector equation, i being a unit vector parallel to the X axis and j being a unit vector parallel to the Y axis. The quantity multiplying these vectors is their length.

In like manner:

$$OB = i\cos\phi + j\sin\phi \qquad (2)$$

(1) and (2) give the amplitude of the vibrations OA and OB. To express the fact that these vibrations vary with time according to the sine law we obtain

$$OA\sin(wt-\theta_1) = \left[i\cos\left(\frac{\pi}{2}+\phi\right)+j\sin\left(\frac{\pi}{2}+\phi\right)\right]\left[\sin(wt-\theta_1)\right] \qquad (3)$$

$$OB\sin wt = [i\cos\phi + j\sin\phi]\sin wt \qquad (4)$$

where $w=$ angular velocity of the time vector
and $\theta_1=$ angular lag of OA with respect to OB

The elliptical vibration is the path traced by the vector sum of these two components:

$$r = \left[i\cos\left(\frac{\pi}{2}+\phi\right)+j\sin\left(\frac{\pi}{2}+\phi\right)\right]\left[\sin(wt-\theta_1)\right] + [i\cos\phi + j\sin\phi]\sin wt \qquad (5)$$

This is the path traced by the tip of the vector r drawn from O as it swings around. Now this motion is altered in two ways, first by passing through the quarter wave plate, and second, by the rays passing through the polarizing prism.

When it passes through the quarter wave plate all the components parallel to its optic axis, or the j components are shifted in phase by ¼ λ or $\frac{\pi}{2}$. In this case we will assume a retardation.

Taking account of this, (5) becomes

$$r = i\cos\left(\frac{\pi}{2}+\phi\right)[\sin(wt-\theta_1)] + j\sin\left(\frac{\pi}{2}+\phi\right)\left[\sin\left(wt-\theta_1-\frac{\pi}{2}\right)\right]$$
$$+\, i\cos\phi\sin wt + j\sin\phi\sin\left(wt-\frac{\pi}{2}\right) \tag{6}$$

The vibration represented by (6) is again altered by transmission through the Nicol prism. This selects all components parallel to PQ, superimposes them upon each other and rejects all transverse components. Projecting the component vectors of (6) upon PQ, dropping the unit vectors and remembering that the projections of the j components are negative, we obtain

$$r\sqrt{2} = \cos\left(\frac{\pi}{2}+\phi\right)\sin(wt-\theta_1) - \sin\left(\frac{\pi}{2}+\phi\right)$$
$$\left[\sin\left(wt-\theta_1-\frac{\pi}{2}\right)\right]$$
$$+\cos\phi\sin wt - \sin\phi\sin\left(wt-\frac{\pi}{2}\right) \tag{7}$$

Transforming and simplifying, we obtain

$$\begin{aligned} r\sqrt{2} &= (-\sin\phi\cos\theta_1 + \cos\phi\sin\theta_1 + \cos\phi)\sin wt \\ &\quad + (\sin\phi\sin\theta_1 + \cos\phi\cos\theta_1 + \sin\phi)\cos wt \\ &= (-\sin(\phi-\theta_1) + \cos\phi)\sin wt \\ &\quad + (+\cos(\phi-\theta_1) + \sin\phi)\cos wt \\ &= A\sin wt + B\cos wt \end{aligned} \tag{8}$$

The amplitude of this vibration

$$= r_m\sqrt{2} = \sqrt{A^2+B^2} = \sqrt{2+2\sin\theta_1}$$

$$r_m = \sqrt{1+\sin\theta_1}$$
$$\text{where } r_m = \text{maximum value of } r \tag{9}$$

Therefore the amplitude of the transmitted light is dependent on the relative retardations of OA and OB only and independent of the angle ϕ. We have thus proved that changing the angular position of the specimen does not alter the transmitted light intensities.

Now θ_1, is the retardation of the *first* quarter wave plate plus that of the specimen, or $\theta_1 = \theta_2 - \frac{\pi}{2}$ where θ_2 = retardation due to specimen alone. Substituting $\theta_2 - \frac{\pi}{2}$ for θ_1, we have

$$\sin \theta_1 = \sin \left(\theta_2 - \frac{\pi}{2}\right) = -\cos \theta_2$$

or from (9)

$$r_m = \sqrt{1 - \cos \theta_2} \tag{10}$$

But $\theta_2 = 2\pi\lambda$ where λ = retardation of one component behind the other in wave lengths

$$r_m = \sqrt{1 - \cos 2\pi\lambda} = \sqrt{2} \sin \pi\lambda \tag{11}$$

We thus obtain the following result:

(1) The amplitude of the transmitted light is independent of ϕ or of the direction of the principal stress lines.
(2) It depends upon the relative retardation of OA with respect to OB according to the sine law.

Negative values of amplitude mean a reversal of phase of the plane polarized light.

The intensity of illumination is proportional to the square of the amplitude, or

$$I = I_m \sin^2 \pi\lambda$$

Where I_m = maximum value of the intensity.

For a graphical representation of the results see Fig. 8.

NOTICES

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Continued from Vol. V page 122, 1921.

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THE PROPAGATION OF LIGHT IN R SYSTEMS*

BY
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The purpose of the present paper† is to investi tions concerning light propagation in a uniforml system, such as the Earth, on both the aether relativity theory.

On both theories we shall have to understand a rotation of our rigid system[1] S with uniform a *relatively to the fixed stars*, or to any other inertial will be shortly referred to as the reference system a specification of rotation is still necessary even i theory, in spite of appearances to the contrary, wi in the sequel where we shall also have the opport out some outstanding difficulties of the relativis theory with respect to the concept of rotation.

With regard to the rotating system S itself, it well to have in mind our own Earth. The more interesting experiment in connection with our su purely terrestrial one.

1. To begin with the aether-theory treatmen (scalar) angular velocity of the Earth (S) relative

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† Paper read December 29, 1920, at the Chicago Meeting of Physical Societies.

[1] It is well known that among the possible motions of a relati (as defined by Born and Herglotz) there is uniform rotation, such from the kinematics of ordinary classical rigid bodies.

stars (S*) and let κ-1 be the rotatory dragging coefficient at and near the surface of the Earth, in other words, let $\kappa\bar{\omega}$ be the relative angular velocity of the aether and the Earth. If the unit vector **k** is taken along the positive axis of rotation, the vector velocity of the aether stream past a point P of S will be

$$\mathbf{u}=\kappa\bar{\omega}\mathrm{V}\mathbf{r}\mathbf{k},$$

r being the vector drawn from the center of the Earth (or from any point fixed on the axis) to the point P; in Cartesians, with $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$,

$$u_x=\kappa\bar{\omega}y,\quad u_y=-\kappa\bar{\omega}x,\ u_z=0,$$

and the resultant velocity, $u=\kappa\bar{\omega}r$, where r is distance from the axis. As to the factor κ, it is not our intention to prejudice its value, which may be any fraction from zero to unity, corresponding to a full drag and to no drag, respectively.

The propagation of light in S* being isotropic, of constant velocity c, the velocity of light in S, always in vacuo, along the wave-normal **n** (unit vector) will be

$$v=c+\mathbf{un}=c\left[1+\frac{\kappa\bar{\omega}r}{c}\cos(\mathbf{u},\mathbf{n})\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1a)$$

Notice that, in the case of the Earth, $\bar{\omega}=2\pi/86164$, the reciprocal length $\bar{\omega}/c$ amounts only to about 2.43×10^{-5}cm.$^{-1}$, so that even if r be of the order of the Earth's radius, the factor $\frac{\kappa\bar{\omega}r}{c}$ is a very small fraction. Such being the case it will be enough to retain in all our formulae $\kappa\bar{\omega}r/c$ itself, rejecting its square and higher powers. Now, rigorously speaking, formula (1) is valid for the *wave-normal* and not for the ray or the tangent to the "light path." But **n** appears in (1a) only in the term multiplied by $\kappa\bar{\omega}r/c$, and since **n** differs from the light ray only by small terms, we have up to higher order terms, simply

$$\frac{v}{c}=1+\frac{1}{c}(\mathbf{up})=1+\frac{\kappa\bar{\omega}r}{c}\cos\gamma,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where **p** is a unit vector along the optical ray, and γ the angle between **p** and **u**.

Such being the expression for the velocity of light along the ray, we can at once find the shape of the ray or light path in S by means of Fermat's law, which it will be enough to work out in detail for the case of a light ray contained in a plane parallel to the equatorial plane.[2] In fact let $d\sigma$ be a line-element of the ray; then Fermat's principle is $\delta\int\frac{d\sigma}{v}=0$, the limits of the integral being fixed. Now, introducing in the said plane polar co-ordinates r, θ, the latter measured positively in the sense of the rotation of S, we have

$$\cos\gamma=-r\,d\theta/d\sigma=r\,\theta'\sin\gamma,$$

so that, by (1),

$$\frac{v}{c}=1+\frac{r^2\tilde{\omega}\kappa\theta'}{c}\sin\gamma,$$

and Fermat's principle becomes, after easy reductions, and considering as corresponding points those having the same r,

$$\int\left\{\frac{\partial}{\partial\theta'}\left[\frac{c}{v\sin\gamma}\right]\right\}\delta\theta'.\,dr=0.$$

Thus, $\frac{d}{dr}\{\}=0$, and the required equation of the light path becomes

$$\frac{\partial}{\partial\theta'}\left[\frac{c}{v\sin\gamma}\right]=\text{const.}$$

Now $\frac{c}{v\sin\gamma}=-\frac{1+r^2\theta'^2}{\sqrt{1+r^2\theta'^2-\frac{\kappa\tilde{\omega}r^2}{c}\theta'}}$. Thus after simple reductions and putting $\gamma=\eta+\frac{\pi}{2}$,

$$\sin\eta+\frac{r\omega(1-2\sin^2\eta)}{(1-\omega r\sin\eta)^2}=\frac{A}{r},\ \text{where}\ \omega=\frac{\kappa\tilde{\omega}}{c},\ A=\text{const.}$$

Rejecting second order terms, the left hand member of this equation can be written $\sin\eta+\omega r$.

Ultimately, therefore, the equation of the light path in S becomes

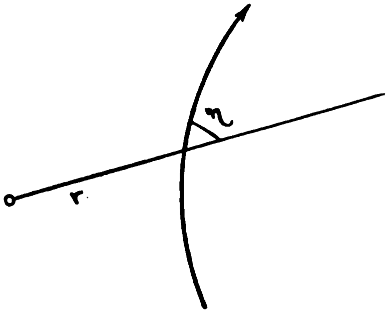
$$r\left(\sin\eta+\frac{r\kappa\tilde{\omega}}{c}\right)=A,\dots\dots\dots\dots\dots\dots(2)$$

---

[2] If the end-points (any two points) of a light path be in such a plane, the whole light path is contained in that plane.

where η is the angle under which the light path cuts the radius vector (Fig. 1).

Fig. 1



Notice that in absence of rotation (2) reduces, as it should, to $r \sin \eta = \text{const}$, this being the equation of a straight line. There is thus, due to rotation, a first-order deviation of the light rays from straight lines.

In order to introduce into the ray equation the polar coordinates r, θ instead of r, η, it is enough to remember that

$$\sin\eta = r\frac{d\theta}{dr} : \sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}.$$

Thus (2) becomes, with $\frac{1}{r} = \rho$, $B = 1+2A\omega$, $\omega = \kappa\tilde{\omega}/c$,

$$-d\theta = \frac{\omega r dr}{\sqrt{Br^2-A^2}} + \frac{A d\rho}{\sqrt{B-A^2\rho^2}},$$

whence, integrating, putting $A/\sqrt{B} = r_0$, and counting $\theta-\theta_0$ from the radius vector $r = r_0$ (in the sense of rotation of S),

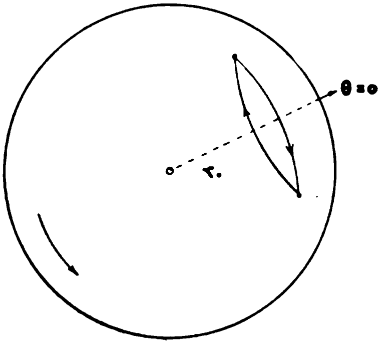
$$\cos\left[\theta-\theta_0 \pm \omega\sqrt{\frac{r^2-r_0^2}{B}}\right] = \frac{r_0}{r}.$$

Here $B = 1+2\omega A$ and $A = r_0 (1+r_0\omega)$, so that rejecting second order terms and with the same right also replacing $\sqrt{r^2-r_0^2}$ by $r_0 \tan\theta$, we have ultimately

$$\frac{r_0}{r} = \cos\left[\theta-\theta_0 \pm \frac{\kappa\tilde{\omega}}{c}\sqrt{r^2-r_0^2}\right] = \cos\left[\theta-\theta_0 \pm \frac{r_0\kappa\tilde{\omega}}{c}\tan(\theta-\theta_0)\right], \quad (3)$$

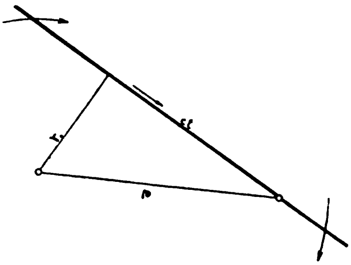
the upper or the lower sign to be taken for light travelling in the sense of rotation of S or in the opposite sense. Or, to put the sign rule in a more convenient form, the light path is convex (or bulges out) always to the left of a person[3] walking in the direction of propagation (Fig. 2).

Fig. 2



This equation of the light ray can also be obtained more directly by transforming the light rays of S^* to our system S by means of the substitution $r = r'$, $\theta' = \theta + \kappa\tilde{\omega}t$ (and $t' = t$). In fact, the equation of a straight line, and such is in S^* every optical ray, can be written

Fig. 3



$$r' \cos\theta' = r_0' = \text{const.},$$

and this becomes, through the said transformation,

$$\frac{r_0}{r} = \cos(\theta + \kappa\tilde{\omega}t).$$

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[3] For whom the rotation of S is clockwise.

In other words the light path is the path of a particle moving with uniform velocity *c* along a straight line which is itself spinning uniformly around O (Fig. 3). If $t=o$ for $r=r_o$ we have $ct=ct'=\sqrt{r^2-r_o^2}$, which gives again

$$\frac{r_0}{r}=\cos\left[\theta-\theta_0\pm\frac{\kappa\bar{\omega}}{c}\sqrt{r^2-r_0^2}\right],$$

as under (3).

Developing the cosine, up to the second order, we can write

$$\frac{r_0}{r}=\cos(\theta-\theta_0).\left[1\mp\frac{\kappa\bar{\omega}r_0}{c}\tan^2(\theta-\theta_0)\right]\ldots(3a)$$

The constants θ_0, r_o can be determined if either the initial direction or any two points, A, B of the light path, say the sending and the receiving stations, are given through two pairs of r, θ. It will be noted that the light path or ray BA (i.e., with B as sending and A as receiving station) does not coincide with AB. The two optical rays AB and BA enclose between them a certain area, having the shape of a biconvex lens. In other words light propagation in S is irreversible. Under appropriate circumstances A will see B without being seen by B.

The last ray-equation can still be simplified. Putting the x-axis along the radius vector $r_o(\theta=\theta_o)$, so that

$$x=r\cos(\theta-\theta_0),\; y=r\sin(\theta-\theta_0),$$

we have $x(1\mp\epsilon y^2/x^2)=r_o$, whence

$$x=r_0\left(1\pm\frac{\epsilon y^2}{x^2}\right),\qquad \epsilon=\frac{r_0\kappa\bar{\omega}}{c}\ldots\ldots\ldots\ldots\ldots(4)$$

In the small term we can write, with the same approximation $x=r_o$, so that the ray equation becomes

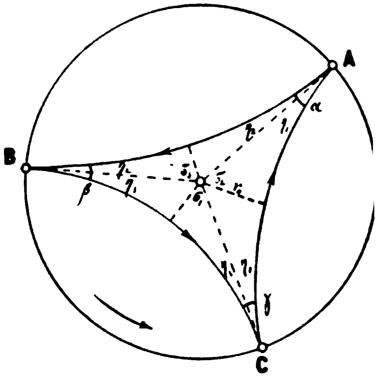
$$x=r_0\pm\frac{\kappa\bar{\omega}}{c}y^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4a)$$

Thus, up to the second order, the optical ray (which more rigorously is a complicated spiral) becomes simply a parabola, with apex in shortest distance (r_o) from O. If the propagation is in the sense of rotation of S, the parabolic ray turns its convex side towards the axis of rotation. In the opposite case it will turn its concave side towards O. Various problems of what may be called optical trigonometry of the rotating system, i.e., problems concerning triangles or polygons built up of optical rays, can

now be easily dealt with, namely by cons
parabolic arcs as light paths and keeping
sense of propagation.

By way of illustration consider an optical
say, in the sense of rotation, and having its corr
circle r = a. Given the angles $2\sigma_1 = BOC$, $2\sigma_2 =$
$(\sigma_1+\sigma_2+\sigma_3=\pi)$, find the sum of the angles α,
triangle ABCA, the order of the letters giving t

Fig. 4



gation. Denote by η_1 the angle between the op
the radius vector at A, and let η_2, η_3 have analog
B and C, as in Fig. 4. Then, since the angle (
η_1, and similarly for the remaining angles, we sh

$$\alpha=\eta_1+\eta_2,\ \beta=\eta_2+\eta_3,\ \gamma=\eta_3+\eta_1, \text{ and}$$
$$\alpha+\beta+\gamma=2(\eta_1+\eta_2+\eta_3)\ldots\ldots\ldots\ldots(5)$$

Now, by (2) and remembering that $A = r_0(1+r_0$

$$r(\sin\eta+r\omega)=r_0(1+r_0\omega).$$

Apply this to the ray CA at the corner A. Then

$$\sin\eta_1=\frac{r_0}{a}(1+\omega r_0)-a\omega\ldots\ldots\ldots\ldots\ldots\ldots($$

and it remains only to find r_0. Now, the radius
the angle COA $=\sigma_2$; thus, applying (3a), with θ_A -

$$\frac{r_0}{a}=\cos\sigma_2.\left(1-a\omega\frac{\sin^2\sigma_2}{\cos\sigma_2}\right).$$

Substitute this in (6) and reject second order
result will be

$$\sin\eta_1=\cos\sigma_2-2\,a\omega\sin^2\sigma_2.$$

Since $\eta_1+\sigma_2$ differs but little from a right angle, put here $\eta_1=\frac{\pi}{2}-(\sigma_2+\delta_1)$; then the last equation will give for the small angle

$$\delta_1=2\,a\omega\,\sin\sigma_2.$$

Similarly the defects of $\eta_2+\sigma_3$ and $\eta_3+\sigma_1$ will be $\delta_2=2\,a\omega\,\sin\,\sigma_3$, $\delta_3=2a\omega\,\sin\sigma_1$. Thus by (5), and since $\sigma_1+\sigma_2+\sigma_3=\pi$,

$$\alpha+\beta+\gamma=\pi-2(\delta_1+\delta_2+\delta_3),$$

i.e., the *defect* of the optical triangle ABCA will be

$$4a\frac{\kappa\bar{\omega}}{c}\,(\sin\sigma_1+\sin\sigma_2+\sin\sigma_3)\ldots\ldots(8)$$

For the optical triangle (or triangular circuit) ACBA we shall have an equal *excess* of the angle sum.

Thus, for instance, for an equilateral triangle, $\sigma_1=\sigma_2=\sigma_3=60°$, the defect, or the excess, will be $4a\frac{\kappa\bar{\omega}}{c}.\ 3\frac{\sqrt{3}}{2}=6\sqrt{3}a\frac{\kappa\bar{\omega}}{c}$. For the Earth (and a triangle parallel to the equatorial plane) even if $\kappa=1$ (no drag) this would amount to $0.''00052$ per kilometer of a, and the difference between the angle sum of ACBA and ABCA would be the double of this. Thus, even for $a=10$ or 20 km. the difference would certainly be too small to be measured directly.

The experimental possibilities with regard to the optical effects of the rotation of the Earth lie in another direction, to wit in the phase retardation in an optical circuit (i.e., closed light path) as in the well-known laboratory experiment of Sagnac[4] with a small spinning interferometer as our system S. The corresponding formula used by Sagnac and before him by Michelson (Phil. Mag. vol. 2, 1904, p. 716–719) who actually proposed but never carried out a terrestrial experiment[5] of the kind here aimed at, can be most simply deduced in the following way. Consider any optical circuit s, traversed by light in the sense of rotation or "positive" circuit, say.[6] By formula (1), the time taken to traverse an element ds of the circuit is, with u_s written for **up**,

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[4] G. Sagnac, "L'ether lumineux démontré par l'effet du vent relatif d'ether dans un intérferomètre en rotation uniforme." C. R. Paris, 157 (1913), p. 708. Ibidem, p. 1410, "Sur la preuve de la réalité de l'ether lumineux par l'experience de l'interférographe tournant." The titles seemed interesting enough to be quoted in full. But Sagnac's experiments (even apart from the question of the reliability of his measurements) by no means decide for the aether as against relativity.

[5] Such a terrestrial experiment was already hinted at by Oliver Lodge, in 1897, Phil. Trans. Roy. Soc. A, vol. 189, p. 151, where also the experiment carried out by Sagnac twenty-five years later is suggested, only with "telescope and observer" instead of a photo camera mounted on a rotating "turn-table."

$$ds.(c+u_s)^{-1}=\frac{ds}{c}\left(1-\frac{u_s}{c}\right),$$

giving a retardation $-\frac{1}{c^2}u_s\,ds$ per line-element, and thus for the whole circuit the phase retardation

$$\Delta\tau=-\frac{1}{c^2}\int uds,$$

as already noticed by Lorentz (Wiss. Abhandlungen, vol. I). Now, by Stokes' theorem, this can be written, if σ be any surface laid through s, and **n** its normal,

$$\Delta\tau=-\frac{1}{c^2}\int \mathbf{n}\ \text{curl}\ \mathbf{u}.\,d\sigma.$$

This holds for any circuit, plane or not, and for any distribution of velocity. Since $-$curl **u** is the double angular velocity, $2\kappa\bar{\omega}.\mathbf{k}$, we have, writing $\bar{\omega}_n=\bar{\omega}\mathbf{kn}$ for the normal component of the spin,

$$\Delta\tau=\frac{2}{c^2}\int \kappa\bar{\omega}_n d\sigma \dots\dots\dots\dots (9)$$

This, the required formula, is valid for any, not necessarily constant value of $\kappa\bar{\omega}$ throughout the surface of integration. In our case, $\bar{\omega}$, and therefore $\bar{\omega}_n$ for a plane circuit, are manifestly constant, but the drag of the aether, if any, may vary from point to point, thus giving rise to a variable coefficient κ.

If the optical circuit is plane and small compared with the dimensions of the Earth, we have simply $\Delta\tau=2\kappa\bar{\omega}_n\sigma/c^2$, or if T be the period of oscillation and $\lambda=cT$,

$$\frac{\Delta\tau}{T}=\frac{2\kappa\bar{\omega}_n}{c\lambda}\sigma\dots\dots\dots\dots(10)$$

where σ is the total area embraced by the circuit. This gives the retardation, in parts of the period, for a positive circuit. As was already mentioned, the same path cannot, rigorously, serve for light propagation in the opposite sense [or, which is the same thing, with inverted normal **n** in (10)]. Thus, for instance, in the case of a triangle (Fig. 5) we have to take for the positive circuit the area σ_1 of the inner, and for the negative circuit the area σ_2 of the outer, convex triangle, so that rigorously we would have for the phase difference of the two beams (say, separated at A by a semi-transparent plate and reflected at B, C) in parts of the period, or for the corresponding shift in fringe widths,

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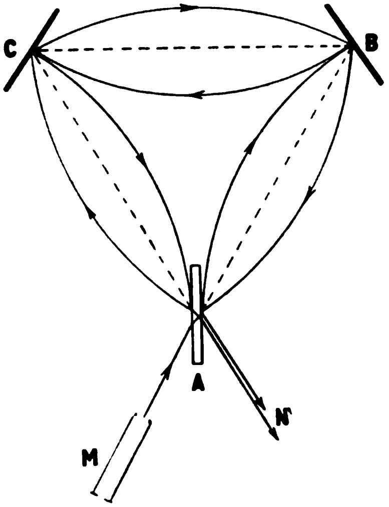
[6] As we already know, the same light path cannot be described in the opposite sense.

Fig. 5



$$\epsilon = \frac{2\kappa\tilde{\omega}}{c\lambda}(\sigma_1+\sigma_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

But the difference of σ_1 and σ_2 is itself small of the first order. Moreover, up to higher terms, the paths AB and BA, etc., are symmetrical with respect to the corresponding (dotted) straights, so that even up to terms of an order higher than the second we can replace $\sigma_1+\sigma_2$ by trice the area (σ) of the rectilinear (dotted) triangle, and similarly in the case of any polygons. Thus:[7]

$$\epsilon = \frac{4\kappa\tilde{\omega}_{\mathbf{n}}\sigma}{c\lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

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[7] Prof. Michelson's paper of 1904, (l.c.) has by a manifest slip the factor 2 instead of our 4.

For horizontally placed circuits we have $\bar{\omega}_n = \bar{\omega} \sin \varphi$, if φ be the geographic latitude at which the experiment is performed. For a latitude $\varphi = 45^\circ$ and the wave-length $\lambda = 5000$ Å. U., formula (12) gives

$$\epsilon = 1.38\kappa \frac{\sigma}{\text{km}^2} \quad \text{(12a)}$$

i.e., about 1.4κ fringe widths per each square kilometer embraced by the circuit. If the drag is complete ($\kappa = 0$) there should be no shift; if there is no drag, we should have the total amount of 1.4 per km², and intermediate values if the Earth in its spinning motion drags the aether, even at its surface, only partially.

In Sagnac's experiment the spinning motion of the (disc-shaped) table bearing the interferometer, the light source, as well as the photographic camera, was reversed, and thus the double of (12) was observed[8] as the shift of the system of interference fringes. No such reversal, of course, is possible in the case of the Earth as the rotating system. But as Prof. Michelson has already pointed out in 1904 (loc. cit.) there is an easy way out of this difficulty, to wit by silvering heavily one, e.g., the upper, half of the dividing glass plate (such as A in the case of Fig. 5 corresponding to three stations; B, C, being mirrors) and leaving the lower part clear or but lightly silvered. A beam of parallel rays from the collimator *M* impinging upon the plate A is here divided into ACBA and ABCA by reflection and transmission respectively. Now, according to Michelson's suggestion, cover the lower half and observe first by reflection from the upper half only (ACBA) when simply the image of the slit is seen. Place the cross-hair of the eyepiece in the center of this image. Next, covering the upper half and leaving the lower half of the plate clear, observe the interference fringes. Then, if there is an effect ϵ, the midpoint of the central fringe will be displaced from the crosshair by ϵ fringe widths. The effect sought for will be easily discernible from undesired accessorial shifts by being proportional to the area σ. Instead of comparing the position of the interference fringes with that of the slit image Prof. Michelson contemplates also the comparison of the two fringe systems given by two considerably differing values of the area σ.

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[8] In Sagnac's case $\kappa = 1$, since such small masses as was his table certainly do not drag the aether, as follows from the widely known older experiments of Sir Oliver Lodge.

It is estimated that the terrestrial experiment could be carried out with the required precision on comparatively small areas, such as 1/10 of a square kilometer. The beams can, of course, be sent around twice or more times making ϵ as many times larger; as many times, however, would the light path be increased which may not be convenient. A more radical way is to leave the circuits simple but to extend the linear dimensions of the circuits; for then the value of ϵ will be increased in the squared ratio. Again the more stations (arranged in a regular polygon) the greater σ for the same light paths. Yet, to avoid too many reflections and other inconvenients, the triangular arrangement as suggested in Fig. 5 or a quadrangular one may turn out to be preferable. But details of a technical kind need not detain us in the present paper.

2. Let us now try to find out what aspect the same problem assumes from the standpoint of the theory of relativity, the special and the generalized one. It goes without saying that with neither of these theories can there be any question of an aether and its being dragged by the Earth in its daily rotation around its axis, or in its annual motion around the sun.

First of all, then, the special relativity theory is wholly incompetent to deal with the problem of light propagation in a rotating system rigorously, simply because it has nothing to do with any reference systems other than the galilean or inertial ones, i.e. the fixed-stars system S^* and those moving relatively to it with uniform translational velocity. The only thing the special theory can do is to treat our problem approximately up to the second order, i.e., rejecting the square (and higher powers) of the ratio β of the velocity of motion[9] to the light velocity. Now up to β^2 the relativistic addition theorem of velocities does not differ at all from that of ordinary, Newtonian kinematics. This amounts simply to a neglect of the Lorentz-Fitgerald contraction. Under these circumstances the propagation of light in S becomes simply a superposition of that in S* and of the reversed spinning motion of S relatively to S* in much the same way as on the aether theory. The only difference is that our previous $\kappa\bar{\omega}$ has now to be replaced by *the full angular velocity* $\bar{\omega}$ of S relatively to S* (fixed stars). Thus formula (1) for the light velocity v will be replaced by

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[9] Of any point of S relatively to S^* say.

$$\frac{v}{c}=1+\frac{1}{c}u_s=1+\frac{\bar{\omega}r}{c}\cos\gamma,$$

and the shift formula (12) by

$$\epsilon=\frac{4\bar{\omega}_n\sigma}{c\lambda}\qquad\text{(12r)}$$

In fact v. Laue, the chief exponent of Einstein's older theory in Germany, gives in his well-known book, in a section on Sagnac's experiment, a formula identical with (12r) at which he arrives by a rather roundabout way, instead of using the simple relation (9), valid for a circuit of any shape.[10]

Similarly all other formulae given above will continue to hold on the special relativity theory with unity written for κ.

From the standpoint of the general relativity theory all questions concerning light propagation (in vacuo) relatively to any system whatever and therefore also to our terrestrial system S will be answered if the four-dimensional line-element $ds^2=\Sigma g_{\iota\kappa}\,dx_\iota dx_\kappa$ belonging to that system be known, to wit, by putting

$$ds=0\qquad\text{(13)}$$

Thus the problem is reduced to building up ds in terms of terrestrial or S-co-ordinates, say x_1, x_2, x_3, $x_4=r$, θ, z, ct, respectively, keeping in mind that in the fixed-star system S* (disregarding in our present connection the extremely minute terms due to the Earth's gravitation) the line-element is

$$ds^2=c^2dt'^2-dr'^2-r'^2d\theta'^2-dz'^2\qquad\text{(14*)}$$

To be faithful to its own leading principles, the relativity theory ought to deduce the terrestrial ds or its coefficients $g_{\iota\kappa}$ as a gravitational effect of the stars and all the remaining matter of the universe rotating around the Earth. This, however, it proved unable to do. As a matter of fact Einstein himself never entered into the details of this important problem of rotation. Thirring[11] tried to solve it by considering a huge massive spherical shell in uniform rotation and evaluating by means of Einstein's approximate integrals of his field-equations the $g_{\iota\kappa}$ thus produced at comparatively small distances from the centre of that gigantic sphere. But the result, though mathematically interesting, was a complete failure,[12] although the treatment of the same problem on Einstein's newest cosmological views (elliptic space and so on) seems more promising. At any rate, the relativity theory is unable to construct the required line-element on its own great principles, and is content to transcribe it from the galilean line-element (14*) by putting simply $r', z', t' = r, z, t$, and

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[10] M. v. Laue, Relativitätstheorie, vol. I, 3rd ed. Braunschweig, 1919, p. 125-127. Laue seems, by the way, to be under the misapprehension that the light rays relative to the rotating table are straight lines, which they are not. As we saw before, their departure from straight lines is a first order effect and does, therefore, by no means disappear though "the Lorentz contraction" be neglected. The influence of the curvature of the rays on (12) or (12r) remains, of course, negligible.

$$\theta' = \theta + at,$$

where a is an appropriate constant. In fact, de Sitter, one of the chief exponents of Einstein's theory, and even Weyl in his interesting book[13] write down without much discussion this simple transformation in order to pass from one to the other system. This gives for the latter,

$$ds^2 = \left(1 - \frac{a^2 r^2}{c^2}\right) c^2 dt^2 - 2r^2 \frac{a}{c} d\theta\, c dt - dr^2 - r^2 d\theta^2 \qquad \text{(14)}$$

The constant a is to be chosen so as to describe correctly the terrestrial laws of physical phenomena, approximately, at least, in our present connection up to the second order only.

Now, if but such an approximation is required, the fundamental principles of general relativity offer us a good test of whether a chosen value of the constant a is or is not the appropriate one. In fact, according to one of these principles $ds=0$ represents light propagation, and according to the other the geodesics of the world, determined by the same line-element, i.e.,

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[11] H. Thirring, Ueber die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie. Phys. Zeitschrift, Vol. 19, 1918, p. 33–39.

[12] Not only that Thirring's "centrifugal force" had also a component along the axis of rotation, but the coefficients of the centrifugal and the Coriolis force, apart from being very unsatisfactory in their structure, bore a wrong numerical ratio to one another.

[13] H. Weyl, Zeit-Raum-Materie, 3rd ed., Berlin, J. Springer, 1920.

$$\delta\int ds=0 \ldots\ldots\ldots\ldots (15)$$

give the equations of motion of a free particle. And since the terrestrial laws of motion are certainly known to that degree of approximation, we can derive from (15) the correct value to be attributed to the constant a.

Now, using (14) in (15) the terrestrial equations of motion of a free particle (always apart from gravitation proper) follow at once. Of these equations it will be enough to write down only that which most interests us here, i.e., the equation corresponding to the variation δr. This is

$$\frac{d^2r}{ds^2}=a^2r\left(\frac{dt}{ds}\right)^2+2ar\frac{d\theta}{ds}\frac{dt}{ds}+r\left(\frac{dt}{ds}\right)^2,$$

and since, up to the second order, $ds^2=c^2\,dt^2$, we can write

$$\frac{d^2r}{dt^2}=r\left(a+\frac{d\theta}{dt}\right)^2.$$

The right-hand member will represent the correct value of the centrifugal acceleration (or "centrifugal force" per unit mass), in size and direction, provided that a stands for the *full* angular velocity of the Earth relatively to the fixed stars, i.e.

$$a=\tilde{\omega}.$$

This then is the value to be substituted into (14) to suit our dynamical experience, and at the same time, as already explained, into the light equation $ds=0$. In short, on the relativity theory the same ds and therefore the same $\tilde{\omega}$ is required for light as for mechanics, while on the aether theory we may have any fraction $\kappa\tilde{\omega}$ of $\tilde{\omega}$ for light, though the full angular velocity $\tilde{\omega}$ is required for mechanics.

Thus, on the relativity theory the terrestrial velocity of light for any direction of the ray will be determined by

$$\left(1-\frac{\tilde{\omega}r^2}{c^2}\right)c^2dt^2-2\frac{\tilde{\omega}}{c}r^2d\theta\,cdt-dr^2-r^2d\theta^2=0 \ldots\ldots\ldots\ldots (16)$$

whence also the form of the optical rays and the circuitous shift formula can be deduced.[14] Or, equivalently and much more simply, we may deduce all these terrestrial optical laws from those of S* (uniform propagation with velocity c) by means of the transformation

$$\theta'=\theta+at=\theta+\bar{\omega}t.$$

Consequently, all our previous formulae for the light velocity, for the shape of rays and the phase retardation in a terrestrial optical circuit, will reappear, with the only difference that the unknown fraction κ will be replaced by the special and definite value 1, i.e., our $\kappa\bar{\omega}$ will be replaced by $\bar{\omega}$. Thus also the shift formula, which from the experimental point of view is the only important thing, will again be

$$\epsilon=\frac{4\bar{\omega}n\sigma}{c\lambda},$$

as on the special relativity theory.

The planned terrestrial experiments with the optical circuit might thus enable us to decide between relativity and aether theory. That is to say, if the result of such experiments will be a full effect (a shift of 1.38 fringe widths under the stated conditions, per square kilometer), there will be no discrimination between the two theories. But should there be either no shift at all or only a fraction κ of the full effect, sensibly different from unity, the relativity theory, special or general, will be irremediably disproved, while on the aether theory we would have only to assume a complete or partial rotational dragging of the aether.

In fine, the optical circuit experiment may easily become crucial and fatal for Einstein's theory especially if it gives a nil-effect. Should it, on the other hand, give a full effect, it will certainly cease to be decisive either way but will even then (as every new positive experiment) be certainly a valuable contribution to our experimental knowledge of the Earth as a physical reference system.

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[14] The only effect of terrestrial gravitation on the optical-circuit experiment would according to relativistic gravitation theory, be represented by the term $\frac{8}{5}\cos^2\varphi\frac{M}{c^2R}$ to be subtracted from the factor 2 of the second term in (16), φ being the geographic latitude, M the mass and R the radius of the Earth. But since $M/c^2=0.45$ cm., the correction term due to gravitation amounts, even at the equator, only to 1.2×10^{-9} which is entirely negligible in presence of 2.

Speculations on a possible rotational drag of the aether by the Earth were made, in connection with the diurnal aberration, by Christian Doppler (1845) and after him by v. Oppolzer.[15] But owing to an almost complete absence of direct observational data on daily aberration, a state of things prevailing even at the present time, Doppler's formula, generalized by Oppolzer, could neither be proved nor disproved. More recently the question of a rotational drag was taken up by Lorentz[16] but it had again to be left unsettled in absence of daily aberration data. If there is anything on the experimental side to make the rotational drag unlikely it is the absence of shift of a star near occultation by Jupiter, as mentioned by Lorentz (loc. cit. p. 413). On the theoretical side the question of the influence of a *spinning* planet on the adjacent aether could not be answered definitely without some special and more or less artificial hypotheses about the properties of that medium in addition to those given to it already by Stokes-Planck. In this respect we can say only that even if there is an almost complete translational (annual) drag of the aether due to its condensation around the planet, there may yet be no appreciable rotational (daily) drag. Again, from Lodge's experiments (loc. cit) one can judge only that there is no such drag by comparatively small spinning masses, such as can be used in a laboratory, but not by such massive bodies as the Earth or other planets. In fine, the only sound way of settling the question would be to carry out the terrestrial experiment with optical circuits embracing as large an area as is technically possible.

ROCHESTER, N. Y.
March, 24, 1921.

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[15] E. v. Oppolzer, Erdbewegung and Aether, Sitzber. Akad. Wien, vol. CXI, IIa, Febr. 1902, pp. 244–254.

[16] H. A. Lorentz, Abhandlungen, Vol. I.

STANDARD WAVE-LENGTHS*

BY

W. F. MEGGERS

I. INTRODUCTION

A train of waves emitted by a source of homogeneous light represents a scale of invariable quality, whose use in making physical length measurements of extraordinary precision is now more or less familiar to everyone. For practical use, it is necessary to choose very narrow spectral lines and to define the lengths represented by their waves and the conditions producing them. Several attempts have been made to define such a system of standard wave-lengths, each successive attempt aiming at higher precision by making use of more satisfactory light sources and more refined methods of measurement. At the present time, it appears likely that the question will be reopened and changes in the pre-war international code of standard wave-lengths have already been suggested. The importance of the fundamental standards to spectroscopy, metrology and precision optics generally, makes it imperative that this subject be brought to the attention of the Optical Society of America. A complete understanding of the present tendency in this field is not possible, however, without some knowledge of the history of standard wave-lengths; this report therefore presents a summary of the past activities in this field as an introduction to the present status and tendency of standard wave-lengths.

II. HISTORICAL REVIEW

1. ROWLAND SYSTEM OF STANDARDS

The first system of standard wave-lengths of importance was the Preliminary Table of Solar Spectrum Wave-Lengths given by Rowland in 1888. For twenty years, practically all spectroscopic measurements were based on these values and even at the present time they are widely used by astrophysicists. The system was based upon an absolute value of the wave-length of D_1 of sodium (5896.156) adopted by Rowland[1] as the weighted mean of various determinations, and consisted of a small number of lines compared with this one by the method of coincidences in diffraction grating spectra, while the remainder were measured by interpolation. Rowland thought that the absolute values were correct to 1 part in 100,000 and that the errors in relative value were not greater than a millionth.

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* 1920 Report of Committee on standard wave lengths.

In 1893, the absolute measurements of Michelson & Benoit on cadmium lines in terms of the standard meter at Paris showed that Rowland's wave-lengths were too large by 1/30,000 but spectroscopists paid little attention to this since there was still no reason to doubt the correctness in relative value of Rowland's numbers. In 1901, Fabry and Perot[2] measured about 30 lines in the solar spectrum between 4643 A and 6471 A by means of their interferometer. This work revealed the fact that systematic errors existed in Rowland's table so that the relative values of wave lengths in the interval which was studied contained errors of nearly 1 part in 100,000. These errors were attributed to the erratic behavior of diffraction gratings and Kayser[3] made some experiments in 1904 with Rowland gratings which led him to the conclusion that the coincidence between orders of spectra with gratings is not to be depended upon. Miss Howell[4] has since shown that the fundamental law holds for good gratings so that the errors in Rowland's values as well as the experience of Kayser may require some other explanation. Jewell[5] has stated that the relative errors in Rowland's Table are due to disturbed apparatus and failure to make any corrections for Doppler effect or for air-pressures and temperatures, while Goos[6] has shown that most of the discrepancies observed by Kayser are explained by slight defects in the adjustment of the grating.

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[1] Phil. Mag. (5), 23, p. 257; 1887, and 27, p. 479; 1889.

[2] Astroph. J. 15, p. 73 and 261; 1902.

[3] Astroph. J., 19, p. 157; 1904.

[4] Astroph. J., 39, p. 230; 1914.

[5] Trans. I. U. S. R., 1, p. 42; 1906.

[6] Zeit. f. Wiss. Phot., 9, p. 173; 1911.

2. INTERNATIONAL SYSTEM OF STANDARDS

The question of standard wave-lengths was discussed at the first meeting of the International Union for Co-operation in Solar Research[7] and the necessity for establishing a new system of standard wave-lengths taken from artificial sources was agreed upon.

At the second meeting[8] of the Union in 1905, the following decisions were reached. (1) The wave-length of a properly chosen radiation shall be taken as a primary standard of wave-lengths. The number which represents this wave-length shall be fixed one time for all; it shall define the unit of wave-length which must differ as little as possible from 10^{-10} meter and shall be called angstrom. (2) There is reason to choose secondary standards separated by not more than 50 angstroms. These secondary standards shall be referred to the primary standard by a method of interference. The luminous source shall be an electric arc of 6 to 10 amperes.

a. *The Primary Standard*

In 1907, a report of the measurement by Benoit, Fabry and Perot[9] of the wave-length of the red radiation from cadmium relative to the standard meter was presented at the third meeting of the International Union and the following resolution was adopted: "The wave-length of the red ray of light from cadmium produced by a tube with electrodes is 6438.4696 Angstroms in dry air at 15° on the hydrogen thermometer, at a pressure of 760 mm of mercury, the value of g being 980.67 (45°). This number will be the definition of the unit of wave-length." The wave-length of the red radiation of cadmium thus defined in terms of the standard meter was claimed to be correct within 1 ten millionth which is also the limit of accuracy attainable in the intercomparison of meter bars. The value differs by less than 1 part in 16 million from the corrected value obtained by Michelson 14 years earlier.

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[7] Astroph. J., 20, p. 313; 1904.

[8] Trans., I.U.S.R., I, p. 234; 1906.

[9] Trans., I.U.S.R., II, p. 109; 1907.

b. *The Secondary Standards*

In 1907, the first determinations of secondary standards were reported (reference 9, p. 138) by Fabry and Buisson who measured the wave lengths of 115 lines in the iron arc 2373–6494 by means of their interferometer method. Similar measurements were made by Eversheim[10] and by Pfund,[11] and the mean values of three independent determinations were adopted as secondary

TABLE I

International Secondary Standards

3370.789	4375.934	5405.780
3399.337	4427.314	5434.527
3445.154	4466.556	5455.614
3485.345	4494.572	5497.522
3513.821	4531.155	5506.784
3556.881	4547.853	5569.633
3606.682	4592.658	5586.772
3640.392	4602.947	5615.661
3676.313	4647.439	5658.836
3677.629	4691.417	5709.396
3724.380	4707.288	5763.013
3753.615	4736.786	5857.759 Ni
3805.346	4789.657	5892.882 Ni
3843.261	4859.758	6027.059
3850.820	4878.225	6065.492
3865.527	4903.325	6137.701
3906.482	4919.007	6191.568
3907.937	4966.104	6230.734
3935.818	5001.881	6265.145
3977.746	5012.073	6318.028
4021.872	5049.827	6335.341
4076.642	5083.344	6393.612
4118.552	5110.415	6430.859
4134.685	5167.492	6494.993
4147.676	5192.363	6546.250
4191.443	5232.957	6592.928
4233.615	5266.569	6678.004
4282.408	5302.315	6750.163
4315.089	5324.196	
4352.741	5371.495	

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[10] Ann. d. Phys., (4), 36, p. 1071; 1911.

[11] Astroph. J., 28, p. 197; 1908.

standards by the International Union in 1910, comprising 54 lines between 4282 A and 6484 A. Additional values of international secondary standards were adopted from time to time,[12] and the complete list is given in Table 1.

The agreement between interference observations of these wave lengths is remarkably good and there is reason to believe that most of the accepted secondary standards have a precision of 0.001 A, i.e., they are correct and reproducible to 1 part in several millions. This is the extreme limit which it is possible to obtain since the most homogeneous lines of iron have a width of about 0.060 A and a precision of 0.001 A therefore corresponds to 1/60 of the width of the line. Some of the differences between values of different observers can probably be accounted for by different arc conditions employed in different laboratories and special investigations on the effect of arc conditions showed that the wave-lengths of certain iron lines which are sensitive to pressure are also affected by current strength, and by the length and portion of arc used.[13] Recognition of these influences led, in 1913, to the adoption of more precise specifications of the iron arc in air[14] as a source of international standards as follows: (1) Length of arc 6 mm; (2) Current of 6 amperes for wave-lengths greater than 4000 A, for wave-lengths less than 4000A, 4 amperes or less; (3) Use direct current with positive pole above the negative and a potential of 220 volts; iron rods of 7 mm diameter; (4) As a source of light use an axial part of about 2 mm in the middle of the arc; (5) Use only iron lines of groups a, b, c, d (Mt. Wilson Classification).

In the past ten years, very little progress has been made in extending this system of secondary standards. The most extensive measurements were made by Burns, who compared directly with the cadmium primary standard, the wave-lengths of 125 iron lines[15] from 5434 A to 8824 A and 100 lines[16] from 2851 A to 3701 A.

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[12] Astroph. J., 32, p. 215, 1910. Ibid. 33, p. 85, 1911; Ibid. 39, p. 93; 1914.

[13] St. John and Ware, Astroph, J., 36, p. 14; 1912 and 39, p. 5; 1914. Goos, Astroph. J., 38, p. 141; 1913.

[14] Trans., I.U.S.R., 4, p. 58; 1914.

[15] Jour. de Phys. (5), 3, p. 457; 1913.

[16] B. S. Bulletin 12, p. 179; 1915.

A few red lines were measured by Eversheim[17] but his values and those of Burns diverge rather widely for the longer waves.

c. *Tertiary Standards, Pole Effect*

In addition to the above determinations of secondary standards, many additional measurements of wave-lengths in the iron spectrum have been made, some with interferometers and others with diffraction gratings, partially to set up a system of tertiary standards and also to investigate further the effect of operating conditions of the arc upon its wave-lengths. Among such investigations the most important in some respects appears to be that of the so-called "pole effect" on arc lines. This has been studied in some detail at the Mt. Wilson Observatory[18] where it is found that certain iron lines which are sensitive to pressure have slightly different wave-lengths in the center of the arc than near the negative pole (one or two millionths), and also that this effect is somewhat reduced when, instead of using two iron electrodes, a carbon electrode is used with an iron one.[19]

III. RECENT DEVELOPMENTS

1. NEW INTERNATIONAL UNION

The world war destroyed the old international co-operation but a new International Astronomical Union, representing several of the allied countries, was organized in Brussels in July 1919. The question of Standard Wave-Lengths was revived by a committee appointed by the American Section of the then proposed International Astronomical Union, meeting at Washington March 8, 1919; from whose report[20] of memoranda for the American delegates to the first meeting of the new Union the following paragraphs are quoted:

> *The Primary Standard.* The remarkable sharpness of the red cadmium line, the extraordinary skill with which the meter was evaluated in terms of this wave-length, and the final definition of the angstrom, as an arbitrary unit, by the International Solar Union at Paris in 1907 would seem to place the primary standard almost beyond the range of question.

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[17] Ann. der Phys., 45, p. 454; 1914.

[18] St. John and Babcock, Astroph. J., 42, p. 231; 1915.

[19] St. John and Babcock, Astroph, J., 46, p. 138; 1917.

[20] Proc. Nat. Acad. Sci., 6, p. 367; 1920

However, the recent perfection of the new method for making end-standards of length, the comparative ease with which the optical length of these end-standards can be determined, and the possible superiority of some other line (such as the bright yellow neon line) over one from solid cadmium as a source of light combine to urge a reconsideration even of the primary standard.

Sources. In order to obtain lines of constant wave-length and constant intensity distribution, it is recommended that the modification of the Pfund arc employed at Mt. Wilson be adopted, namely, the Pfund arc operated under 120–240 volts, with 5 amperes or less, at a length of 12 millimeters, and used over a central zone, at right angles to the axis of the arc, not to exceed 1¼ mm in width.

These sections are the only ones which suggest fundamental changes in the erstwhile International System of Standard Wave Lengths, and their validity can be determined only by careful consideration and decision that the expected improvements will justify both the loss of enormous labor which produced the present system and the additional task of building up a new one.

2. THE PRIMARY STANDARD

Exact reproducibility and high homogeneity are the most important characteristics of a primary standard wave length for purposes of precision measurement. There is little or no reason to doubt the reproducibility of the wave-length of the cadmium red line. As nearly as can be determined the same wave-length has been observed from tubes of various designs, with or without electrodes and operated with different currents,[21] pressures and purity of metal.

The total width of a spectral line, assuming that it is accounted for by the Doppler-Fizeau effect, has been shown to be

$$\Delta = 0.82\times 10^{-6}\times\lambda\sqrt{\frac{T}{M}}$$

where λ represents the wave-length, T the absolute temperature and M the molecular weight of the radiating gas or vapor. The reciprocal of the relative value $\frac{\Delta}{\lambda}$ defines the limiting order of interference N which the width of the line permits, and may be called the sharpness or fineness of the line.

$$N = 1.22\times 10^{6}\sqrt{\frac{M}{T}}$$

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[21] Trans., I.U.S.R., 2, p. 24; 1907.

This has been verified experimentally by Buisson and Fabry.[22] Michelson[23] actually determined the widths of many spectral lines when searching for a homogeneous radiation with which to compare the meter, and the red line in the spectrum of cadmium was the narrowest line then known. Only in recent years have sharper spectral lines been found. This is in consequence of the discovery and study of the so-called rare or inert elements which exist as gases not only at ordinary and high temperatures but even at very low temperatures. These gases have a range in molecular weights from 4 to 222 and can be made to emit rather complicated spectra of great intensity.

If a line from one of the rare gas spectra replaces the cadmium line as a fundamental standard, it should be chosen from one of the heavy gases. At the temperatures usually employed in their production, the neon lines are not as sharp (N = 320,000) as the cadmium line (N = 500,000). The green line 5570 A in the spectrum of krypton, however, has a width of only 0.006 A and its limiting order of interference has been observed as 600,000 at ordinary temperatures and almost a million near the temperature of liquid air. Theoretically, the lines in the spectrum of niton are nearly twice as sharp as those of krypton. Perhaps the chief obstacle to the present use of such lines is that the heavy gases are extremely rare and Geissler tubes of them are practically unobtainable. The recent disclosure of relatively enormous quantities of helium gas in natural gas wells suggests the possibility of also finding larger quantities of the heavier gases. At the present time it is perhaps doubtful if the possible increase in precision is so necessary as to warrant the adoption of a light source which is almost inaccessible and which would require repeating the difficult experiment of comparing a new wave-length with the meter.

3. SECONDARY STANDARDS

Since the spectral lines from Geissler tubes of the inert gases are exceptionally sharp, reproducible and convenient to use, their wave-lengths can be determined very accurately and may serve excellently as standards. For several years the wave-lengths of helium and neon lines have been in continuous use at the Bureau of Standards for the accurate length measurements of precision end standards and munition gages and for a study of the doubtful permanency of such material standards.[24]

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[22] J. de Phys., April 19, 1912.

[23] Phil. Mag., (5), 31, p. 338; 1891.

The wave-lengths of many lines in the spectra of helium, neon, argon, krypton and xenon have already been compared with the primary standard (6438.4696A) by interferometer methods and it would appear advisable to adopt such lines as additional international standards in all cases where three observers are in good agreement. A summary of these data will now be given.

TABLE 2

Standard Wave Lengths from Helium

Bureau of Standards[25]	Rayleigh[26]		Eversheim[27]	Ignatieff[28]
	a	b		
2945.104	....	....	....	....
3187.743	....	....	....	....
3613.641	....	....	....	....
3705.003	....	....	....	....
3819.606	....	....	....	....
3888.646	....	....	....	....
3964.727	....	....	....	....
4026.189	....	....	....	....
4120.812	....	....	....	....
4143.759	....	....	....	....
4387.928	....	....	....	....
4471.477	(.478)	.480	.493	....
4713.143	(.171)	.142	.154	....
4921.929	.925	.928	.934	....
5015.675	.680	.678	.683	....
5047.736	....	....	....	....
5875.618	.616	.623	.639	....
6678.149	.144	.147	.151	....
7065.188	.189	.197	.207	....
7281.349	....	....	....	....
....	....	....	....	10829.11
....	....	....	....	10830.32
....	....	....	....	20581.31

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[24] Peters and Boyd, American Machinist, Sept. 30 and Oct. 7, 1920.

[25] Merrill, B. S. Bulletin 14, p. 159; 1917.

[26] Phil. Mag. (6) 11, p. 685; 1906. Ibid (6) 15, p. 548; 1908.

[27] Zt. f. wiss. Phot., 8, p. 148; 1909.

[28] Ann. d. Phys (4) 43, p. 1117; 1914, relative to 5015.6815.

TABLE 3

Standard Wave-Lengths from Neon

Bureau of Standards[29]	Meissner[30]	Bureau of Standards[31]
3369.904	....	....
3417.906	....	....
3447.705	....	....
3454.197	....	....
3460.526	....	....
3464.340	....	....
3466.581	....	....
3472.578	....	....
3498.067	....	....
3501.218	....	....
3515.192	....	....
3520.474	....	....
3593.526	....	....
3593.634	....	....
3600.170	....	....
3633.664	....	....
5330.779	....	....
5341.096	....	....
5400.562	.564	....
5764.419	....	....
5820.155	....	....
5852.488	.4875	.4862
5881.895	.896	.8958
5944.834	.834	.8344
5975.534	.534	....
6029.997	.999	....
6074.338	.337	.3383
6096.163	.163	.1608
6143.062	.061	.0600
6163.594	.594	.....

[29] Burns, Meggers and Merrill, B. S. Bulletin 14, p. 765; 1918.

[30] Ann. d. Phys. (4) 58, p. 333; 1919, and 60, p. 414; 1919.

[31] Priest, B. S. Bulletin 6, p. 573; 1911. Ibid. 8, p. 539; 1912.

TABLE 3 (*Continued*)

Standard Wave-Lengths from Neon

Bureau of Standards	Meissner[30]	Bureau of Standards[31]
6217.280	.279	...
6266.495	.495	.4948
6304.789	.788	.7929
6334.428	.428	...
6382.991	.991	.9882
6402.245	.2460	.2392
6506.528	.527	...
6532.883	.881	...
6598.953	.953	...
6678.276	.275	...
6717.043	.042	...
6929.468	.465	...
7024.049	...	...
7032.413	.410	...
7051.292	7051.314	...
7059.109	7059.119	...
7173.938	7173.938	...
7245.166	7245.165	...
7438.899	7438.885	...
7488.872	...	...
7535.784	7535.786	...
7544.050	7544.061	...
...	7937.010	...
7943.182	7943.193	...
...	8082.460	...
...	8118.554	...
8136.408	8136.423	...
...	8259.392	...
...	8266.092	...
8300.369	8300.338	...
8377.606	8377.630	...
...	8418.447	...
8495.358	8495.359	...
...	8591.266	...
...	8634.668	...
...	8654.380	...

TABLE 4
Standard Wave-Lengths from Argon

Bureau of Standards[32]	Meissner[33]	Bureau of Standards[32]	Meissner[33]
3948.980	....	7147.042	....
4044.419	....	7206.986	....
4158.591	....	7272.935	....
4164.180	....	7353.316	....
4181.884	....	7372.119	....
4190.714	....	7383.979	978
4191.027	....	7503.867	868
4198.316	....	7514.651	648
4200.676	....	7635.106	107
4251.184	....	7723.758	760
4259.362	....	7724.210	210
4266.286	....	7948.175	177
4272.169	....	8006.156	158
4300.101	....	8014.784	786
4333.561	....	8103.693	691
4345.168	....	8115.307	310
4510.733	....	8264.522	525
4522.325	....	8408.210	216
4596.096	....	8424.646	650
4628.445	....	8521.443	....
4702.317	....	....	....
6032.127	....	....	....
6416.307	....	....	....
6677.282	....	....	....
6752.831	....	....	....
6871.290	....	....	....
6937.666	....	....	....
6965.429	432	....	....
7030.250	....	....	....
7067.217	218	....	....

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[32] Meggers, To be published in B. S. Bulletin.

[33] Ann. d. Phys. (4) 51, p. 95; 1916.

Table 5

Standard Wave-Lengths from Krypton

Bureau of Standards[34]	Buisson and Fabry[35]
4273.9696	....
4282.967	....
4318.552	....
4319.580	....
4355.478	....
4362.6422	....
4376.122	....
4399.969	....
4453.9174	....
4463.690	....
4502.354	....
4807.065	....
5562.224	....
5570.2872	.2908
5870.9137	.9172
6456.290	....
7587.414	....
7601.544	....

Table 6

Standard Wave-Lengths from Xenon

Bureau of Standards[36]

4500.978
4524.680
4582.746
4603.028
4624.275
4671.225
4697.020
4734.154
4807.019
4829.705
4844.333
4923.246

The elegance and refinement of interferometer methods for wave-length comparisons are truly remarkable, and most of the above values may be considered to be correct to 1 part in several millions. This is evidenced by the agreement between entirely independent determinations and also by the frequency differences of many of the rare gas spectral lines belonging to combination series. There are a few cases where differences of 0.004 A or more exist and the reasons for these are not often apparent. More work of this kind on the part of different observers is very much to be desired so that an extended system of accurate secondary standards will ultimately be produced. The principal sources of error which require attention in such investigations lie in (1) determining the order of interference, (2) correcting for the dispersion of phase change on reflection,[37] (3) maladjustments of the optical system and spectrograph, (4) operating conditions of the light sources, (5) attempts to measure close or complex lines, and (6) in the corrections to wave-lengths measured in air under other than normal conditions.

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[34] Meggers, To be published in B. S. Bulletin.
[35] Comptes Rendus 156, p. 945; 1913.
[36] Meggers, To be published in B. S. Bulletin.

4. CORRECTIONS TO WAVE LENGTHS MEASURED IN AIR

The international wave-length standards are specified in dry air at 15° C and 760 mm of mercury pressure. Variations in the density or index of refraction of the air appreciably affect the absolute values of the wave-lengths and when such measurements are made in air whose temperature is not 15° C, or whose pressure is not equal to 760 mm, corrections must be applied to reduce them to their values under normal conditions. Furthermore, wave lengths measured in air can be changed to their values in a vacuum only by an exact knowledge of the indices of refraction of the air for these wave-lengths.

The Rowland system of wave-lengths was defined in air at 20° C and 760 mm and correction tables to reduce such wave-lengths or oscillation frequencies to vacuum values were used as given in Kayser's Handbuch 2, p. 514. Since the new International System specifying wave-lengths at 15° C has come into general use, there has been no uniformity in such corrections. The resulting confusion has been pointed out by Birge[38] who recommends that the recent work of the Bureau of Standards[39] on the index of refraction of air for wave-lengths from 2218 A to 9000 A be referred to for these corrections in all future work. The correction tables prepared by the Bureau of Standards are based upon some 1200 measurements of the index of refraction of air (0°, 15° and 30°C) for wave-lengths from the extreme ultra-violet, through the visible spectrum, and in the infra-red. International adoption and use of these tables will exclude all further possibility of ambiguity or question about the conversion of wave-lengths to standard air condition or to vacuum when such wave-lengths are not actually observed under these conditions.

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[37] Meggers, B. S., Bulletin 14, p. 2; 1915.

[38] Astroph. J., 50, p. 72; 1918.

BUREAU OF STANDARDS
March 25, 1921

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[39] B. S. Bulletin 14, p. 697; 1918.

AN UNFAMILIAR ANOMALY OF VISION AND ITS RELATION TO CERTAIN OPTICAL INSTRUMENTS

BY

W. B. Rayton

Any optical instrument provided with a prism erecting system should satisfy the condition of complete erection of the image, that is, vertical and horizontal lines of the object should appear vertical and horizontal in the image within certain small limits. One one occasion the writer attempted to judge the character of this adjustment of a field glass by employing a well known method often used for making an approximate determination of magnifying power, viz., the method of simultaneous observation of an object by one eye through the telescope and by the other without the telescope. Choosing in this case a flag staff as the most suitable object, the two images did not appear parallel and the glass was condemned as imperfectly adjusted. Subsequent examination with the apparatus usually employed for the purpose proved that the adjustment was satisfactory. Curiosity aroused, the former method was then applied to a telescope without a prism erecting system and again the two images were found to be inclined to each other. Tests were next made on nearly all of the Scientific Staff of the Bausch & Lomb Optical Company and without exception all observed the same effect.

Reference to the literature disclosed the fact that the phenomenon is well known to physiological opticians but the knowledge seems not to have spread far out of their ranks. The subject was very thoroughly investigated by A. W. Volkman in 1864, whose measurements are extensively quoted by Helmholtz in his Handbook of Physiological Optics.

It appears, according to Volkman, Helmholtz and other more recent writers, that a horizontal line appears horizontal to both eyes in the great majority of cases but that a vertical line does not in general appear vertical to either eye. In single binocular vision the two images are fused into a single vertical line but if the fusion be destroyed by any means so that the two images are seen separately they appear to converge downwards or upwards according as the convergence of the eyes is less or more than is required for fusion. Consideration is limited in this discussion to subjects who are not afflicted with pathologic cyclophoria nor with extraordinary degrees of astigmia.

In addition to the method described of looking at a vertical line through a telescope with one eye and alongside the telescope with the other, there are other methods of producing the two separate images necessary for the observation. One such method consists in inserting a prism before one eye with base in, that is, with the base turned toward the nose. If placed before the eye with base out the eyes will generally converge to accomplish fusion of the images unless the deviation of the prism be so great as to make observation difficult for other reasons. If instead of a single prism a bi-prism be placed before the one eye with edge vertical, three images of a single vertical line will be seen which cannot all be fused into a single one. Again, holding a pencil vertically in the line of sight look at the wall beyond it. Two images of the pencil will be seen. Under these circumstances the two images of the pencil will appear to diverge upwards.

It is significant that the amount of the divergence is generally about equal to the angle subtended at the gound by the interpupillary distance. It is equally significant that if, in the experiment with the pencil, instead of holding it vertical the top is tipped away from the observer so that it lies approximately at the inclination of a book in reading, the two images appear parallel. These facts lead to the conclusion that the phenomenon may be due to some effect of experience.

Now if the effect is due to some process of education it seems that children, whose experience in life has not been so extensive, should not exhibit it or if they do that it should be to a less degree. Six children were tested using the telescope and the pencil tests as described. Five, ranging from nine to twelve years of age, were certain they saw the two images as parallel. One girl of thirteen was uncertain thinking that at times she saw them parallel and at other times as diverging slightly. It is recognized that in a purely qualitative test, such as this, consideration must be given to the lack of training of children's powers of observation. In order to test to some degree the value of their observation, however several pairs of lines were drawn, each pair on a separate sheet of paper, some pairs being parallel while others were of varying degrees of divergence. The children tested had no hesitation in detecting divergence of considerably less than 1° which is less than half the usual divergence seen by an adult. More precise tests on a greater number of subjects are contemplated. The experiment, incomplete as it is, tends to establish the truth of the education hypothesis.

As to the nature of the product of this hypothetical educational process, it must be one of two things; either, as suggested by Stevens, there comes to be established a slight cyclophoria which is manifest when the eyes are at rest but which is overcome whenever single binocular vision is obtained, or else it is purely a matter of interpretation.

Most of our work is done at comparatively short ranges and calls for continued effort on the part of the muscles which produce convergence. So much is this so that most of us, while possessing strong power of convergence, have practically lost the power to diverge the lines of sight. Very young children and savages possess both to about equal degree. It is worth noting in this connection that single binocular vision is one of the latest developments in the process of evolution. The prolonged tension to which the muscles of the eye are subjected possibly results in a position of the eyeball which, while abnormal with respect to condition at birth, let us say, is normal in the sense that it is peculiar to practically all civilized adults. The cyclophoria hypothesis may find support in the fact that the phenomenon under discussion varies in its character and amount if the head is tipped forward and the eyeballs rolled upward. A position of the head can generally be found which makes the images parallel but that position is not by any means the natural one.

On the other hand, there are two experiments which seem to cast doubt upon the validity of the cyclophoria hypothesis. If a distant vertical line be regarded with both eyes and the head in normal position, of course a single line is seen. Now by quickly interposing a prism, of about four prism dioptries preferably, with its base turned toward the nose, a double line is immediately seen, the lines being inclined to each other. If fusion can be accomplished they merge into a single line. If by voluntary effort the eyes can be held in their original position and the prism be suddenly withdrawn the line immediately appears single. On the basis of the cyclophoria hypothesis it seems necessary to assume that the introduction of the prism and the consequent destruction of fusion permits the eyes to assume a position of rest and that to get single binocular vision again it must be assumed that they rotate slightly on their anterior-posterior axes. With full recognition of the possibilities of motor responses to stimuli which have not yet reached the brain it seems impossible to believe that the brain does not ultimately recognize the stimulus especially if watching for it. It seems reasonable, therefore, in this experiment, to expect that the two lines first seen when the prism is interposed before the eye would appear parallel and that they would ultimately take a position of upward divergence. Conversely, when the prism is suddenly removed from the eye we should expect to see two crossed lines which by rotation about their point of intersection ultimately fused into one. Furthermore, it does not appear entirely unreasonable to expect that the mutual rotation of the image could be detected although this point is more open to question. A little practice is required before the image can be seen without momentary confusion, but after a little experience the single or the double image can be instantly recognized and the change from one to the other appears to occur instantaneously.

Again, regard two vertical parallel lines; hold a pencil vertical before the eyes and adjust distances so that the pencil appears tangent to one of the lines when seen by the right eye alone and tangent to the other line when seen by the left eye alone. Now fix on the plane of the two parallel vertical lines. The two parallel lines appear parallel but the two pencil images appear to diverge upwards. Inasmuch as the lines on the back-ground appear single, vertical, and parallel, it must be assumed that the eyes have assumed the hypothecated position for single binocular vision, and if the phenomenon with which we are concerned is due to a departure of the eyes from this position when the required stimulus is absent, that we have now supplied the stimulus which should bring the eyes into working position. Under these conditions the two pencil images should appear to be parallel, but they do not and it seems impossible, therefore, that the phenomenon can be ascribed to cyclophoria. It seems much easier to believe that it is a matter of interpretation in the same class, possibly, as the interpretation of the inverted images of our retinas.

The phenomenon has a direct bearing on all optical instruments which employ a prism system for erecting the image such as prism binoculars, binocular microscopes, stereoscopic range finders, etc. In any of these instruments, unless single binocular vision is obtained, there is the impression that the instrument is out of adjustment on account of this apparent divergence of lines which should appear parallel. Furthermore, it is quite unsafe to attempt to discover by means of the expedient described above whether the image formed by a prism telescope is completely erected.

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SOFT X-RAYS FROM ARCS IN VAPORS*

BY

F. L. MOHLER AND PAUL D. FOOTE

The present paper is a preliminary report of a study of the potentials required to excite radiations in the frequency range between the regions accessible to the vacuum grating spectroscope and the long wave-length limit of the x-ray crystal spectrometer. This region is of the utmost importance in the development of a theory of atomic structure, for it is in this range that the transitions from x-ray to ordinary arc and spark spectra take place. We will use the word x-ray to denote all radiation from the inner part of the atoms as distinguished from that due to the displacement or removal of outer valence electrons alone.

For this portion of the spectrum the photo-electric effect of the radiation offers about the only means of observation. The method of measuring the potential required to excite radiation by observing its photo-electric effect has been previously applied in studying radiating potentials below the first ionization point. The apparatus used in the search for higher critical potentials is similar to that employed for the determinations of ionization and resonance potentials.[1]

The element studied is boiled in a vacuum tube containing cylindrical electrodes having the following arrangement. The hot wire cathode at the center is surrounded by a small grid of fine wire gauze. A second gauze grid is at a relatively large distance from this and a cylindrical-plate collecting-electrode outside and close to the outer grid. The inner grid is put at the variable potential $+V$, which maintains the arc. The outer grid at a potential $-V_1$ shields the collecting electrode from all electrons and negative ions. With the plate at a potential $+V_2$ greater than V positive ions also are kept away so that any current to the plate is due to photo-electrons emitted by the outer grid when illuminated by the radiation from the arc. The ionization can be measured by making the collecting plate negative. Special care had to be taken with the electrical insulation on account of the high temperatures. The shielding of the plate from all ions both positive and negative can be tested readily and is an important precaution.

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* Published with the permission of the Director of the Bureau of Standards.

[1] Cf. for example, Bureau of Standards Scientific Papers Nos. 400 and 403.

In obtaining readings the current from the cathode as well as the current to the collecting electrode was recorded for every setting of the voltage V. Results were plotted with current to electrode divided by the cathode current as ordinates and V as abscissae. It happens that the photo-electric effect of the radiation when so plotted gives a straight line relation with a change in slope at each critical potential. The physical significance of this very striking linear relation is not known, as yet. The value of such a relation in determining the critical points is, however, evident. Corrections for initial potential were made by comparing the observed ionization point with the value for this point known from our earlier experiments.

The following results have been obtained:

Potassium—Radiation curves show a sharp increase in radiation starting at 23±1 volts. Curves from 23 volts to over 100 show a straight line relation. As the vapor pressure is increased a lower break in the curve at 20 volts, as well as 23 volts, is seen. Ionization curves under some conditions show a break at 23 volts but the latter measurement is not nearly as certain.

Sodium—A definite break in the radiation curve at 35 volts. The point is not nearly as sharp as with potassium. A break is also evident near 17 volts. As a trace of hydrogen is always present with sodium vapor this point may be due to hydrogen but consideration of the results with other elements suggests that it is a sodium point. Ionization curves likewise show a break in the curves at 35 volts

Magnesium—A definite break in the radiation curves at 46±1 volts. Evidence of a point near 34 volts. Curvature below this indicates other lower radiation potentials as is to be expected from the enhanced spectra. The data available were not sufficient to separate these points. Experimental difficulties due to high temperature are considerable with magnesium.

Phosphorus—A sharp break at 126 ± 1 volts. Definite breaks at 162 and near 99 volts. Evidence of lower points which were not measurable with any accuracy.

Sulphur—The radiation curves show breaks at 152 ± 5 and 121 ± 2 volts. The breaks became definite only at high vapor pressure under which conditions the currents were rather unsteady.

These various critical potentials in the range 20 to 160 volts correspond by the quantum relation to a wave length range from 600A to 80A.

In the case of the alkali metals with one valence electron it is evident that critical potentials above the first ionization potential must be due to ejection of electrons from the x-ray rings. The potential required to eject the electron is determined by the limiting frequency of an x-ray series and this potential is then both an ionization and radiation point.

In the case of polyvalent atoms we may expect to find potentials above the first ionization giving multiple ionization, as well as those giving x-radiation. Thus in the case of magnesium we can compute from spectral data that the potential required to eject both valence electrons is 22.6 volts, while 14.95 volts is required to eject the second electron from an ionized atom. As noted before, the present measurements were not sensitive enough to show these latter points clearly.[2] Nothing is known of the enhanced spectral series of the non-metallic elements here considered, so that there are no means for computing the values of the multiple ionization potentials.

From x-ray data on the K series, it is possible to compute the limiting frequencies of the L series down to atomic number 12 from the relation:[3]

$$La_1 = Ka - K\alpha_1$$

La_1 is a limiting frequency of an L series and gives the potential required to eject an electron from this L ring. In Fig. 1 the upper line shows the Mosely relation between the square root of the frequency and atomic number for La_1. The dots are points computed from the above relation. The circles are computed from the observed potentials; sodium 35, magnesium 46, phosphorus 126, sulphur 152. The sulphur point was known to be inaccurate. The phosphorus point falls on the straight line, not on the computed point. With magnesium the agreement is exact. However, the sodium point falls 5 volts above the extrapolated line and the difference in this case is far beyond any probable experimental error.

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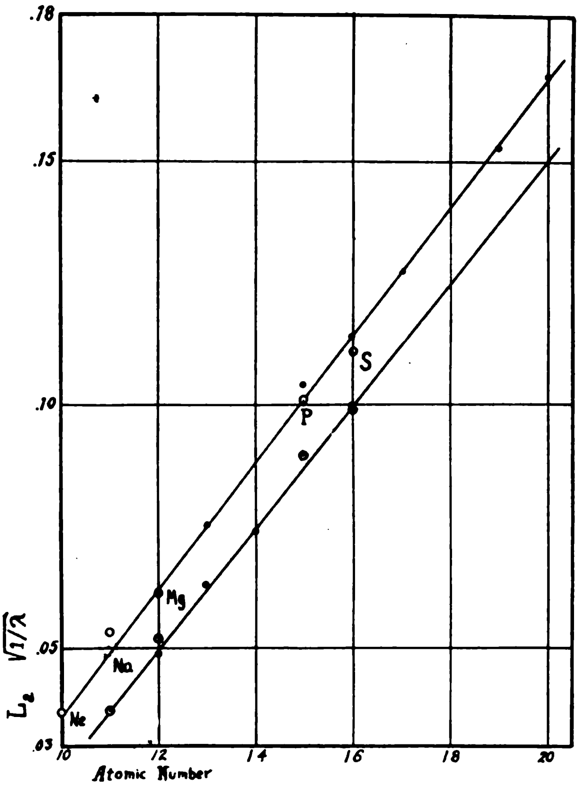
[2] The spectral changes at these potentials will be discussed in detail in a paper by Foote, Meggers, and Mohler, now in preparation.

[3] Duane and Shimizu, Phys. Rev. 14, p. 67; 1919.

FIG. 1



L—Limits for Low Atomic Numbers

Since obtaining this result, Professor Millikan has announced that he has found two isolated lines in the extreme ultraviolet spectrum of sodium at λ = 372 and 376A. These are undoubtedly the $L\alpha$ lines. They correspond to a potential of 33 volts. Consideration of the accepted model of the sodium atom shows that these lines must be due to the valence electron falling into the L ring, with a loss in energy equivalent to 33 volts. The potential required to eject an L electron and excite these lines must then exceed the potential corresponding to the emission lines by the ionization potential, or 5 volts. The potential corresponding to the L limit is then 38 volts, while we observed 35. There is evidently a sharp departure from Moseley's relation at the lower end of the L line.

The combined results show also another series of critical potentials which when plotted in this manner fall on a straight line. The lower line in Fig. 1 is drawn through points computed from the following observed values: sulphur 121, phosphorus 99, magnesium 34, sodium 17. The last point is of doubtful origin and all except sulphur were inaccurately located on account of faintness, but a straight line relation seems to fit. It is possible to compute this line from x-ray data. For all the light elements and for these alone, apparently, there are faint lines $K\alpha_3$, $K\alpha_4$, $K\alpha_5$ and in some cases $K\alpha_6$. We have computed the frequency $K\alpha - K\alpha_5$ for these elements as indicated by the solid dots on the lower line of the figure. $K\alpha_6$ would fit as well where it is given but the terms α_3 and α_4 would not be satisfactory.

The potassium points are doubtless related to the M series but it is impossible to compute or extrapolate the M series to potassium accurately.

We have added to Fig. 1 the first ionization potential of Neon, atomic number 10, at 16.7 volts.[4] It falls on the extrapolated $L\alpha_1$ line. A similar relationship has been pointed out by Kossel. In Sommerfeld's book,[5] however, this value is plotted on the $L\alpha$ straight line, whereas probably the value for the lowest resonance potential should be used, as the ionization potential should correspond to an x-ray limit as given by Fig. 1.

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[4] Horton and Davies, Proc. Roy. Soc., 98 p. 124, 1920.

[5] Sommerfeld. Atombau und Spektrallinien. Second Edition, p. 548.

The agreement between computed and observed points on the upper line of Fig. 1 indicates that this new method of observing the limiting frequency of x-ray series is well justified. The accuracy attained in the case of phosphorus for example is about one per cent. The data for the lower line of Fig. 1 are much less satisfactory and further work must be done in this regard. This is a new x-ray series or group of series which, judging from the K-lines, only extends through a small range of light elements. The authors hope to apply this method in tracing out the beginnings of the M and N series in a similar manner.[6]

BUREAU OF STANDARDS, APRIL 27, 1921
WASHINGTON, D. C.

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[6] Recently Kurth has reported before the Physical Society results of experiments on characteristic X-rays *from solids*, by a photo-electric method. (Abstract Phys. R. 17 p. 528, 1921. Also Washington meeting of Am. Phys. Soc.) The same type of experiment was also carried out by us independently, with results which in some cases confirm the beautiful data of Mr. Kurth, and which will be extended and published later. The present work was undertaken to supplement this by using metals which are easily vaporized and also gases and non-metallic vapors. This is the first example of the excitation of x-rays in vapors. Many other observers have applied the photo-electric method in a search for low-voltage characteristic radiation from solids but without definite results.

Millikan has been able to identify many x-ray lines of wave-length longer than 130 Å in "hot spark" spectra. His results in sodium were noted above. This method of photographing the lines directly over this narrow spectral range and the extreme end of the x-ray series greatly adds to the value of determinations in which the critical voltages have been observed.

Note added Aug 8. Additional data are as follows: Chlorine 198 and 157 volts; Carbon 272 volts; Nitrogen 374 volts; and Oxygen 478 volts. The last [illegible] are K limits.

NOTE ON MR. T. SMITH'S METHOD OF TRACING RAYS THROUGH AN OPTICAL SYSTEM

BY

C. W. Woodworth

Professor Southall has commented under the above title[1] upon the new English method of lens calculation, raising the doubt, however, whether there was actually any shortening of the process, and suggesting the comparison of a similar method based on the use of central perpendiculars. A still better method in my opinion is the use of lines perpendicular to the axis as proposed by me in a previous number of this *Journal*[2] applicable for general use but urged particularly for the purpose of avoiding serious inaccuracies of other methods in the special case of a ray with a very narrow θ angle passing from a curved to a flat surface.

The use of such perpendiculars does indeed simplify Smith's equation but no more so than the direct calculation of the foci based on the equation

$$n'c' \sin\theta' = n\,c \sin\theta$$

which expands at once into

$$\frac{n\,c}{n'c'} = \cos\alpha' \cos\alpha + \sin\alpha' \sin\alpha + \frac{\cos\theta}{\sin\theta} (\sin\alpha' \cos\alpha - \cos\alpha' \sin\alpha).$$

The equation by the other method using the parameter b normal to the axis can likewise be obtained in a similar form by expanding the equation

$$n'b' \cos\theta' = n\,b \cos\theta$$

which becomes

$$\frac{n\,b}{n'b'} = \cos\alpha' \cos\alpha + \sin\alpha' \sin\alpha - \frac{\sin\theta}{\cos\theta} (\sin\alpha' \cos\alpha - \cos\alpha' \sin\alpha)$$

After obtaining the value of c′ or of b′ by the second of one of these pairs of equations one can proceed to obtain the function of the θ' angle by the corresponding first equation.

The relation between the α and θ functions is shown by the equation

$$r \sin\alpha = -c \sin\theta = b \cos\theta$$

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[1] Journal of the Optical Society of America. Vol. 5, pp. 14–21.

[2] *Ibid.*, Vol. 4, pp. 286–293.

For flat surfaces b and b′ are identical since the parameter must be measured along the surface; and since $\alpha=\theta$,

$$n' \sin \theta' = n \sin \theta.$$

In the case of paraxial rays through flat surfaces the angular functions disappear and

$$n'/u' = n/u.$$

The adoption of Smith's method at least when modified as above certainly results in a material reduction of the labor of calculating through a lens system. Even in theoretical work where one may desire greater accuracy than afforded by the four place sine-cosine table one can obtain the desired functions on the calculating machine by the equation $\cos \theta = \sqrt{1-\sin^2 \theta}$ with a facility that I think will compete favorably with any other method of equal accuracy. It may be observed that it is not at all necessary actually to determine the successive foci or parameters but only the functions of the successive angles, thus eliminating two steps in the calculation of each intermediate surface.

UNIVERSITY OF CALIFORNIA

THE BLACKENING OF A PHOTOGRAPHIC PLATE AS A FUNCTION OF INTENSITY OF MONOCHROMATIC LIGHT AND TIME OF EXPOSURE, ON THE BASIS OF HURTER AND DRIFFIELD'S AND ROSS' FORMULAE

BY
P. S. HELMICK

In a previous contribution,[1] making use of a modification of Hurter and Driffield's formula,[2] a method was given of expressing the density of a photographic plate exposed to monochromatic light as a function of intensity of light and time of exposure to light. This expression readily lends itself to the calculation of the variation of Schwarzschild's exponent p[3] with intensity of light.

It has also been pointed out,[4] that this method gives results which fit the facts fairly well for densities of plates between 0.2 and 5. For densities less than 0.2, the method gives values which are greater than the experimental values.

By making use of other density expressions, it is possible to obtain agreement between observed and computed values of blackenings within the limits of experimental error, and to compute with the same ease as before, the variation of Schwarzschild's exponent p with intensity of light.

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[1] Phys. Rev., 17, 135; 1921.

[2] Journ. Soc. Chem. nd., 9, 455; 1890.

The particular expression used was:

$$D = \frac{1}{a} \log_e \left[b - (b-1)e^{-t10^{A+B\log_{10} I + C\log^2 I}} \right],$$

where D is the density $\left(\text{density equals } \log_{10} \frac{\text{Light incident on plate}}{\text{Light transmitted by plate}} \right)$ of the plate exposed for a length of time t to a monochromatic light of intensity I. a, b, A, B, and C are parameters dependent upon the wave-length of light and the characteristics of the plate.

[3] Astrophys. Journ., 11, 89; 1900. The expression states that for equal blackenings, $It^p = \text{constant}$.

[4] Phys. Rev., 17, 411; 1921.

In order to contrast these several methods of expressing the blackening of a photographic plate exposed to various intensities of monochromatic light for different times of exposure, a set of experimental data as previously obtained[5] for a Seed 23 plate exposed to monochromatic light of wave-length 550μ μ for lengths of time from 1 second to 15 minutes will be utilized, and the blackening equations found according to the four following modifications of Hurter and Driffield's[6] or Ross'[7] equations:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-10^A\,I^{B+C\log_{10}I_t}}\right]\dots\dots\dots\dots(1)$$

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A\,I^{B+C\log_{10}I})\frac{1}{k}}\right]\dots\dots\dots\dots(2)$$

$$\mathrm{D}=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1}e^{-r^s10^A\,I^{B+C\log_{10}I_t}}\right]\dots\dots\dots\dots(3)$$

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1}e^{-r^s(10^A\,I^{B+C\log_{10}I_t})^{\frac{1}{k}}}\right]\dots\dots\dots\dots(4)$$

(1) and (2) reduce to Hurter and Driffield's equation, and (3) and (4) reduce to the equivalent of Ross' equation, upon setting $B=1$ and $C=O$ for (1) and (3), and setting $B=1$, $C=O$, and $k=1$ for (2) and (4).

THEOREM FOR DETERMINING COEFFICIENTS IN BLACKENING EQUATIONS BY SEMI-GRAPHICAL METHODS

An expression will be developed at this point, which will be of use in the semi-graphical determination of the parameters occurring in each of the equations.

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[5] *Loc. cit.*

[6] *Loc. cit.*

[7] JOURN. OPTICAL SOC. OF AMERICA, 4, 261; 1920.

Ross' original formula is:

$$D=d_m\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1}e^{-kr^sIt}\right],$$

where D is the density of a plate exposed for a time t to light of intensity I, d_m is the density for infinite exposure, k is the sensitivity factor, and r is the common ratio of the sensitivities of the various groups of grains in the plate.

Let the density D of a photographic plate exposed to monochromatic light of intensity I be expressed in the form:

$$D=\frac{1}{a}f\,(c\,t),$$

where t is the time of exposure, a is a constant of the plate, and c is some function of I and t.

Consider a blackening curve for a plate exposed to monochromatic light of constant intensity, plotted on tracing paper to a $\log_{10}D-\log_{10}t$ scale (cf. one of the curves of Fig. 3).

Suppose that it is possible to make this curve coincide with some curve $y=\Phi(x)$, orienting the curves so that the y and the $\log_{10}D$ axes, and the x and the $\log_{10}t$ axes are parallel.

Let l and m be the respective x and y intercepts of the $\log_{10}D$ and the $\log_{10}t$ axes.

Then

$$x=l+\log_{10}t,$$

and

$$y=m+\log_{10}D.$$

If x and y are so chosen that

$$y=\log_{10}aD,$$

and

$$x=k\log_{10}ct;$$

then

$$k\log_{10}ct=l+\log_{10}t,$$

and

$$\log_{10}aD=m+\log_{10}D$$

From whence follows that

$$c=10^{\frac{l}{k}}\,t^{\frac{1}{k}-1},$$

$$l=A+B\,\log_{10}I+C\,\log_{10}^2 I,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

or if

$$c=10^{\frac{1}{k}\,(A+B\,\log_{10}I+C\,\log^2_{10}I)}\,t^{\frac{1}{k}-1},\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

or

$$c=(10^A\,\mathrm{I}^{B+C\,\log_{10}I}\,t^{1-k})^{\frac{1}{k}};\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6')$$

and as

$$a=10^m,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7$$

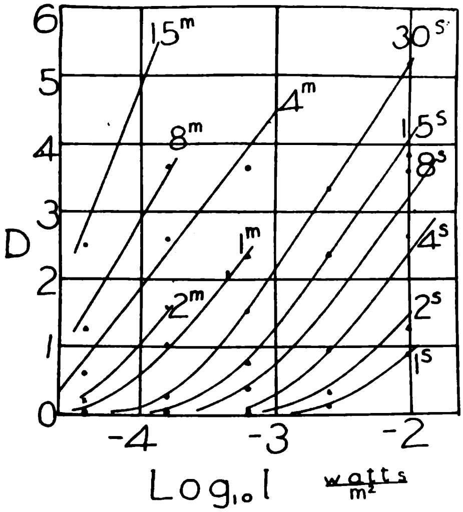
so

$$D=10^{-m}f\,(10^A\mathrm{I}^{B+C\,\log_{10}I})^{\frac{1}{k}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Thus if a curve $D=f\ (ct)$ can be found such that an experimental $\log_{10}D-\log_{10}t$ curve with intensity constant, fits it when the respective axes are parallel, m can be determined, and the l can be found for the particular intensity used to obtain the $\log_{10}D-\log_{10}t$ curve. Various l's can now be determined for different I's using other $\log_{10}D-\log_{10}t$ curves, and from this set of values of l; A, B, and C can be obtained. m, A, B, C, k, and the form of the f-function being known, by substitution in (8), the density of the plate is now expressible in terms of the intensity of the monochromatic light and the time of exposure.

This method will now be used to determine the constants of (1), (2), (3), and (4) from the data represented by the points of Fig. 1.

FIG. 1



Experimental data to be fitted by blackening curves. Points represent experimental data, curves are drawn free-hand through the points. Seed 23 Plate. $\lambda=550\ \mu\mu$.

EQUATION 1

$$D=\frac{1}{a}\left[b-(b-1)e^{-10^{A}I^{B+C\log_{10}I}t}\right].$$

A method of obtaining the constants of this equation has already been described in a former paper (loc. cit.), but the process will again be outlined, in order that the results of this equation may be contrasted with the results of the other equations.

In order to represent the data of Fig. 1 according to equation (1), set

$$c = 10^{A} I^{B+C \log_{10} I}.$$

(6) then becomes identical with this last expression if the k of (6′) is set equal to unity.

(1) will now reduce to

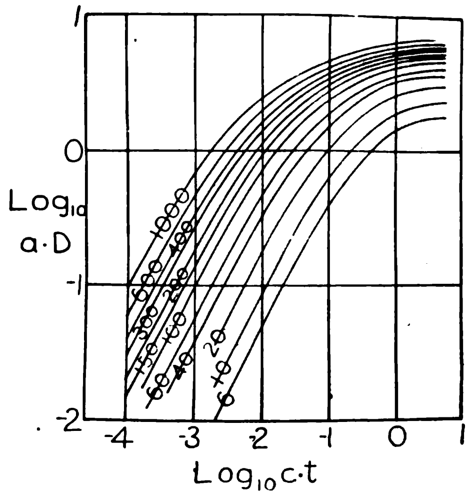
$$D = \frac{1}{a} \log_e \left[b - (b-1) e^{-ct} \right] \dots\dots\dots (9)$$

an expression of the form, $D = \frac{1}{a} f(ct)$,

where $f(z) = \log_e [b - (b-1) e^{-z}]$.

From (9), $\log_{10} aD$ is now plotted against $\log_{10} ct$, as in Fig. 2, with b a parameter,—values of b ranging from 6 to 1000. A

FIG. 2



Monographic chart for determination of parameters in the blackening equation:

$$D = \frac{1}{a} \log_e \left[b - (b-1) e^{-10^{A} I^{B+C \log_{10} I} t} \right].$$

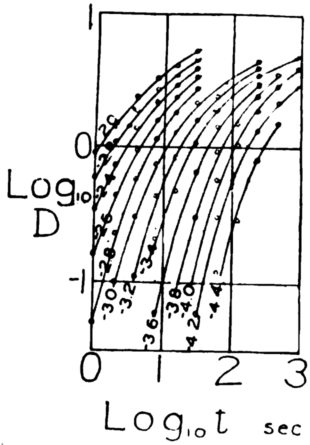
separate $\log_{10} aD$—$\log_{10} ct$ curve will, of course, be obtained for each distinct value of b. The next step is to obtain a set of curves in which the experimental data are plotted to a $\log_{10} D$—$\log_{10} t$

scale for various intensities of light,—one curve for each separate intensity chosen.

If the experimentally determined points of Fig. 1 are used as they stand, it will be possible to draw only five $\log_{10}D-\log_{10}t$ (intensity being constant) curves for this set of data, for but five different values of intensity were used.

From these five curves, a value of l can be obtained for each of the five intensities.

FIG. 3



Data from the curves of Fig. 1 plotted in a form suitable for the graphical determination of parameters in the four blackening equations.

From (5), five equations

$$l_1=A+B\log_{10}I_1+C\log_{10}^2I_1,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$l_5=A+B\log_{10}I_5+C\log_{10}^2I_6,$$

will then result, from which A, B, C, and l can be determined by the method of least squares.

If it is possible to obtain a much greater number of equations than five, the A, B, and C can be determined by a method of Campbell,[8] which is considerably simpler than the standard method of least squares.

Accordingly, free-hand curves representing various times of exposure from 1 second to 15 minutes were drawn in Fig. 1.

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[8] Phil. Mag., 39, 177; 1920.

From the intersection of these curves with the $\log_{10}I$ ordinates $-2.0, -2.2, \ldots, -4.4$, Fig. 3 is obtained. This figure shows the relationship between $\log_{10}D$ and $\log_{10}t$ for thirteen different values of intensity.

The next step is to determine some value of b which gives the best fit to these thirteen curves, and also some constant value of $\log_{10}aD$ which gives an abscissa along which one can slide Fig. 3, keeping the $\log_{10}D=0$ intercept on the $\log_{10}aD$ axis constant. A trial shows that the curve $b=200$, and an intercept $m=-0.2$ gives the best fit. That is to say, Fig. 3 will be moved over Fig. 2, keeping the axis $\log_{10}D=0$ of Fig. 2 always a distance of 0.2 below the axis $\log_{10}aD=0$ of Fig. 3, and the values l of the $\log_{10}D$ intercept on the $\log_{10}aD$ axis will be found for each of the thirteen values of $\log_{10}I$, i.e., whenever any of the various $\log_{10}I$ equals constant curves: $-2.0, -2.2, \ldots, -2.4$, coincide with the curve $b=200$.

Performing this operation gives values for l as found in Table 1, Column l_1.

TABLE 1

$\log_{10}I$	Values of l for Equations (1), (2), (3), and (4)			
	l_1	l_2	l_3	l_4
−2.0	−2.36	−1.64	−0.30	0.06
−2.2	−2.53	−1.86	−0.50	−0.14
−2.4	−2.73	−2.02	−0.68	−0.30
−2.6	−2.92	−2.22	−0.88	−0.51
−2.8	−3.12	−2.42	−1.06	−0.72
−3.0	−3.36	−2.65	−1.30	−0.94
−3.2	−3.25	−2.84	−1.50	−1.14
−3.4	−3.76	−3.05	−1.70	−1.35
−3.6	−3.90	−3.24	−1.88	−1.52
−3.8	−4.08	−3.44	−2.06	−1.71
−4.0	−4.22	−3.60	−2.25	−1.87
−4.2	−4.42	−3.80	−2.47	−2.13
−4.4	−4.63	−4.02	−2.64	−2.32

From (5),

$$l=A+B\log_{10}I+C\log_{10}^2I,$$

so the following equations are now at hand for the determination of A, B, and C:

$$\left.\begin{aligned}-2.36&=A-2.0B+\ 4.00C,\\-2.53&=A-2.2B+\ 4.84C,\\-2.73&=A-2.4B+\ 5.76C\\-2.92&=A-2.6B+\ 6.76C,\end{aligned}\right\}\quad\text{(I)}$$

$$\left.\begin{aligned}-3.12&=A-2.8B+\ 7.84C,\\-3.36&=A-3.0B+\ 9.00C,\\-3.56&=A-3.2B+10.24C\\-3.76&=A-3.4B+11.56C,\end{aligned}\right\}\quad\text{(II)}$$

$$\left.\begin{aligned}-3.90&=A-3.6B+12.96C,\\-4.08&=A-3.8B+14.44C,\\-4.22&=A-4.0B+16.00C\\-4.42&=A-4.2B+17.64C,\\-4.63&=A-4.4B+19.36C.\end{aligned}\right\}\quad\text{(III)}$$

In determining approximate values of A, B, and C by Campbell's[9] "Method of the Zero Sum," the equations are divided into three sets, (I), (II), and (III), and the coefficients of each of the terms are added together, the theory assuming that with a *large* number of equations, the sum of the errors in each sum of coefficients approaches zero. With the small number of equations used here, it is justifiable to use this method as a rough approximation only.[10] Performing such additions, the following three "normal" equations result:

$$\begin{aligned}-10.54&=4A-\ 9.2B+21.36C,\\-13.80&=4A-12.4B+38.64C,\\-21.25&=5A-20.0B+80.40C.\end{aligned}$$

From these equations, the values of the constants can be determined as follows:

$$\begin{aligned}A&=0.24\pm0.03,\\B&=1.42\pm0.03,\\C&=0.075\pm0.008.\end{aligned}$$

As we have already found $b=200$, and $m=-0.2$, and from (7) $\frac{1}{a}=10^{-m}$, from (1),

$$D=10^{0.2}\log_e\left[200-(200-1)e^{-10^{0.24}I^{1.42+0.075\log_{10}I}\,t}\right].$$

---

[9] *Loc. cit.*

[10] Roeser's method of fitting curves to observations taken at equal intervals of the independent variable would be very applicable to these data. Its use is equivalent to

For the present purpose it is scarcely worth while to simplify this formula further, for the most direct way of calculating values of D for various values of I and t is to compute the value of l corresponding to any value of I from

$$l=0.24+1.42\log_{10}I+0.075\log_{10}^2 I,$$

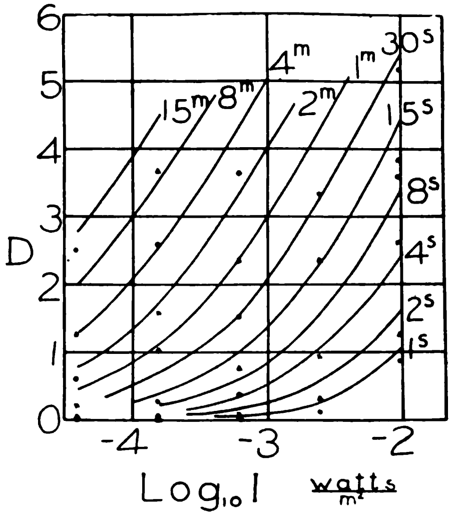
and then knowing l, b, a, and t, to read off the value of $\log_{10}D$ from Fig. 2.

In this manner the curves of Fig. 4 were obtained, the points representing the experimental values as used in Fig. 1.

EQUATION 2

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^{B+C\log_{10}I}t)^{\frac{1}{k}}}\right].$$

FIG. 4



Agreement between experimental values (points) of the data of Fig. 1 and blackening curves computed from the expression:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-10^A I^{B+C\log_{10}I}t}\right]$$

Note the poor agreement between observed and computed values for small densities.

---

the ordinary method of least squares which assumes that the independent variable is not subject to error. Of course, this assumption would not be strictly true for these data. See Sci. Papers of Bur. Standards, No. 388; 1920 for details of this method.

From Fig. 4 it can be seen that the agre experimental and the computed values of densi small blackenings are considered. The exp computed values of density can never agree f because for small times of exposure, the slop Figs. 2 and 3 are different, making coincidence o sible. Calculation and also actual measurem matter what the value of b, the slopes of the c equal to unity when $t=0$, while the slopes o curves are equal to 1.67 for $t=0$.

These considerations lead to the addition $\frac{1}{k}$ to (1), thus forming a new equation, (2), t exponent being to increase the slope of the curv

In order to reduce (2) to the form $D=\frac{1}{a}f\,(ct$

$$c=(10^{A}I^{B+C\log_{10}I}t^{1-k}),$$

whence from (2),

$$D=\frac{1}{a}\log_{10}e\left[b-(b-1)e^{-ct}\right]$$

and so the intercepts l and m of the experimen curve on the $\log_{10}aD-k\log_{10}ct$ curve suffice to the constants.

The $\log_{10}D-\log_{10}t$ curve is the one already d

It is necessary to plot a set of $\log_{10}aD-k\log_{10}$ different value of k which it is thought desirab shows such a set of curves plotted for $k=0.6$. I the slopes of the experimental curves of Fig. 3. Fig. 5 are the same for small values of t, so b may now be expected for small times of exposur

Trial shows that the best fit between Figs. 3 when $b=1000$ and $m=0.08$. In order to find t the experimental curves of Fig. 3, Fig. 3 is sup 5, so that the axis $\log_{10}D=0$ is always a distan axis $\log_{10}aD=0$. Fig. 3 is then shifted along

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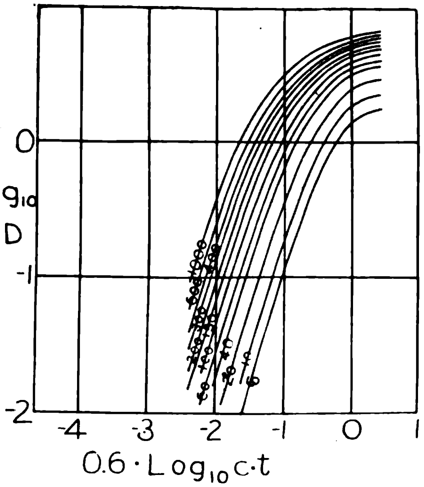
[11] $1/0.6=1.67$, which is the slope of the curves of Fig. 3.

FIG. 5

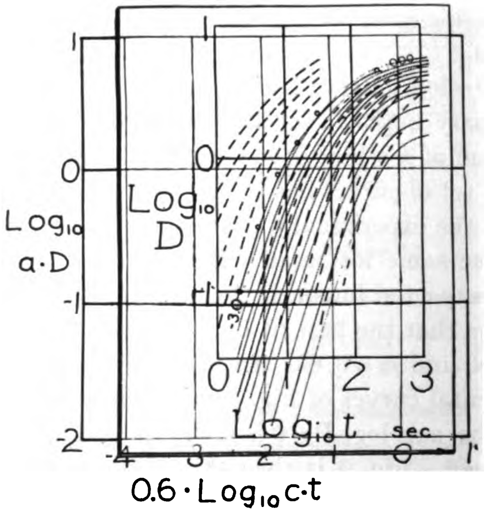


Monographic chart for determination of parameters in the blackening equation:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^{B+C\log_{10} I}\,t)^{\frac{1}{k}}}\right].$$

$k=0.6$

FIG. 6



Method of obtaining parameters of curve -3.0 of Fig. 3 by superposition of Fig. 3 on Fig. 5. $b=1000$. x and y intercepts give $l=-2.65$, $m=0.08$.

the same distance between the axes, and whenever a curve of Fig. 3 coincides with the curve $b=1000$ of Fig. 5, the l will be read off as the distance between the axis $\log_{10}t=0$ and the axis $\log_{10}ct=0.6$.

Fig. 6 is an attempted representation of this process. It shows the two curves placed so the $\log_{10}D=0$ axis is a distance 0.08 higher than the $\log_{10}aD$ axis, and pictures a sample "coincidence" of the curve $\log_{10}I=-3.0$ of Fig. 3 with the curve $b=1000$ of Fig. 5. The circles and the heavy line marked *−3.0* is the curve $\log_{10}I=-3.0$, and the faint line marked *1000* cutting the line $\log_{10}I=-0.3$ is the line $b=1000$. This illustration is chosen to show what is probably the worst fit in the set. The value of l for this particular curve $\log_{10}I=-3.0$, or the distance between the axes $\log_{10}D=0$ and $\log_{10}aD=0$, is -2.65.

Proceeding in this way, the various values of l corresponding to the thirteen different curves $\log_{10}I=-2.0$, etc. of Fig. 3 are found as set down in the column l_2 of Table 1. A, B, and C can now be determined by the "Method of the Zero Sum," as given under Equation 1.

The "normal" equations resulting from a division of the equations $l=A+B\log_{10}I+C\log_{10}^2I$ into the three groups I, II, and III as before, are:

$$\begin{aligned}-\ 7.74&=4A-\ 9.2B+21.36C,\\ -10.96&=4A-12.4B+38.64C,\\ -18.10&=5A-20.0B+80.40C.\end{aligned}$$

A solution of these equations gives:

$$\begin{aligned}A&=\ 0.50\pm0.02,\\ B&=\ 1.09\pm0.02,\\ C&=0.016\pm0.005.\end{aligned}$$

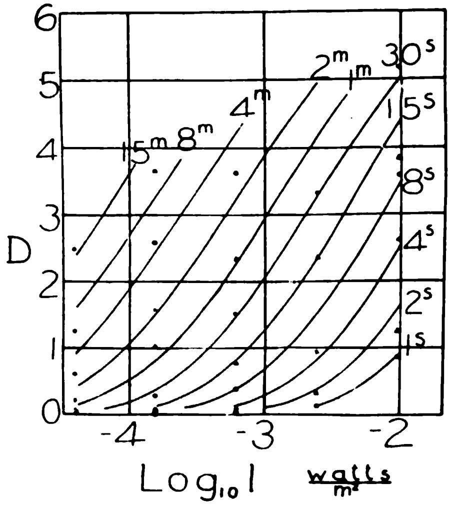
The blackening equation is accordingly deduced by simplifying the expression:

$$D=10^{-0.08}\log_e\left[1000-(1000-1)e^{-(10^{0.50}I^{1.09+0.016\log_{10}I}t)^{\frac{1}{0.6}}}\right].$$

This equation, by a semi-graphical solution similar to that described under Equation 1, gives a representation of the data as shown in Fig. 7, the curves giving the computed values by the formula, and the points the experimental values as before.

The accuracy of fit is within the limit of experimental error of the data.

FIG. 7



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-\left(10^A I^{B+C\log_{10} I}\,t\right)^{\frac{1}{k}}}\right].$$

EQUATION 3

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s 10^A I^{B+C\log_{10} I}\,t}\right].$$

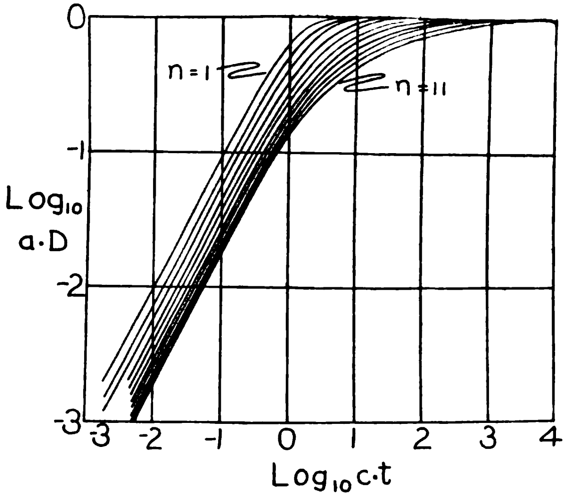
(3) is transformed into the desired form $D=\frac{1}{a}f\,(ct)$ by putting $c=10^A I^{B+C\log_{10} I}$ and as c is then equivalent to (6′) where the k of (6′) is set equal to unity, a, A, B, and C can be obtained from the values of the intercepts l and m by (5) and (7), and u and r can be determined by the particular curve of the $D=\frac{1}{a}f\,(ct)$ family which gives the best fit to the experimental $\log_{10}D-\log_{10}t$ curves.

The family of curves to be plotted on $\log_{10}aD-\log_{10}ct$ co-ordinates is given by

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s ct}\right].$$

Fig. 8 shows a family of eleven such curves[12] plotted, with the parameter n having eleven different values as shown,—$n=1$, $n=2$, $n=3$, . . . , $n=11$, with r having the value $\frac{1}{2}$.

FIG. 8



Monographic chart for determination of parameters in the blackening equation:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s 10^A I^{B+C\log_{10} I} t}\right].$$

An attempt to fit the curves of Fig. 3 to those of Fig. 8 shows that the values $m=-1.05$, and $n=11$ give the best results.

By shifting the curves over each other, always keeping the axis $\log_{10}D=0$ on the line $\log_{10}aD=-1.05$, values of l are found as given in Table 1, Column l_3.

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[12] Cf. Ross., Fig. 4, *loc. cit.*

The "normal" equations for the determination of A, B, and C, are accordingly:

$$\begin{aligned} -\ 2.36&=4A-\ 9.2B+21.36C,\\ -\ 5.56&=4A-12.4B+38.64C,\\ -11.30&=5A-20.0B+80.40C, \end{aligned}$$

from whence

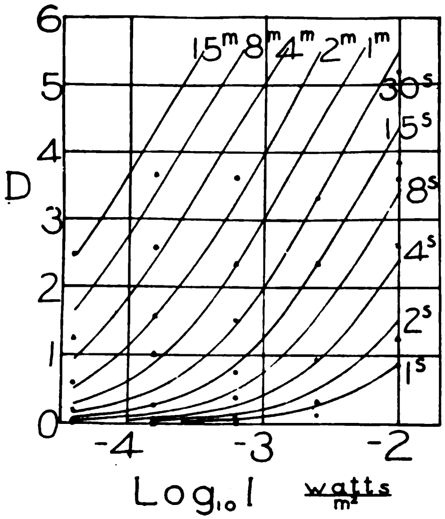
$$\begin{aligned} A&=\ 1.85\ \pm0.02,\\ B&=\ 1.10\ \pm0.02,\\ C&=\ 0.019\pm0.004, \end{aligned}$$

and so the equation of blackening is found to be:

$$D=10^{1.05}\left[1-\frac{1}{11}\sum_{s=0}^{s=10} e^{-(\frac{1}{2})^s(10^{1.85}I^{1.10+0.019\log_{10}I}t}\right]$$

This equation gives values for blackening as shown in the curves of Fig. 9, the points representing the experimentally determined values. There is a noticeable departure between the calculated and the observed values for low densities.

FIG. 9



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s10^AI^{B+C\log_{10}I}t}\right].$$

Note the poor agreement between observed and computed values for small densities.

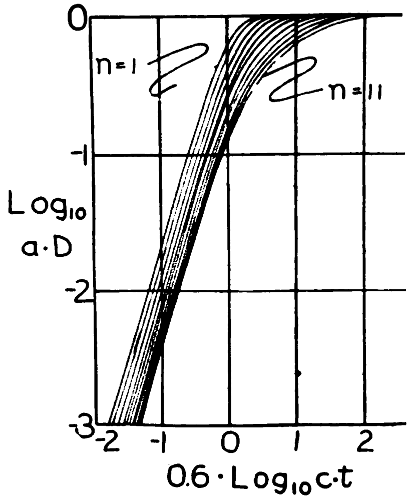
EQUATION 4

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^A I^{B+C\log_{10} I}\,t)^{\frac{1}{k}}}\right]$$

After having observed the poor fit of Equatio
ties, it is natural to try to modify the equatio
accurate representation of the data.

In the curves of Fig. 8, the parameter *r* was s
were given some other value,[13] the shape of th
altered. For small blackenings, the slope of the
in Fig. 8 is equal to unity, while the slope of
curves is equal to 1.67. Calculation shows th
the values of *r* or of *n*, the slope of the theoretic
will be always equal to unity for $t=0$.

FIG. 10



Monographic chart for determination of parameters in the blackening eq

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^A I^{B+C\log_{10} I}\,t)}\right.$$

$k=0.6$

---

[13] Ross states that the field of practical emulsions should be v
and assigning values of *n* from 1 to 10.

Hence an accurate expression for small blackenings becomes impossible with Equation 3, but a suitable modification,—or Equation 4, will give the same slope to each set of curves.

To convert (4) into the form $D=\frac{1}{a}f\,(ct)$, put

$$C=(10^{A}I^{B+C\log_{10}I}t^{1-k})^{\frac{1}{k}},$$

an expression which is equivalent to (6′).

D then becomes:

$$D=\frac{1}{a}\left[1-\sum_{s=0}^{s=n-1}e^{-r^{s}ct}\right],$$

and so a family of these curves with $r=\frac{1}{2}$ and n ranging from 1 to 11 is plotted on a $\log_{10}aD-k\log_{10}ct$ scale, as in Fig. 10. The best fit is given by the curve $n=11$, with $a=-0.75$.

Placing the $\log_{10}D=0$ axis of Fig. 3 on the line $\log_{10}aD=-0.75$ of Fig. 10, and sliding Fig. 3 over Fig. 10 in a horizontal direction gives values of l as shown in Column l_4 of Table 3.

A, B, and C are then determined from the "normal" equations:

$$\begin{aligned}-1.01&=4A-9.2B+21.36C\\-4.15&=4A-12.4B+38.64C\\-9.55&=5A-20.0B+80.40C,\end{aligned}$$

and their solution gives:

$$\begin{aligned}A&=2.05\pm0.03\\B&=1.02\pm0.03\\C&=0.007\pm0.007.\end{aligned}$$

The expression for blackening according to Equation 4 is as a consequence:

$$D=10^{0.75}\left[1-\frac{1}{11}\sum_{s=0}^{s=10}e^{-(\frac{1}{2})^{s}(10^{2.05}I^{1.02+0.07\log_{10}I}t)^{\frac{1}{0.6}}}\right].$$

Fig. 11 shows the agreement between the experimental values (points) and the computed values (curves).

SCHWARZSCHILD'S EXPONENT p

Schwarzschild[14] states that It^{p} must be constant for equal blackenings. The form of equations (1), (2), (3), and (4) shows that for equal blackenings $I^{B+C\log_{10}I}t$ must be constant, or $It^{(B+C\log_{10}I)^{-1}}$ must be constant.

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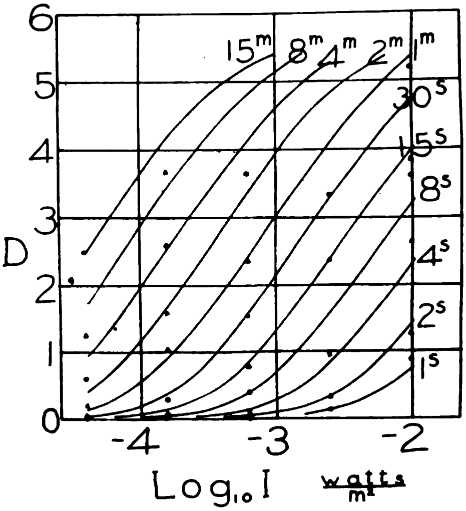
[14] *Loc. cit.*

Hence the expression[15] $(B+C\log_{10}I)^{-1}$ is the equivalent of Schwarzschild's p.

FIG. 11



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^A I^{B+C\log_{10}I}t)^{\frac{1}{k}}}\right].$$

Therefore one of the advantages of using equations (1), (2), (3), and (4), lies in the fact of the resulting simplicity attached to finding the value of p, and any variation[16] in the value of p with the intensity of light.

As $p=(B+C\log_{10}I)^{-1}$, the B and the C should be identical for each of the four equations (1), (2), (3), and (4). The values of B in the four equations were found to be: 1.42, 1.09, 1.10, and 1.02; and the values of C were found to be: 0.075, 0.016,

---

[15] If l be expressed in the form

$$l=A+B\log_{10}I+C\log_{10}{}^2I+D\log_{10}{}^3I+\ldots\ldots\ldots\ldots+N\log_{10}{}^nI,$$

then $p=\mathrm{B}+C\log_{10}I+D\log_{10}{}^2I+E\log_{10}{}^3I+\ldots\ldots\ldots\ldots+N\log_{10}{}^nI.$

[16] On variations of p with intensity of light cf. Astrophys. Journ., 29, 154; 1909; and Ann. der Phys., 41, 751; 1913.

0.019, and 0.007. As (2) and (4) give better fits of experimental values to computed values than do (1) and (3), weights 2 and 1 may be respectively assigned. With these conservative values of weights, the most probable value of B is 1.12 ± 0.05, and the most probable value of C is 0.023 ± 0.012.

If it now be assumed that the numbers 1.12 and 0.023 give the correct values of B and C for the four equations, the following most probable values of the constants A, B, and C are found, the subscripts denoting the equation to which the constant is attached:

$A_1 = -0.17 \pm 0.04$	$B_1 = 1.12 \pm 0.04$	$C_1 = 0.023 \pm 0.009$
$A_2 = 0.51 \pm 0.03$	$B_2 = 1.12 \pm 0.03$	$C_2 = 0.023 \pm 0.007$
$A_3 = 1.86 \pm 0.02$	$B_3 = 1.12 \pm 0.02$	$C_3 = 0.023 \pm 0.005$
$A_4 = 2.22 \pm 0.03$	$B_4 = 1.12 \pm 0.03$	$C_4 = 0.023 \pm 0.008$

So the best values of B and of C from the equations (1), (2), (3), and (4) as weighted are:

$$B = 1.12 \pm 0.03,$$

and

$$C = 0.023 \pm 0.007.$$

With these values it is possible to calculate the variation of p when $\log_{10} I$ varies from -2.0 to -4.4

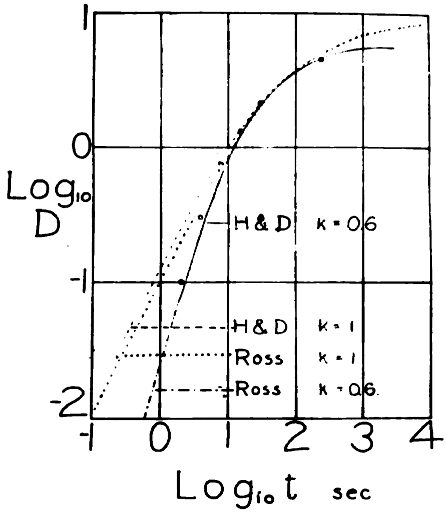
$$\text{For } \log_{10} I = -2.0,\ p = 0.93 \pm 0.03,$$
$$\text{For } \log_{10} I = -4.4,\ p = 0.98 \pm 0.04.$$

It can be calculated that the chances are 26 to 100 that the value of p remains constant when the intensity of light changes by a factor $1:10^{2.4}$, or 1:250, so it is probable that the p of Schwarzschild is not a constant in this case.

COMPARISON OF THE FOUR EQUATIONS

From the figures showing the fit of the empirical curves to the experimental data, it is easy to see that for the data at hand, equations (2) and (4) give the best fit, but accurate statistical methods must be applied before it can be said that one of these equations is superior to the other. The data given here are probably not accurate enough to be adjusted to a degree which would disclose which of the two equations gives the best fit.

FIG. 12



Curves showing agreement to be expected between experimental data (curve −3.0 of Fig. 3) and curves computed from modifications of Hurter and Driffield's and Ross' formulae. Circles represent experimental data, curves represent computed values.

Fig. 12 shows the fit of the four different equations to the experimental curve $\log_{10}D = -3.0$. The circles give the experimental values, and the curves of equations (1), (2), (3), and (4) are respectively labelled: H & D, k = 1; H & D, k = 0.6; Ross, k = 1; and Ross, k = 0.6.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA,
April 13, 1921

A PORTABLE VACUUM THERMOPILE*

BY
W. W. COBLENTZ

I. INTRODUCTORY STATEMENT

The object of this paper is to describe a vacuum thermopile mounting which is rendered portable by attaching a calcium evacuator permanently to the radiometer container.

By this simple means one can maintain a vacuum without being obliged to keep the radiometer attached to a pump.

One of the chief complaints of workers in thermal radiometry is the uncertainty of the measurements, caused by unsteadiness of atmospheric conditions. Radiometers are necessarily of small heat capacity and, as a consequence, are greatly affected by air currents. The loss of heat by convection is very large. The sensitivity of a modern thermopile, (that is, its rise in temperature when exposed to radiation) is doubled and that of a thermocouple of fine (0.01 mm) wire is quadrupled by placing it in an evacuated receptacle.[1]

The container for a vacuum bolometer or thermopile differs from that of an evacuated, glass-bulb, incandescent lamp in that it cannot be thoroughly heated to expel the adsorbed water vapor and occluded gases. The radiometer container is usually provided with a fluorite or rock salt window which cannot be heated without risk of breakage, and hence is attached with a cement that may give off vapors. Furthermore the radiometer receiver is covered with lamp black, etc., containing absorbed gases which slowly escape into the evacuated container.

As a result of these difficulties attempts to seal off the vacuum radiometer permanently from the evacuating device have not been successful.[2] Consequently it is necessary to provide a vacuum pump or other means for removing the hydrocarbon vapors coming from the stop-cock grease and the gases disengaged from the walls or leaking through the joints of the vacuum chamber.

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* Published with the permission of the Director of the Bureau of Standards.

[1] B. S. Bulletin 11, pp. 132 and 613; 1914.

[2] Buchwald, Ann. der Phys., (4), 33, p. 928; 1910.

In a previous paper[3] a simple radiometric attachment was described for maintaining a vacuum by means of metallic calcium, which has the property of chemically combining with all atmospheric gases except argon. It is a simplification of the method used by Soddy,[4] in that the calcium is contained in a quartz tube which is heated to a low red by either a Bunsen burner, an alcohol lamp, or electrically. The calcium, being covered with a thin coating of oxide, does not combine with the quartz container unless it is heated to a very bright red color.

Portable vacuum thermopiles in which the vacuum is maintained by means of calcium have been in use by the writer for nearly seven years. One of the vacuum thermocouple containers (No. 7) used in stellar radiometry[5] was provided with a new quartz tube and calcium on March 22, 1919. On January 19, 1921 there was found only a slight decrease in vacuum, (total pressure perhaps 0.5 mm). This was easily remedied by warming the quartz tube containing the calcium. Container No. 6 used in stellar radiometry was heated Dec. 10, 1917, for the first time after August, 1914, and then again the second time on January 19, 1921, i.e., twice in seven years. This vacuum container which began to leak soon after it was finished, was easily repaired by applying a coat of shellac. However, as mentioned in the previous paper, sufficient air had entered to impart to the Geissler tube discharge the characteristic blue color of argon. In the most recent attempt to remove this residual gas (pressure less than 1 mm after standing three years) prolonged heating of the calcium evacuator had no effect in diminishing the argon.

The linear vacuum thermopile, to be described presently, has been in use for about four years. The first container in which it was mounted always leaked seriously. Nevertheless, after closing the stop-cock (in 1917) and sealing it with Chatterton wax, the vacuum was maintained by occasionally heating the calcium.

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[3] B. S. Bulletin 11, p. 185; 1914.

[4] Soddy, Proc. Roy. Soc., 78, p. 429; 1907. Baly's Spectroscopy, new Ed., p. 433. Also magnesium is reported to be good for "vacuum cleanup."

[5] B. S. Bulletin, 11, p. 624, Fig. 3, 1914.

II. Description of the Thermopile

The thermopile is constructed of bismuth wire, 0.08 mm in diameter, (0.1 mm would be less fragile and hence adapted for general use) and silver wire, 0.031 mm in diameter. The individual receivers are of tin, 1.7 by 1.1 by 0.018 mm. The total length of the receiver of the completed instrument of 12 elements is 12.5 mm, and the reistance is 6.7 ohms. The bismuth wire is cut to the proper length (4 mm) and pressed flat between glass plates to increase thermal conduction where it is in contact with the receivers and thus facilitate the attainment of thermal equilibrium.[6] The base upon which the thermoelements are mounted may be of ivory or of red fiber (shown in Fig. 1) which has been boiled in paraffin to shrink it and to expel the air.

The rate of rise of emf of the thermopile, when exposed to radiation, is so rapid that a galvanometer swing of 2 seconds is not retarded. In fact, when using a galvanometer having a single swing of 2 seconds, the response of the thermopile is too rapid for making observations comfortably. For example, 90 per cent (that is 18 cm) of a total deflection of 20 cm is obtained in about 1 second.

III. Description of the Vacuum Thermopile Container

Radiometer containers which can be evacuated have been described in previous papers.[7]

The thermopile in its new vacuum mounting (20 cm in height) is shown in Fig. 1, which is a rear view. This mounting is in the form of a cup made out of one piece of glass, which is covered with a thick glass window, *W*. The inset, in the lower left-hand corner is a side view of a similar vacuum thermopile container used in stellar radiometry. The quartz tube, (standard size quartz test tube) containing the calcium *Ca*, is attached to the glass tube by means of Khotinsky cement, *K*. Pyrex glass tubing may be used instead of quartz and joined directly to the glass, thus

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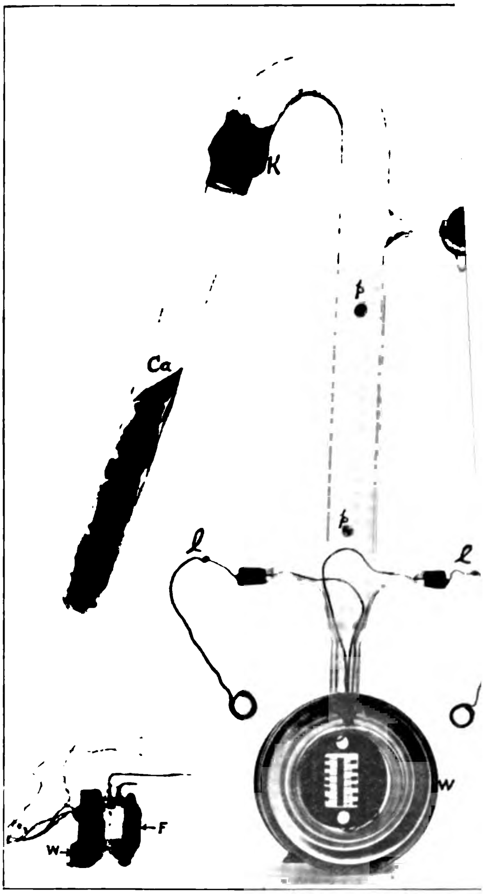
[6] B. S. Bulletin 11, pp. 132 and 613; 1914.

[7] B. S. Bulletin, 9, p. 39, 1911; 13, p. 444, 1916.

dispensing with the cement, but it is likely to
tal overheating.

FIG. 1



VACUUM THERMOPILE

The electrodes, *p*, of platinum wire 0.3 to 0.4
are used for testing the evacuation. For this pur
coil, or a 2,000 to 10,000 volt transformer is used

The glass window, *W*, covering the rear of
7 mm thick. Experience shows that a thick

necessary in order to prevent warping by atmospheric pressure and conseqent leaking through the ground joint. The latter is closed with a combination of beeswax and tallow (stop-cock grease) and the edge is covered with Chatterton wax.

The fluorite window, covering the front of the container, is about 3 mm in thickness, which is sufficient to prevent warping when placed over the slot (8 by 16 mm) in the fine-ground face of the glass receptacle. It is joined to the glass by means of stop-cock grease and the outer edge is covered with Chatterton wax.

The lead wires to the thermopile, *l*, are of copper, sealed to the glass with a platinum wire. This refinement is not necessary, though it is desirable to reduce the number of places where leakage may occur.

The glass receptacle is mounted in a brass box which slides in vertical ways as illustrated in previous papers.[8]

In concluding this description of the thermopile container it is relevant to add a few remarks concerning the operation of the calcium evacuator. The calcium is out into small fragments with a chisel or by other means and placed in the quartz tube, which is then attached to the thermopile container. The device is then evacuated with an oil pump. While the pumping is in progress the calcium is heated with a Bunsen flame in order to cause it to combine with the water vapor, the residual air and the acetylene gas which is generated from the water vapor and the calcium carbide which is formed in previous use. Keeping the calcium hot, fresh air is allowed to enter and the pumping is continued. This operation should be repeated several times to sweep the water vapor from the walls of the container, after which the stop-cock is permanently closed.

With reference to the efficiency of the device it may be stated that, after the gases have been absorbed by the calcium, the vacuum is so high that the discharge from an induction coil passes through the 5 cm air space between the electrodes *p*, *p*, Fig. 1, in preference to passing through the evacuated tube.

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[8] B. S. Bulletin 11, p. 156, 1914; 13, p. 446, 1916 ("Mounting for Vacuum Thermopiles").

As mentioned elsewhere, an electric heater
it requires close attention to avoid overheating
It is not absolutely necessary to use a pump to
tion. However, explosions may result from th
absorbed water vapor, acetylene, etc., and the
in gas pressure, when the calcium is first heate
at atmospheric pressure. Hence provision m
increase in gas pressure during the initial heati
in a closed tube containing air at atmospheric pr
remembered that the chemical action begins at
is most effective at a low red heat.

The first samples of the vacuum chamber, v
cium evacuator, described in the foregoing pag
about 7 years ago and taken 3,200 miles to an ob
mishap. This experience showed that equip
vacuum radiometers of this type one can go
station without taking a pump.

A further improvement will come from th
cement for securing the windows which will no
and thus increase the efficiency of the device.

IV. CONCERNING RADIATION SENSITIVIT VACUUM RADIOMETERS

It is beyond the scope of this paper to describe
of calibration of radiometers and the various me
be employed to test the variation in their radiat

In this connection it is to be remembered th
of the auxiliary Thomson galvanometer varies
change in terrestrial magnetism, etc. The c
may therefore be tested by a simple device desc
papers.[9]

The radiation sensitivity of a bolometer and
thermopile (because of its greater heat capacity
the kind and the pressure of the residual gas. T

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[9] B. S. Bulletin 10, p. 19, 1913; 11, p. 171, 1914. (Specific
Sensitivity of a Thermopile).

bolometer is 2.5 as great in hydrogen as in air at the same gas pressure. It has been found by Reinkober[10] and also by the writer that the radiation sensitivity of the Ruben's thermopile (of wire 0.15 mm in diameter) in a vacuum is only about 1.5 times that in air. The radiation sensitivity of the present thermopile is doubled in a vacuum. In this instrument the sensitivity varies but little during the course of a day's work. In making transmissivity (and reflectivity) measurements it is sufficient to have the radiation sensitivity remain constant while making the two sets of measurements, viz., (1) the intensity of the radiation transmitted through the substance, and (2) the intensity of the direct radiation.

If it is desired, the variation in radiation sensitivity of the vacuum thermopile (and of vacuum radiometers in general) may be determined by means of a standard incandescent lamp,[11] or other suitable means.

In conclusion it is relevant to emphasize the fact that the increase (say 50 to 100 per cent) in radiation sensitivity by placing the ordinary laboratory radiometer (thermopile) in a vacuum is relatively small and that the chief gain lies in elimination of unsteadiness of the galvanometer reading by elimination of convection currents. This is easily accomplished by keeping the air pressure less than 0.1 mm.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
Mar. 4, 1921.

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[10] Reinkober, Ann. der Phys., (4) 34, p. 343; 1911. Lebedew (Ann. der Phys., (4) 9, p. 209; 1902) found that above 5 mm (also below 0.01 mm) a variation in gas pressure has but little effect upon the sensitivity.

[11] McCauley, Astrophys. J., 37, p. 164; 1913.

THE ORIENTATION OF THE GRAINS IN A DRIED PHOTOGRAPHIC EMULSION*

BY

L. SILBERSTEIN

In my first paper on this subject the number of grains (small plates) whose inclination in the initial state falls within the limits θ and $\theta+d\theta$ was, through an obvious inadvertency, assumed to be $\frac{2}{\pi} N d\theta$, as if the plate-normals were confined to a plane. The directions of these normals being haphazardly distributed in *space*, the number in question is manifestly equal N times the ratio of the area of the spherical zone $d\theta$ to that of the hemisphere, *i.e.*,

$$N \sin\theta \, d\theta \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

This then has to replace the first (and only) expression on p. 171.

The remainder of p. 171, as well as the whole of p. 172 and p. 173 up to section 1 are unaffected, while the formulae concerning the orientation in the final, dry state of the emulsion are to be modified as follows:—

1. By the "sine-law," $\sin\theta' = k \sin\theta$, formula (1), we have, as before, $\cos\theta' = \sqrt{1-k^2\sin^2\theta}$, and therefore, by ($a$), for the efficiency coefficient κ, as defined on p. 171,

$$\kappa = \int_0^{\pi/2} \sin\theta\sqrt{1-k^2\sin^2\theta}\, d\theta$$

or, putting $\cos\theta = u$,

$$\kappa = \int_0^1 \sqrt{1-k^2+k^2u^2}\, du.$$

The integral is easily evaluated and gives the efficiency coefficient

$$\kappa = \tfrac{1}{2} + \frac{1-k^2}{4k} \log \frac{1+k}{1-k} \ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{I})$$

or, developing the log,

$$\kappa = 1 - \frac{k^2}{1.3} - \frac{k^4}{3.5} - \frac{k^6}{5.7} - \ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{I}')$$

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* Corrigenda to a paper of equal title published in the March number of this Journal, pp. 171–177, 1921.

of which, the contraction ratio k being a small fraction, the first two terms will suffice in practice. For $k=0$ we have $\kappa=1$, an obvious result, and for $k=1$ (initial state) $\kappa=\frac{1}{2}$, as it should be. The whole range of κ is thus contained between $\frac{1}{2}$ and 1.

If the thickness ϵa of the grain-plates is taken into account (p. 174), the efficient area is increased by $a\epsilon k.\ \sin^2\theta=\frac{1}{2}a\epsilon k$, and the correction term $\frac{1}{2}\epsilon k$ is to be added to the value of κ given above.

2. By the alternative assumption on the contraction process, the "tangent-law," $\tan\theta'=k\tan\theta$, we have $\cos^2\theta'=1:(1+k^2\tan^2\theta)$, as before, and, by ($a$), substituting $u=\sin^2\theta$,

$$\kappa=\int_0^{\pi/2}\frac{\sin\theta\cos\theta\,d\theta}{\sqrt{1-(1-k^2)\sin^2\theta}}=\frac{1}{2}\int_0^1\frac{du}{\sqrt{1-(1-k^2)u}}.$$

Thus the efficiency coefficient corresponding to the tangent-law is simply

$$\kappa=\frac{1}{1+k}\qquad\text{(II)}$$

This gives for $k=1$ and $k=0$ the values $\kappa=\frac{1}{2}$ and 1, as above, while for intermediate k the values of κ are smaller than those given by (I). For contractions such as $k=\frac{1}{10}$ the two κ-values differ only by about 9 percent. The following table (replacing that of p. 175) gives κ on either assumption for a few values of k:

k	=1.0	0.5	0.3	0.2	0.1	0.0
κ(I)	=0.5000	0.9125	0.9695	0.9866	0.9967	1.0000
κ(II)	=0.5000	0.6667	0.7692	0.8333	0.9091	1.0000

(The remainder of p. 175 is unaffected.) On the sine-law the total range of inclinations θ' in the dry film is 0° to $a=\arcsin k$, and on the tangent-law, 0° to 90° (as before).

Finally, concerning the distribution through these angle ranges (p. 176), if $N'd\theta'$ be the number of grains whose inclinations in the final state fall within the interval θ' to $\theta'+d\theta'$, we have

$$N'd\theta'=N\sin\theta\,d\theta.$$

Thus $\left(\frac{d\theta}{d\theta'}\right.$, being as on p. 176$\left.\right)$ we have, on the *sine-law*, the final distribution law

$$N'd\theta'=N\frac{\cos\theta'\sin\theta'\,d\theta'}{k\sqrt{k^2-\sin^2\theta'}},\qquad 0<\theta'<\arcsin k\qquad\text{(4)}$$

and, on the *tangent-law*,

$$N'd\theta' = N\,\frac{\sin\theta' d\theta'}{k^2\cos^2\theta' + \sin^2\theta'},\qquad 0<\theta'<\frac{\pi}{2}\,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4^t)$$

Formula (4^s) gives, for the number $n'(\theta')$ of grains whose inclinations fall within the interval 0 to θ',

$$n'(\theta') = \int_0^{\theta'} N'd\theta' = \frac{N}{2k}\int_0^{u} \frac{du}{\sqrt{k^2-u}},$$

where $u = \sin^2\theta'$, *i.e.*,

$$n'(\theta') = N\left[1-\sqrt{1-\frac{\sin^2\theta'}{k^2}}\right] = N\,[1-\cos\theta],\quad 0<\theta'<a\,\ldots\ldots\ldots\ldots(5^s)$$

as it should be, for all these grains belonged originally to the range 0 to $\theta = \arcsin\,(\frac{\sin\,\theta'}{k})$. Similarly, in the case of the tangent-law,

$$n'(\theta') = N\,[1-\cos\theta],\qquad \theta = \arctan\left(\frac{\tan\theta'}{k}\right),\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5^t)$$

valid for any θ' between 0 and $\frac{\pi}{2}$.

By (5^s) we have, for example, for $\sin\theta' = \frac{1}{2}k$,

$$n' = N\left(1-\frac{\sqrt{3}}{2}\right),$$

i.e., 13.4 per cent of all grains (the remaining 86.6 per cent being contained between $\arcsin\frac{k}{2}$ and $\arcsin k$). For small k formula (5^s) can be written, approximately,

$$n'(\theta') = \frac{1}{2}N\left(\frac{\theta'}{k}\right)^2.$$

The tangent-law formula (5^t) gives for $\theta' = 45°$, for instance, when $\tan\theta = 1/k$,

$$n'\left(\frac{\pi}{4}\right) = N\left[1-\frac{k}{\sqrt{1+k^2}}\right].$$

Thus, if $k = \frac{1}{10}$,

$$n'(45°) = 0.905N,$$

that is to say, 90½ per cent of the grains have inclinations contained between 0° and 45°, and the remaining 9½ per cent between 45° and 90°.

EASTMAN KODAK CO
ROCHESTER, N. Y

SOME MAJOR PROBLEMS IN PHOTOMETRY*

BY

E. C. CRITTENDEN and J. F. SKOGLAND

I. INTRODUCTION

The purpose of this paper is to present a general statement of certain important problems in practical photometry. The questions raised are not new, but have been under discussion for many years. Notable progress has been made, but some difficult problems still remain. For workers in optics outside of this special field it appears worth while to give a brief statement of the present situation as a reminder that these problems still exist and that investigators in related fields may be able to make material contributions to their solution.

In some respects photometry is now on a very satisfactory basis, particularly in its industrial applications, so that it may perhaps be considered fairly advanced as an art, if not so far along as a science. The value of the fundamental *unit*, the candle, is firmly established and undoubtedly can be maintained for a long time with a high degree of certainty. A group of about fifty carbon-filament incandescent electric lamps now maintains this unit at the Bureau of Standards. These lamps are carefully preserved and are burned only at such times as it is considered necessary to recalibrate the secondary standards which are used for routine tests. Consequently there is little danger of any appreciable drift in their mean value, and it is believed that the unit can be maintained for a very long time to within one-tenth per cent for the average of all, and to within two-tenths per cent for individual groups of from ten to twelve lamps. By comparison with such standards the more common sources can be measured with an accuracy which is fairly satisfactory.

From another point of view, however, more in line with scientific considerations, the whole field of light measurement is in an

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[1] Presented at the December, 1920, meeting of the Optical Society of America. Published with the permission of the Director of the Bureau of Standards.

unsatisfactory state. In the first place there is no really reproducible fundamental standard. Even if it be granted, that such a standard is not vital to successful photometric measurements, and that an arbitrary unit well maintained meets the requirements of practice, there remains a second difficulty in that no general agreement has been reached regarding the methods which should be used in measuring lights which differ in color from that given by the standard source.

There are therefore two problems presented, which in their broader sense embrace all photometry. The first is the establishment of a reliable and reproducible primary standard of light; the second, the specification of the methods to be used in comparing other lights with that of the standard. For the present purpose it will suffice to outline the actual status of these problems without attempting to indicate what may be their ultimate solutions.

II. PRIMARY STANDARDS

As a preliminary to the discussion of standards it may be well to emphasize the distinction between the *unit* of measurement and the *standard*. Some years ago all the important nations except the Germanic group came to an agreement on a common unit, and the relation between this unit and the German Hefnerkerze was established as the simple ratio of 10 to 9. Incidentally in all countries the name given to the unit is *candle*, or an equivalent word, but no country has actually retained the candle as a standard. In passing from the candle as a standard, however, each country has followed an independent course, and consequently the nominal basis of the common unit is different in each of the four great nations, France, Great Britain, Germany, and the United States. The first three of these adopted different flame lamps as standards. If the units were actually redetermined from these standards which are nominally accepted as fundamental, it is therefore probable that differences would appear among the units thus derived.

In France the classic basis for the value of the candle is Violle's platinum standard, but no one ever succeeded in reproducing

Violle's results, and it is now recognized that this standard is far from being exactly reproducible. In practice Violle's determination of the unit was applied by using the Carcel lamp with the value which he assigned to it, and until recently this lamp has been accepted as at least an intermediate reference standard. The law of April 2, 1919, while referring to the Violle standard, specifically says that the unit, the Bougie Décimale, is represented practically and in a permanent manner by means of incandescent (electric) lamps deposited in the Conservatoire Nationale des Arts et Métiers. In Great Britain the candle was superseded by the Harcourt 10-candle pentane lamp, but since it is at least difficult to reproduce such lamps exactly, the standard most generally accepted is not the pentane lamp in general, but a particular lamp of this kind at the National Physical Laboratory. Moreover, the standards actually used at the National Physical Laboratory are electric incandescent lamps. Values were originally assigned to these lamps by comparison with the pentane lamp mentioned, but published records indicate that only two series of such comparisons have been made, so that in effect the unit is maintained by the electric standards. In Germany the Hefner lamp is presumably still the generally recognized standard. Its extreme simplicity gives it an exceptional degree of reproducibility, but with so simple a lamp one can obtain only a very yellow light of low intensity, and in transferring values to standards of considerably different color and intensity such as are necessary for practical use grave difficulties are introduced. These difficulties have become progressively greater as efficiencies of ordinary illuminants have been improved, with a consequent greater departure from the color of the flame lamps, particularly the Hefner.

In addition to the uncertainties inherent in these various flame standards, all of them show systematic variations resulting from atmospheric conditions. Consequently in every case where definite values have been established, the actual light output of the flame standards under different conditions has been determined by means of incandescent electric lamps, which are not affected by these conditions. The electric lamps have proved so much more reliable and convenient than the flames that there has been a marked tendency to use them as the real standards, comparisons with the flames being rarely made and then usually only as approximate checks. In fact the international agreement mentioned has been obtained and is maintained by means of comparisons between groups of electric standards.

This procedure is in part analogous to that followed in the case of the international ohm, which is maintained by means of wire resistance standards whose value is derived from mercury ohms set up at infrequent intervals, but there is the very important difference that there exists no reproducible primary standard of light analogous to the mercury ohm. In the adoption of the unit of candlepower in the United States this difficulty was faced, and it was decided not to adopt any of the existing nominal primary standards, but to base all values on a large group of electric incandescent standards pending agreement on a more satisfactory primary standard.

Some fourteen years have passed since this decision was made. During that time extensive experimental studies of the various flame standards have been made, and the net result of these investigations has been a confirmation of the wisdom of the course taken. It appears that the flame lamps at best can serve only as a comparatively rough check to show any considerable drift of the electric standards. Tradition and national pride make it difficult for any country to give up its old standards, but so far as the United States is concerned the field is entirely open, and internationally the time appears favorable for the consideration of new proposals in this field.

Without attempting to review the many proposals and the relatively few direct experimental contributions made to this subject during the past decade, it may be said that progress in related fields of work has undoubtedly brought nearer the possibility of two solutions of the problem, either of which would be satisfactory. One of these is the precise specification of the conditions under which some radiator, such as a black body, will give light of a definite intensity; the other is the accurate measurement of radiant power of specified quality incident on a surface. The first appears the more promising, but it yet remains to be shown experimentally that the conditions of operation of the radiator can be controlled so as to reproduce light intensities with a certainty of one per cent or better. At the present time any procedure which would promise this degree of certainty would receive serious consideration, but as indicated above, in order to replace the present electric lamps as the ultimate basis of reference, a standard must be capable of reproducing values within a few tenths of one per cent over a long period of time.

III. GENERAL PHOTOMETRIC METHODS

1. *Significance of "Brightness."* When a unit of light has been established and standards obtained which maintain or reproduce this unit with the required accuracy, questions still arise regarding the meaning of "measurements" of light, and the significance of the results obtained by different methods.

So long as the lights dealt with are of about the same color it is easy to determine quantitative relations between them, and it is generally agreed that measurements shall be made by methods which consist essentially of obtaining an equality of brightness on two similar surfaces. But when we have to deal with lights differing considerably in color it becomes doubtful what is meant by equal "brightness." Although it is convenient to describe lights (or colors) by reference to three properties which may be called hue, saturation and brightness, this analysis is more or less arbitrary, since sensations are not really separable into components. In assigning values to such an assumed property of a sensation we are at best on rather uncertain ground. Moreover it is certain that no one numerical value can represent the relative usefulness or effectiveness of two lights which differ in color, since their relative values must depend on the purposes to be served and on the conditions under which they are to be used. But for the very purpose of studying relative usefulness or effectiveness of different lights under different conditions it is necessary to have some means of describing them quantitatively. If radiometric measurements were not so difficult to make there would be decided advantages in comparing lights on the basis of radiant power, supplemented by suitable qualitative specifications. In practice, however, the most convenient and useful way to compare lights quantitatively is by a measurement of the somewhat indefinite property called "brightness." The unsettled question is, by what method this comparison is to be made.

2. *Ordinary Photometric Measurements.* As a natural development the same types of photometers used for measurements involving no difference of color have been commonly used also for measurements in which color differences occur. If this procedure gave satisfactory and consistent results it would have been accepted without question, but such serious difficulties rise when the color differences are considerable that the results obtained are inconclusive and unsatisfactory, and it appears that if any systematic and general method of photometry is to be established it must be something other than the method of direct comparison by the usual photometer.

It should be recognized that the ratio of the brightness of two lights which differ in color is not a definite and single-valued quantity, but depends upon the conditions of observation as well as upon the characteristics of the individual observer's eye. In particular, in order to make the results definite it is necessary first to specify more or less precisely the size and brightness of the photometric field, which determine the parts of the retina used and its level of adaptation. This condition is easily fulfilled. It is also necessary to include observations by a sufficient number of individuals to approximate the results which would be obtained by the average, or normal, observer. This last condition is difficult to realize in practice in any one laboratory when the ordinary photometer is used. In the first place consistent results can be obtained only by selected observers who have had considerable practice in the particular kind of measurements to be made, and no laboratory can be expected to have many observers who have sufficient experience to make satisfactory measurements. There is also a natural tendency in selecting and training observers to bring the new ones into agreement with the more experienced, and thus to perpetuate any peculiarity shown by the original staff. Experience has shown that groups of observations which are apparently very reliable because of their high precision may be decidedly inaccurate when referred to values established by larger numbers of observers. This difficulty can be obviated to some extent by the collaboration of a number of laboratories; and for some kinds of measurements, such as the comparison of different types of incandescent electric lamps, fairly satisfactory results can be obtained by establishing secondary reference standards of the different types. This course has already been followed to some extent in international comparisons through which approximate agreement on certain tungsten filament standards has been obtained, thus supplementing the original agreement on a basic unit preserved by means of carbon filament lamps. The values for these secondary standards are originally determined by comparison with the fundamental standards, but the time and labor required to obtain a satisfactory agreement are so great that values once determined and agreed upon are likely to be considered as practically independent. Such multiplication of standards, while serving to meet the more urgent commercial needs, is limited in its application. There is always doubt as to the certainty with which the values can be reëstablished by reference to the fundamental standards, even though the color differences bridged are the relatively small ones represented by incandescent lamps of different efficiencies.

3. *Flicker Photometry*. As a more general solution it has been proposed to use the flicker photometer, which presents the two photometric surfaces alternately in succession in the same place instead of simultaneously side by side. Its advantages depend upon the fact that the surfaces can be interchanged at such speeds that the disturbing hue differences are no longer seen while differences of brightness still show in the form of a perceptible flickering of the light in the field. Thus a setting for equal brightness may be made by finding the point at which the flicker disappears. Whether the "brightness" thus determined is exactly the same thing as that found by the ordinary photometer is a disputed question, but since brightness must be rather arbitrarily defined in either case this particular question does not appear to be of much importance. At any rate, a comparison of values obtained by these two methods indicates that approximately the same quan-

tities have been evaluated, and the differences are not so great as those which appear, for example, between acuity and equal-brightness results.

A choice of conditions giving exact agreement between the two methods would, of course, be most desirable. Ives, who has done most to develop the possibilities of the flicker method, decided from his experiments that if a small field (2°) and a fairly high brightness (2.5 millilamberts) were used on each photometer the same results would be obtained on the two. Such independent evidence as is available seems to corroborate this conclusion, but the conditions specified are considerably different from those of the ordinary standard photometer, and results obtained with it differ accordingly. If the new conditions proposed are adopted, some values already established under the older conditions will presumably have to be modified slightly, but the changes required are of the same order of magnitude as the uncertainty which would be involved in repeating the old determinations.

The flicker instrument is more complicated and consequently more difficult to keep in good working order. Measurements with it are also somewhat slower and considerably more fatiguing. It is essentially an instrument for the research or standardizing laboratory, rather than for general use. Its great advantages are that definite and fairly precise measurements can be made on lights of any color, and that erratic variations of individual judgment are reduced, so that from an observer's results in one kind of measurement his performance in another kind can be accurately predicted. This makes it possible to select observers systematically to represent proper average characteristics. Such averages or normals for any desired color can easily be established by large numbers of observers, thus avoiding the assumption that the work of a few specially selected observers is a fair representation of average visual characteristics. Although full experimental data are lacking, experience so far available indicates that, with the flicker photometer properly used, different laboratories may readily obtain good agreement in measurements involving color differences of any degree.

4. *Use of Visibility Curves.* In a sense the ultimate result of any method of comparing the intensity of lights of different colors is expressed in a "visibility curve," that is, a curve showing the relative visual effect of radiant power of different wave lengths throughout the spectrum. Given such a curve and the spectral transmission curve of any absorbing screen, the determination of the total transmission becomes merely a matter of calculation or of graphic determination.

Visibility curves differ considerably for different conditions of measurement, as well as for different individuals under given conditions. Curves obtained by different observers under the same conditions scatter over a range much greater than the range of the average curves obtained by different methods. Moreover, the experimental difficulties in determining these curves are great, and systematic errors of considerable magnitude may occur in the best determinations. Priest has therefore proposed as a solution of the whole problem of heterochromatic photometry the adoption of an arbitrary normal visibility curve to be used especially in calculating the transmission of quartz-nicol combinations which can be chosen to equalize almost any color difference, thus reducing all photometric observations to a color match.

This procedure is undoubtedly practicable, and such use of visibility curves already gives results in some cases more reliable than those obtained by direct measurements of the usual kind, but it is hardly to be expected that direct measurements in general will be discarded or given a secondary position. Instead of this, a visibility curve which may be adopted as normal will presumably be adjusted to obtain results in concordance with direct measurements, and this adjusted curve will be used within the limits of its applicability. For example, such a curve when established will also constitute the transmission curve of solutions for use with non-selective radiometers as physical photometers. The basis of photometry must, however, continue to be evaluation by the sense of sight, and this is the fundamental argument for the adoption of a visual method as the basic one.

5. *Choice of Basic Method.* At present there appear to be three possible methods of meeting the difficulties of heterochromatic

photometry. First, the usual standard photometer may be retained as the basis of all measurements, the effects of its unsatisfactory features being in part avoided by the establishment of secondary standards for lights of various colors and by calculations from a visibility curve in extreme cases. Second, the flicker photometer may be adopted as the standard method for all color-difference measurements, visibility curve calculations being used as auxiliary only. Third, by some adjustment of conditions all three methods may be brought into approximate agreement.

The last is of course the solution to be desired, but it is not clear how it can best be brought about. It is certain that for some time to come the question of the use of the flicker photometer will be a live one, and every photometric laboratory may well acquire some acquaintance with this instrument, which in our experience has gained greater confidence in regard to certainty of results and adaptability to all measurements as its use has become more common in the laboratory.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A REMEASUREMENT OF THE RADIATION CONSTANT, h, BY MEANS OF X-RAYS[1]

BY

WILLIAM DUANE, H. H. PALMER and CHI-SUN YEH

Since the discovery of the fact that the short wave-length limit of the continuous x-ray spectrum obeys the quantum law,[2] $Ve = h\nu$, this phenomenon has been used by a number of experimenters[3] to determine the value of h. The most accurate measurement of h in this laboratory was made by Blake and Duane,[4] who obtained the value $h = 6.555 \times 10^{-27}$, with $e = 4.774 \times 10^{-10}$, (Millikan's value).

Some years ago our laboratory purchased 200 high resistance manganin wire coils for the purpose of measuring the voltages applied to x-ray tubes by comparing them directly with the electromotive force of a standard cell by the simple potentiometer principle. We hoped by this means to measure the voltages with such accuracy that we could neglect the errors in them in comparison with those in the other quantities entering into the quantum equation. It now appears that we can do this, and we are able, therefore, to test the quantum law under various experimental conditions and measure the value of h with greater precision and accuracy than before.

In addition to the voltage measurements improvements have been made in the measurements of the frequency, ν. We have used a new x-ray spectrometer with scales that appear to be somewhat better than the old ones (there being no appreciable eccentricity) and with a new calcite crystal. Further, the x-ray tube had a long side arm attached to it with a very thin mica window W at its end. This extended out toward the spectrometer (Fig. 1). The fact that the mica allowed a much larger fraction of the x-radiation to pass through enabled us to use a narrower slit (Slit 1), which reduced the correction that must be applied for the slit's width.

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[1] A paper presented to the American Physical Society at its Washington meeting, April, 1921.

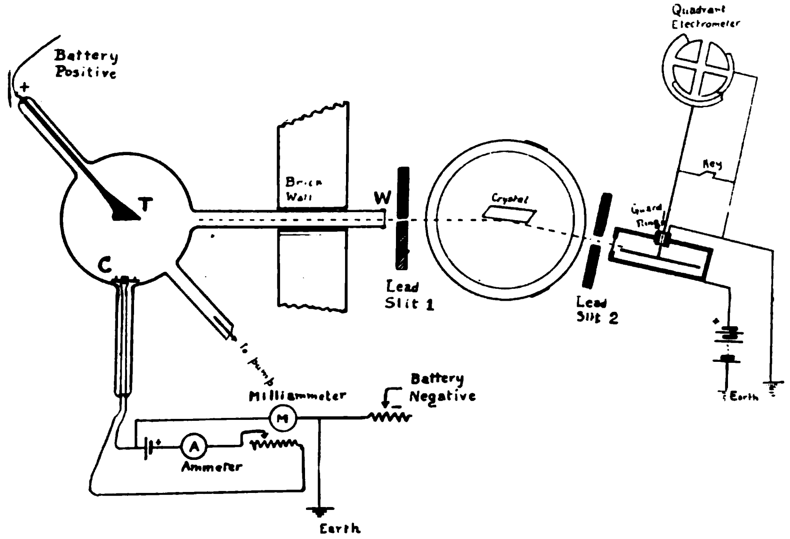
[2] Duane and Hunt, Phys. Rev., Aug., 1915, p. 166.

[3] For references etc., see reports in the Jahrbuch der Radioaktivität for Dec., 1919 by E. Wagner and for Dec. 1920 by R. Ladenburg and also a report on Data Relating to X-Ray Spectra by William Duane, published by the National Research Council.

[4] Phys. Rev. Dec. 1917, p. 624.

In estimating the difference of potential, V, through which the electrons in the x-ray tube pass we must take account of the following considerations. The x-ray tube was constructed in our laboratory, and was permanently connected to a series of three pumps—a mercury vapor diffusion pump, a Gaede rotary mercury pump and a rotary oil pump. We had no difficulty in exhausting the tube so that no appreciable current could be ascribed to the ionization of the residual gas in it. The x-ray tube contained a Coolidge cathode, which consists of a spiral tungsten wire that can be heated by means of a current coming from a small auxiliary storage battery. The electrons emitted by this wire, when hot, carry the current through the tube between the anode, T, and the cathode, C, (Fig. 1). Owing to the heating current flowing through the spiral wire the points on it are not all at exactly the same potential. An examination of the electrical connections represented in Fig. 1 shows that the points on the spiral wire

FIG. 1



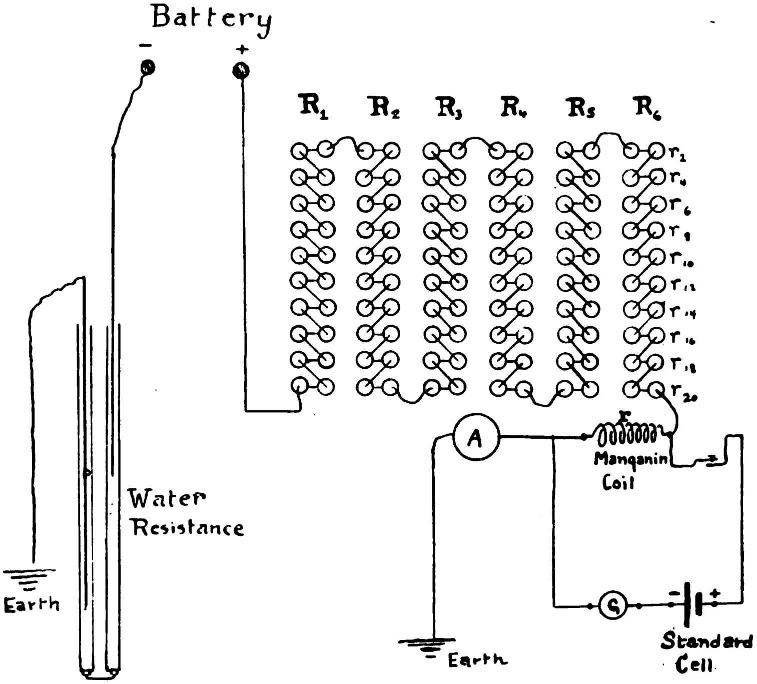
had potentials above that of the gas pipe (marked earth in the figure), to which the electrical circuits were connected by a soldered junction. Since in the quantum relation we deal with the maximum frequency of the x-rays in the continuous spectrum, we must estimate the maximum difference of potential between the anode and any point on the cathode. This point is the end of the spiral nearest to the wire connecting it to earth. Measurements by means of a high resistance voltmeter indicated that this point had a potential about 0.7 volt above that of the earth under the conditions during the experiments. We have neglected this difference of potential in comparison with that applied to the x-ray tube, which amounted to 24,413 volts. We have also neglected the effect of any Volta difference of potential that may have existed between the anode and the cathode, and of the fact that the electrons leave the hot tungsten spiral with velocities due to its high temperature.

Neglecting the corrections mentioned in the preceding paragraph it appears that the value of V to be used in the quantum equation is the difference of potential between the wire connected to the anode of the x-ray tube and the earth. This difference of potential we measured by means of the simple potentiometer represented in Fig. 2. We used a high potential storage battery to excite the x-ray tube, and connected the wire joining its positive pole to the anode of the tube with the earth through the various resistances R_1, R_2, etc., the x-ray tube and the potentiometer being, therefore, in parallel circuits.

In insulating the various circuits between the positive pole of the battery and the earth we used the same precautions as are employed in making ionization measurements with an electrometer or an electroscope. All instruments, ammeter, galvanometer, standard cell, etc. rested on hard rubber supports the surfaces of which we cleaned carefully by means of sandpaper. No wires touched the tables or walls of the room etc. They were supported on silk threads. The various resistance coils, r, r_1, r_2, etc. hung from shellacked glass rods, which in turn rested in six wooden frames, forming six sections, R_1, R_2, etc., of the main resistance. These six wooden frames hung by silk threads from a broad, horizontal board, which served not only as a support, but also to keep the dust off of the insulating surfaces. We tested carefully the insulation. In particular, on closing the circuit to the high potential battery no permanent deflection of the galvanometer, G, occurred, if the key in its circuit was open. A transient deflection occurred, doubtless due to electrical induction.

In order to compare the difference of potential applied to the x-ray tube with the emf of the standard cell we do not have to know the absolute values of the resistances. Their ratios suffice. To determine these ratios we proceeded as follows. We measured the resistance of r_{20} by means of a standard Wheatstone bridge, and found it to be $r_{20} = 50{,}214.6$ ohms. We do not have to examine the accuracy of this value. We assume it to be correct, and compare the resistances of the other coils with it. To do this we formed a Wheatstone bridge with three of the coils of section R_5,

FIG. 2



and substituted in it, as the fourth arm, each one of the coils of section R_6 in succession. The galvanometer had a sensitiveness of 10^{-11} ampere per mm of scale deflection, and we used as much of the high potential battery as would send a current of 4.0 milliamperes through each arm of the bridge. (About 4 milliamperes passed through the potentiometer circuit during the experiments with x-rays). With each of the coils of section R_6 in the bridge we determined the resistance that had to be added to it in order to balance the bridge. These resistances could be measured to within about 0.1 ohm, and, as they were to be added to resistances of over 50,000 ohms they did not have to be known with great accuracy. From these values we calculated the numbers of ohms that must be added to r_{20} to make the sums equal to the other resistances in section R_6 respectively. These resistances appear in Table 1.

TABLE 1

Resistances of Coils

Values of $r_x - r_{20}$ in ohms

+304.9	+271.4	+203.8	+321.4
+101.6	+174.6	+175.4	+ 82.3
+133.4	− 35.8	+264.5	+ 39.1
− 51.1	+161.7	+330.8	+ 24.2
+151.8	−220.7	+122.4	
			+2555.8

From these values we compute the total resistance of section R_6 to be

$$R_6 = \sum_{x=1}^{x=20} r_x = 20 \times r_{20} + 2555.8 = 1{,}006{,}848,$$

and this value must be correct to within a few ohms, assuming always that r_{20} has the above mentioned value.

To measure the resistances of the other sections R_1, R_2, etc. we formed Wheatstone bridges with the several sections as arms, and by substituting R_6 and each of the other sections in them we determined the resistance that must be added to R_6 to equal the resistance of each of the other sections. Table 2 contains these values.

TABLE 2

Resistances of Sections

Values of $R_y - R_6$ in ohms

−866.4	+899.8	+452.3
−378.6	−1691.	
		−1584.

They could be measured to within about one ohm, so that the total resistance of all six sections amounts to

$$R = \sum_{y=1}^{y=6} R_y = 6R_6 - 1584 = 6{,}039{,}504 \text{ ohms},$$

and this appears to be correct to within a few 10's of ohms. In these experiments, also, the current through each arm of the bridge was maintained at about 4 milliamperes.

It remains to determine the resistance of the coil r, to the ends of which are attached the terminals of the standard cell circuit. The ratio of r to r_{20} we measured by means of a Wheatstone bridge in which the other two arms consisted of standard coils of 50 and 10,000 ohms respectively. The values of these coils were certified by the Bureau of Standards, and further their ratio was carefully checked in Professor H. N. Davis' laboratory by comparison with a number of other certified standards. The value of the ratio we used is 199.998. Except r, all the coils we used in our measurements were several years old and it does not seem unreasonable to suppose that they remained sufficiently constant during the course of our experiments. We wound r, however, expressly for these experiments, and we did not feel justified in assuming that it would remain constant. We therefore determined for each of our measurements of h the number of ohms that had to be added to r_{20} in order to make its ratio to r equal the ratio of the standard coils as given above. These resistances, called x, appear in the third column of Table 3. The fourth column contains the values of r calculated from x, assuming that $r_{20} = 50{,}214.6$ ohms, as above. In these experiments we found that, if the coils r and r_{20} had been heated by a current of 4 milliamperes for some time, the value of x was 1 ohm larger than when cold. We recorded the larger value.

TABLE 3

Resistances and Differences of Potential

Date	Temperature	Resistance x	Resistance r	Emf. of Standard Cell	Difference of Potential, V
March 15	21°	185.1	252.014	1.01863	24412.4 ± 2.0
March 21	22°	185.3	252.015	1.01863	24412.3
March 30	20°	183.8	252.007	1.01864	24413.3
April 5	19°	183.8	252.007	1.01864	24413.3

By taking the ratio of $(R+r)$ to r we get the ratio of the voltage applied to the x-ray tube to the emf of the standard cell under the condition that no current passes through the galvanometer circuit. During the course of an experiment one observer watched the galvanometer and kept changing the variable water resistance (Fig. 2) so as to keep the galvanometer reading as near zero as possible. Deflections of several mm could not be avoided, but, as the galvanometer had a sensitiveness of 10^{-11} ampere and the resistance in its circuit was not very large, these variations did not correspond to appreciable fluctuations of the voltage.

With the above described measurements and procedure it seems reasonable to assume that we knew the value of the voltage and maintained it constant during an experiment to within an accuracy represented by the accuracy with which we know the emf of a standard cell. We had two Weston unsaturated cells at our disposal each with a certificate from the Bureau of Standards, in which the accuracy was stated to be one part in ten thousand. We compared these two cells with each other during each experiment by means of a potentiometer. In each case the difference between them agreed with the difference between their certified values to within the accuracy claimed in the certificates. In estimating the emf of the cell actually used in a measurement we gave equal weights to the certificates. The mean value of this emf appears in column 5 of Table 3. The sixth column contains the differences of potential, V, applied to the x-ray tube. We think these values are correct to within about two volts.

In determining the position of the short wave-length limit of the continuous spectrum (the value of ν to be substituted in the quantum equation) we proceeded as in previous experiments except that we used a slightly different method of estimating the slit correction. While one observer watched the galvanometer, G, and kept the voltage constant another measured the rate of deflection of the quadrant electrometer, which determines the current in the ionization chamber (Fig. 1). Curves A and B in Fig. 3 represent these ionization currents (measured on both sides of the zero of the instrument) as functions of the readings of the verniers on the scale that fixed the positions of the reflecting crystal. The horizontal portions of these curves correspond to the natural leak of the instrument and to that due to stray radiation etc. The inclined portions correspond to x-radiation reflected from the crystal. The inclined portions recorded in the figure occupy angular breadths of 2′ and 3′. Since the glancing angle, θ, amounts to about 290′, this means that the part of the radiation actually measured in this experiment lay within about one per cent of the end of the continuous spectrum. The limit of the spectrum appears, therefore, very sharply defined. We believe that we can estimate the minimum value of θ in a good experiment to within about 7″ of arc.

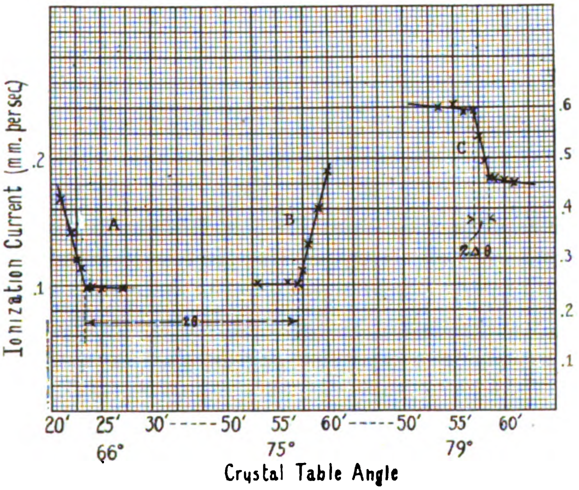


FIG. 3

Owing to the fact that the rays passing through the slit 1 are not all exactly parallel to each other a small correction has to be added to the angle as measured from curve A to curve B. The correction to be added to the double glancing angle, 2θ, equals the greatest angle between any of the rays in the x-ray beam. Slit 2 being wide enough to allow the whole reflected radiation to enter the ionization chamber, the breadth of the source and the width of the slit 1 determine this angle. Its magnitude equals the angular breadth of the slit 1 as seen from the target plus the angular breadth of the focal spot on the target as seen from slit 1.

In our experiments the angular breadth of the focal spot as seen from the slit 1 was small. We determined its magnitude as follows. Before the tube was constructed the surface of the target was ground very smooth. After the tube's construction, but before it had been put in place, we estimated by reflecting a beam of light from the smooth surface of the target that the plane of this surface would pass through a point about 5 mm from the centre of the mica window, W. Since the distance from the target to the window was about 720 mm, this means that the rays that passed through the centre of the slit 1 left the target making an angle of about 24′ with its surface. From the slight marks on the target we estimated that the breadth of the focal spot was 8 mm. Since from slit 1 this is seen at an angle of 24′ its apparent angular breadth is about 15″.

We determined the width of slit 1 by the method described in the Physical Review for December, 1917, p. 630. In one experiment we found that we had to decrease the width of the slit .313 mm before the current in the ionization chamber ceased, and this is the effective width of the slit. Dividing it by the distance from the slit to the target, 760 mm, we find that the angular width of the slit as seen from the target is 1′ 25″. Adding the angular width of the focal spot we get 1′ 40″ as the correction, $2\Delta\theta$, to be added to the double glancing angle, 2θ, in this particular case.

The geometry of the tube and spectrometer determines the correction as calculated above. It might well be that other causes, such as irregularities in the crystal planes, would introduce errors. It becomes, therefore, desirable to have another method of estimating the correction,—if possible, one that would give us an upper limit for its value, since the geometry determines a lower limit. We have examined some of the emission lines in the L series of tungsten to see if their widths corresponded with the geometry of the slit and source. In every case the peak on the ionization curve that represented an emission line appeared to be somewhat broader than would be expected from the widths of the slit and source. On putting ethyl-bromide into the ionization chamber, however, and on examining the rise in the ionization curve that corresponded to the critical ionization wave-length of bromine we found that the breadth of this rise almost exactly equalled the geometrical estimate of the slit and source correction. In the above mentioned experiment, for instance, it amounted to 1′ 42″. The difference between this and 1′ 40″ is much less than experimental errors; so that we cannot say that a critical ionization has any perceptible breadth of its own. Curve C in Fig. 3 represents a rise in an ionization curve at the critical ionization wave-length.

We have made experiments on the critical ionization of bromine during each one of our measurements of h. We have, therefore, data from which, incidentally, we can calculate a very accurate value of its wave-length. In the four experiments recorded in our tables we have obtained values of the corresponding glancing angle that do not differ from their mean value, 8° 43′ 7″, by more than 5″. This gives for the wave-length of the critical ionization of bromine $\lambda = .91796 \times 10^{-8}$cm.

Table 4 column 2 contains the values of the glancing angle, θ, as measured by curves similar to A and B (Fig. 3).

TABLE 4

Glancing Angles and Radiation Constant

Date	θ (Uncorrected)	$\Delta\theta$	θ(Corrected)	$V\sin\theta$	$h \times 10^{27}$
March 15	4° 46′ 43″	47″	4° 47′ 30″	2039.2	6.5539
March 21	4° 46′ 53″	43″	4° 47′ 36″	2039.9	6.5561
March 30	4° 46′ 53″	51″	4° 47′ 44″	2041.0	6.5594
April 5	4° 46′ 48″	45″	4° 47′ 33″	2039.6	6.5552

Column 3 contains the correction, $\Delta\theta$, to be added to θ, determined in each case as described above, and column 4 contains the corrected values of θ. In determining the value of h we calculated the wave length, λ, by means of the equation $\lambda = 2d\sin\theta$, with $2d = 6.056\times10^{-8}$cm, then the frequency, ν, from the equation $\lambda\nu = c = 2.9986\times10^{10}$ and finally h from the equation $Ve = h\nu$, with $e = 4.774\times10^{-10}$.

Evidently what we measure is the product $V\sin\theta$, which, by the way, is not an absolute constant of calcite, for it depends upon the distance between the crystal planes, and this in turn depends upon the temperature, etc. Column 5 in Table 4 contains the values of $V\sin\theta$, the average of which is $V\sin\theta = 2039.9$ at about 20° C, with an estimated error of about $\pm .9$. The grating constant 2d has been calculated for 20° C. The product $V\lambda = 12{,}354$ is independent of the temperature.

The values of h appear in the 6th column of Table 4. The mean value is $h\times10^{27} = 6.556$ with an estimated error of precision (i.e. without taking account of errors in d, c and e) of $\pm .003$. If we introduce the errors in d, c and e, the probable error in h comes out about $\pm .009$.

This value of h agrees with that previously published by Blake and Duane, (l.c.) but is a fraction of one per cent larger than that recently obtained by E. Wagner in a careful series of measurements.

In the experiments described above the x-rays left the target in a direction at right angles to the line of motion of the cathode particles. An interesting question has been raised recently as to whether the limit of the continuous spectrum would be altered, if the rays came off at some other angle.[5] To test this point with the accurate method of measuring the voltage which we now have, we made a series of experiments with an ordinary Coolidge tube (tungsten target) placed so that the x-rays that passed through the spectrometer's slit left the target at an acute angle of about 45°. If there is any appreciable Doppler effect, this should decrease the value of h as computed by the above equations. The results of these measurement appear in Table 5. As the x-ray tube had no thin mica window, the accuracy is probably somewhat less than before. The value of V sinθ, however, does not differ from that for rays at right angles to the cathode stream. There does not appear to be a Doppler effect for the short wave length limit of the spectrum that amounts to as much as one part in two thousand. This agrees with Wagner's results.

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[5] See E. Wagner (l.c.) and G. Zecher, Ann. d. Physik, Sept., 1920, p 28.

TABLE 5

Glancing Angles and Radiation Constant

Date	θ(Uncorrected)	$\Delta\theta$	θ(Corrected)	V sinθ	$h \times 10^{27}$
April 6	4° 46′ 35″	1′	4° 47′ 35″	2039.8	6.556
April 6	4° 46′ 33″	1′	4° 47′ 33″	2039.6	6.555
April 12	4° 46′ 53″	48″	4° 47′ 41″	2040.7	6.558
April 27	4° 46′ 43″	47″	4° 47′ 30″	2039.3	6.554

Difference of Potential Applied to Tube = 24413 volts.

HARVARD UNIVERSITY,
CAMBRIDGE, MASS.

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9. *Reflectometry:* L. R. Ingersoll, C. H. Sharp, A. H. Pfund, I. G. Priest, and A. H. Taylor, Nela Research Laboratories, Nela Park, Cleveland, Ohio.
10. *Refractometry:* H. G. Ott, and I. C. Gardner, Bureau of Standards, Washington, D. C. (incomplete).
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13. *Spectroradiometry:* F. E. Fowle, L. R. Ingersoll, H. M. Randall, W. Weniger, and W. W. Coblentz, Bureau of Standards, Washington, D. C.
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15. *Visual Sensitometry:* H. M. Johnson, L. A. Jones, P. G. Nutting, C. F. Prentice, and Prentice Reeves, 3 Kodak Park, Rochester, N. Y.
16. *Wave Lengths:* Henry Crew, K. Burns, A. H. Pfund, H. M. Randall, and W. F. Meggers, Bureau of Standards, Washington, D. C.

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DISPERSION IN OPTICAL GLAS

BY

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In the first paper of this series on dispersion
attention is directed to certain linear relations w
the partial dispersions of optical glasses; namel
one of a series of optical glasses any partial
n_F-n_C, be plotted against any other partial
$n_D-n_{A'}$, the plotted points fall on a straight lin
was found to hold within one or two units in
place for all optical glasses and for most colorl
the basis of this relation a series of empirical di
with two and three constants were deduced
dispersion in optical glasses with a high degre
the same paper the average refractive indices
of optical glasses are also listed; the two sets of
index values are:

$n_{A'}$	n_C	n_D	n_F
1.539909	1.543168	1.545958	1.5526
1.588807	1.593565	1.597767	1.6082

The equation which expresses the linear rel
two partial dispersions, such as n_y-n_x and n_F

$$n_y-n_x=a(n_F-n_C)-b$$

in which a and b are constants and y and x
spectral lines.

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[1] J. Opt. Soc. Am. *4*, pp. 148–159, 1920.

[2] J. Opt. Soc. Am. *4*, p. 154, 1920. In this paper a ty
which should be corrected. For the expression $e^{1'\lambda^2}$ substitute e^{1}

With this equation and the foregoing average refractive indices as a basis the values of table 2 were computed. The average partial dispersions deduced from these refractive indices are listed in table 1. In this table the values of the constants *a* and *b* of equation (1) are given for the case that, in place of the partial dispersion n_y-n_x, the partial dispersions $n_C-n_{A'}$, n_D-n_C, n_F-n_D, and $n_{G'}-n_F$ are substituted, and n_F-n_C is retained on the right hand side of the equation.

TABLE 1

List of Average Partial Dispersions of Optical Glasses

	n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$
	.009448	.003259	.002790	.006658	.005378
	.014636	.004758	.004202	.010434	.008794
a		.288936	.272167	.727833	.658443
b		.000529	.000219	−.000219	−.000843

In table 2 the increment of the mean dispersion, n_F-n_C (column 1) is 0.0001 from line to line. The corresponding partial dispersions, $n_C-n_{A'}$, n_D-n_C, n_F-n_D, $n_{G'}-n_F$, are given in columns 2, 3, 4, and 5, respectively. By arithmetical interpolation the mean dispersion and the corresponding partial dispersions can be read off directly from the table to the fifth decimal place.

TABLE 2

Table of Average Partial Dispersions of Optical Glasses

n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$	n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$
.00500	.00197	.00158	.00342	.00245	.01000	.00342	.00294	.00706	.00574
510	200	161	349	252	1010	345	297	713	581
520	203	163	356	258	1020	348	300	720	587
530	206	166	364	265	1030	351	302	728	594
540	209	169	371	271	1040	353	305	735	600
550	212	172	378	278	1050	356	308	742	607
560	215	174	386	284	1060	359	310	750	614
570	218	177	393	291	1070	362	313	757	620
580	220	180	400	298	1080	365	316	764	627
590	223	182	407	304	1090	368	319	771	633
.00600	.00226	.00185	.00415	.00311	.01100	.00371	.00321	.00779	.00640
610	229	188	422	317	1110	374	324	786	647
620	232	191	429	324	1120	377	327	793	653
630	235	193	437	331	1130	379	329	800	660

TABLE 2 (*Continued*)

n_F-n_C	n_C-n_A'	n_D-n_C	n_F-n_D	$n_G'-n_F$	n_F-n_C	n_C-n_A'	n_D-n_C	n_F-n_D	$n_G'-n_F$
640	238	196	444	337	1140	382	332	808	666
650	241	199	451	344	1150	385	335	815	673
660	244	202	458	350	1160	388	338	822	679
670	246	204	466	357	1170	391	340	830	686
680	249	207	473	363	1180	394	343	837	693
690	252	210	480	370	1190	397	346	844	699
.00700	.00255	.00212	.00488	.00377	.01200	00400	.00349	.00851	.00706
710	258	215	495	383	1210	403	351	859	712
720	261	218	502	390	1220	405	354	866	719
730	264	221	509	396	1230	408	357	873	726
740	267	223	517	403	1240	411	359	881	732
750	270	226	524	410	1250	414	362	888	739
760	272	229	531	416	1260	417	365	895	745
770	275	231	539	423	1270	420	368	902	752
780	278	234	546	429	1280	423	370	910	759
790	281	237	553	436	1290	426	373	917	765
.00800	.00284	.00240	.00560	.00442	.01300	00429	.00376	.00924	.00772
810	287	242	568	449	1310	431	378	932	778
820	290	245	575	456	1320	434	381	939	785
830	293	248	582	462	1330	437	384	946	791
840	296	251	589	469	1340	440	387	953	798
850	298	253	597	475	1350	443	389	961	805
860	301	256	604	482	1360	446	392	968	811
870	304	259	611	489	1370	449	395	975	818
880	307	261	619	495	1380	452	397	983	824
890	310	264	626	502	1390	455	400	990	831
.00900	.00313	.00267	.00633	.00508	.01400	.00457	.00403	.00997	.00838
910	316	270	640	515	1410	460	406	1004	844
920	319	272	648	521	1420	463	408	1012	851
930	322	275	655	528	1430	466	411	1019	857
940	325	278	662	535	1440	469	414	1026	864
950	327	280	670	541	1450	472	417	1033	870
960	330	283	677	548	1460	475	419	1041	877
970	333	286	684	554	1470	478	422	1048	884
980	336	289	691	561	1480	481	425	1055	890
990	339	291	699	568	1490	483	427	1063	897
.01000	.00342	.00294	.00706	.00574	.01500	.00486	.00430	.01070	.00903
.01500	.00486	.00430	.01070	.00903	.02000	.00631	.00566	.01434	.01233
1510	489	433	1077	910	2010	634	569	1441	1239
1520	492	436	1084	917	2020	637	572	1448	1246
1530	495	438	1092	923	2030	639	574	1456	1252
1540	498	441	1099	930	2040	642	577	1463	1259
1550	501	444	1106	936	2050	645	580	1470	1266
1560	504	446	1114	943	2060	648	583	1477	1272

TABLE 2 (*Continued*)

n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$	n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$
1570	507	449	1121	949	2070	651	585	1485	1279
1580	509	452	1128	956	2080	654	588	1492	1285
1590	512	455	1135	963	2090	657	591	1499	1292
.01600	.00515	.00457	.01143	.00969	.02100	.00660	.00593	.01507	.01298
1610	518	460	1150	976	2110	663	596	1514	1305
1620	521	463	1157	982	2120	665	599	1521	1312
1630	524	466	1164	989	2130	668	602	1528	1318
1640	527	468	1172	996	2140	671	604	1536	1325
1650	530	471	1179	1002	2150	674	607	1543	1331
1660	533	474	1186	1009	2160	677	610	1550	1338
1670	535	476	1194	1015	2170	680	613	1557	1345
1680	538	479	1201	1022	2180	683	615	1565	1351
1690	541	482	1208	1028	2190	686	618	1572	1358
.01700	.00544	.00485	.01215	.01035	.02200	.00689	.00621	.01579	.01364
1710	547	487	1223	1042	2210	691	623	1587	1371
1720	550	490	1230	1048	2220	694	626	1594	1377
1730	553	493	1237	1055	2230	697	629	1601	1384
1740	556	495	1245	1061	2240	700	632	1608	1391
1750	559	498	1252	1068	2250	703	634	1616	1397
1760	561	501	1259	1075	2260	706	637	1623	1404
1770	564	504	1266	1081	2270	709	640	1630	1410
1780	567	506	1274	1088	2280	712	642	1638	1417
1790	570	509	1281	1094	2290	715	645	1645	1424
.01800	.00573	.00512	.01288	.01101	.02300	.00717	.00648	.01652	.01430
1810	576	515	1295	1107	2310	720	651	1659	1437
1820	579	517	1303	1114	2320	723	653	1667	1443
1830	582	520	1310	1121	2330	726	656	1674	1450
1840	585	523	1317	1127	2340	729	659	1681	1456
1850	587	525	1325	1134	2350	732	661	1689	1463
1860	590	528	1332	1140	2360	735	664	1696	1470
1870	593	531	1339	1147	2370	738	667	1703	1476
1880	596	534	1346	1154	2380	741	670	1710	1483
1890	599	536	1354	1160	2390	743	672	1718	1489
.01900	.00602	.00539	.01361	.01167	.02400	.00746	.00675	.01725	.01496
1910	605	542	1368	1173	2410	749	678	1732	1503
1920	608	544	1376	1180	2420	752	681	1739	1509
1930	611	547	1383	1186	2430	755	683	1747	1516
1940	613	550	1390	1193	2440	758	686	1754	1522
1950	616	553	1397	1200	2450	761	689	1761	1529
1960	619	555	1405	1206	2460	764	691	1769	1535
1970	622	558	1412	1213	2470	767	694	1776	1542
1980	625	561	1419	1219	2480	769	697	1783	1549
1990	628	564	1426	1226	2490	772	700	1790	1555
.02000	.00631	.00566	.01434	.01233	.02500	.00775	.00702	.01798	.01562

TABLE 2 (*Continued*)

n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$	n_F-n_C	$n_C-n_{A'}$	n_D-n_C	n_F-n_D	$n_{G'}-n_F$
.02500	.00775	.00702	.01798	.01562	2760	850	773	1987	1733
2510	778	705	1805	1568	2770	853	776	1994	1740
2520	781	708	1812	1575	2780	856	779	2001	1746
2530	784	710	1820	1582	2790	859	781	2009	1753
2540	787	713	1827	1588	.02800	.00862	.00784	.02016	.01759
2550	790	716	1834	1595	2810	865	787	2023	1766
2560	793	719	1841	1601	2820	868	789	2031	1773
2570	795	721	1849	1608	2830	871	792	2038	1779
2580	798	724	1856	1614	2840	873	795	2045	1786
2590	801	727	1863	1621	2850	876	798	2052	1792
.02600	.00804	.00730	.01870	.01628	2860	879	800	2060	1799
2610	807	732	1878	1634	2870	882	803	2067	1805
2620	810	735	1885	1641	2880	885	806	2074	1812
2630	813	738	1892	1647	2890	888	808	2082	1819
2640	816	740	1900	1654	.02900	.00891	.00811	.02089	.01825
2650	819	743	1907	1661	2910	894	814	2096	1832
2660	821	746	1914	1667	2920	897	817	2103	1838
2670	824	749	1921	1674	2930	899	819	2111	1845
2680	827	751	1929	1680	2940	902	821	2118	1852
2690	830	754	1936	1687	2950	905	824	2125	1858
.02700	.00833	.00757	.01943	.01693	2960	908	827	2132	1865
2710	836	759	1951	1700	2970	911	830	2140	1871
2720	839	762	1958	1707	2980	914	833	2147	1878
2730	842	765	1965	1713	2990	917	836	2154	1884
2740	845	768	1972	1720	.03000	.00920	.00838	.02162	.01890
2750	847	770	1980	1726					

The foregoing refractive indices and partial dispersions may serve as a datum surface to which to refer the partial dispersions of any given glass. Each one of the partial dispersion values of table 1 is the arithmetical mean of the values of 158 different glasses listed by Chance Bros., by Parra-Mantois, and by Schott und Genossen. In these lists the predominating influence of the flint glasses cannot easily be avoided, but for general purposes this is not a drawback.

In order to test the accuracy of table 2 and at the same time to interpret the partial dispersions of a given glass in terms of the average dispersions, all the partial dispersions of the glasses listed by Parra-Mantois were compared with the corresponding partial dispersions of table 2. The results are given in table 3. In this table there is listed in column 1, the refractive index of the glass; in column 2, its mean dispersion, $n_F - n_C$; in column 3, the difference between the partial dispersions, $(n_C - n_A')$ glass and $(n_C - n_A')$ table 2 corresponding to the given mean dispersion, $n_F - n_C$, of column 2; in columns 4, 5, and 6 the corresponding differences between the partial dispersions, $n_D - n_C$, $n_F - n_D$, and $n_{G}' - n_F$ of glass and table 2 are listed. These differences represent the numerical departures of the given glass from the average type of glass of table 2.

TABLE 3

Partial Dispersion Differences

n_D 1.	$n_F - n_C$ 0.0	$C-A'$ $x10^5$	$D-C$ $x10^5$	$F-D$ $x10^5$	$G'-F$ $x10^5$
Fluor crown					
4933	0706	−2	−2	+2	+9
4711	0702	−4	−4	+2	+12
4637	0707	−4	−3	+3	+12
Average departures		−3	−3	+2	+11
Borosilicate crown					
5021	0755	5	−2	1	4
4958	0755	1	−1	0	6
4950	0765	3	−2	2	3
5012	0776	5	0	0	5
4936	0766	5	0	0	6
4718	0721	−2	−1	1	6
4760	0728	5	−2	2	5
4728	0727	−4	−1	1	9
5176	0801	7	0	0	6
5102	0794	1	−1	1	8
5146	0811	−1	0	0	2
Average departures		+2	−1	+1	+6
Crown glasses					
5054	0783	6	−1	1	5
5261	0854	0	−1	1	2
5303	0869	−1	−2	2	0
5214	0856	0	−1	1	2
5442	0926	1	−1	1	2
5007	0822	−5	2	2	6
5035	0839	−4	−4	4	6

n_D 1.	$n_F - n_C$ 0.0	$C-A'$	$D-C$	$F-D$	$G'-F$
5117	0851	−1	−2	2	3
5102	0856	−2	−2	2	6
5122	0865	−2	−2	1	7
5101	0865	+5	−2	1	6
5179	0859	+2	−2	2	6
5220	0868	−1	−2	2	2
5129	0839	−2	−2	2	7
5131	0853	−3	−2	2	4
5102	0852	−5	−4	4	5
5197	0870	−1	−3	3	3
5262	0883	−2	0	0	3
5158	0873	−4	−2	2	7
5134	0872	−1	−4	4	6
5171	0885	−3	−1	0	5
5203	0898	−5	0	0	3
5138	0889	−3	−2	2	4
5196	0903	−3	−2	3	7
5151	0906	−3	−1	1	6
5106	0890	−2	−2	2	5
5199	0909	−5	−3	3	7
5150	0917	−3	−1	1	3
5010	0886	1	0	0	1
5129	0932	0	−3	3	8
5179	0951	2	0	0	6
5142	0947	−5	−1	1	3
5202	1014	1	−1	1	4
5211	1025	−2	−2	2	3
5230	1035	−2	−2	1	2
5251	0959	0	−2	0	5

TABLE 3 (*Continued*)

n_D 1.	n_F-n_C 0.0	$C-A'$	$D-C$	$F-D$	$G'-F$
5266	0971	−3	−1	1	4
5307	0999	−1	0	0	4
5310	1003	−1	−3	3	2
5353	1014	−3	−1	1	1
5327	1015	−4	−1	0	1
5377	1037	−2	−2	2	1
5365	1042	0	−2	2	4
5481	1122	−2	−1	2	5
Average departures		−2	−2	+2	+ 4
		Barium crown			
5400	0893	0	−1	1	1
5379	0898	1	−1	1	0
5387	0904	1	−3	3	−1
5697	0988	3	3	−3	−4
5707	0993	−1	0	0	−5
5738	1000	0	1	−1	−5
5682	0994	0	0	0	−4
5734	1003	−1	0	0	−4
5778	1021	−1	4	−4	−3
5888	0966	10	1	−1	−8
5901	0971	3	1	−1	−8
6082	1036	5	1	−1	−10
6087	1041	9	1	−1	−9
6126	1048	3	4	−2	−11
6115	1047	6	2	−2	−10
6110	1047	7	2	−2	−10
6129	1052	3	2	−2	−11
6123	1067	7	2	−2	−8
6103	1071	4	1	−1	−9
6087	1070	4	2	−2	−9
5999	1059	3	2	−2	−5
6087	1075	6	1	−2	−8
6123	1084	7	1	−1	−11
6137	1087	4	2	−2	−10
6138	1089	3	2	−2	−9
6044	1073	4	1	−1	−8
6028	1072	2	1	−1	−8
6081	1083	6	1	−1	−8
6098	1092	3	1	−1	−8
6115	1100	1	1	−1	−10
6145	1108	1	2	−2	−11
6150	1113	1	1	−1	−8
6028	1202	3	−1	1	−5
6065	1234	1	−1	1	−7
6246	1098	5	2	−1	−12
6314	1165	4	2	−2	−15
6377	1194	5	0	0	−11
6227	1177	2	2	−1	−10
6169	1181	4	0	0	−9
6337	1239	5	2	−2	−10
6237	1390	0	0	0	−8
6495	1393	3	4	−4	−13
6491	1418	0	2	−1	−10
6576	1469	−3	1	−1	−11
Average departures		+3	+1	−1	−8
		Flints			
5174	0913	3	2	−2	0
5075	0933	2	−3	0	−1
5210	0962	3	0	0	0
5220	0970	7	−2	2	0
5274	0980	4	1	−1	2
5207	1006	3	0	0	−2
5230	1019	7	−2	2	−2
5293	1073	5	0	0	−1
5339	961	−1	−3	2	0
5408	1133	0	−1	2	3
5495	1199	0	−2	2	2
5585	1230	−1	−2	2	1
5510	1216	0	0	0	3
5619	1288	−3	0	0	−1
5683	1308	2	−1	1	2
5679	1327	1	−1	1	2
5750	1346	−2	0	1	0
5781	1366	1	−1	1	0
5877	1401	1	0	0	2
5870	1433	−1	−2	2	2
5742	1340	1	−1	1	3
5841	1382	−1	1	−1	1
5795	1379	0	−1	1	−1
5768	1380	−2	0	0	1
5828	1401	1	−1	1	−1
5808	1388	0	0	0	0

TABLE 3 (*Continued*)

n_D 1.	n_F-n_C 0.0	$C-A'$	$D-C$	$F-D$	$G'-F$
5762	1390	0	−1	1	2
5838	1416	4	−1	1	4
5863	1370	2	1	−1	−1
5783	1409	0	1	−1	0
5821	1437	−1	−1	1	−2
5843	1450	−2	−3	3	0
6020	1493	7	1	−1	−3
6049	1583	3	−1	1	−2
6063	1591	−2	−1	1	−2
6093	1619	1	−2	2	−2
6096	1623	0	−1	1	−4
6117	1635	2	0	0	−1
6132	1658	3	0	0	−1
6173	1669	0	1	−1	1
6185	1641	1	1	−1	−4
6207	1692	1	−1	1	0
6258	1709	2	0	0	−3
6235	1706	−1	0	0	−2
6277	1722	0	1	−1	−1
6284	1728	3	1	−1	−3
6303	1740	−1	2	2	−2
6143	1671	0	0	2	1
6171	1691	0	1	−1	0
6187	1701	1	1	−1	0
6210	1721	−1	1	−1	−2
6229	1734	2	−1	1	−1
6492	1878	2	1	−1	−3
6456	1898	1	0	0	0
6461	1911	1	−1	1	1
6487	1921	2	1	−1	1
6514	1951	−1	−1	+1	−1
6660	2009	−1	0	0	−2
6923	2219	−2	2	−2	1
6958	2243	1	−1	1	1
6960	2253	−2	0	0	3
7184	2488	−3	0	0	2
7453	2675	−2	0	0	10
7540	2803	−9	−1	1	14
7784	2929	−4	−1	1	11
Average departures		1	0	0	0

Borosilicate flints

n_D 1.	n_F-n_C 0.0	$C-A'$	$D-C$	$F-D$	$G'-F$
5401	1050	9	0	0	−9
5348	1047	10	2	−2	−6
5465	1085	1	0	0	2
5524	1122	9	0	0	−4
5571	1149	11	0	0	−4
5680	1216	2	0	0	0
5805	1311	3	1	−1	1
Average departures		+5	0	0	−3

Barium flints

n_D 1.	n_F-n_C 0.0	$C-A'$	$D-C$	$F-D$	$G'-F$
5469	1022	−1	−2	2	2
5484	1029	0	−1	1	1
5518	1039	−2	−2	2	3
5503	1041	−4	−1	2	3
5486	1044	−2	0	0	1
5516	1051	−1	−3	3	1
5637	1016	−8	3	−3	−4
5684	1029	−5	0	0	−4
5812	1082	0	−1	1	−3
5680	1067	−3	0	0	−3
5682	1078	−4	−1	1	−2
5731	1104	−2	0	0	−3
5546	1071	0	−1	1	0
5667	1106	0	−1	1	2
5641	1119	−2	−2	2	1
5779	1168	−4	0	0	1
5561	1159	−6	0	0	4
5802	1265	−3	−1	2	−1
6050	1376	−3	−1	−1	−3
6057	1388	−1	1	−1	−3
6032	1438	−1	1	−1	−2
6250	1610	3	0	0	0
6259	1621	−1	−1	1	−3
6605	1827	−1	2	−2	−4
Average departures		−2	0	0	−1

In table 3 the several types of optical glasses are treated separately in order to indicate their individual traits clearly. Thus the ordinary flint glasses depart but little from the average type and the partial dispersions of table 2 are valid for flint glasses within a few units of the fifth decimal place. The borosilicate flints on the other hand show an increased partial dispersion in the red and a decreased partial dispersion in the blue end of the spectrum. The barium flints exhibit small decreases both in the red and in the blue. Both barium flints and borosilicate flints are characterized by higher refractive indices than the ordinary flints for the same mean dispersion. The barium crowns show an increase in refringence relatively in the red and an appreciable decrease in the blue. In the borosilicate crowns there is a slight increase both in the red and in the blue. In the ordinary crowns there is a slight decrease in the red and a slight increase in the blue. The fluor crowns are remarkable for the relatively large increase in the blue with slight decrease in the red.

The data in table 3 demonstrate clearly the conclusions stated in the first paper of this series, namely that there is a general course of dispersion throughout the visible spectrum which is characteristic of colorless substances, especially glasses, such that if any two refractive indices are given, or even a single partial dispersion, the dispersion curve is fixed for the entire visible spectrum. Slight departures from this curve due to peculiarities in chemical composition occur, but these are commonly restricted to the fifth decimal place. So long as the glass remains colorless there is little hope of changing these relations.

The chief optical differences in glasses are not differences in the character of the dispersion, but in the fact that two glasses may have the same dispersion relations and yet have appreciably different absolute refractive indices. In a dispersion formula both refringence and dispersion should be specifically recognized; the first as a constant giving the datum level of refringence and the second as a function of one or more terms expressing the run of dispersion throughout the visible spectrum. These conclusions follow directly from the analysis of the linear relations which have been shown to exist between the partial dispersions of a series of optical glasses. They prove that a rise in the partial dispersion at any part of the spectrum is accompanied by a corresponding rise in partial dispersions over the entire visible spectrum.

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ABOLITION OF ASTIGMATISM OF OBLIQUE BUNDLES IN CASE OF A SINGLE LENS

BY

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1. SO-CALLED "PUNKTAL" SPECTACLE LENS

The determination of the form and dimensions of a single lens which, for a prescribed refracting power (F) and for a given position of the object, shall be free from astigmatism of oblique bundles of rays, is an optical problem of considerable practical importance for a number of purposes; on which therefore much has been written, particularly of recent years in connection with the investigation of the most satisfactory forms of modern spectacle lenses. Among those who have made notable contributions to this special phase of the problem may be mentioned Tscherning, Percival, Ostwalt, von Rohr and Gullstrand. Perhaps now more than ten years ago, the Zeiss Optical Company placed on the market a form of spectacle lens called the "Punktal" calculated and designed by von Rohr; which is also made in this country by the Bausch & Lomb Optical Company.

The curvatures and thickness of the "Punktal" lens depend on the power of the glass, which means that it is expensive to manufacture and consequently has not yet come into very extensive use. There is no particular theoretical difficulty about designing a lens of this type; in fact the necessary formulae have long been known. The chief difficulty in the way of its practical realization consisted perhaps in recognizing the essential fact that the centre of rotation of the eye (and not, for example, the centre of the pupil) was the point of reference in this particular optical combination which was the effective point for the determination of the chief rays of the oblique bundles which it was the business of the lens to render stigmatic before they entered the eye.

For positive lenses of powers higher than between 7 and 8 dioptries, the theory shows that no form of "Punktal" lens is possible; and for such higher powers of positive spectacle lenses as are needed, for example, in cases of very high hypermetropia or of aphakia (when the eye has been operated on for cataract), Gullstrand has designed a lens with an aspherical surface which is made by Zeiss under the name of "Katral" lens. As heretofore comparatively little has been published in English about the so-called "Punktal" lens (or about similar types of lenses made by other manufacturers), the following notes may be of interest, especially if they lead to criticism and discussion on the part of such workers in the optical research laboratories as have been engaged recently in studying the properties of this important type of spectacle lens.

In ordinary direct vision the eye is mobile and turns almost involuntarily to look at the point of fixation which is sharply focused on the retina in the *fovea centralis*. The lines of fixation (which show the direction in which the eye is gazing) all pass through a fixed point called the eye-pivot or centre of rotation of the eye (designated always hereafter by Z') which may be assumed to lie on the optical axis of the eye at a distance of 13 mm beyond the vertex of the anterior surface of the cornea. In general, therefore, the optical axis of a mobile ametropic eye corrected by a spectacle glass will not coincide with the optical axis of the correction-glass; and consequently a spectacle glass which may be all right when used centrally will usually show astigmatism when the eye turns in its socket to look obliquely through the lens. In order to see distinctly through the glass, no matter how the eye turns behind it, the form of the lens must be specially designed so that the astigmatism of oblique bundles of rays is abolished. Practically, the problem amounts to computing the form of the lens for the case of a small rear stop placed beyond it with its centre at the place where the centre of rotation of the eye would be when the lens is in use in its right position in front of the eye; so that within a certain field of fixation corresponding to the angular movement of the eye-ball the astigmatism of oblique cylindrical bundles of incident rays will be abolished by the lens and delivered to the eye as if they emanated, really or virtually, from a point lying approximately at least on the so-called "far point sphere" of the mobile eye.

The distance between the eye and the nearer surface of a correction-glass (measured from the vertex of this surface to the vertex of the anterior surface of the cornea) should ordinarily be not less than about 12 mm. Thus, if the vertices of the two surfaces of the lens are designated by A_1 and A_2, then the distance A_2Z' should generally be not less than about 25 mm; although it may be as great as 30 mm. However, within these limits, a smaller value of this distance is usually better for two reasons, first, because the astigmatism of a lens with a rear stop is more easily corrected in this case, and , second, because the nearer the centre of rotation of the eye is to the correction-glass, the smaller the diameter of the lens will have to be for a given angular excursion of the eyeball; which means that the spectacles may be made both smaller and lighter, as is always desirable.

2. NOTATION

The form of the required lens (that is the curvatures R_1, R_2 of its two surfaces and its axial thickness $d = A_1A_2$) will depend on its refracting power (F), the place of the object, the distance of the stop, and of course also the index of refraction of the glass (n). If

$$F_1 = (n-1)R_1, \quad F_2 = -(n-1)R_2,$$

denote the refracting powers of the two surfaces of the lens, then

$$F = F_1 + F_2 - c\,F_1\,F_2, \qquad (1)$$

where $c = d/n$ denotes the "reduced" thickness of the lens. If the principal points of the lens are designated by H, H′, then $A_1H = c\ F_2/F$ and $A_2H = -c\ F_1/F$. Moreover, let z, z' designate the positions on the axis of another pair of conjugate points, one of which is to be regarded as the centre of the small stop. If the lens is a spectacle correction-glass for an ametropic eye, the point Z′ (as already explained) must be chosen to coincide with the centre of rotation of the eye, and in this case the calculation must be made, therefore, for a lens with a rear stop with its centre at Z′. Now if we put $z = 1/Z = \mathrm{HZ}$, $z' = 1/Z' = \mathrm{H'Z'}$, then $Z' = Z + F$. The distance of the object from the lens may be denoted by $u_1 = 1/U_1$; and since it will be assumed that the object is at an infinite distance (as is always the case for a so-called correction-glass in spectacle optics), we shall have $U_1 = 0$.

From a point of the object which is not on the axis there will proceed a narrow cylindrical bundle of incident rays, the chief ray being the one which is directed towards the point Z. After passing through the lens this chief ray will cross the axis at a point Z″ which will usually be not far from the point Z′ and which would actually coincide with Z′ if the lens were free from spherical aberration with respect to the pair of conjugate axial points Z, Z′. After emerging from the lens the narrow bundle of rays will generally be astigmatic, that is, the so-called meridian (or tangential) rays will intersect ("really" or "virtually") at a point P′ on the chief ray, whereas the sagittal rays will focus at another point Q′ on this same ray. The interval P′Q′ is called the "astigmatic difference" of the bundle; and in the special case when Q′ coincides with P′, so that the astigmatic difference vanishes, the bundle of emergent rays will be stigmatic or monocentric; and the lens may be said to give a punctual image of the object. This is what is meant here by abolishing the astigmatism.

3. CASE OF AN INFINITELY THIN LENS

As is always the case with a problem of this kind, the investigation will be greatly simplified provided it is permissible to neglect entirely the thickness of the lens; that is, to regard the points designated above by A_1, A_2, and H, H′ as all coincident at the optical centre of the lens. In a spectacle lens this simplifying assumption is not very far from correct, although it is less permissible in modern meniscus forms of lenses than in the older forms of spectacle lenses. Now according to Seidel's famous approximate expressions for the spherical aberrations of an optical system,[1] the condition of the abolition of the astigmatism of oblique bundles of rays for an infinitely thin lens and an infinitely distant object may be expressed by an homogeneous equation of the 3rd degree in which the variables are the magnitudes denoted above by the symbols F, F_1, and Z' and the coefficients are functions of the index of refraction (n). This equation may be written as a quadratic in F_1, as follows:

$$\alpha F_1^2+\beta F_1+\gamma=0, \quad (d=0,\ U_1=0); \qquad (2)$$

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[1] See, for example, Southall's *Mirrors, Prisms and Lenses* (The Macmillan Company, New York, 1918), Article 193.

where

$$\alpha=\frac{n+2}{n(n-1)^2}F, \quad \beta=-\left\{\alpha F+\frac{2(n+1)}{n(n-1)}Z'\right\}F,$$

$$\gamma=\left(\frac{F}{n-1}+Z'\right)^2 F.$$

Accordingly, for given values of F and Z', two values of F_1 may be found which will satisfy this equation, namely:

$$F_1=\frac{(n+2)F+2(n^2-1)Z'\pm\sqrt{(n+2)(2-3n)F^2-4(n-1)^2(n+2)FZ'+4(n-1)^2Z'^2}}{2(n+2)}$$

The refracting power of the other surface will be $F_2=F-F_1$. In the case of a spectacle correction-glass, the actual values of F_1 are found to be positive always and those of F_2 negative; so that the form of the required lens (whether convex or concave) is of the meniscus type (except in one special case mentioned below, when the lens is planoconcave).

For a given value of Z' (for example, when $z'=25$ mm or $Z' = 40$ dioptries) the quadratic equation with variables F and F_1, referred to a system of rectangular axes with these variables as coördinates, will be found to represent an ellipse which is tangent to the F-axis (abscissa-axis) at the point $F=-(n-1)Z'$, $F_1=0$. The form of lens corresponding to this special point on the curve is therefore planoconcave. Except at this point, the ellipse lies wholly above the abscissa-axis, and hence (as was stated above, assuming that $Z'>0$, as it must be for the case of a rear stop), the form of the lens is invariably that of a meniscus. The coördinates of the centre of the ellipse are:

$$-\frac{2(n-1)^2}{3n-2}Z', \quad +\frac{2n^2}{(n+2)(3n-2)}Z';$$

and the squares of its semi-axes, major and minor, will be found by dividing $2n^3(n-1)^2Z'$ by

$$(n+2)(3n-2)\left\{n+(n+2)(n-1)^2\pm\sqrt{n^2+(n-1)^2(n+2)(n^3-4n+4)}\right\}.$$

Thus, for a given value of Z', there may always be found two real values of F_1 corresponding to each value of F comprised between the limits of the two extreme values of F, namely:

$$F=\frac{2(n-1)}{3n-2}\left\{-(n-1)\pm n\sqrt{\frac{n}{n+2}}\right\}Z';$$

the upper sign corresponding to the highest possible positive value of F and the lower sign to the extreme negative value. For example, for $n = 1.52$, the two extreme values are $F = +0.19452\,Z'$ and $F = -0.6170\,Z'$; which shows that the range on the negative side extends more than three times as far as on the positive side. Putting $Z' = 40$ dptr., the limiting values of F are found to be $+7.78$ dptr. and -24.68 dptr. It may happen that the required power of the correction-glass is greater than this extreme positive value, as was stated above; and in such a case no form of so-called "Punktal" spectacle lens is to be had, and a "Katral" lens with an aspherical surface will have to be employed. On the other hand, the extreme negative value of F is more than sufficient to satisfy the requirements of a correction-glass with punctual imagery for the worst cases of myopia.

As to the two solutions of the quadratic equation in F_1, one of them gives a meniscus lens of shallower form than the other, and it is this shallower type that corresponds to the modern "Punktal" spectacle lens. The other more concave form is similar to the so-called "periscopic" eye-glass which was recommended by Dr. Wollaston more than a century ago.

If the magnitude denoted by Z' is regarded as a variable parameter, the equation $\alpha F_1^2 + \beta F_1 + \gamma = 0$ will represent a family of ellipses of similar character, as has been pointed out by H. Boegehold in a paper published in 1920 in *Zs. f. ophthalm. Optik*, 8, p. 10–16. These ellipses all have a pair of common tangents which intersect at the origin of the rectangular system of coördinates. One of these tangents (as mentioned already) is the abscissa-axis itself which touches each ellipse at the point for which $F = -(n-1)Z'$, corresponding always to a comparatively high negative value of F. The equation of the other common tangent to this family of ellipses is $F_1 = n^2 F$. This straight line touches each ellipse at a point whose coördinates are:

$$F = \frac{n-1}{n(n+1)-1} Z', \quad F_1 = n^2 F.$$

4. GENERAL CASE; WITH NUMERICAL EXAMPLES

If, however, the thickness of the lens is taken into account, Seidel's expression for the condition that a single lens of unit refracting power ($F=1$), of refractive index n and of thickness $d=\mathrm{n}\,c$, shall be free from astigmatism of oblique bundles for the case of an infinitely distant object, leads to the following formula arranged here as an equation of the 5th degree in F_1:

$$\begin{aligned}
c^2F_1^5 &- \{\mathrm{n}c(\mathrm{n}+2)-2cS_1+1\}cF_1^4\\
&+\{c\,S_1^2-2(\mathrm{n}^2c+\mathrm{n}\,c+1)S_1+\mathrm{n}(2\mathrm{n}^2c+\mathrm{n}\,c+2\mathrm{n}+4)\}c\,F_1^3\\
&-\{(\mathrm{n}^2c+2\mathrm{n}\,c-\mathrm{n}^2+1)c\,S_1^2+2\mathrm{n}(\mathrm{n}^2-2\mathrm{n}^2c-\mathrm{n}\,c-\mathrm{n}-3)c\,S_1\\
&\qquad\qquad+\mathrm{n}(\mathrm{n}^3c^2+4\mathrm{n}^2c+2\mathrm{n}\,c+\mathrm{n}+2)\}F_1^2\\
&-\{\mathrm{n}(2\mathrm{n}^2-2\mathrm{n}^2c-\mathrm{n}\,c-2)c\,S_1^2+2\mathrm{n}(\mathrm{n}^3c^2-\mathrm{n}^3c+3\mathrm{n}^2c+\mathrm{n}\,c-\mathrm{n}^2+1)S_1\\
&\qquad\qquad-\mathrm{n}^3(2\mathrm{n}^2c+2\mathrm{n}+1)\}F_1\\
&-\mathrm{n}^2\{(\mathrm{n}^2c^2-2\mathrm{n}^2c+2\mathrm{n}\,c+\mathrm{n}^2-2\mathrm{n}+1)S_1^2\\
&\qquad\qquad-2\mathrm{n}(\mathrm{n}\,c-\mathrm{n}+1)S_1+\mathrm{n}^2\}\;=0,\quad (F=1,\ U_1=0). \qquad (3)
\end{aligned}$$

In this equation $S_1=1/s_1$ where $s_1=\mathrm{A}_1\mathrm{Z}$ denotes the distance of the point designated by Z from the first surface of the lens. The equation above does not contain the magnitude F, because it applies to the case of a lens of unit power. In order to obtain the corresponding solution for a lens of any power F, positive or negative, all the linear magnitudes (denoted by the small italic letters) must be divided, and all the reciprocal magnitudes (denoted by the capital italic letters) must be multiplied, by the given value of F.

This equation may be arranged also either as a quadratic in S_1, namely:

$$\begin{aligned}
&\{(F_1^3-X)c^2+(\mathrm{n}-1)(\mathrm{n}^3-\mathrm{n}F_1-F_1^2+\tfrac{X}{\mathrm{n}})c-\mathrm{n}^2(\mathrm{n}-1)^2\}S_1^2\\
&\quad+2\{(F_1^3-X+\mathrm{n}F_1^2)c^2F_1-(F_1^3-X+Y\,F_1)c+Y\}S_1\\
&\quad-X(1-c\,F_1)-c(1-c)(F_1^3-X)F_1=0,\quad (F=1,\ U_1=0), \qquad (4)
\end{aligned}$$

or as a quadratic in c, namely:

$$\begin{aligned}
&\{(F_1^3-X)(S_1+F_1)^2+2\mathrm{n}\,S_1F_1^3\}c^2\\
&\quad-\{(\mathrm{n}-1)(F_1^2+\mathrm{n}F_1-\mathrm{n}^3-\tfrac{X}{\mathrm{n}})S_1^2+2(F_1^3-X+Y\,F_1)S_1+F_1(F_1^3-2X)\}c\\
&\quad-\mathrm{n}^2(\mathrm{n}-1)^2S_1^2+2Y\,S_1-X=0,\ (F=1,\ U_1=0); \qquad (5)
\end{aligned}$$

where the symbols X and Y are employed as abbreviations for the following expressions:

$$\begin{aligned}
X&=\mathrm{n}\{(\mathrm{n}+2)F_1^2-\mathrm{n}(2\mathrm{n}+1)F_1+\mathrm{n}^3\},\\
Y&=\mathrm{n}(\mathrm{n}-1)\{(\mathrm{n}+1)F_1-\mathrm{n}^2\}.
\end{aligned} \qquad (6)$$

Equation (4) implies that if a lens is given completely (that is, if the magnitudes denoted by n, c and F_1 are known and $F=1$), two positions of the narrow diaphragm may be computed for which the optical system will be free from astigmatism of oblique bundles for an infinitely distant object.

Similarly, for a known value of n and for a given curvature of the first surface of the lens and for a fixed position of the stop with respect to this surface, equation (5) implies that there will be two values of the thickness ($d = \text{n}\ c$) of a lens of unit power which will satisfy the condition of freedom from astigmatism of oblique bundles for an infinitely distant object.

If the absolute term of the quadratic in c, equation (5) above, is put equal to zero, the equation obtained is equivalent to the condition given by equation (2) for the case of an infinitely thin lens of unit power.

(1) A numerical example will illustrate better than anything else the method of using these formulae. Suppose that it is required to determine the form of a spectacle-lens of refracting power $F = 4$ dptr., made of glass of index $\text{n} = 1.52$, which shall be free from astigmatism of oblique bundles for a prescribed distance of the centre of rotation of the eye from the correction-glass. This latter distance A_2Z', as we saw, must lie between 25 and 30 mm, but the difficulty here consists in the fact that until the form of the lens has been ascertained, it is impossible to assign precisely the position on the axis of the corresponding point designated by Z; because what we really need to know is the value of $s_1 = A_1Z$ in order to substitute in equation (3) the value of $S_1 = 1/s_1$. If we assume that $z' = H'Z' = 30$ mm, that is, $Z' = 1/z' = 33\frac{1}{3}$ dptr., then, since $Z = Z' - F$, we shall have $Z = 29\frac{1}{3}$ dptr. Now if we put S_1 equal to this value of Z, we shall obtain finally a value of A_2Z' which for a suitable lens-thickness will be comprised between the limiting values 25 and 30 mm. It remains only to choose the value of the thickness of the lens which we shall take here as 3 mm ($d = \text{n}\ c = 0.003$ m). All these values are for a lens of refracting power 4 dptr. Hence, before substitution in equation (3) the value of c above must be multiplied, and that of S_1 divided, by 4; thus, we obtain for substitution: $\text{n} = 1.52$, $S_1 = 7\frac{1}{3}$ and $\log c = 7.8973377 - 10$. Evidently, the numerical

work is rather long and laborious. If equation (3) is written in the form

$$F_1^5+A\,F_1^4+B\,F_1^3+C\,F_1^2+D\,F_1+E=0,$$

we find finally with the above data:

$\log A=2.1334844-$, $\log B=2.6949651-$, $\log C=4.8645063-$,
$\log D=5.7627989+$, $\log E=6.0085579-$.

The cöefficients A, B, C, D and E having been determined, it remains now to ascertain the one root of this equation of the 5th degree which is applicable for our purposes. Under ordinary circumstances this would be a tedious task; but luckily it is comparatively easy to find an approximate value of F_1 which will enable us to locate the exact value without much difficulty. For this purpose all we have to do is to substitute in equation (2) the values $F=1$, $Z'=(33\tfrac{1}{3})/4=8\tfrac{1}{3}$ dptr.; whence we obtain for the shallower type of meniscus lens the approximate value: $F_1=2.78$ dptr. ($F=1$, $d=0$). With this clue to aid us, it is now a comparatively simple matter to ascertain the root of equation (3) which we are trying to find and which turns out to be:

$$F_1=2.74162 \text{ dptr.}, (F=1 \text{ dptr.}).$$

Accordingly, for a lens of power 4 dptr., we obtain finally the following system of values:

$F=+4.00000$ dptr., $n=1.52$, $d=3.0000$ mm, $A_1Z=+34.0909$ mm;
$F_1=+10.96648$ dptr, $F_2=-7.10423$ dptr.;
Radii: $r_1=+47.4172$ mm, $r_2=+73.1959$ mm;
$A_1H=-3.5054$ mm, $A_2H'=-5.4111$ mm.; $A_2Z'=+27.2704$ mm.

The field of fixation of the mobile eye has an angular extent of from 50° to 60°, say. Consider a narrow cylindrical bundle of incident rays inclined to the axis of the lens at an angle of 28° as representing the outermost bundle. The chief ray directed towards the point Z emerges from the lens and crosses the axis at a point Z″ such that $Z''Z'=+0.78$ mm; and the astigmatic difference of this outside bundle of rays as found by exact trigonometric computation is $P'Q'=-3.35$ mm.

The results of similar calculations by means of these formulae are given below for several other cases.

(2) For the following arbitrarily assigned values, namely:

$F=+4.250$ dptr., $n=1.52$, $d=3.5294$ mm, $A_1Z=+33.6129$ mm;

the computation gives:

$F_1 = +11.3645$ dptr., $F_2 = -7.2780$ dptr.;
Radii: $r_1 = +45.8275$ mm, $r_2 = +71.4479$ mm;
$A_1H = -3.9765$ mm, $A_2H' = -6.1941$ mm, $A_2Z' = +26.2118$ mm.

The spherical aberration along the optical axis for the chief ray of slope 28° is found to be $Z''Z' = +0.7918$ mm, and the astigmatic difference on this ray: $P'Q' = -3.2294$ mm.

(3) For the following arbitrarily assigned values, namely:

$F = +6.0000$ dptr., $n = 1.52$, $d = 3.0000$ mm, $A_1Z = +36.5853$ mm;

the computation gives:

$F_1 = +17.3130$ dptr., $F_2 = -11.7133$ dptr.;
Radii: $r_1 = +30.035$ mm, $r_2 = +44.394$ mm;
$A_1H = -3.853$ mm, $A_2H' = -5.695$ mm, $A_2Z' = +26.849$ mm.

The spherical aberration along the optical axis for the chief ray of slope 28° is found to be $Z''Z' = -0.297$ mm, and the astigmatic difference on this ray: $P'Q' = -0.62$ mm.

(4) For the following arbitrarily assigned values, namely:

$F = +6.0000$ dptr., $n = 1.52$, $d = 5.0605$ mm, $A_1Z = +41.1523$ mm,

the computation gives:

$F_1 = +14.0540$ dptr., $F_2 = -8.4493$ dptr.;
Radii: $r_1 = +37.0002$ mm, $r_2 = +61.5432$ mm;
$A_1H = -4.6884$ mm, $A_2H' = -7.7983$ mm, $A_2Z' = +28.1539$ mm.

The spherical aberration along the optical axis for the chief ray of slope 28° is found to be $Z''Z' = -0.0412$ mm, and the astigmatic difference on this ray: $P'Q' = -2.701$ mm.

(5) For the following arbitrarily assigned values, namely:

$F = +6.0000$ dptr., $n = 1.52$, $d = 6.1122$ mm, $A_1Z = +41.6667$ mm;

the computation gives:

$F_1 = +14.0540$ dptr., $F_2 = -8.5363$ dptr.;
Radii: $r_1 = +37.0002$ mm, $r_2 = +60.9165$ mm;
$A_1H = -5.7210$ mm, $A_2H' = -9.4190$ mm, $A_2Z' = +27.4779$ mm;

The spherical aberration along the optical axis for the chief ray of slope 28° is found to be $Z''Z' = +0.7731$ mm, and the astigmatic difference on this ray: $P'Q' = -3.010$ mm.

(NOTE. As a matter of fact, numbers (4) and (5) above were computed by a different method from that explained under number (1); which amounted to assigning in advance the value of F_1 and calculating the corresponding value of d by solving the quadratic equation in c, equation (5) above.)

(6) For the following arbitrarily assigned values, namely:

$F = -8.5000$ dptr., $n = 1.52$, $d = 1.7647$ mm, $A_1Z = +23.5294$ mm;

the computation gives:

$F_1 = +3.0627$ dptr., $F_2 = -11.6038$ dptr.;
Radii: $r_1 = +169.7828$ mm, $r_2 = +44.8130$ mm;
$A_1H = +1.5849$ mm, $A_2H' = +0.4183$ mm, $A_2Z' = +27.3945$ mm.

The spherical aberration along the optical axis for the chief ray of slope 28° is found to be $Z''Z' = -1.7592$ mm, and the astigmatic difference on this ray:

$$P'Q' = +1.0746 \text{ mm}.$$

Lack of space prevents any discussion of these results, but it is hoped that they will be useful not only as illustrating the method of calculation but also as affording a basis of future calculation of lenses of this character.

5. CURVATURE OF THE STIGMATIC IMAGE

The so-called "Punktal" spectacle lens is a correction-glass which is designed to give a punctual image of the infinitely distant object-plane. The image of this plane in the lens is a curved surface which crosses the axis at the second focal point F′ of the glass, which is likewise the position of the far point of the passive ametropic eye. The centre of curvature of this surface is a point on the axis which may be designated by K, such that $F'K = -n f$, where $f = 1/F$ denotes the focal length of the lens (whose thickness is assumed here to be negligible). The centre of the so-called "far point sphere" is at the centre of rotation of the eye, its radius being F′Z′. The image-surface is tangent to the far point sphere at F′, but in general these two surfaces will not have any other point in common. The condition that the two surfaces shall coincide is that their centres K and Z′ shall be coincident; which means that we must have $f = -AZ'/(n-1)$, where A designates the optical centre of the infinitely thin lens. For $n = 1.52$ and $AZ' = 30$ mm, this condition gives $f = -57.7$ mm or $F = -17.3$ dptr. Accordingly, it may be concluded that the stigmatic image surface of a "Punktal" lens will coincide with the far-point sphere only for an unusually strong negative correction-glass. In general, therefore, the eye will have to make a little effort of accommodation in order to see distinctly in oblique directions through a "Punktal" correction-glass.

6. Merely as an additional note, it may be remarked here that the condition that an infinitely thin lens of refracting power F, placed in front of the eye at a distance $z' = 1/Z'$ from the centre of rotation of the eye, shall give a punctual or stigmatic image of an *object lying in the primary focal plane of the lens* (as in the case of an ordinary magnifying convex lens or of a spectacle lens for a very aged person who has lost entirely the power of accommodation) is as follows ($U = U_1 = -F$): .

$$\left(\frac{n}{n-1}F - Z'\right)^2 + \frac{n+2}{n(n-1)^2}F_2^2 - \frac{2n+1}{(n-1)^2}F_2F + \frac{2(n+1)}{n(n-1)}Z'F_2 = 0.$$

Negative values of F are excluded here, because if the object is in front of the eye, the lens must be positive. This equation also represents an ellipse which touches the F-axis at the point $F = (n-1)Z'/n$. The maximum value of F for $n = 1.52$ is $F_{max} = 0.38158\ Z' = 12.72$ dptr. for $Z' = 33\frac{1}{3}$ dptr.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY, NEW YORK, N. Y.
July 9, 1921.

ON TRACING RAYS OF LIGHT THROUGH A REFLECTING PRISM WITH THE AID OF A MERIDIAN PROJECTION PLOT

BY

F. E. WRIGHT

In many optical instruments, especially for military purposes, prisms are used to change the directions of incident rays of light and thus to enable the observer to see around a corner or over obstacles, as in periscopes; or to invert or erect an image, as in field glasses; or to bring two images in juxtaposition, as in eyepiece prisms for range-finders. In many cases it is not an easy matter to follow the paths of the rays through a prism or a train of prisms, chiefly because the rays are reflected along different directions in space and these directions may become confused in the mind of the observer. Experience has shown that, as an aid to the visualization of spacial relations of this kind, projections are helpful and are in constant use in many branches of science, such as astronomy, geodesy, crystallography, and crystal optics. In the present paper the usefulness of projection methods in the study of reflecting prisms is emphasized and the mode of their application is illustrated on some of the standard types of reflecting prisms.

There are a number of different types of projection available; but for the present purpose the projection plot of Fig. 1 is probably the best. In this Postel or angle meridian projection both great and small circles of the sphere become oval curves resembling hyperbolas. They are drawn 2° apart. This projection is used in map work to represent the hemisphere; in it latitude and longitude, or polar distance (co-latitude) and azimuth are plotted as in other types of projection. The angle projection is preferable to the stereographic, the orthographic, the gnomonic, and other types of projection because its distortion over the entire field is less and the relative accuracy of different parts of the plot is more uniform.

To apply this projection to prism work, consider each prism face to be represented by its normal, and this normal to pass

through the center of a sphere (projection sphe
Each prism face is thus reduced to a line (rad
and this in turn becomes a single point in the pro
a ray of light is represented by a point in the

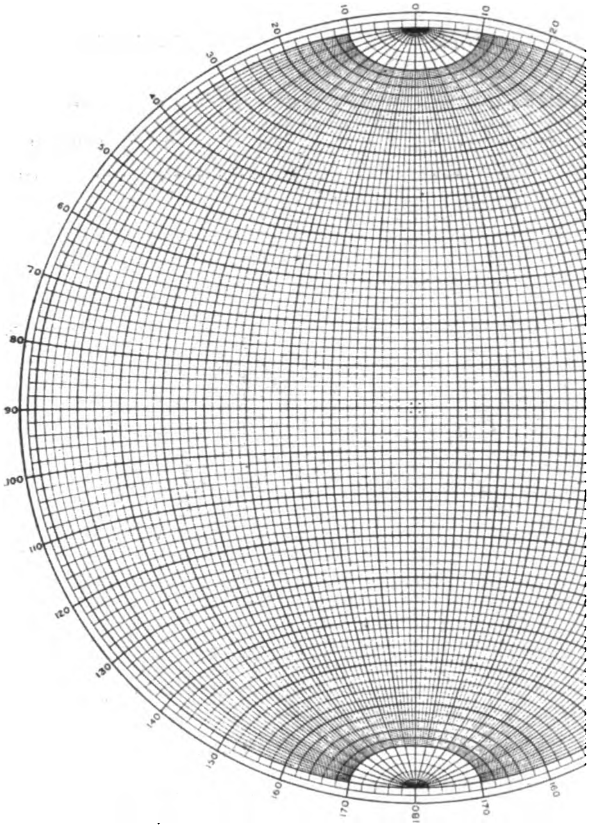

FIG. 1. The Postel or angle meridian projection. In this equidistant
distances are directly proportional to the circular arc ρ and are plotted as x =
polar angles is nil throughout, while that of the tangential azimuthal angl
projection at the center to $\pi/2$ to 1 at the margin. In this plot the angul
successive great circles or small circles is 2°.

angle of incidence is then the angle included be
in the projection; the angle of reflection is e
of incidence, and the reflected ray is in the plan
the two given points (great circle in the project

The usual method of using the projection plot (Fig. 1) is to place a sheet of thin paper over the plot and to make all measurements on this sheet which is held in position by a needle point passing through the center of the projection plot. About this needle as axis the sheet of paper can be rotated; the angular distance between any two points can thus be read off directly on the great circle passing through the two points.

The mode of application of the projection plot is best shown by concrete examples of the different types of prisms, such as prisms with one reflecting face, with two reflecting faces, and with three or more reflecting faces.

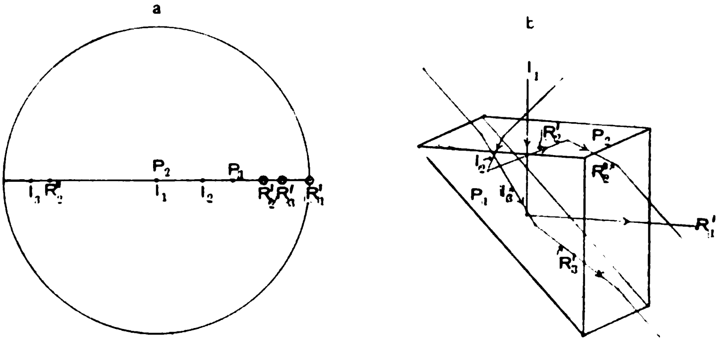

FIG. 2. Perspective drawing (*b*) and corresponding projection plot (*a*) illustrating paths of rays through a right angle reflecting prism. Thus the ray I, incident at an angle of 45° with the reflecting face P_1 emerges along R_1' which is perpendicular to the ray I_1; the refracted ray I_2 incident upon P_1 is reflected along R_2'; the refracted ray I_3 incident upon P_1 is reflected along R_3' to P_2 whence it is reflected along R_3'' which is perpendicular to I_3 (fig. 2b). In the projection a point represents a ray traveling toward the center of the projection sphere; a small circle represents a ray traveling away from the center.

PRISM WITH SINGLE REFLECTING FACE

Let the direction of the ray incident on the reflecting face be the zenith or center I_1 in the projection (Fig. 2b). Let the normal to the reflecting plane be represented by the point P_1 of Fig. 2a. The plane of incidence is then the east-west plane and the angle of incidence is the angle I_1P_1; the angle of reflection is then $P_1R_1' = I_1P_1$ and this can be read off and plotted directly on the thin paper above the projection plot of Fig. 1. In this plot the angular distance between two successive great or small circles is 2°. In Figs. 2 to 8 the normal to a reflecting surface and also the direction of a ray proceeding toward the center of projection are indicated by a single point; the normals to reflecting surfaces which point below the horizon and also the directions of rays traveling away from the center of projection are indicated by small circles.

In case the angle of incidence is 45°, the reflected ray traverses the horizontal axis of the projection and is perpendicular to the incident ray. In case rays from an object of appreciable angular extent are reflected, the angle of incidence varies within certain limits, but for each ray its angle of incidence is equal to its angle of reflection. This is simply another form of statement for the fact that the reflecting plane functions as a plane of symmetry between incident rays and reflected rays. From these relations it is possible to deduce in the case of multiple reflections the exact relations between original object and each intermediate reflected image.

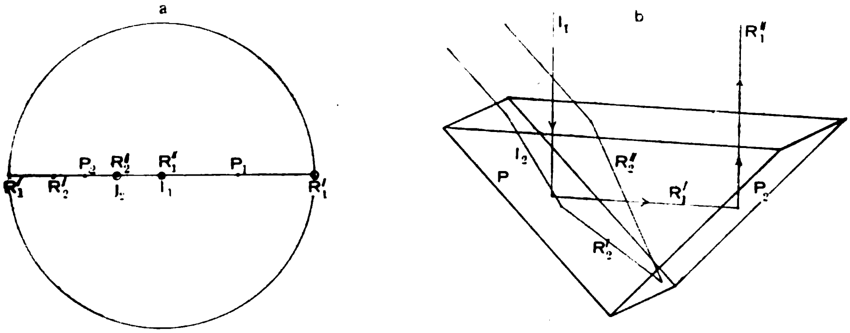

FIG. 3. Perspective drawing (*b*) and corresponding projection plot (*a*) illustrating paths of rays through a right angle reflecting prism of the Porro type. The incident ray I_1 is reflected by P_1 along R_1' and this in turn is reflected by P_2 along R_1'', which coincides in direction with I_1. Similarly for the incident ray I_2 and the reflected rays R_2' and R_2''. This prism is of the constant deviation type and has the effect of rotating the image through 180°.

PRISM WITH TWO REFLECTING SURFACES

I. INCIDENT RAY CONTAINED IN THE PLANE OF THE TWO FACE NORMALS

(a) *Deviation* 0°. *Porro prisms.*—Let us consider first the cases in which the incident ray is contained in the plane of the two normals to the reflecting faces. (Fig. 3b). Let the angle between the normals to the two faces be 90°. Then the first reflection at P_1 changes the direction of the incident ray I_1 to R_1'. The incident ray I_1 proceeds toward C, the center of the projection sphere, the reflected ray R_1' travels away from the center of the sphere; its direction in this case is from west to east (Fig. 3a). The same ray functions as an incident ray when it impinges on the face P_2; in the projection this is indicated by taking the opposite end of the diameter $R_1'\ C\ R_1'$. The ray $R_1'\ C$ impinges then from the west on the face P_2 and is reflected in the direction R_1'' which coincides with the direction of incidence of the original ray. The image does not coincide with the object, however, but appears to be turned on an axis normal to the plane of incidence through an angle of 180°. In the case of vertical reflection from a single face, there is no rotation possible by which the image can be made to coincide with the object. In this case the single mirror functions as a plane of symmetry between object and image and the two are in general enantiotropic to each other, i.e., as the right hand is to the left hand. From Fig. 3a it is evident that irrespective of the angle of incidence the second reflected beam R_2'' is always parallel with the incident beam I_2 providing the angle between the reflecting surfaces is a right angle.

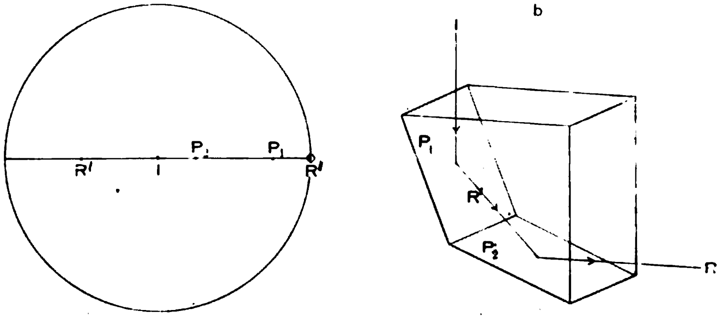

FIG. 4. Perspective drawing (*b*) and projection plot (*a*) illustrating paths followed by a ray through a Wollaston prism. The incident ray I is reflected at P_1 along R' which in turn is reflected by P_2 along R''. R'' is normal to I and the image is rotated through 180°.

(b) *Deviation 90°.*—Let the angle between the normals to the reflecting faces be 45°.

Wollaston prism. In this prism (Fig. 4), for which the angle between the normals to the reflecting faces is 45°, the second reflected beam R'' is perpendicular to the incident beam I irrespective of the angle of incidence. In this prism, as in the Parro prism, the image is similar to the object, but appears, with reference to it, to be turned through an angle of 180° on an axis normal to the plane of incidence.

Penta prism. In this prism the angle between the normals to the reflecting faces is 135° (Fig. 5). The emergent ray R'' is perpendicular to the incident ray I and the image bears the same relation to the object as does the image in the Wollaston prism.

The right angle reflecting prism as a constant deviation prism. In this prism the angle between the incident ray I_2 and the emergent ray R_2'' is 90° (Fig. 2). The image is similar to the object but is inverted as in the foregoing twice reflecting prisms.

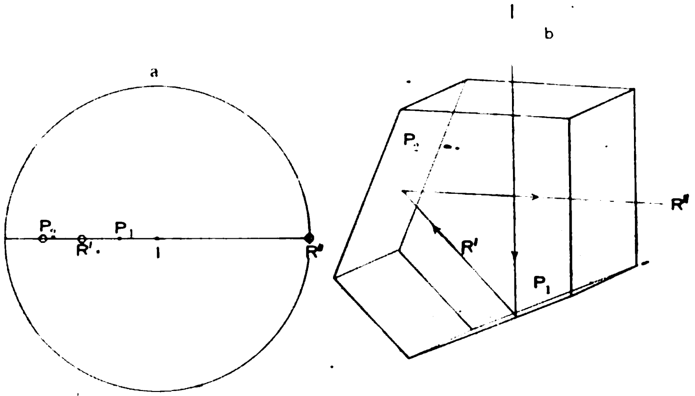

Fig. 5. Perspective drawing (*b*) and projection plot (*a*) showing the paths followed by a ray through a penta prism. The incident ray I is reflected by P_1 along R' to P_2 whence it proceeds along R'' at right angles to I. The image is rotated through 180° as a result of the two reflections.

In view of the fact that in this and other constant deviation prisms in which the surface at which the incident ray enters (first refraction surface) includes with the second refraction surface an angle equal to twice that between the normals to the reflecting surfaces, the refraction angle at the first refracting surface is equal to the refraction angle at the second refraction surface; from this it follows that the angle between the ray (I) incident upon the first reflecting surface (first refracted ray) and the second reflected ray (R'') is equal to the angle between the incident ray and the emergent ray. This relation obtains for any angle between the reflecting faces, providing the entrance and emergent faces are placed as indicated in figs. 3 to 6.

In all constant deviation prisms of this group the two reflecting faces may be considered to function as a unit while the two refracting surfaces which include an angle equal to twice that between the two reflecting surfaces function as a second unit. In the prism of Fig. 3 the angle between the two reflecting surfaces is 90°; the angle between the two refracting surfaces is therefore 180°; in other words, they coincide and are one and the same plane.

(c) *Constant deviation of any angle.*—From the foregoing comments it is evident that a constant deviation prism of any deviation angle between 0° and 90° or still greater can be made, and that for a constant deviation prism of given shape the deviation is fixed and independent of the refringence and dispersion of the glass of which it is made

II. INCIDENT RAY NOT IN THE PLANE OF THE NORMALS TO THE TWO REFLECTING FACES

The relations between incident rays and emergent rays are much more complex in this case because the rays at each reflection proceed along a different direction in space, and the mind does not readily visualize the relations without graphical aid of some kind. The Figs. 6 and 7 indicate that only for a few types of prisms do the incident and emergent rays bear a relation one to another which can be utilized in practical optics.

(a) *Roof-edge prism.* Let the angle between the reflecting surfaces be 90°. It can be shown with the aid of the graphical plot (Fig. 1) that in this case any ray incident in the plane perpendicular to that of the normals to the reflecting faces emerges in the same plane and in a direction equally inclined with, but on the opposite side of, the plane of the normals. In case the ray enters along the direction I_{10}, it emerges along the direction R_{10}'' (Fig. 6a). It is because of these relations that the roof-edge pair of reflecting faces can be substituted for any single reflecting face and an inversion of the image thereby obtained. (Fig. 7). Thus in prism binoculars an inversion of the image can be obtained by the use of two Porro prisms, or by a roof-edge, right angle prism in combination with a penta prism or a wollaston prism; or by a roof-edge reflecting unit combined with any two reflecting faces equally inclined to the axis, the first reflecting the light into the roof-edge prism, the second restoring the original line of propagation, as in the Brashear-Hastings prism and similar types of prisms used in telescopes in place of a lens erecting system (erecting eyepiece).

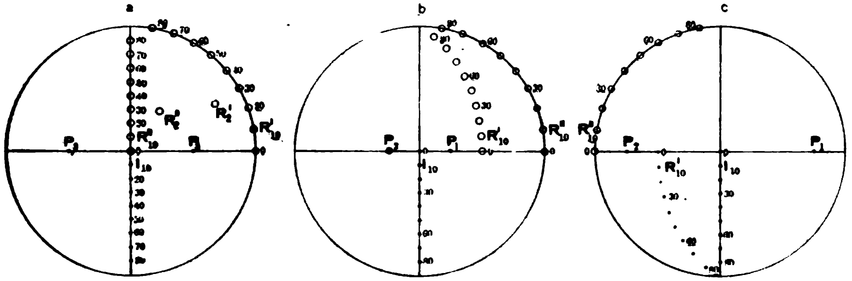

FIG. 6. Projection plots illustrating the paths of rays followed in a roof-edge (fig. 6) prism or a prism in which the angle between the normals to the reflecting faces is 45° or 135° (fig. 6*b* and 6*c*). (a) Incident rays I including angles of 0°, I_{10}, 20°—80°, respectively, with the bisector of the roof edge angle P_1P_2, are reflected by P_1 along the directions 0°, R_{10}', 20°—80° and proceed to P_2 whence they travel along 0, R_{10}', 20°—80° respectively. The plot shows that the rays R'' after reflection at P_2 are contained in the vertical plane and include an angle with the bisector of the roof-edge angle P_1P_2 equal to that between the incident ray and this bisector. A ray incident along I_2 is reflected by P_2 along R_2'', a direction which includes the same angle with the bisector of the angle P_1P_2, but is on the opposite of the plane P_1P_2, as though this plane functioned as a plane of symmetry between incident ray I_2 and the second reflected ray R_2''. Figs. 6*a* and 6*b* are of the same general construction as fig. 6*a*. The angle between the normals of the reflecting faces is, however, different (45° and 135°) and hence different relations are found between incident and second reflected rays. Obviously the method of projection is not limited in its application to prisms of these particular angles but can be applied to prisms of any angles and any number of reflecting face s.

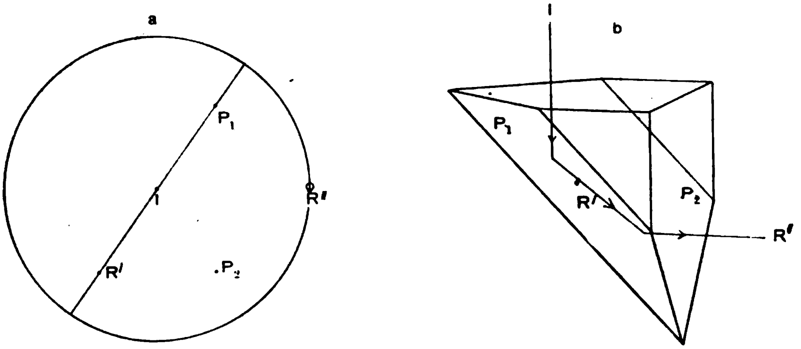

FIG. 7. Perspective drawing (*b*) and projection plot (*a*) illustrating the paths followed by a ray though a roof edge reflecting prism. The incident ray I is reflected at P_1 along R_1, which in turn is reflected at P_2 along R''.

(b) Let the angle between the normals to the reflecting faces be 45° or 135° (Figs. 6b, 6c). In this case the emergent ray R'' resulting from a ray I incident in the optical north-south plane (Figs. 6b and 6c) is contained in the horizontal plane and includes an angle with the plane of the normals to the reflecting faces equal to that included by the incident ray with the bisector of the angle between the two reflecting planes (zenith of projection). For rays not contained in the north-south vertical plane of Fig. 8 the emergent ray does not bear a symmetrical relation to the principal planes. This type of prism is therefore restricted in its usefulness.

THREE REFLECTING FACES MUTUALLY AT RIGHT ANGLES

In Fig. 8 let P_1, P_2, P_3 be the projection points of three reflecting faces mutually at right angles. Let I be the projection point of a ray impinging on P_1; after reflection it takes the direction R'; is then reflected by P_2 and travels in the direction R'' to P_3 whence it is reflected in the direction R''' which coincides with I. This type of prism, which is used especially in the adjustment of range-finders with self-contained base and in long-distance signalling, has the peculiarity that any entrant ray is reflected back upon itself or along its own path. The image produced by the thrice reflected rays is inverted and enantiotropic with respect to the

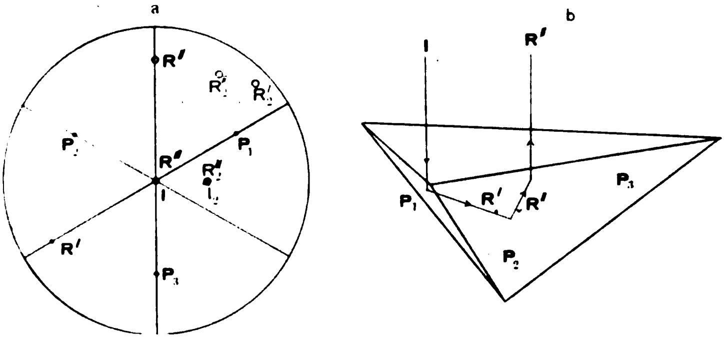

FIG. 8. Perspective draw'ng (*b*) and corresponding projection plot (*a*) illustrating paths followed by a ray on transmission through a prism having three reflecting faces $P_1P_2P_3$, mutually at right angles. The incident ray I is reflected by P_1 along R', which in turn is reflected by P_2 along R'' and this in turn is deflected by P_3 along R''', a direction which coincides with I. The third reflected ray is thus deflected back on itself. In fig. 3*a* the construction shows that for any incident ray I_1 the third reflected ray R''' coincides with the incident ray but travels in the opposite direction. This type of prism is therefore useful for signalling purposes because the ray of light is always reflected back upon itself irrespective of the exact adjustment of the prism.

object; in other words, the uplifted right hand appears after the three reflections as the left hand pointing downwards.

These examples suffice to indicate the usefulness of the projection method in the study of reflecting prisms. Incidentally, it may be noted that the projection method, especially the gnomonic projection, affords the simplest and most direct method for the drawing of prisms of complex shape. It would go too far in the present note to consider these methods which are in general use by crystallographers.

SUMMARY

Prisms are used in optical instruments to change the courses of the paths of transmitted rays of light; prisms are of different shapes and sizes and are used either singly or in combination. In many prisms it is difficult to visualize the paths followed by the rays on transmission without graphical aid of some kind. In the present paper the angle meridian projection plot is suggested as suitable for the purpose. Its application to the study of prisms is illustrated by a series of examples from the more common types of reflecting prisms.

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WASHINGTON, D. C.
June, 1921.

THE COINCIDENCE TYPE SELF-CONTAINED RANGE FINDER

BY

I. C. GARDNER

The modern horizontal base self-contained range finder determines the range by a method of triangulation. The base line is contained in the instrument and by reason of its extreme shortness in comparison with the range to be determined, the accuracy and precision required in the determination of the angles at the two ends of the base line is very great. The triangle solved by the range finder is shown in Figure 1. AB is the base of length *b*, *c* the target and AC the range *r*. In the usual construction, the base length and angle at A are maintained constant in value. The scale upon which the other angle is read may then be graduated to read directly in range.

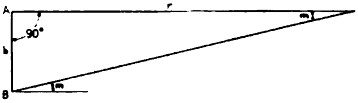

FIG. 1. Fundamental triangle

Since the angle *m* is very small, we may write with sufficient accuracy $\frac{b}{r} = \mathrm{m}$. Differentiating and transposing:

$$dr = -\frac{\mathrm{r}^2}{\mathrm{b}}\, dm \qquad (1)$$

and we have the relationship existing between the range error *dr* and the corresponding angular error *dm*. In the horizontal base range finder, as used by the Field Artillery, the base length is one meter. At a range of 4000 meters the total value of the angle to be measured is 50 seconds and if an accuracy of 2% is to be secured, the value of this angle must be determined to within one second.

In an attempt to obtain a high degree of precision and accuracy in an instrument expected to withstand the rough usage of service conditions, the design and construction has been very carefully worked out along novel lines which differentiate this instrument completely from all other angle-measuring instruments. It would

be impossible to obtain the accuracy required
from the two ends of the base line were ma
observers or successively by a single observ
two observations from the two ends of the ba
taneously by a single observer, the fields of
telescopes being viewed through a single ocul
been suitably combined by an ocular prism.

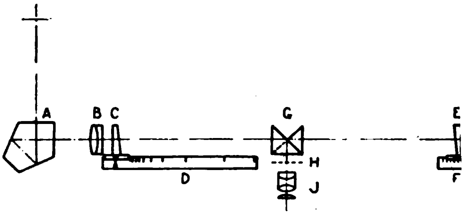

FIG. 2. Optical parts of range finder

Figure 2 is a diagrammatic sketch showing
parts of the range finder. The details of const
changed in some instances in order that the oper
clearly indicated for the reader, but the esse
represented. A and A′ are two penta prisms
base line, by which the rays of light are directed
prism shown diagrammatically at G. B and
objectives. The ocular prism functions in such
H there is a "divided" field, that is, the field is
parts by a sharp boundary line. The image i
field is that produced by rays traversing pe
objective B, while in the other part the imag
A′ and B′. This composite field is viewed throu

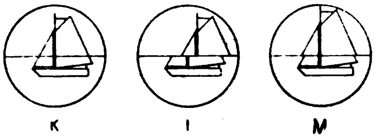

FIG. 3. Erect field

Figures 3 and 4 show two types of such a di
shown in Fig. 3 is known as the erect type.
shown when "coincidence" is secured. Except
of a dividing line, the field appears as though f

objective. At L the same type of field is shown when the range finder is not adjusted for coincidence. In Fig. 4 the inverted type of field is represented in which the objects in the upper half of the field are inverted. When coincidence is secured as at N, the image in the upper half is the same as would be obtained if the image in the lower half were mirrored in the dividing line. At O the field is not adjusted for coincidence.

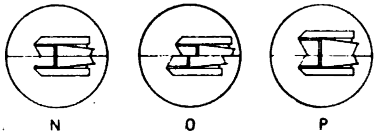

FIG. 4. Inverted field

The erect type of field is usually preferred when the target is of a regular geometrical shape with well-defined boundaries and it is, therefore, commonly used by the Navy and Coast Artillery Corps. The inverted type is considered to have advantages when the target is poorly defined or very irregular in shape and is consequently used by the Field Artillery and Infantry where the objects sighted upon may be trees, trench parapets, etc.

Returning to Fig. 2, if prisms C and E are not present and if the instrument is symmetrically and perfectly constructed so that the deviations of the rays at all reflecting and refracting surfaces have their nominal values, the image of an infinitely distant object presented by the two objectives will be the same and in the field we shall have coincidence as indicated at K or N, Figs. 3 and 4. If at *C* and *E* we insert two prisms of glass of very small angle and with their thicker edges turned oppositely, we shall have both images shifted slightly in the focal plane in the same direction and the coincidence will not be affected. If now an object is viewed which is within the useful working range of the instrument, the two images in the two portions of the divided field will not be in coincidence. If in particular the range finder is rotated in a horizontal plane until the image of the object presented by A′ lies in the center of the field, the image presented by A will be displaced from the center, the distance $(\tan \frac{b}{r})f$, or since the angle

b is small, the displacement will be $\frac{b}{r}f$ where b is the base length, r the range to be determined and f the focal length of the objective B. If the prism at C bends the ray through the angle i by shifting the prism in the appropriate direction through a distance s so that

$$s \tan i = \frac{b}{r} \cdot f \qquad (2)$$

or with sufficient exactness-

$$s\,i = \frac{b}{r} \cdot f \qquad (3)$$

the image may be brought back into coincidence. The length s through which the prism C is moved is then a measure of the range and the scale along which C moves may be graduated to read directly in range. The graduations will be marked according to the formula:

$$s = \frac{b\,f}{i} \cdot \frac{1}{r} \qquad (4)$$

where s is the distance of the graduation for range r from the original position of the prism which is marked infinity. The size of the instrument and requirements of portability usually limit the value which may be given to b and f; so that by making i the angle of deviation produced by the prism C, very small, the distance between successive graduations may be increased and the scale made as open as may be desired.

It will be noted that the scale is a reciprocal one, and as r increases, the distance between successive graduations for a given increment in r decreases. If the scale were graduated to read the angle subtended at the target by the base of the instrument, the formula for determining the graduations is:

$$s = \frac{f}{i} \cdot m \qquad (5)$$

where m is the angle to be measured.

The wedge at E provides the so-called infinity adjustment. The effect of slight deformations due to stresses originating from temperature changes and other conditions of service is to introduce errors much greater than the instrumental errors of the range finder. If the instrument is trained on an object at a known range, with the prism *C* set opposite the corresponding reading on the scale at *D*, there should be coincidence between the two images in the field. If this is not the case, coincidence may be secured by shifting the prism *E*. The range finder is now adjusted so as to be accurate for this particular range, the errors due to the slight deformations having been compensated by the shift of *E*. After the scale has been justified at this one point, if the instrument has been correctly constructed and designed, it will indicate ranges accurately over the entire scale.

The known range commonly selected is an "artificial infinity" secured by the use of a "measuring lath" or some type of internal adjustment and for this reason the adjustment of prism *E* is usually referred to as the infinity adjustment. On some types of instruments the infinity adjustment is provided with a scale as at F graduated in entirely arbitrary divisions which are numbered. This enables one to make successively several independent infinity adjustments and to select the mean of the several readings on the scale *F* as denoting the correct adjustment.

The particular methods of construction utilized in the different makes of range finders vary. In general, the infinity adjustment is not accomplished by a prism as shown at *E* shifted along the axis of the instrument. Instead the prism is commonly placed between *B′* and *A′* and is so placed that the plane in which the deviation takes place does not lie in the plane of Fig. 2 but at an angle to it. Then, by rotating the prism about the axis of the instrument, the component of the deviation lying in the plane of the drawing (the part which affects the coincidence) may be varied and the infinity adjustment thus secured. In the Barr & Stroud range finder, the window placed in front of *A′*, not shown in Fig. 2, is slightly wedge-shaped and may be rotated for securing the desired correction.

In the Bausch & Lomb range finder, the longitudinal movement of *C* is obtained by connecting *C* with a spiral thread cut on the interior of a collar which revolves about the entire optical tube of the instrument. The range scale is then cut in a spiral on the outside of this collar. This enables the scale to be made very long and open. On the Barr & Stroud instrument the scale *D* is attached rigidly to the prism and moves past a fixed index. A window in the outside tube protected by glass enables the scale to be read.

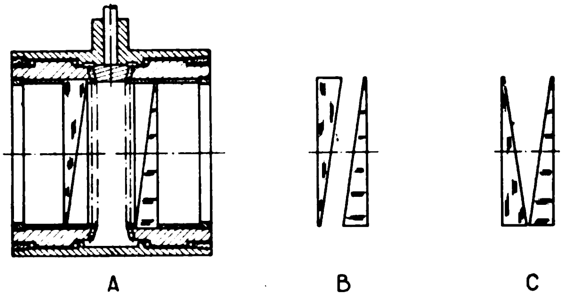

FIG. 5. Rotating prisms

In some range finders a prism of the type shown at *C* in Fig. 2 is not employed for securing coincidence. Instead, two similar prisms of small angle are mounted, one before the other, as shown in section in Fig. 5. They are connected by gearing in such a manner that the observer in securing coincidence, causes the two prisms to rotate through equal angles in opposite directions. When the prisms are oriented with respect to each other as at *B*, the deviation produced by one prism is neutralized by the other and there is no deviation. When they are as at *C*, the maximum deviation is produced and for intermediate positions the deviation has intermediate values. Such a combination of prisms then gives a deviation which is constantly in one plane and which may be caused to vary from zero to a maximum value.

It is very difficult to make *B* and *B′* so that they have precisely the same equivalent focal length. If they differ, coincidence will not be independent of the position of the object on the dividing line. Frequently, one objective, say *B*, is made of longer focal length than *B′*, and a second lens termed an "equal magnification lens" is placed between *B* and the focal plane. By varying the distances between the two lenses, the equivalent focal length of the two combined may be continuously varied until equal to that of *B′*.

There are numerous varieties of ocular prisms in use. In Fig. 6, there is shown a drawing of a type of prism in the one meter base invert type range finder, as used by the Field Artillery. Two views of the prism are shown. In the first, the different components are separated in order that the different reflecting surfaces may be seen more clearly. The part *Q* receives the light from the left hand objective and after two reflections, the light passes upwards to the eyepiece at an angle of approximately 60 degrees. The rays of light from the right hand objective after two reflections in the rhomb *R* are reflected forward and upwards by prism *S*. It should be noted that one reflection takes place on the lower surface of *S*, a portion of which is silvered. It is the boundary of this silvered region which forms the dividing line, the rays which are not intercepted and reflected by the silver going downward and never entering the ocular. On the other hand, the rays from the left hand objective which strike the silver are reflected downward and do not enter the ocular. The prism *T* cemented on the inclined portion of *S* is traversed by light from both objectives. Its purpose is to equalize the length of light path for different portions of any beam in such a way as to obviate spectral dispersion.

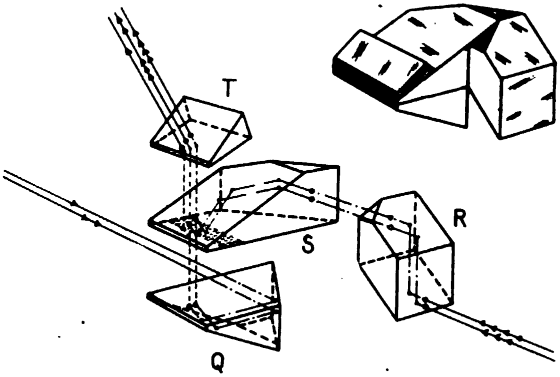

FIG. 6. Ocular prism

In order to prevent, so far as possible, any relative motion between the two objectives and the ocular prism, these are commonly mounted in a substantial inner tube termed the optical tube. This is designed very carefully and made from especially selected homogeneous material in order that the detrimental effect of temperature changes may be nullified so far as possible. Slight displacements of the penta prisms do not so seriously affect the accuracy of the instrument and they are, therefore, carried by the outer tube. The outer tube is covered with heat insulating material, in order to protect the inner optical tube from non-uniform temperature changes. In order that slight deformation of the outer tube may not subject the optical tube to mechanical stress, the two are connected by a three-point suspension. A slight displacement of one tube relative to the other may lead to an appearance in the field as shown in Figures 3 and 4 at M and P, where the dividing line cuts the two images at different heights. This is usually spoken of as an error in the halving adjustment. Means are provided for bringing the dividing line back to its correct position by shifting the inner tube relative to the outer. By inspection of the figures referred to above, it is apparent that if any object accurately perpendicular to the dividing line is used for determining the coincidence as for example, the mast of the ship in the drawing, a small error in the halving adjustment introduces no error in the determination of coincidence. If, however, a sloping line as the edge of the sail is used for setting coincidence, the error is considerable.

Long base range finders are frequently provided with an attachment termed an astigmatiser. Two small cylindrical lenses are arranged to be swung into the paths of the rays from the two objectives by means of external levers. When in position, the image of any point is drawn out into a short line perpendicular to the dividing line. During the advance of a hostile fleet at night, the best target available may be a searchlight. Without the astigmatiser, range taking on a searchlight would require that the image be kept exactly on the dividing line, a task which would prove difficult. With the astigmatising lenses in position, the image of the searchlight is a short line and the training of the range finder does not have to be done so accurately.

The large number of prisms in the optical train of a range finder gives rise to very great absorption of light. The variation of absorption in different instruments is large as so much of the light path is in glass that a relatively slight falling off of the transparency of the glass increases the absorption greatly. Range finders which have been measured show a transmission as great as 40% in some cases, while in particularly poor instruments the transmission falls as low as 17 to 20%. In general, the absorption for the two ends of a single instrument differs greatly as is shown by the fact that one of the fields is usually visibly darker than the other.

An ingenious modification of the range finder, as described above, has been invented by Eppenstein and is known in the Army as the "azimuth type" of instrument, in order to distinguish it from the more usual form which in contra-distinction is termed the "fixed base" type. Suppose that both prisms indicated in Fig. 2 are omitted and that one of the objectives has a slightly longer focus than the other. As both objectives serve a common eyepiece, the magnification of the two ends of this instrument will be different. Now, if the instrument is so constructed that all refractions and reflections take place in the nominal manner, the image of an object at an infinite distance will still remain in coincidence at the center of the field. In general, there will not be coincidence, except at the center as the two images are represented to different scales by reason of the difference in magnification. If the entire instrument is rotated in azimuth, the image in one half of the field will move faster than in the other half since the magnifications are different. Therefore, if the instrument is rotated as a whole in the proper direction, one image will overtake the other and coincidence will be secured. It will be shown below that the position on the dividing line at which coincidence is secured is a function of the range. Consequently, a scale may be etched on the prism along the dividing line and graduated to read range directly. This scale will appear in the field of the instrument and to determine the range, it is only necessary to rotate the instrument as a whole in azimuth until coincidence is secured. Then, on the scale opposite the point on the dividing line, where coincidence is obtained, the correct range may be read.

The method of graduating the scale is as follows: Let f and $f+\Delta f$ be the focal lengths of the two objectives. Assume that the instrument is so oriented that the image of the target falls in the center of the field of the objective, the focal length of which is $f+\Delta f$. Then, if the base length is b and the range is r, the image in the other half of the field is displaced from the center in the focal plane through the linear distance $\frac{bf}{r}$. If we turn the range finder in azimuth thru the angle k, the one image in the focal plane moves a linear distance $(f+\Delta f)k$, the other $f\,k$. (The angle k is so small that the angle is substituted for tan k). If in particular k is so chosen that—

$$(f+\Delta f)\,k=f\,k+\frac{bf}{r} \tag{6}$$

the one image will have overtaken the other and we shall have coincidence. Transposing and collecting terms, equation 6 becomes:

$$\frac{\Delta f}{bf}\cdot k=\frac{1}{r} \tag{7}$$

$(f+\Delta f)\,k$ is the distance from the center at which coincidence is obtained. If we denote this distance by l, we have:

$$l=(f+\Delta f)\frac{bf}{\Delta f}\frac{1}{r} \tag{8}$$

This then is the formula by which we can construct our scale. l is the distance measured from the center of the field for the graduation corresponding to any range r. It will be noted that the scale is a reciprocal one and that for a given base length the distance between graduations increases as Δf is decreased. The actual length of the scale is limited by the total length of the dividing line, that is, visible through the eyepiece. In an instrument constructed as described, only half of this length could be utilized for a scale as the infinity point from which the graduations begin is in the center of the field. In actual construction, the infinity adjustment is so set that the scale begins at the extreme edge of the field.

The advantages of this instrument lie in its simplicity and comparative freedom from moving parts as the moveable prism and its mechanism are eliminated. This also probably enables a more waterproof construction to be utilized. No estimate of its accuracy as compared with the fixed base type of instrument is available. One possible disadvantage lies in the fact that the center of the field cannot be used for all ranges. In particular, in making the infinity adjustment, coincidence is secured at the extreme edge of the field when conditions are most unfavorable, as regards aberrations. Nearly all of the 80 cm base range finders supplied to the Infantry are of the azimuth type.

The more important errors to which the range finder is subject may be classified under the two following heads: (a) Accidental errors arising from inability to determine coincidence precisely. which introduce errors proportional to the square of the range and to which the theory of errors is applicable; (b) systematic errors which introduce errors proportional to the square of the range and which can be completely compensated by a suitable setting of infinity adjustment.

(a) *Accidental errors arising from inability to determine coincidence precisely and which introduce errors proportional to the squrea of the range.*—The accidental errors depend fundamentally upon the limitations of the eye and the quality of optical performance of the range finder. Under the most favorable conditions, when two halves of an image separated by a dividing line, are presented to the eye as by the range finder, it is commonly considered that the probable error in determining coincidence will be of the order of ten or fifteen seconds. In order to secure round numbers, assume that the angular error is .00005 radians which approximately corresponds to 10 seconds. This is the error in the apparent field. If the magnification of the instrument is a, the true angular error in the fundamental triangle of measurement corresponding to the apparent angular error in the field of view of the ocular is $\frac{.00005}{a}$ radians. It should be emphasized that the value as given above gives the value of dm_1 under the most favorable conditions, that is, a trained observer, a well illuminated, sharply-defined target, and an instrument giving excellent definition. The value of dm_1 which will be obtained in practice is a function of the following elements: the training of observer, the character of target, the condition of atmosphere and the optical performance of the range finder. A bright, well-illuminated target, presenting well-defined contrasting lines on which the setting may be made will be much more favorable for range finding than a target lacking these features. When the air is "boiling," the irregular refraction makes exact coincidence difficult to obtain. Excellent definition, high light transmission and a large exit pupil may all be considered favorable to the accurate determination of coincidence.

It will be noted that the value of dm_1 is independent of the value of m which, of course, depends upon the range. The error in range corresponding to any particular value of dm_1 is, however, not independent of the range, but is related to it as shown by the following equation:

$$dr_1 = -\frac{r^2}{b}\,\frac{dm_1}{206\,000} \qquad (9)$$

The 206000 is introduced in the denominator in order that dm_1 may be expressed in seconds instead of radians. The magnifying power of the smaller range finders is generally from 12 to 15, depending upon the particular type. The larger range finders have two eyepieces, one of which gives a magnification of 28 and can only be used under very favorable conditions.

(b) *Systematic errors which introduce errors proportional to the square of the range and which can be completely compensated by suitable setting of the infinity adjustment.*—If we have an error of .00005 radians in the apparent field of a 15 power range finder, the error in the true field is only .000003, or approximately six tenths of a second. This implies a remarkable degree of accuracy and it is not surprising that under service conditions the parts of the instrument exposed to mechanical shock and heating and cooling are subject to relative displacements and deformations which, although slight when considered according to ordinary standards, are nevertheless sufficiently great to introduce systematic errors too great to be neglected. Many of these deformations are such that there is introduced an error dm_2 in the measurement of m, which is independent of the value of m and will, therefore, be compensated if a corresponding opposite error is introduced by a movement of the adjusting prism *E*, shown on Fig. 2. For example, let it be supposed that the penta prism A' is non-uniformly heated with a resultant change in the angle between the two reflecting surfaces such that the deviation due to penta prism A' is 90 degrees $+dm_2$ instead of the normal value of 90 degrees. If now the prism C is brought opposite a graduation corresponding to any particular range, and the range finder directed at an object at that distance, we shall in general not have coincidence in the field of the ocular because of the incorrect deviation produced by A'. This can be compensated for by appropriately shifting E along the scale F, so that an equal and opposite deviation is arbitrarily introduced. Since the error to be compensated is solely due to A' and does not vary with the range of the target, the error when compensated for one range is perfectly compensated for all other ranges. It is readily seen that if there are several errors arising from different sources, each of which, like the one dealt with above, is a constant angular error independent of range, the net result of all the errors can be compensated by a single shift of E. Such errors may be introduced by many causes, among which are flexure of the optical tube, relative displacement of the objectives at right angles to their common axis, displacement of ocular prism or unequal heating of the optical components. The adjustment of the prism E may, as outlined above, be made by selecting an object at a known distance and making the adjustment. As, however, objects at a known distance are not always available, recourse is usually had to some ingenious method of obtaining two parallel pencils of light. The prism C is then set opposite the infinity mark and the prism E adjusted until coincidence is obtained. This adjustment is, therefore, termed the infinity adjustment. If the infinity adjustment is imperfectly made, there is a constant error dm_2 introduced and to this there corresponds a range error:

$$dr_2 = -\frac{r^2}{b}\frac{dm_2}{206000} \qquad (10)$$

For any particular faulty infinity adjustment, introducing a constant angular error dm_2, the corresponding error dr_2 will always have the sign and magnitude indicated by the above equation. It will be noted that the error dr_1 unlike dr_2 is an accidental error and is as likely to be positive as negative. Therefore, under the most favorable conditions, the reading for a range r may be expected to lie between $r+dr_1+dr_2$ and $r-dr_1+dr_2$.

It is apparent that a range finder must be carefully adjusted if accurate readings are to be secured. Furthermore, the accuracy of angular measurement required of the range finder is so great that a method of adjustment in the field is necessary which, if possible, should be sufficiently convenient to permit adjustment immediately before the instrument is used.

Adjustment by means of a known range is the simplest method. If the distance to a suitable target is given, the range scale on the range finder is set to this known value. If the two halves of the target do not coincide, they are brought into coincidence by means of prism *E*, Fig. 2. The range finder is then in correct adjustment. Under service conditions, a known range is not ordinarily available.

A variant of this method depends upon the use of a celestial body, for which the moon is the most favorable. The range scale of the range finder is set at infinity and coincidence secured, as before, by movement of prism *E*, Fig. 2. The sun should never be used as a target for adjusting a range finder. Even though one should be provided with a filter sufficiently dense to protect the eye, the image of the sun is focussed upon a cemented surface of the ocular prism and is likely to damage the instrument. If a star is selected as the target, the astigmatiser should not be used unless previous tests show that the range finder gives the same readings with or without the astigmatiser in the light path.

Both of the methods of adjustment outlined require special conditions for their application. To overcome this objection, four methods of adjustment depending upon the use of an "artificial infinity" have been developed and applied to range finders.

The most simple and older method of adjustment is by means of the adjusting lath or stadia. One of the chief disadvantages of this method lies in the fact that the lath is not self-contained in the instrument. Also, it is not always convenient to obtain the range required when the lath is to be used. Accordingly, three types of internal adjustment have been devised which form an integral part of the instrument and permit adjustment to be made immediately before use.

The adjusting lath is commonly used with the smaller base range finder, probably because of its portability, simplicity and low cost. It consists of a simple bar carrying at either end a target which bears a well-defined black line upon a white background. These black lines should be the same distance apart as the actual base length of the range finder. The method of use is illustrated in Fig. 7.

In Fig. 7 *A* and *B* represent the two penta prisms at the ends of the range finder, and *C* and *D* the black lines on the targets at the ends of the adjusting lath. If the base length of the range finder is the same as the length of the adjusting lath, and if *CD* is parallel to *AB*, the paths *AC* and *BD* of the beams of light are accurately parallel. Consequently, if the scale of the range finder is set at the infinity mark and the adjusting prism *E*, Fig. 2, moved until coincidence is secured between the image of the right end of the lath in one field and that of the left end in the other, the correction is accomplished. If the distance *CD* does not equal *AB*, an angular error should be introduced, the value of which will be $\frac{AB-CD}{206\ 000\ AC}$ seconds. It is apparent that this error decreases

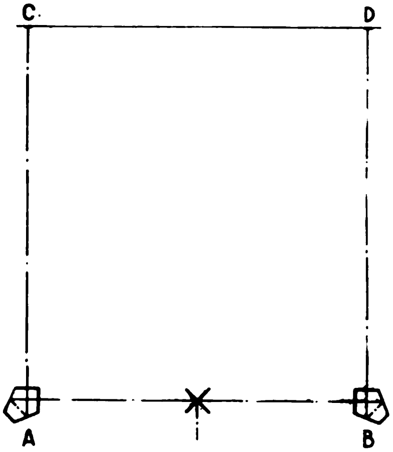

FIG. 7. Adjustment by lath

as *AC* is increased and the instructions of the manufacturer are that the adjusting lath shall be set at a distance of 200 meters or more from the range finder. However, it is not improbable that conditions may arise in service when it is impracticable to use as great a distance, and accordingly some consideration will be given to the use of an adjusting lath at distances of 50 and 100 meters.

The difference in length may be an apparent one due to lack of parallelism between *AB* and *CD* or to an actual difference of length which may be due to an initial difference of length between *AB* and *CD* or to a difference of length brought about by temperature changes. In order to avoid the foreshortening, the lath should be provided with an efficient sighting device by which parallelism can be secured to within at least one degree. By reference to a table of cosines, it can be seen that the error will then be negligible for a range finder of one or one and one-half meters base length. In addition, each adjusting lath should be fitted to the particular range finder with which it is to be used. To a first approximation it seems that the alteration in base length of the range finder with temperature will be due to the increased separation of the penta prisms and a result of the expansion of the outer tube. If this is true, effects due to a common rise of the temperature of range finder and lath will be perfectly compensated by making outer tube of range finder and the measuring lath of the same material.

Below is given a table showing the difference in length between *AB* and *CD* which will produce an error of one second in the adjustment for three distances of adjusting lath from range finder.

Distance of Adjusting Lath from Range Finder	Length Producing Error of 1 second
50 meters	.25 mm
100 meters	.50 mm
200 meters	1.00 mm

It will be noted that the angular error produced is entirely independent of the base length. The range error is connected with angular error by equation 10.

The value of the range errors for 5000 and 10000 meters corresponding to an error of 1 second for a one-meter base range finder, are respectively 125 and 500 meters. The magnitude of these errors makes very apparent the need for great care in the adjustment of the lath to the particular instrument with which it is to be used.

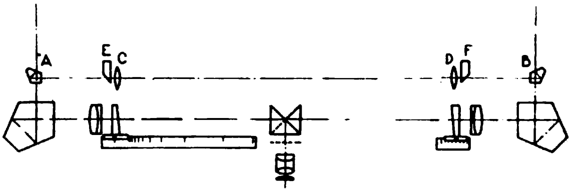

Fig. 8. Internal adjustment, first type

Figure 8 illustrates a type of internal adjuster essentially similar to that used on some of the Barr & Stroud Range Finders. *A* and *B* are two small penta prisms mounted directly in front of the large prism. *C* and *D* are two lenses, the focal lengths of which are identical and equal to the separation *CD*. The inner face of each lens has etched upon it a vertical line. The small prisms *E* and *F* communicating with windows in the outer tube serve to illuminate the etched marks.

Rays proceeding from the mark on lens *C* are rendered parallel by lens *D* and enter the optical system of the range finder proper after having been bent through ninety degrees by the small penta prism at *B*. Since the bundle of rays were rendered parallel by lens *D*, the image of the mark is seen sharply defined when viewed through the range finder in the same manner as though the range finder were actually trained on an object at a great distance. In a similar manner rays from the mark on lens *D* are rendered parallel by lens *C* and enter the range finder at the other end. Consequently, on looking into the range finder one sees the one mark in the upper field, the other in the lower. If prisms *A* and *B* actually deviate the rays through ninety degrees, the two bundles of rays entering the end prisms of the range finder are parallel. Therefore, if the range finder scale is set at infinity, the images of the two marks should coincide. If not in coincidence, the instrument is out of adjustment and the adjusting prisms must be altered until coincidence is secured.

It should be noted that the function of any in
is to deliver to the range finder two bundles o
be parallel to each other under all conditions. B
it is meant that the parallelism must be mai
possible regardless of the distortion of the sys
mechanical stresses and temperature changes to
ment is inevitably subjected in service. So fa
optical system of the internal adjustment mus
sating. Otherwise, the conditions which put
out of adjustment also renders the internal a
as a standard by which to readjust.

The self-compensating feature of this type c
in the fact that each lens bears the mark which
for the other lens. If, for example, lens *C* is
the direction of the bundle of rays entering
changed due to the displacement of the vertica
C. But at the same time the direction of th
prism *A* is deviated the same amount since th
pass through *C*. Therefore, parallelism of th
rays is maintained independently of any sligh
lens *C* or *D*. If then the deviations produced b
do not change, the adjuster will function satisfa
are penta prisms, a small movement will not aff
Non-uniform heating of either of these prisms n
error. In regard to this point, the manufacturer
are so small that appreciable non-uniform heat
result.

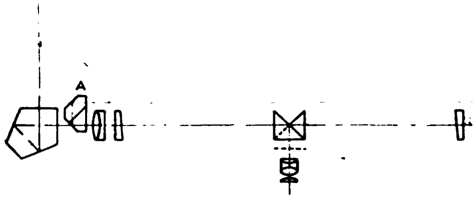

FIG. 9. Internal adjustment, second type

Another type of internal adjustment is shown
the range finder is to be adjusted, prisms *A*

normally outside the path of the light rays, are swung into the position shown in the drawing by levers on the outside of the instrument.

A special type of ocular prism is employed which bears a small index mark that appears in the field of view. A window is provided in the outer tube by which this mark is illuminated. Rays proceeding from this index mark pass to the right through the wedge and the objective. The rays are then deviated through 180° by the triple mirror. A second similar triple mirror returns the rays to the left hand objective and an image of the index mark is formed in the field of view. The course of the rays is indicated by the arrows. On looking into the eyepiece one sees the index mark in one field and its image in the other. Adjustment is accomplished by setting the range at infinity and bringing image and index mark into coincidence by means of the adjusting prism.

The self-compensating feature of this adjuster lies in the fact that the deviation produced by a triple mirror is always 180° irrespective of the orientation of the prism. If the triple mirrors were non-uniformly heated to such an extent that the deviation produced varies from the normal value, the internal adjuster does not function correctly. Furthermore, it will be noted that when adjustment is made, the two penta-prisms are not in the path of light. Consequently, any error due to the non-unifrom heating of these prisms or gradual deformation due to original strain would not be compensated by an adjustment accomplished by the internal adjuster. It is hoped that tests now in progress will show whether the errors arising from these sources are appreciable in the smaller range finders.

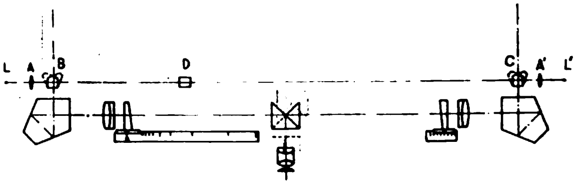

FIG. 10. Internal adjustment, third type

The large range finders used in the Coast Defenses are equipped with an absolute self-contained adjusting apparatus, shown in schematic form in Fig. 10. *B* and *C* are two small penta prisms which are brought into position in front of the penta prism of the range finder when adjustment is to be accomplished. At *L* there is a small electric lamp, the rays from which are rendered parallel by the collimator objective at *A*. This portion of the system is so located that half the rays from the collimator are intercepted by the small penta prism at *B* and enter the large prism of the range finder. The other half passes over the prism at *B* and is intercepted by the rhomboid prism. This rhomboid prism brings the beam down into the plane of the penta prism at *C* and the half of the beam finally enters the large penta prism to the left. On looking into the eyepiece one sees an image of the light at *L* in both fields. The range scale is set at infinity and coincidence secured. If now both prisms *B* and *C* deviated the rays through ninety degrees, adjustment would be completed. But in general, this is not the case.

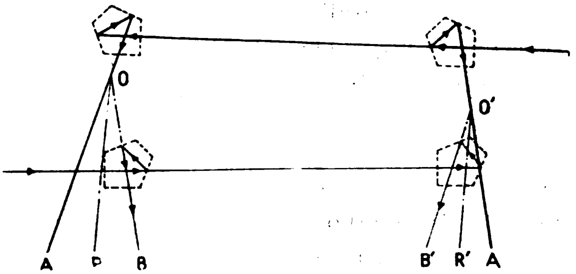

FIG. 11. Operation of third internal adjustment

To meet this difficulty, a second collimator indicated by the dotted lines is placed at the other end of the range finder. The light at *L′* is turned on and the two penta prisms are swung into the positions indicated by the dotted lines. The rhomboid prism at *D* is also swung through 180 degrees. Light now proceeds from *L′* into the two ends of the range finder and adjustment is again made. In general, the setting of the adjusting prism for the two adjustments will not be the same. The adjusting prism is provided with a scale and a setting intermediate between the two settings gives the correct adjustment for the range finder.

The accuracy of this adjustment does not depend upon the magnitude of the deviation produced by the two small penta prisms nor upon the parallelism of the two collimator system. That this is so is apparent on reference to Fig. 11. Using the right hand collimator and with the penta prisms in the position indicated by the full lines, the course of the rays is indicated by the heavy lines. With the left hand collimator illuminated, the prisms are turned as shown by the dotted lines and the course of the rays is indicated by the light lines. The lateral separation of the two positions of the prism is introduced in order that the lines of the drawing may not be confusing. Also by intention, the path followed by rays proceeding from the two collimators are represented considerably out of parallel. With one setting of the range finder, adjustment is made on rays *OA* and *O'A'*, from the other setting on *OB* and *O'B'*. It is evident that neither of these two pairs of lines are parallel. Therefore, each adjustment is incorrect. But the intermediate setting of the adjusting prism is equivalent to setting on the bisectors of angles *AOB* and *A'O'B'*. These two bisectors are parallel in all cases as is evident from the geometry of the drawing. The only condition which can affect the accuracy of this adjustment is a change in the deviation produced by any component which should develop between the two successive adjustments. As the total time required for both adjustments is only a few minutes, this lies outside the bounds of probability. This method of adjustment can therefore be truthfully termed an absolute adjustment. This self-contained adjusting apparatus is much too complicated for introduction into the smaller instruments, but is used in the permanently emplaced long base range finders used by the Coast Artillery Corps.

ORDNANCE DEPARTMENT, U. S. A.
FRANKFORD ARSENAL, PHILADELPHIA, PA.
July 13, 1921.

SILICA-GLASS PRISM FOR REFRACTOMETRY OF LIQUIDS AT ELEVATED TEMPERATURES

BY

F. R. v. BICHOWSKY AND H. E. MERWIN

Silica-glass plates about 1.5 to 2.0 mm in diameter and 3 mm thick were beveled together to make a prism having angles of 45°, 60°, and 75°. The edges were then sealed in an oxy-gas flame. During sealing the outer surfaces of the plates required protection to prevent dimming by the corrosive action of the flame gases. The plates were firmly set in grooves of hard graphite and protected by thick graphite pieces worked into proper shape. It was necessary to use the hottest small flame available, and to work for only a few seconds at a time, so that only the part actually worked was hot. Unless this was done, the entire prism and its graphite holder heated up, and the silica plates, which it is difficult to obtain in an entirely unstrained condition, warped slightly, especially if the holder subjected them to stresses.

One prism was made with a silica tube fused into the top. Such a stem forms a good support for holding the prism in a spectrometer furnace, and for the thermoelement. In case colored liquids are to be used the prism should be especially perfect at the refracting edge.[1]

The electrically heated furnace, which had to be designed to fit a particular goniometer (spectrometer) and prism, will be briefly described. It consisted of a copper cylinder (3 cm diameter) wound with asbestos covered nichrome wire, and held within a cylindrical copper box (5 cm diameter). A little fan mounted on a vertical shaft in hard graphite bearings below the prism was rotated by an elastic belt from a small motor. The openings in the outer box, except those closed by selected microscope cover-glasses, were covered by removable flaps of asbestos sheet, and by asbestos collars. By hand regulation of the heating current, temperatures within the prism as determined with a potentiometer and gold-constantan thermoelement, could be held constant to better than 0.1° even at temperatures near 300° C.

Measurements were made on four liquids, namely: Squibb's glacial acetic acid, 99.5 per cent; Kahlbaum's α-monochloro-

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[1] The joining of glasses which anneal at lower temperatures has been described by Parker and Dalladay (Faraday Soc. Trans. *12*, p. 305) and by Fairchild (J. Opt. Soc. Amer. *4*, p. 496; 1921).

TABLE 1

Sulfuric Acid

	D*	G′*	Hg lines		
Temp. °C	589.3	434.1	578.0	435.8	404.7
28.4	1.4245	1.4326	1.4250	1.4325	1.4351
49.1	1.4198	1.4279	1.4201	1.4277	1.4303
85.8	1.4119	1.4196	1.4123	1.4194	1.4221
113.8	1.4058	1.4136	1.4062	1.4133	1.4160
173.9	1.3924	1.4001	1.3930	1.4000	1.4923
206.8	1.3848	1.3922	1.3852	1.3919	1.3945
264.2	1.3709	1.3780	1.3715	1.3779	1.3800
307.5	1.3591	1.3663	1.3594	1.3662	1.3684

At 28.4° Li = 1.4218; F = 1.4292*

Chloronaphthalene

	D*	G′*	Hg lines		
Temp. °C	589.3	434.1	578.0	435.8	404.7
23.9	1.6330	1.6763	1.6348	1.6754	1.6942
41.3	1.6245	1.6672	1.6263	1.6664	1.6850
72.2	1.6100	1.6517	1.6117	1.6509	1.6691
93.6	1.5999	1.6410	1.6015	1.6403	
119.6	1.5874	1.6275	1.5891	1.6271	
148.1	1.5739	1.6130	1.5756	1.6122	
189.3	1.5534	1.5910	1.5549	1.5904	
251.9	1.5210	1.5565	1.5225	1.5557	1.5710*

At 23.9° Li = 1.6232; F = 1.6554*; Hg_{191} = 1.6538

Acetic Acid

	D*	G′*	Hg lines		
Temp.	589.3	434.1	578.0	435.8	404.7
25.5	1.3695	1.3778	1.3699	1.3777	1.3806
40.4	1.3632	1.3713	1.3636	1.3711	1.3740
62.1	1.3542	1.3621	1.3544	1.3620	1.3651
90.0	1.3424	1.3501	1.3427	1.3500	1.3527
112.2	1.3327	1.3402	1.3332	1.3401	1.3429
113.7	1.3221		1.3323		1.3421

At 25.5° Li = 1.3672

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* Columns and values marked thus * are interpolated from dispersion curves and from curves of refractive index against temperature.

naphthalene; Baker's 95–96 per cent sulfuric Results on the first three appear in the accomp as regards dispersion are probably accurate

As regards refractive index, n_D corrected tc agrees within 0.001 with recent determinatior pure acid; chloronaphthalene is 0.002 higher th measurement we have found recorded;[3] sulfuri within ±0.001 with measurements by Le Blar considered in relation to the older ones of Van Gladstone,[6] whose values are respectively 0.004

All three of these observers agree in finding a mum in the refractive index-concentration acid and water near, or at, 84.5 per cent sulf the composition of what has been called th sulfuric acid. Pickering[7] has discussed this dispersion maximum (Van der Willigen's) as ev in solution. But when note is taken of (1) the al maximum here, and (2) the lower relative sulfuric acid side of the maximum, then it is evi in atomic refraction have occurred.

These measurements cover a temperature large to make necessary a correction for the din index of the air in the furnace at the higher te sulfuric acid the maximum calculated correcti chloronaphthalene 0.00017, and for acetic a this basis the observed values were diminishe representing the refractive indices of the liq temperatures compared with air at 20°.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.
May, 1921.

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[2] Hubbard, Phys. Rev. *30*, p. 759, 1910, 1.3719; Eisenlol p. 590, 1911, 1.3727; Schwers, J. Chem. Soc. *101*, p. 1892, 1 1.3718.

[3] K. Arndt, Diss. Basel 1897, quoted in Landolt-Börnstei

[4] Z. phys. Chem. *4*, p. 557; 1889.

[6] As given in Rec. Données Numer., Opt., p. 403.

[5] Phil. Trans. *160*, p. 28; 1890.

[7] J. Chem. Soc. *63*, p. 99; 1893.

ATMOSPHERIC CORRECTIONS FOR THE HARCOURT STANDARD PENTANE LAMP[1]

BY

E. B. Rosa, E. C. Crittenden, and A. H. Taylor

1. Introduction

In several papers on flame standards and the photometry of flame lamps of various kinds the authors have had occasion to discuss the effect of atmospheric conditions on flames.[2] Particular attention has been paid to the Harcourt 10-candle pentane lamp because of its excellence as a practical standard of candlepower. Remarkably consistent results have been obtained on this lamp, and yet there has been a persistent, unexplained difference between some of the correction factors obtained abroad and those found at the Bureau of Standards. The purpose of this paper is to review the more recent work and to give an explanation of some of these differences, particularly with regard to the effects of moisture and temperature.

2. Previous Observations on Effect of Humidity

Water vapor in the air reduces the intensity of flames; the effect has been found to be practically proportional to the amount of

Table 1

Water-Vapor Correction Factors for Pentane Lamp

(The numbers given are the percentage of decrease in candle-power caused by an increase of 1 liter of water vapor per cubic meter of air.)

Authority	Factor
Paterson[3]	0.66
Dow[4]	.71
Butterfield, Haldane and Trotter[5]	.625
Bureau of Standards,[2] 1910	.57
Bureau of Standards,[2] 1911–12	.57
Paterson and Dudding[6]	.63
Takatsu and Tanaka[7]	.64

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[1] Presented at the December, 1920, meeting of the Optical Society of America; published by permission of the Director of the Bureau of Standards.

[2] Bulletin Bur. Stds., *10*, p. 391; 1914; (Sci. Paper No. 216). 10, p. 557, 1914; (Sci. Paper No. 222).

Trans. Illum. Eng. Soc. *5*, p. 753; 1910; *6*, p. 417; 1911; *8*, p. 410; 1913; *10*, p. 843; 1915.

water present, but various observers have differed considerably with regard to the magnitude of the proportionality factor. The results of the more important determinations are shown in Table 1.

The consistency of the Bureau results on the seven lamps used in the earlier investigations is shown in Table 2.

TABLE 2

Water-Vapor Correction Factor For Pentane Lamps

	No. of meas.	Factor derived	Range of water-vapor
Chance, No. 116......	220	0.0569	4.0–27.0
Chance, No. 118......	81	0.0572	14.1–26.8
Sugg. No. 171.........	165	0.0561	4.0–27.0
Mean..............		0.0567	
American, No. 25......	42	0.0577	5.0–14.3
American, No. 74......	43	0.0558	5.0–27.0
American, No. 157.....	46	0.0570	18.2–26.8
American, No. 162.....	31	0.0562	13.7–26.0
Mean..............		0.0567	
Mean of all.........	628	0.0567	

Equally consistent results were later obtained on many other lamps. The average Bureau factor was 0.567 for every group of lamps, while the lowest British factor was 0.63, the average of the two determinations at the National Physical Laboratory being 0.645. The differences are in themselves not very important, but the apparent accuracy of the determinations is such that the discrepancies can not be attributed to purely accidental varia-

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[3] Electrician (London), 53, p. 751, 1904; Jour. Inst. Elect. Engrs., 38, p. 271, 1906-7; Jour. Gas Light, 99, p. 232, 1907; Nat'l Phys. Lab., Collected Researches, 3, p. 49, 1908.

[4] Electrical Review (London), 59, p. 496, 1906.

[5] Jour. Gas Light., 115, p. 228, 1911.

[6] Phil. Mag., 29, p. 63, 1915; Proc. Phys. Soc. London, 27, p. 263, 1915

[7] Report No. 12, Electro-technical Laboratory, Department of Communications, Tokyo, Oct. 1917. Sci. Abst. A, 21, Abst. 259, 1918.

tions. For example, if the British set of correction factors had been used in calibrations made at the Bureau of Standards in the summer, the normal or certified values for the lamps tested would often have been in error by 1.5 per cent, thus falling entirely outside the range of observations made under the normal atmospheric conditions.

It is practically certain that some condition more or less systematically connected with the humidity has affected the measurements. The most obvious condition which might be involved is temperature, for in general the seasons of high humidity have also a high temperature, and the suggestion has repeatedly been made that probably the so-called humidity factor was really a combined humidity-temperature factor.

3. TEMPERATURE EFFECTS ON THE PENTANE LAMP, AND THE TRUE HUMIDITY FACTOR

In the case of the pentane lamp there has always been a belief that the candlepower must vary with the temperature of the air, but there are several ways in which air temperature affects the operation of the lamp and there has been difference of opinion even as to the sign of the probable temperature effect. Several observers have sought to show this effect, but none has been able definitely to do so. In the early work of the present authors such effects were sought for by admitting to the laboratory alternately cold outside air and warm air from the heating system of the building. The range of temperatures thus attainable was small, and there was considerable uncertainty in the results because of the effects of drafts and of possible variations in the purity of the air in the photometer room. It was concluded that over the ordinary range of our laboratory temperatures (18 to 30° C), temperature variations did not affect the candlepower of the lamps appreciably, although at lower temperatures there seemed to be a definite rise in candlepower, amounting to one per cent or more at 10°C.

Furthermore, such relation as existed between temperature and absolute humidity was a matter of seasonal variation. Over any short period there would usually be apparently no definite relation between the two variable conditions: Water vapor corrections, computed on the assumption that temperature had no effect, were found to be practically the same for observations made within such short periods as for observations extending throughout the year, which corroborated the belief that temperature effects were small.

It was remarked, however, in an earlier paper[8] that "the results obtained can be represented about equally well by assuming that temperature has no effect or by making a small correction for temperature and using a correspondingly different factor for water vapor." Since the matter was considered unimportant no numerical values were mentioned; the factors which had been used were, respectively, 0.5 per cent change in candlepower per unit change of humidity and only 0.1 per cent change in candlepower per degree (Centigrade) change in temperature, both being negative in the sense that the candlepower was found to *decrease* with rise of temperature, as well as with increase of humidity.

Paterson and Dudding, on the contrary, from an analysis of observations in which both temperature and humidity varied considerably, came to the conclusion that there was a large *increase* of candlepower with rise of temperature (0.76 per cent per degree), that the real humidity factor was very large (1.87 per cent per liter of water vapor content), and that the so-called humidity factor represented only the difference between the two effects. On this basis they proposed a possible explanation of the outstanding difference between the various English observers on the one hand and the Bureau of Standards on the other. They were obliged to assume, however, that at the Bureau there was a larger variation in temperature corresponding to a given range of humidity than was the case at the National Physical Laboratory, while in fact the opposite is true.

The publication of the National Physical Laboratory data makes possible a reliable separation of the two effects by comparison of the results obtained in the two countries. For the periods during which the Bureau's published determinations of the so-called humidity factor were made there was on the whole a rise of 0.6° C in the laboratory temperature for each liter of increase in humidity. A characteristic set of observations taken at intervals extending over nearly a year is represented in Fig. 1. Similar data for several other years have been examined, and the general relation between temperature and humidity has been found to be very nearly the same for each year, the general seasonal change in temperature for each unit of variation in humidity varying between 0.5° and 0.7° C. These observations of course refer to laboratory conditions as controlled by the heating system during the colder seasons; and the marked differen from British conditions is in part due to differences in heating.

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[8] Crittenden and Taylor, Bull. Bur. Stds. *10*, p. 391, 1914; (Sci. Paper No. 216); See p. 410.

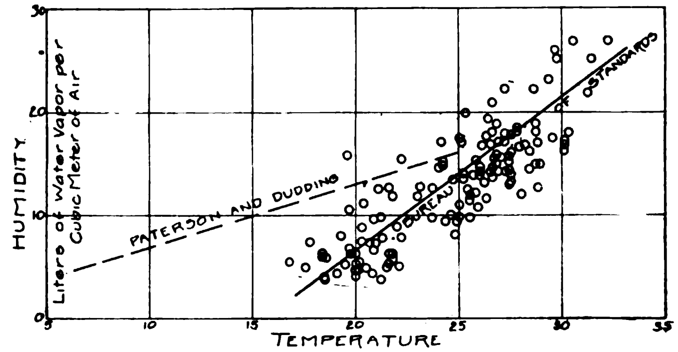

FIG. 1.—SEASONAL VARIATION OF HUMIDITY AND LABORATORY TEMPERATURES

Each circle represents the average of a day's observations in connection with the calibration of pentane lamps at the Bureau of Standards during 1911. The dashed curve is the average of several years' observations at the National Physical Laboratory.

At the National Physical Laboratory the corresponding change reported by Paterson and Dudding was 1.6°C. This difference in conditions not only explains the discrepancy between the values found for the "humidity" factor, but gives a determination of the real temperature coefficient which, on account of the care taken in the measurements at both laboratories and the large number of observations, should be very reliable. The mean factor found at the National Physical Laboratory was 0.645, while the result at the Bureau was 0.567. The difference, 0.08, is the percentage decrease in candlepower per degree rise of temperature, because the corresponding temperature changes differed by exactly one degree (1.6–0.6).

Assuming this temperature correction one can calculate the true humidity factor, which is found to be 0.52 per cent per liter of water vapor.

4. DIRECT DETERMINATION OF TRUE HUMIDITY FACTOR

By means of air tanks and auxiliary apparatus which were built for special investigations of atmospheric effects and have been previously described[9] it was possible to make a direct determination of the effect of humidity with negligible changes in temperature, and such a determination had been made before the calculations above were carried out. The observations are represented in Fig. 2. Measurements were made alternately at low and at high humidities; in obtaining the points shown the whole cycle of change from dry to moist air and back to dry, was carried through five times. The slope of the curve represents a decrease of 0.523 per cent in candlepower for each part per thousand of water vapor in the air. That is, this direct separate determination agrees with the value calculated above within one per cent of the factor; and since the total correction for humidity practically never exceeds ten per cent, the corresponding

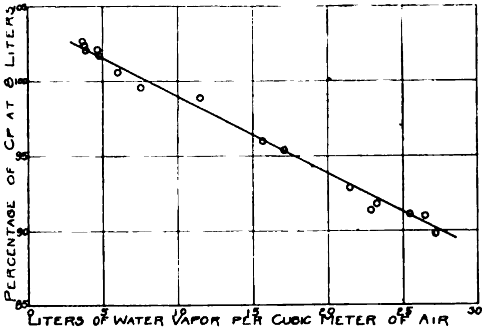

FIG. 2.—TRUE VARIATION OF CANDLEPOWER OF PENTANE LAMP WITH HUMIDITY

Results of determinations in special apparatus at practically constant temperature. The slope of the curve is 0.52 per cent per unit of humidity.

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[9] Trans. Illum. Eng. Soc., *10*, p. 843; 1915.

uncertainty in the corrected candlepower would be in the extreme case 0.1 per cent.

The direct separate determination of the temperature coefficient is more difficult, and since the above results were so definite and were consistent with the conclusions previously reached at the Bureau, it was not considered worth while to construct the special apparatus and to give the time needed for a highly accurate determination. Some tests were made, however, by admitting different proportions of cold air and of hot air from the heating system. The new photometer room used has special provision for forced ventilation, and a more rapid change of temperatures was obtained than could be gotten in the earlier tests mentioned. Measurements were first made at a fairly high temperature, then cold air was admitted until the temperature fell about 10 degrees, and finally the cold air was excluded so that the temperature rose to approximately its original value. Four such runs were made and the results are shown in Fig. 3. The decrease in candlepower per degree rise of temperature was found to be 0.11 per cent, which, within the uncertainty of this determination, is a confirmation of the value derived above.

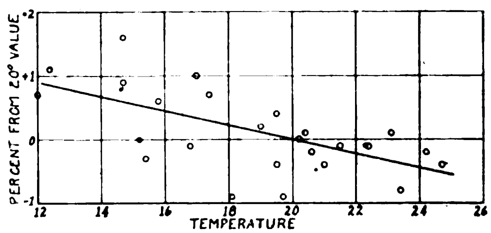

FIG. 3.—DIRECT DETERMINATION OF EFFECT OF TEMPERATURE ON CANDLEPOWER OF PENTANE LAMP
This rough determination checks satisfactorily the results obtained indirectly by analysis of measurements made under varying natural atmospheric conditions.

Small changes of humidity occurred during these runs, (the amount of variation being about 2 liters per cubic meter of air), and the candlepowers plotted were corrected for these variations using the factor 0.52 per cent. Consequently the value here found for the temperature coefficient depends to a slight degree on the humidity factor. It should be noted, however, that the data include series of measurements in which the variation of temperature for a change of one liter in the water vapor was 5°, 1.6°, and less than 0.1°, and that all these are brought into practically exact agreement by using the two factors given above; namely, 0.52 per cent decrease in candlepower for each liter of water vapor per cubic meter of air, and 0.08 (or in round numbers 0.1) per cent decrease per degree rise of temperature.

It may also be noted that this negative temperature coefficient affords an explanation for the fact that pentane lamps which have been measured with electric standards at both the National Physical Laboratory and the Bureau of Standards have always given slightly lower values at the Bureau, for the average temperatures in the latter laboratory are decidedly higher. This naturally suggests the desirability of establishing a standard temperature, as well as a standard humidity and barometric pressure, for these lamps. Since the pentane lamp is the basis of the British candle which has been transferred to electric lamps by measurements made at the National Physical Laboratory, the normal temperature would naturally be that which best represents conditions at that laboratory. Paterson and Dudding used 12°C as a normal temperature because on the average this temperature coincides with the previously fixed normal humidity of 8 liters per cubic meter, but this is distinctly below the average temperature at which their measurements were made. The adoption of 15° as the normal would be more consistent with practice in other measurements and apparently would also approximate the average conditions at the National Physical Laboratory.

5. MORE RECENT JAPANESE RESULTS

All of the above work was done before the war but publication was deferred in order to complete certain details. Meantime Takatsu and Tanaka of the Electrotechnical Laboratory, Department of Communications of Japan, have published a new determination of the humidity factor giving the value 0.638 (See Table 1). Mr. Takatsu has kindly furnished his temperature observations which were not published, and from these it is found that the variation of temperature was 1.5° per unit variation of humidity, so that, assuming a temperature factor of 0.08, the real humidity factor is found to be 0.52 or identically the same as that recorded above. Because of the similarity of climatic conditions indicated in Japan and in England comparison with the Japanese data can hardly be considered as giving an additional derivation of the results stated, but it is gratifying to find that the results obtained in both countries can be brought into harmony with our own.

6. CONCLUSION

In view of the close agreement between the values found by special experiments and those derived by comparison of measurements made under natural conditions in Great Britain, Japan, and the United States, it seems certain that the factors here given represent very exactly the effects of the atmospheric conditions considered. So far as consistency of results in any one laboratory is concerned it makes little difference whether the new factors are adopted or the old practice retained, but in the interest of uniformity and of conformity to facts it is believed that the new results should be used.

BUREAU OF STANDARDS
WASHINGTON, D. C.
December, 1920.

NOTE ON THE DETERMINATION OF THE RELATIVE EXPANSIONS OF GLASSES

BY

F. E. Wright

The most direct method for the determination of the state of strain or annealing in a glass sample is that of polarized light. This method is in general use and is in every way satisfactory both for laboratory and for works purposes; it can be learned in a short time by a factory hand and is readily applied. The apparatus required is not complicated nor easily put out of adjustment. There are, in fact, few methods of test which compare with this for simplicity and effectiveness.

It may not be generally realized that the method can be applied with equal success to the determination of the relative expansion of two glass samples. A test for relative expansion is useful in case it is desired to fuse two different kinds of glass together; thus, in the manufacture of bifocal spectacle lenses a button of medium flint glass is fused to a lens of spectacle crown glass; in chemical apparatus different types of glass are joined together or a metal filament is carried through a glass seal as in electric lamp bulbs. In the case of cords or striae in glass it is important for the glass-maker to ascertain the cause of the cords, whether pot-solution, or surface film, or lack of homogeneity in the batch, etc; he is able, by ascertaining the relative refringence and the relative expansion of cord and surrounding glass, to infer with some degree of assurance the chemical composition of the cord. In the present paper the mode of applying the strain-birefringence method to problems of this sort is outlined briefly.

Let us consider first the case of a cord in glass. The cord is a streak of slightly different composition and of correspondingly different expansibility. If the cord has a greater expansion coefficient than the enclosing glass, then, on cooling down from a high temperature, it tends to contract faster than the main glass mass; this tendency is, however, opposed by the more slowly contracting adjacent glass, with the result that the cord reaches room temperature in a state of radial tensional stress. Vice versa, a cord with lower expansibility than the adjacent glass is compressed by it during the cooling and reaches room temperature in a state of radial compression. The effect exerted on transmitted light-waves by a body under radial compression is different from that exerted by the same body under radial tension. From this optical effect it is possible to determine the state of stress in a body and thus, in the case of a cord embedded in glass or of two glasses fused together, to infer their relative expansibilities for the temperature interval 20° C to the upper limit of the annealing range (400° to 650° C, depending on the type of glass).

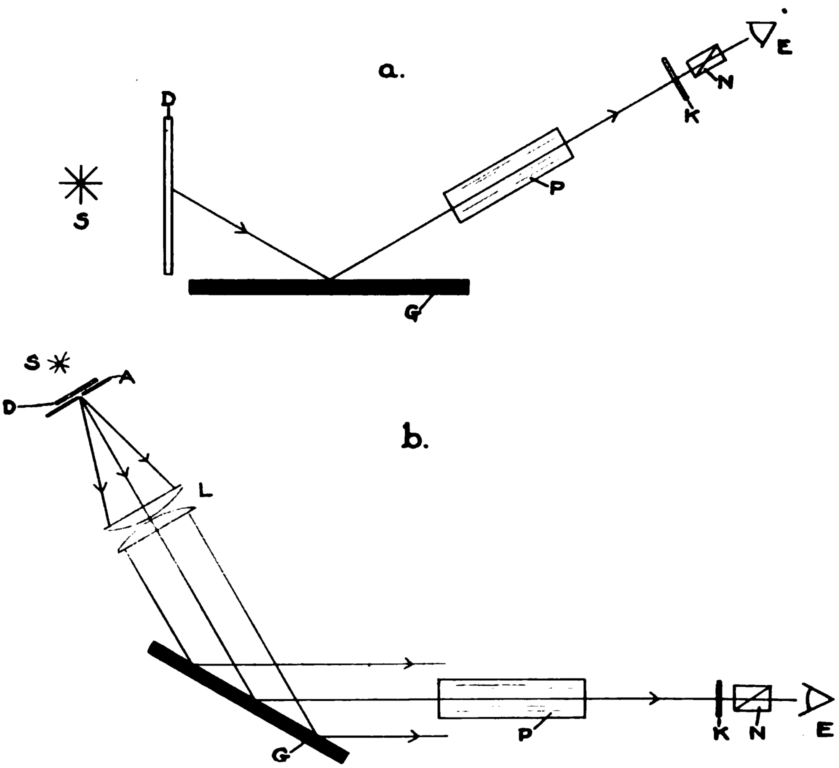

FIG. 1. Apparatus suitable for the inspection for strain in optical glass plates. In Fig. 1*a* a diffusion screen of ground glass illuminated from the rear by an electric lamp, is used; in Fig. 1*b* the diffusion screen is replaced by a condenser lens system. In both arrangements the lamp is properly housed in a sheet iron box. (a) *S* is the source of light; *D*, a diffusion screen of ground glass; *G*, a plate of opaque glass; *P*, plate of optical glass; *K*, sensitive tint plate; *N*, nicol prism; *E*, the eye of the observer. (b) *D* is a diffusion plate; *A*, pin-hole aperture; *L*, a condenser lens; *S*, *G*, *P*, *K*, *N* and *E* are the same as in Fig. 1*a*.

The apparatus required is essentially that employed in tests for the state of annealing of glass, and is illustrated in Fig. 1, a and b.

To set up the apparatus place the opaque reflecting plate *G* in a horizontal position or so that the plane of incidence for the rays from the source of light *S* is the vertical plane. Sight through the analyzer *N* toward *S*, as seen reflected from *G*. Rotate the analyzer about the line of sight as axis to the position for which the intensity is a minimum. Without further rotating the analyzer, raise or lower the line of sight (change the angle of incidence and angle of reflection) to the position at which the illumination is a minimum. The correct adjustment of the apparatus is more readily accomplished by this method than by actually measuring the Brewster angle of incidence, the tangent of which is the refractive index of the reflecting opaque glass plate.

Insert now the sensitive tint plate *K* (of quartz or of selenite) with its normal in the line of sight and rotate it about the line of sight as axis to the position of maximum intensity of illumination (diagonal position); note that at 45° on either side of this position the plate extinguishes completely and has then no apparent effect on the transmitted rays of light.

Place now the plate *P* in the line of sight (Fig. 1) and observe that the interference colors change as the plate is rotated. In a well annealed plate but little change is seen in the interference colors.

As an experiment hold a well annealed plate *P* (Fig. 1) so that its flat surfaces are parallel with the line of sight, but inclined at 45° with the vertical plane. Press now firmly against opposite edges of the plate and observe the changes in the interference colors. The uniform purple color of the sensitive tint plate is seen to change near the points of application of the pressure either to red and orange or to blue. In case the plate is stretched instead of compressed the interference colors change either to blue or to red and orange; in other words, in a direction opposite to that of the first. This practical test can be repeated at any time with a given piece of glass and the direction of shift, either up or down the Newton color scale, can be ascertained for a given kind of load, either compressional or tensional. The observation can be made in a few seconds and suffices to determine whether the strain at a given point in a strained piece of glass is the result of radial compression or of radial tension. In the case of a glass tube or rod it suffices to bend the tube slightly and to note the interference colors on the concave (compression) and convex (dilatation) sides of the bent tube.

A cord richer in silica than the surrounding glass has commonly a smaller expansibility than the adjoining glass. On cooling down from a high temperature, it tends to oppose the more rapid shrinking of the glass and reaches room temperature in a state of radial compression; tested in polarized light it exhibits the

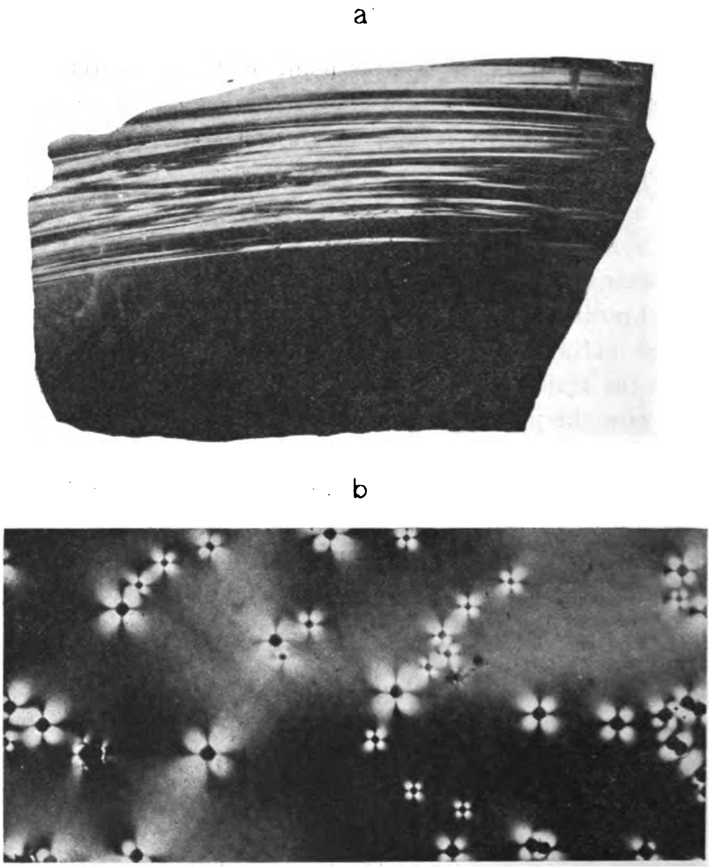

FIG. 2. (a) Strain shown by cords in a well annealed block of glass. (b) strain aureoles in glass surrounding "stones" as shown in polarized light.

distribution of interference colors characteristic of glass under compression (Fig. 2, a). A cord richer in alumina (pot solution), on the other hand, may have a greater expansibility than the enveloping glass and hence cools in a state of radial tension.

An interesting example of strain distribution is exhibited in the glass adjacent to a "stone," especially a crystallization body in glass. The crystalized material may contract at a faster rate than does the glass; the glass adjacent to the stone is in a state, then, of radial tension and the characteristic distribution of interference colors in the four quadrants for radial tensional stresses can be seen. (Fig. 2, b). The interference colors rise in two opposite quadrants and fall in the two remaining quadrants, the central cross exhibiting the purple color of the sensitive tint plate.

No rule has been given thus far for ascertaining the character of the stress from the interference colors alone. A statement of the rule has been purposely avoided because a number of factors are involved and the practical test can be carried out so easily that in many instances it is better to make an actual test by pressing or bending a glass plate rather than to trust to the memory for the exact wording of the rule. Suffice it to state that glass under compressional load behaves optically as a negative uniaxial crystal, the principal axis of the crystal coinciding with the direction of the applied load. A plate held so that the polarized light is transmitted along a direction normal to the acting pressure offers different degrees of resistance to transmitted waves of light, depending on the direction of their vibrations whether in, or normal to, the direction of the applied load. Thus for most glasses the waves whose vibrations are normal to the line of pressure (principal axis of the optically negative, uniaxial crystal) travel more slowly in the glass than do the waves whose vibrations are parallel with the axis. As a result the faster wave emerges from the glass a short distance ahead of the second wave; the distance between the two waves is called the path-difference. Brewster discovered over a century ago[1] that in glasses the relation between the load and the resulting path difference is linear, so that the optical path-difference per unit length glass path may be taken as a direct measure of the internal stresses. The path-difference is measured by means of a plate or wedge of a crystal in which the two directions of vibration and the relative speeds of propagation of the two transmitted waves of light are definitely known. On inserting this plate (sensitive tint plate K, Fig. 1) or wedge in the line of sight, the path difference produced by the strained glass plate is increased or decreased; from the rise or fall of the interference colors thus obtained, the relative speeds of the two waves transmitted through the strained glass plate can be determined. This operation can be done in a moment and the desired information ascertained regarding the relative expansion of the two glasses.

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[1] Philosophical Transactions, 1813.

The determination of the relative expansions of two glass samples.—For the solution of this problem by the strain-birefringence method it is necessary to fuse or weld the two samples together; in case the expansibilities of the glasses are greatly different the surface of fusion contact must be small, otherwise fracturing results. The glass samples may be in the form of rods, tubes, plates or irregular fragments. The mode of procedure differs somewhat, depending on the shape.

Let one of the glass samples be a glass tube 2 or 3 mm in diameter. Insert a small fragment of the second glass sample, 0.5 to 2 mm in diameter, into the tube to a point several millimeters from the end. Insert the end of the tube into a Meker or blast lamp flame and fuse it to a rounded end with the small glass fragment included in the molten glass drop. Allow the glass to cool slowly through the annealing range by holding it first in a yellow smoky flame and then by cooling it in layers of cotton. After the rod has cooled down to room temperature it can be examined for strain. In this connection a small liquid immersion cell filled with a liquid of about the same refractivity as the glass may be used to eliminate the disturbing surface effects of the glass rod. The rod is immersed in the liquid and examined in polarized light. Note the distribution of interference colors after insertion of a sensitive plate in the diagonal position. If the included glass fragment has the smaller coefficient of expansion, it is then in a state of compression and exhibits the interference colors indicated in Fig. 3, a.

In case the second glass sample is in the form of a glass rod or tube draw it out to a thin rod or thread and insert a short length in the first tube. Fuse down tube and rod and note the distribution of interference colors. The state of strain is indicated in Figs. 3, c and 3, d.

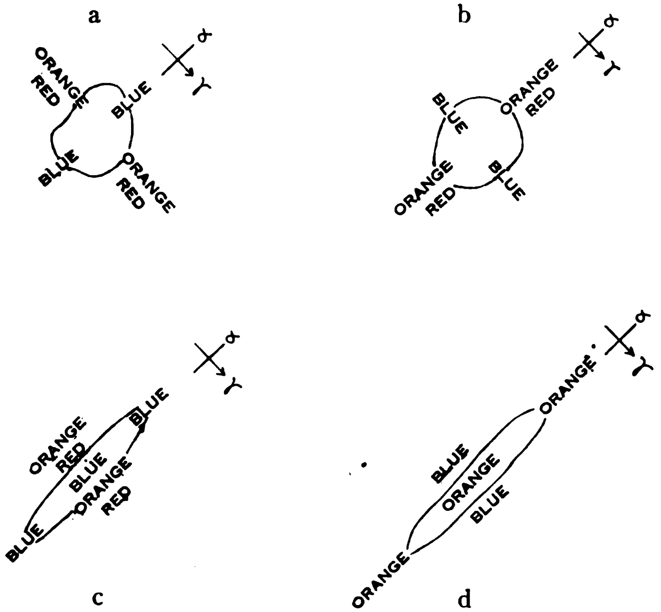

FIG. 3. (a) Diagram illustrating the distribution of interference colors observed in polarized light with the aid of a sensitive tint plate about a grain of glass or other material embedded in a glass of higher expansibility. (b) Same as Fig. 3*a* except that the surrounding glass is of lower expansibility. (c) Distribution of interference colors about an included thread of glass of higher expansibility. (d) Same as Fig. 3*c* except that the included thread has a lower expansion coefficient.

If two glass rods are available weld them together either end to end or side to side; if side to side draw out the molten double layer end with a pair of pincers to a long thread and note that the double layer thread, on cooling, curls up with the glass of greater expansibility on the inside.[2] Examined in polarized light the glass with the lower expansibility (convex side of thread) is seen to be in a state of tangential compression; the concave side (greater expansibility) is in a state of tangential tension. From the degree of curvature of the thin, double layer thread or from the interference colors produced, the relative expansibilities of the two glasses can be inferred directly.

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[2] O. Schott; Verein zur Beförderung des Gewerbefleisses. 4 April, 1892, p. 11; See also H. Hovestadt "Jena Glass." Translation by J. D. and A. Everett, p. 225 London, 1902; D. E. Sharp; J. Am. Ceram. Soc. *4*, p. 219-223, 1921.

Similar interference phenomena are produced when two different glass plates or lenses are welded together, as in the bifocal spectacle lenses.

The approximate numerical value of the coefficient of expansion of an unknown glass or of a small crystal fragment or of a metal wire can be determined by means of a set of glasses of measured expansibilities.[3] Small fragments of the material of unknown expansibility are welded or fused into a glass piece of known expansibility and the relative expansibility is compared by the strain-birefringence method.

Summary.— In this paper attention is called to the application of the methods of polarized light to the study of the relative expansion of two glasses. For the purpose polarized light produced either by reflection from a polished, opaque piece of glass at the polarizing angle or by a polarizing prism is employed. The plate under test is examined through a sensitive tint plate and an analyzer sighted toward the polarizer. From the observed change in interference colors the state of radial compression or of radial tension of the glass sample can be inferred and with it the relative expansion coefficients of the two pieces of glass, or of a piece of glass and an included fragment of glass, of crystallized material, such as a stone, or of a metal wire. The disturbing effects of irregular boundary surfaces can be largely eliminated by immersion of the glass samples in a liquid of the same refractivity.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
June, 1921.

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[3] See D. E. Sharp, J. Am. Ceram. Soc., *4*, p. 222; 1921.

SOME PROBLEMS OF PHYSIOLOGICAL OPTICS[1]

BY

F. K. RICHTMYER

Members of the Optical Society:

I wish to express to you again my appreciation of the honor which you have conferred upon me in electing me to this high office in the Society. For many years it has been customary for the retiring presidents of our scientific societies to deliver addresses which may be looked upon as key notes for after activities or as a summary of knowledge in that particular science up to date. In conformance with this practice I have chosen to speak briefly this afternoon upon the subject of physiological optics, not that I am an expert in that field—for at best I am only an amateur with, however, an amateur's enthusiasm for the subject—but because I feel that it is one of the most important problems to which this Society can direct its attention.

It is hardly necessary to emphasize, before an audience of this kind, the importance of further study of visual phenomena. The importance of good vision in making increased efficiency in all lines of human endeavor to say nothing about the general well being of society is universally recognized, and furthermore it is quite fitting that the Optical Society of America should consider the study of vision and of research therein as well within its province. We are interested as a society in optical instruments of all kinds, and is not the eye the most wonderful of all optical instruments.

The problems which must be considered when one is discussing visual phenomena begin with the physical and chemical processes involved in the production of light and extend through the distribution of light, its passage through the optical system of the eye, its transformation into a visual sensation in the retina and the reception of this sensation in the brain centers. Thanks to the work of our sister society, the Illuminating Engineering Society, a large number of the problems in the production and distribution of light have been cleared up during the past two decades. I think it is not going too far to say that if you wish an illumination of any quality, color and distribution, judged by physical standards, a competent illuminating engineer could be found who would be willing to undertake to meet these specifications. Comparatively speaking, therefore, we may regard the problems of light production and distribution as in a satisfactory stage of development.

The next series of processes involved are those connected with the passage of a beam of light through the optical system of the eye until it falls upon the retina. Thanks to the work of the oculist and the optometrist the physical constants of eye media and the geometry of the optical system of the eye are sufficiently well known to enable the capable oculist to prescribe such corrective lenses as will produce upon the retina a sharp image of the field of view.

At this point, however, begin a series of processes, little if at all understood, which result in a visual sensation. We have made little progress in theories of vision since the time of Helmholtz. And yet this lack of progress cannot be said to be due to lack of interest or of activity. One has only to glance over the literature of the subject of vision to realize that a vast amount of work has been done in recent years. But what does impress one in looking over this literature is the lack of unity underlying theories of the visual processes. One well known authority has recently estimated that there are some sixty theories of vision. One may well raise the question: why this diversity?

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[1] Presidential address before the Optical Society of America at the annual meeting, Chicago, December 27, 1920. Abbreviated.

The answer to this question is to be found by comparing the development of sciences with the development of any other human activity. When society was young it was possible for primitive man to supply all of his own simple needs of food and shelter. Each individual was then an economic unit independent of his fellows. But as society grew men began to gather into tribes and it was found that some individuals could do one thing better than others in the tribe. Thus began industry and very soon we hear of "the butcher, the baker, and the candlestick maker," each one supplying some one of the community's needs.

When industry was young each worker likewise was an independent unit by himself. Thus the wagon maker of one hundred years ago cut the trees, very likely from his own forest, worked the trees up into lumber and the lumber up into wagons. These he then sold after which he proceeded to cut more lumber and start the cycle over. But as industry grew it was found profitable to differentiate activities. At the present time a whole army is involved directly or indirectly in the making of the wagon. Each individual in this highly complex organization has only a simple part to play. He perhaps paints only the left front wheel!

When science was young it was possible for an Aristotle to know all of the existing sciences and to be well versed in practically all of the then existing branches of knowledge. But as science grew it became more and more impossible for any one person to comprehend it all. Specialization became increasingly necessary. Science split up into divisions and these into sub-divisions, so that at the present time a man may no longer be a chemist. If he would succeed he must be an inorganic or a physical chemist. No longer may a Helmholtz leave his name in several branches of science. He must specialize in some particular part of one of the many branches of science.

Herein lies the reason for the present unsatisfactory status of the subject of physiological optics, particularly as regards the processes taking place in the retina. This subject represents one of many subjects occupying a middle ground between two or more major divisions of science but belonging to neither of them. In the transformation of radiant energy into a visual sensation are involved the sciences of psychology, physiology, physics, and chemistry. Any single individual who attempts research in this general field of vision must therefore have a good working knowledge of each of these four sciences. And half a century ago such a composite knowledge was possible. Hence the work of Helmholtz. But, great as he was, Helmholtz was by no means a super-scientist. There are probably many scientists today who in latent ability are his equals. But the field of any one science has grown so vast and has become so intrinsically attractive that very few workers have either the courage or the interest to take up these middle ground problems. Or if they do attempt work in such a field as physiological optics each one carries on his investigations with his own science as a background, be he chemist, physicist, physiologist or psychologist. The result is inevitably lack of unity and coordination.

Furthermore our best research talent has been attracted elsewhere by the burning questions which each investigator finds in his own field. Thus the physicist during the past two decades has devoted a very large part of his time and attention to that most illusive of problems, the structure of the atom, and as the solution has seemed to be nearer and nearer more and more have entered the field. But I wish to predict that as much fame awaits the man who finally evolves a satisfactory and lasting theory of vision as awaits the man who unravels the mysteries of atomic structure.

The need of a central working theory to advance our knowledge of the visual process is peculiarly emphasized by comparison with the problem of atomic structure. The great advances in our knowledge of the atom which have been made during the past two decades have been undoubtedly due to the fact that on certain major theories practically all of the physicists of the world have been in substantial agreement. The work of the investigator in Australia could be compared with that from Chicago. No one has ever seen an atom to say nothing of an electron, and yet atoms and electrons have become very real things even to the elementary student. The fact that we cannot observe directly what goes on in the retina should not deter us from attempting to find out the true nature of these mysterious but wonderfully efficient processes. A central working theory so that we may correlate not only research done in Australia with that done in Chicago but work in physics with that in psychology will go a long way toward solving this highly important problem.

As I see it, therefore, the hope of the future lies in cooperative effort. In some way we should organize our research facilities so that groups of individuals representing physics, chemistry, physiology, and psychology may work together in the same laboratory, under one roof, with a common goal in sight each bringing his particular preparation to the work and being able thereby to assist his fellow workers. We need furthermore to develop physiological optics as a separate subject so that no longer will it be necessary to speak of it as occupying middle ground, which simply means that it is without a home anywhere. This organization for more effective and cooperative research and for the development of a separate subject I believe to be by far the most important problem in the field of physiological optics today.

One might even with profit take a still broader view. The subject of physiological optics or, if that term be too restrictive, of vision, is an illustration of a place where several sciences meet. Another subject which seems remote but which really is very closely connected with vision is that of the effect of X-rays upon human tissue. Both have to do with interrelations of radiant energy and living matter. Some philanthropist could do no greater good than to found at some university an institute for the study of all such problems of biophysics.

Coming now to a discussion of the specific problems in the field of physiological optics, I do not feel competent to do more than mention a few. An amateur does not discuss critically.

One of the problems which has always attracted my attention is that of the time factor in visual sensations. This enters in various ways. What happens, so far as sensation is concerned, when the eye previously dark adapted is suddenly exposed for several seconds, let us say, to a luminous field of definite brightness? There is the initial stage when the sensation is growing from zero (approximately) to the final permanent value. Many years ago Broca and Sulser investigated this part of the time-sensation curve and found that the sensation reached a maximum after a few hundredths of a second after which it decreased to a final permanent value. These results are at least in qualitative agreement with those reported by Shelford Bidwell. But it is probable that much more work must be done on this subject before anything like satisfactory quantitative data is available.

Regarding the next section of the time-sensation curve, little need be said in this discussion. It is with the end of the curve that we are particularly concerned. I refer to the well known phenomena of after images, both positive and negative. Shelford Bidwell in his interesting little book, "Curiosities of Light and Sight," gives diagramatically the progress of this curve from the instant of exposure to light until the last after-image has died away. One cannot but feel that a thorough investigation of this whole curve from beginning to end would throw a great deal of light upon the visual processes.

Another very important part which time plays in a visual sensation is the well known phenomenon of adaptation, both light adaptation and dark adaptation. The recent work of Hecht in analyzing the dark adaptation curve and in giving evidence of the taking place within the retina of a bi-molecular reaction is very suggestive. In this connection one must also mention the work of Ives on the study of the flicker photometer.

A very great deal of information and data has been collected in regard to color vision, and among other things it is becoming recognized that we cannot arbitrarily divide individuals into the two classes: color-blind and normal, any more than we can divide people into three classes—dwarfs, normals, and giants. Just as there are all gradations in stature from the Tom Thumbs to the Goliaths so there are apparently all gradations in vision and color vision from those who are totally blind to both intensity and color to those who possess so called normal vision. We need more information of a statistical nature. We need to study the peculiarities of those who, for example, are extremely blue sensitive but who, judged by ordinary methods of test, possess the ability to distinguish colors. We need to know more about the way in which various factors such as age, health, conditions of work, etc., influence the color sense.

One must also mention in connection with problems of vision the very interesting work of Dunlap and of Magnusson and Stevens in studying the effect of an alternating magnetic field upon visual sensation. These investigators found, for example, that the sensation of flicker as produced by the alternating magnetic field persisted to frequencies far higher than when the retina is stimulated by a flickering light. If this result be confirmed it means that the damping out of flicker as observed in the ordinary way takes place in the retinal processes rather than in the optic nerve and brain center, for it is to be observed that the magnetic field stimulates a visual sensation by generating an electric current directly in the optic nerve or in the nerves leading from the end of the optic nerve to the rods and cones. Further study of this phenomenon is needed.

Another class of subjects upon which more information is needed is the effect, perhaps physiological, perhaps psychological, of various colored lights on vision. The question is frequently asked: which is better for night-time work, the kerosene oil lamp, the ordinary electric incandescent lamp, or the so called daylight lamp, assuming of course intensity and distribution to be the same in either case. There are those who insist that they work with the greater degree of comfort when using the oil lamp or the old fashioned open flame gas burner. These say that the comparative harshness of the daylight lamp is both objectionable and injurious to their eyes. And there are others who insist that they work with the greatest degree of comfort when using the daylight lamp. Realizing the possibility of physiological differences in the eyes of these two classes of people it is quite possible that each is correct. But what I wish to emphasize is that what we need is fact rather than opinion. At the present time this whole class of questions is settled almost entirely by opinion. I may give it as my opinion that *if* conditions of intensity and distribution are identical in the three classes mentioned above, a worker will experience no more discomfort from one than from the other. But if I were to say so it should be emphasized that I give ***my opinion***. I have no facts save possibly my own inadequate experience upon which to base an answer. It must be admitted of course that investigation in this particular field is extremely difficult and discouraging, but nevertheless the need for some investigations is very great.

Perhaps after all the greatest progress would be made if out of the existing chaos there could be developed a unified theory of visual processes. One needs only to mention a few items involving theories absolutely opposed to each other. For example, are the rods the visual preceptors at low intensities according to the older theory or, according to Edridge-Green, is the function of the rods solely to secrete the visual purple which, being exuded, produces a visual sensation in the cones by some kind of a photochemical process?

Again, does the dark pigment act as a sort of a curtain to screen a part of the light sensitive area of the retina from too great a light intensity, or according to a recent suggestion is its function to act as an absorber of energy and thereby to give the sensation of brightness, the sensation of color coming from the cones? Then we have a number of theories as to the real nature of the visual process. Is it photochemical, and if so what photochemical reactions are involved? Or are the rods and cones, as recently postulated, miniature wireless receivers which resonate with the frequency of the incoming light wave?

One must admit that the whole subject is extremely difficult and perhaps more complex than almost any other field of investigation which confronts the scientist. Nevertheless there are few if any subjects before the scientist of greater importance, and if the importance of the subject bids us enter upon this field of investigation much more are we attracted by the challenge which visual phenomena offer. The more difficult and involved the problem the greater is the satisfaction in solving it, and it is to be hoped that the Optical Society, which is perhaps the only organization in which all of the various contributory sciences can be found, will lend its prestige and its enthusiasm for further inquiry into this field which involves phenomena of the utmost importance to the human race.

CORNELL UNIVERSITY,
ITHACA, NEW YORK

NOTICES AND REVIEWS

Bibliotheca Chemico-Mathematica, compiled and annotated by H. Z. and H. C. S. 2 vols. pp. xii and 964, with 127 plates containing 247 portraits and facsimiles; Henry Sotheran and Co., London, 1921.

This historical catalog of scientific books, including biographical and historica references, with a remarkable collection of portraits and facsimiles, is worthy of a place on the shelves of every scientific man interested in the development of modern scientific thought. The two beautiful volumes have a very considerable human interest, apart from their being an annotated bibliography of mathematical, physical, chemical and engineering science. More than 17,000 titles of books are listed for sale, easily made available by an exhaustive subject index giving date of publication of each book listed.

There are one hundred and twenty-seven plates, giving facsimile reproductions of title pages, diagrams and cuts taken from famous books. There is a reproduction of the title page of Copernicus' "De Revolutionibus Orbium Coelestium," from the excessively rare original edition of this epoch-making work, published in 1543. This book marks an epoch in human thought, but was never seen by its author. When a copy arrived at his bedside he could only touch it, for his faculties and memory were gone and he died within a few hours.

A copy of Newton's "Opticks" with manuscript corrections in Newton's own handwriting is offered. The facsimile of the title page of the first issue of the first edition of the "Principia" reads, "Imprimatur S. Pepys, Reg. Soc. Praeses, Julii 5, 1686," and the compiler of the catalog remarks, "The idea of Mr. Samuel Pepys, P. R S. giving the Society's *imprimatur* to Newton's 'Principia' is tickling."

The reproduction of the obverse of the first leaf of the first (1482) printed edition of the "Euclidis Elementa" in Gothic type with geometrical diagrams on the margin is noteworthy, the text being a translation from the Arabic by Adelard of Bath. Two copies of the first edition of the original Greek text are listed. This edition (1533) is interesting on account of its dedication by Simon Grynaeus, professor of Greek at Basel, to Cuthbert Tunstall, Bishop of Durham. Of a copy of the first complete edition (1546) offered for sale, the compiler of the catalog says: "The title and first leaf of the text bear the stamp of the clerical censor, who thought it needful to cut out the three preliminary leaves,—an interesting instance of the rigidity of ecclesiastical censure at the period of the Reformation."

There is an illustration showing Huyghen's pendulum clock, a page from the "Traitè de la Lumiere" (1690) where he deduces the equality of the angles of incidence and reflection, with the original of the diagram, now familiar in all text books on optics, and a plate showing the phases of Saturn from the "Systema Saturnium" (1659). From Kepler's "Dioptrice" (1611) are shown two pages with diagrams showing the formation of an image by a convex lens. The "Dioptrice" contains the first account of the invention of the astronomical telescope. A plate from the "Micrographia" of Robert Hooke shows his compound microscope. This book (1665) is one of the most suggestive in the history of science—containing the earliest investigations of the colors of thin plates, the first notice of the black spot in soap films, a theory of light as a "very short vibrative motion," an explanation of the twinkling of stars, as well as the invention of the compound microscope. The 38 copper plates were engraved by the author himself. Hooke was Boyle's assistant and Mr. Sotheran offers for sale Bishop Burnet's copy of the first edition of the "New Experiments, touching the Spring of the Air," which gives a description of Boyle's air pump as improved by Hooke.

A very interesting plate in this collection of striking illustrations from old books is that from a book on distilling, originally published at Strasburg in 1519 and translated into English under the following curious title: "A Vertuose Boke of Distyllacyon of all Maner of Waters of the Herbes, with the Fygures of the Styllatoryes, fyrst made and compyled by the Thyrte Yeres Study and Labour of the most conynge and famous Master of Phisyke, Master Jherom Bruynswyke, and now newly translated out of Duyche into Englisshe by Lawrence Andrewes. Imprynted at London in the Flete Strete by me Laurens Andrewe, in the sygne of the Golden Crosse, MCCCCCXXVII, the XVIII daye of Apryll." Only 3 copies, two in German and one in English are for sale.

Engineers are not generally aware that a practical electric telegraph was invented by Sir Francis Ronalds and described in his book "Descriptions of an electric telegraph," 1823. The apparatus used is shown in one of the illustrations in this catalog. Signals were transmitted by Ronalds from his house at Hammersmith (occupied later by William Morris, the poet, and known as "Kelmscott House") over a system of wires 8 miles long, by observing at one end the divergence of a pair of pith balls when a charge of electricity was sent into the other. The invention was offered to the Admiralty, and was refused on the ground that as the French war was over telegraphs of any kind were totally unnecessary.

The examples chosen are only a very few of the many striking and interesting books listed in the catalog. The old books are, of course, out of date for practical purposes, but they are landmarks in the history of the human spirit, as it slowly works itself out in the progress of the race. The history of the great discoveries in physical and mathematical science is most interestingly shown in this unique catalog.

J. FRANKLIN MEYER

Radioactivity and the Latest Developments in our Knowledge of the Chemical Elements. K. Fajans, 3rd Edition *1921*, No. 45 in the series "Sammlung Vieweg Tagesfragen aus den Gebieten der Naturwissenschaften und der Technik." Viegweg und Sohn Braunschweig; 124+VIII pp.; 12 cuts and 11 tables; price 7 marks. The third Edition of this excellent treatise brings the subject up to December 1920, including a comprehensive review of Aston's experiments. To compress the treatment of radioactivity into 124 pages has required the most careful consideration by the author. The book may be commended as a short text for graduate students and is an extremely interesting account of the important phases of this subject especially from the standpoint of atomic structure.

PAUL D. FOOTE

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Continued from Vol. V page 291, 1921

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A PROPOSED STANDARD METHOD OF COLORIMETRY

BY

HERBERT E. IVES

The problem of the accurate measurement and specification of color is not one which has, as yet, received a generally acceptable solution. The choice is between simple and scientifically attractive methods of measurement on the one hand, which are not sufficiently accurate for technical purposes, and, on the other, methods which are undesirably complicated or rest on empirical bases. More specifically, in the first group lie the methods of measurement and specification in terms of three elements, namely, the three color additive method, the three color subtractive method, the spectrum hue, brightness and purity method. These all suffer in accuracy from the fact that their renderings differ from observer to observer by amounts greater than can be tolerated in technical color reproduction. They require, for their successful utilization, the employment of observers of carefully selected normal or average color vision or the development of methods of correction applicable to observers of known characteristics of vision.[1] In the second group lie the various color charts or scales with their hundreds of sample colors, each of which must ultimately be measured and specified by some standard method, and whose reliable reproduction in quantity is a stupendous practical color problem in itself.[2]

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[1] Assuming that the adoption of adequate methods of test and selection of observers would bring the three-element methods under serious consideration for the selection of one as a standard method, it would appear that the additive three-color (red, green and blue mixture) method presents the most advantages. The single setting used to obtain a measurement calls for an exact match, both of brightness and color, which from the photometric standpoint is ideal. The measurement when obtained can be transformed into color sensation, or spectrum hue, brightness and purity values, if so desired, by comparatively simple steps. The spectrum hue, brightness and purity method as a method of measurement suffers under the serious disadvantage that one of the several settings necessary for a complete measurement is that of brightness with a

As the present writer pointed out in a discussion of color measurement some years ago,[3] the only unique specification of a color is by means of its spectrophotometric analysis, in which paradoxically, the measurement of *color* as such does not enter. The conclusion then drawn was that the most promising generally acceptable solution of the color measurement problem was to be sought in the development of simple and accurate methods of spectrophotometry and methods of expressing spectrophotometric results.

The objections to the use of the spectrophotometer for practical color measurement are obvious. It is an expensive and delicate instrument. A large number of measurements are necessary to specify a single color. The color cannot be reproduced for study or comparison by setting the indicator on the instrument to the

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large color difference, thus introducing a fertile field for wide discrepancies. The three-color subtractive method shares with the additive method the merit that the setting which yields the measurement is an exact match. It is, however, less simple, for the reason that the transmission, purity and hue of the absorbing "wedges" all change with depth. Unless the white light source which is modified by the absorbing wedges consists of spectrally narrow red, green and blue bands, the light transmitted by the three superposed wedges changes in intensity and purity at the same time. This necessitates the use of a neutral tint wedge or other device to complete the match, whereby the system becomes one of four elements instead of three. Furthermore, the reduction of the subtractive measurements to sensation, or hue, brightness and purity values is exceedingly difficult.

[2] An intermediate position is occupied by the polarization color scale as used in the Arons Chromoscope. Here the matter of reproducibility is well taken care of, being a question of angles of setting and thicknesses of quartz. The colors produced are spectrally different in character from the ones measured so that some difference of setting between different observers will occur, although this difference is much less than with the three color mixture method. The measurements when obtained are on scales having no obvious relation to wave-length or hue, and their reduction to a color sensation scale is a much lengthier process than the reduction of three color mixture measurements.

[3] "The Transformation of Color Mixture Equations from One System to Another." Journal Franklin Institute, Dec. 1915, p. 673.

measured values. The results—plots of readings against wave-lengths—mean little to anyone except a specially trained expert.

These and other objections may be granted at once. On the other hand, there is no ground for believing that satisfactory color measurement will ever be done by anything except high-grade and hence expensive apparatus, or that really accurate color specification can be made simple. If the conclusion is valid that spectrophotometric analysis is the only method of color measurement which will ultimately prove satisfactory, the obvious next step is to decide on the simplest and most practical way to work it out, having in mind that *color* rather than *spectral distribution* is the object of the measurement. This was the problem whose

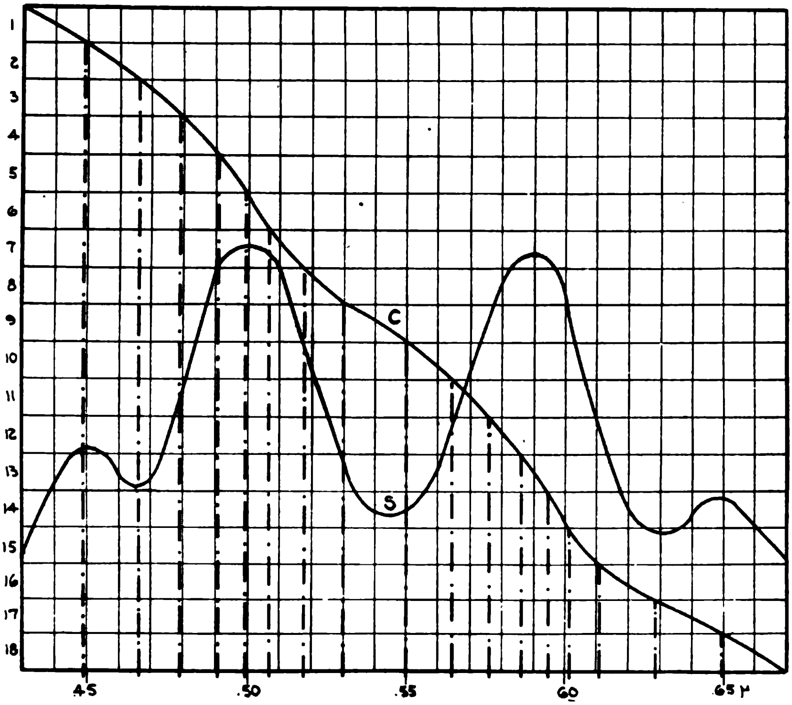

Fig. 1

Hue sensibility (S), and hue scale of spectrum (C). The hue scale is here shown divided into 18 equal parts, and the wave-lengths corresponding to the limits of these equal parts are obtained by projection on the wave-length axis (dot and dash lines).

study led to the method proposed below, which may be roughly described as a method in which the hue scale rather than the wave-length scale is employed and in which the whole spectrum is covered whatever the fineness of the scale used.

In considering the problem of simplifying spectrophotometric measurements for color specification, the following three requirements appeared:

(1) The measurements should be close enough together on the *hue scale* of the spectrum so that the resultant values completely and uniquely specify the color.

(2) The individual measurements should be made with portions of the spectrum narrow enough so that there is no color difference in the spectrophotometer field at each setting.

(3) The entire spectrum must be included by the measurements, so that the effect of narrow absorption or emission bands or abrupt changes of intensity with wave length will not be overlooked.

These three requirements are not entirely independent; their significance is perhaps best grasped by proceeding at once to the scheme proposed for meeting them. This consists, in brief, *in measuring adjacent patches of the spectrum, each patch being of width inversely as the hue perception sensibility of the eye at that point and sufficiently narrow so that no significant color difference can enter due to differences of intensity distribution in the spectra of the compared colors.*

The object of using adjacent (juxtaposed) patches of the spectrum is of course to meet requirement (3) obviating the necessity for measuring an infinite number of points. The choice of patches so narrow as to show no color difference in the photometric field is primarily in order to eliminate errors due to differences in color vision. Variation in width of patch with the hue sensibility (Fig. 1)[4] is in order to hold to a minimum the number of patches necessary. It will be clear that, given the instrumental means to make a spectrum measurement of the kind postulated, each setting will be a simple equality setting with no distinct color or significant spectral composition difference, thus avoiding the sources of error in the three-element methods of analysis. The complete measurement will consist of the minimum number of settings by which the (subjective) color element can be unambiguously represented, thus having an advantage over an ordinarily spaced spectrophotometric series of settings, in which the number of points of setting would be larger than necessary in the region of slow hue change. Finally, since the whole spectrum eventually enters the instrument, any type of color of any extreme of spectral composition such as a line source can be measured, which is not the case with the spectrophotometer as ordinarily used with settings at isolated wave-lengths.

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[4] See Nutting, "A Method for Constructing the Natural Scale of Pure Color," Bull. Bureau of Standards, Vol. 6, No. 1.

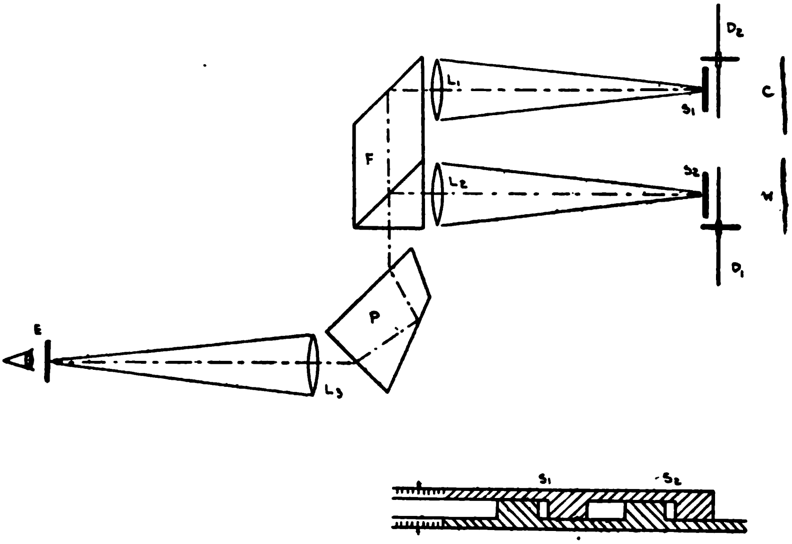

Fig. 2

Hue scale patch spectrophotometer
C, surface whose color is to be measured
W, standard white surface
S_1, S_2, adjustable slits
L_1, L_2, collimator lenses
F, photometric prism
P, dispersing prism
L_3, telescope lens
E, viewing slit.

As an instrumental means to achieve the measurements it is proposed to utilize the principle of the Maxwell color box, furnishing the box with special slits with movable jaws to facilitate the selection of adjacent spectral patches. A possible arrangement of apparatus is shown in Fig. 2. *C* is the surface whose color is to be measured. *W* is a standard white surface under the same illumination. At S_1 and S_2 are adjustable slits, whose design will be discussed below. L_1 and L_2 are two lenses placed at their focal distances from S_1 and S_2. *F* is a compound photometric prism designed to reflect the light from L_1 through one part of the field as viewed from *E*, and the light from L_2 through another part, the dividing edge being as fine as possible. *P* is a dispersing prism or grating, L_3 the telescope lens, and *E* the narrow viewing slit. The eye, placed at *E*, sees the photometric surface *F* in light of the color of the portion of the spectrum received at *E*.

The two slits, S_1 and S_2, are placed at corresponding positions in the two spectrometers formed by L_1 and L_2, and are arranged to move laterally across the focal planes of L_1 and L_2 at the same rate. The slits are made according to a plan utilized by the writer some years ago in studying the summational capacity of the flicker photometer.[5] Each slit consists of a preceding and a following jaw. The two jaws are first placed in contact at a given wave-length, which is the limit to one of the standard spectral patches; then the preceding edge is moved to the other limiting wave-length, and a measurement made. Then the following edge is brought up to contact and the process repeated. By mounting the two corresponding jaws on a single sliding piece the adjustment of both slits may be made simultaneously.

In order to make a photometric match a variable aperture sector D_1 is placed before the white light slit (a compensating simple sector D_2 on the test color side is necessary to match the transmission of the variable sector at its fullest opening). The process of making a measurement consists in isolating the successive monochromatic patches, and for each one varying the sector transmission until photometric match is obtained.

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[5] "The Addition of Luminosities of Different Color," Phil. Mag., Dec. 1912, p. 845.

A pictorial plot of such a measurement is shown in Fig. 3. Such a plot constitutes a necessary and sufficient record of any type of color. It is to be noted that this plot may be constructed from an ordinary spectrophotometric series of measurements equally well if the latter is available, and that this plot may be used as a spectrophotometric one, to derive color sensation values.

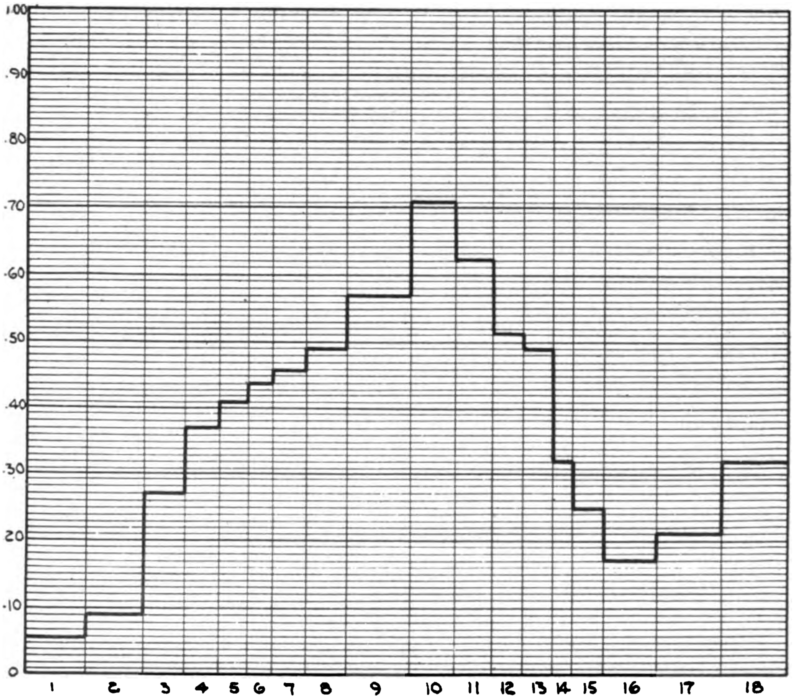

Fig. 3

Plot of measurement, consisting of a photometric setting, recorded in per cent, for each monochromatic patch of equal hue scale width.

Some practical questions which would arise in the construction of the hue scale spectrophotometer may be noted. Obviously the spectrum should be as long as possible in order to achieve practically the sharp demarkation of the spectrum patches. The longer the collimators, the greater the actual slit width corresponding to a given wave-length interval. The longer the telescope, the narrower will the eye-slit be compared to the collimator slit, and hence the more nearly will it approach the geometrical line which is called for to avoid overlapping of the patches. Similarly the greater the dispersion of the prism the longer will be the spectrum, and the easier to insure its accurate subdivision. For the full realization of the scheme, an apparatus of at least two meters optical length should be used. The illumination of the test and standard surfaces would have to be as intense as possible. Probably sunlight would be the most practical illumination; although, since any continuous spectrum source will do, high power tungsten lamps might suffice.

The question naturally comes up as to the number and spacing of the monochromatic patches to be used. As to the number, the upper limit for measuring purposes, to avoid color difference settings, is that fixed by each patch being half a least perceptible difference step. With patches this narrow, any color whatever, including isolated spectral lines, could be measured by exact no-color-difference settings. This would mean probably about 250 patches. It may be confidently stated, however, that for the vast majority of colors, far fewer patches would be necessary. The minimum which would permit the recognition of the five minima and four maxima exhibited by the hue scale (Fig. 1) would be nine. The most useful and practical number will be a matter for experiment to decide. It is indeed quite likely that the method could be standardized on the basis of several different patch numbers, each representing a different degree of refinement of measurement, the proper patch number being determined by the spectral complexity of the color and degree of accuracy required. If these numbers were each fractional parts such as ½ or ¼ of the most refined scale, they could all be plotted on the same coördinate paper. For instance a 36 patch scale could be used to plot 18 and 9 patch observations.

The spacing of the color patches should be according to the hue sensibility curve for the *average* eye (Fig. 1). This curve, of course, varies from one individual to another, which will mean that with wide patches the approximation to no-color-difference in the photometric setting will vary. This will, however, only have a second order effect on the result.

Thus far the discussion has been entirely confined to the problem of measuring the color. An equally important feature of a system of colorimetry is means for reproducing the measured color. To do this with the instrument described, the procedure will be to open both slits to their full extent, and to place in the plane of S_2 a sector disc (Fig. 4) so cut that its opening at the radial distance corresponding to each wave-length shall be proportional to the ordinate of Figure 3 at that wave length, thus synthesizing the color according to a method due to Abney

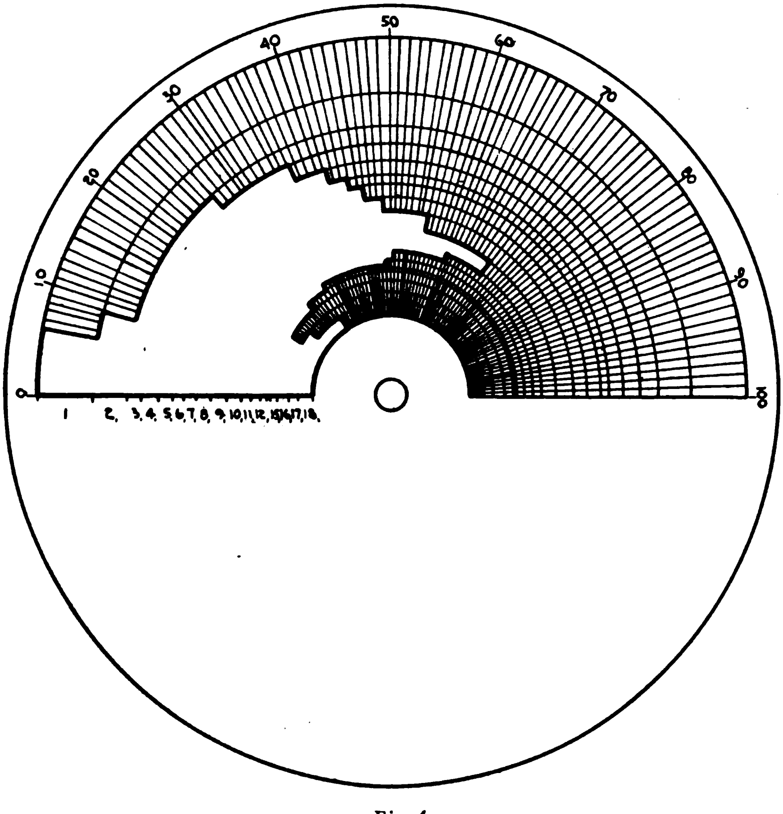

Fig. 4

Spectrum template disc for synthesis of color plotted in Fig. 3.

already utilized by the writer.[6] Such a set of sector templates will constitute, when used with the appropriate instrument, a set of working color standards exactly specified. To insure interchangeability of these templates, all hue-scale spectrophotometers should be manufactured of standard collimator focus and with identical prisms. If this is done blanks for cutting the template discs could be stamped with patch limit and percentage opening marks ready for cutting.

SUMMARY

A method of color measurement is suggested which consists essentially of the spectrophotometry of adjacent patches of the spectrum, each patch of a width fixed by the hue scale, and the number of patches dictated by the kind of color and degree of accuracy required. An instrument is described by which the measurements can be made, and in which the color can be reproduced for study.

RESEARCH LABORATORIES,
THE AMERICAN TELEPHONE AND
TELEGRAPH COMPANY, AND
WESTERN ELECTRIC COMPANY, INC., NEW YORK
June 28, 1921.

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[6] Ives and Brady, "An Apparatus for the Spectroscopic Synthesis of Color." Jnl. Franklin Institute, July 1914, p. 89.

A NEW X-RAY DIFFRACTION APPARATUS

BY

WHEELER P. DAVEY

The use of monochromatic X-Rays for the determination of crystal structure is well known. It is the purpose of this article to describe an apparatus by which the characteristic X-Ray diffraction patterns of fifteen powdered crystals may be taken at once. Each pattern is recorded on a strip of photographic film 1⅞ x 16 inches, placed in a cassette which automatically holds the film on the arc of a circle of 8 inch radius, having at its center the glass tube containing the specimen of powdered crystal. These cassettes are placed radially on a circular horizontal table at a fixed distance from a vertical water-cooled Mo-target Coolidge tube. The fifteen specimens thus lie along the edges of a horizontal polygon whose center is on the axis of the Coolidge tubes. The Coolidge tube is excited by a transformer placed under the table. In this way, the whole apparatus, exclusive of the switchboard, occupies a space 3 feet in diameter and about 4½ feet high (see Fig. 1).

The time required to obtain a satisfactory record of the diffraction pattern varies from 5 to 12 hours for substances like NaCl to 48 to 70 hours for substances like CuCl and CsI. The diffraction pattern may be interpreted directly in terms of the interplaner distances in the crystal by means of a metal scale which is calibrated in Ångstrom units. If the crystal happens to belong to the cubic, tetragonal, or hexagonal systems its structure may be interpreted directly from these interplaner distances by means of graphs described below.

THE TABLE

The table-top consists of a disk of steel ½ inch thick, provided with 15 radial guides for the cassettes. A hole 12 23/64 inches in diameter is cut in the center. In this hole is fitted a steel cylinder 11 9/16 inches high which acts as a support for the slit system, and at the same time, because of the long wave length of X-Rays employed, provides X-Ray protection for the investigator. Fifteen rectangular openings at the level of the slit system allow the rays to reach the glass tubes containing the specimens. Under each of these openings is another opening which may be used when desired for calibrating the photographic film. It is, of course, imperative, that all openings not covered by cassettes be blocked off by sheet lead or by 1/8 inch sheet brass to avoid danger to the investigator. The lead or brass should overlap the holes by at least half an inch.

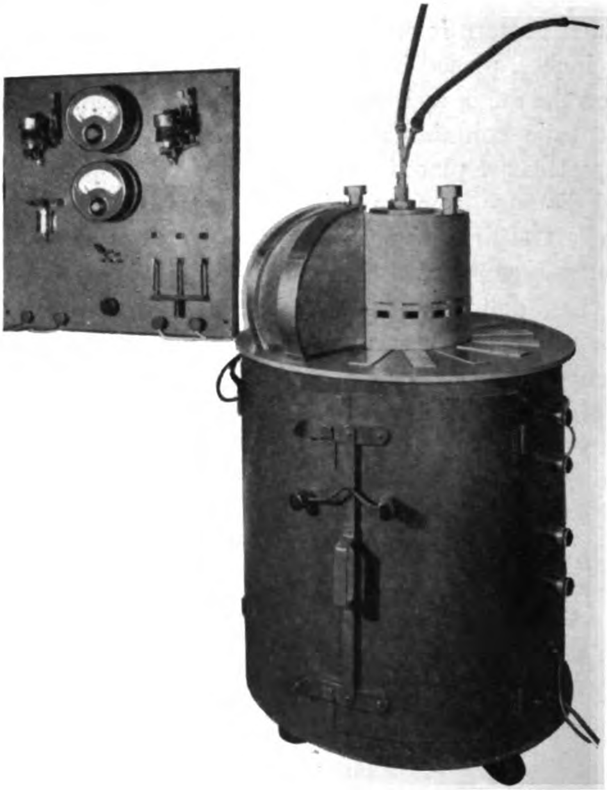

Fig. 1A—X-Ray Diffraction apparatus (doors closed)

The top of the cylinder is closed by a flat brass cover which also acts as a rigid support for the anode end of the Coolidge tube. The cathode end of the tube hangs freely. The cylinder and cover are machined so that the axis of the tube automatically coincides with the axis of the cylinder. A set-screw allows a vertical adjustment of the tube along the axis of the cylinder.

The under part of the table is enclosed by a cylinder of heavy sheet iron, reënforced with angle iron, and provided with doors to give access to the high-tension transformer which sits inside (see Fig. 1 B). The primary circuit of the transformer is carried across the front of these doors in such a way that the doors can not be opened without opening the primary circuit (see Fig. 1 A). In this way, when the X-Ray tube is in operation, the operator is protected from accidental contact with the high potential.

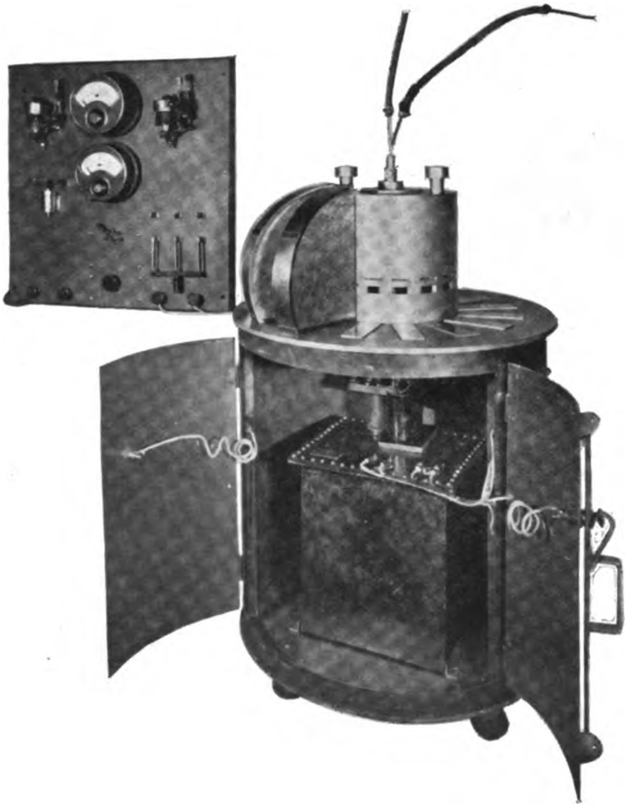

Fig. 1B. X-Ray Diffraction apparatus (doors open)

The whole table is mounted on heavy castors so that it may be easily moved from place to place.

THE TUBE AND ITS ACCESSORIES

The X-Ray tube is of the usual Coolidge type, but with a special water-cooled anode. This anode consists of a hollow copper rod at the end of which is fastened a molybdenum button. The shape of the rod is such as to provide an easy heat-path from the button to a large surface of water, so that local boiling is avoided. The face of the Mo button is perpendicular to the axis of the tube, so that X-rays may be taken off all the way around the anode.

The cathode is the standard 30 ma. cathode commonly found in the Radiator Type Coolidge tubes supplied to roentgenologists. Because of the large anode-cathode distance and the low voltage employed (30 kv_{RMS}) the tube shows space-charge characteristics, that is, the voltage drop immediately in front of the cathode filament is so low that when the filament operates at normal heating current (say 4.5—4.75 amperes), fewer electrons are dragged from the cathode to the anode than are evaporated from the filament. For this reason, at a definite voltage across the tube the current is independent of the filament temperature provided that the temperature exceeds a certain minimal value. A tube operated so as to show space—charge characteristics appears to be more efficient in crystal analysis work than one having the characteristics of the ordinary Coolidge tube, in which the space-charge characteristics are noticeable at lower voltages, but in which a higher percentage of the electrons is utilized at 30 kv. There is, besides, this additional advantage, that the tube load is nearly constant even though the filament is heated from a variable source of potential, provided that the variation in voltage on the filament circuit is less than 5%.

The maximum allowable filament current for 1000 hours life is 4.75 amperes. The maximum tube voltage for efficient production of Mo characteristic rays is 30 kv_{RMS}. At this voltage the tube current should be about 30 ma, but will vary from 28 to 35 ma from one tube to another.

Dimensions of the tube are shown in Fig. 2.

The radiation from the tube is filtered through two filters of ZrO_2, each having .05 g. of ZrO_2 per cm^2. One filter is built into the slit system,—the other into the cassettes. The first filter eliminates most of the "white" (general) radiation, and a large fraction of the Mo β doublet (.63 Å). The second filter makes the rest of the Mo β doublet negligible, and eliminates a large part of such characteristic rays as may be given off by the specimen itself.[1] The diffraction pattern, as recorded on the photographic film, is therefore caused by the Mo α doublet (average wave length .712 Å).

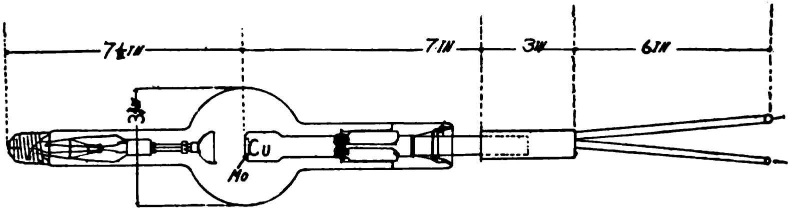

Fig. 2. Diagram of water cooled Coolidge tube

The optimum D. C. voltage for the production of Mo α is 28–30 kv,[1] but operation on A. C. at 30 kv $_{\text{MAX}}$ makes the time required for a satisfactory film so long that it is prohibitive for ordinary work. Experience shows that 30 kv $_{\text{RMS}}$ is about the maximum voltage at which the "white" radiation may be easily filtered from the Mo characteristic rays. For this reason it was adopted as the standard operating voltage for this apparatus.

The Coolidge tube is operated directly from a transformer without any additional rectifying device. This necessitates a special design of transformer, for, although the secondary must supply power only on one half of the wave, the voltage of the inverse half-wave must not greatly exceed that of the useful half-wave. Otherwise there will be considerable trouble from corona during the inverse half-wave. The transformer has only one developed terminal, the other end of the secondary being grounded through the milliammeter. A protective spark-gap is provided across the milliammeter circuit so that the meter is always safe to handle even if one of its connections is broken. The filament of the Coolidge tube is heated by an extra coil at the high-potential end of the secondary. Both ends of this coil, as well as the high-potential end of the secondary, are brought out through the high-potential terminal to a Kearsley stabilizer,[2] which automatically lowers the filament temperature in case of excessive discharge currents through the tube. Fig. 3 gives a diagram of the connections. From the Kearsley stabilizer, the high-potential filament circuit goes directly to the cathode of the Coolidge tube which hangs directly above it. The anode end of the tube is in electrical contact with the table top, as described above, so that the circuit is completed by the grounded metal table, and the milliammeter back to the transformer. In this way the high potential is entirely surrounded by grounded metal.

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[1] A. W. Hull, "A New Method of Crystal Analysis," *Phys. Rev.* 10, p. 661, 1917.

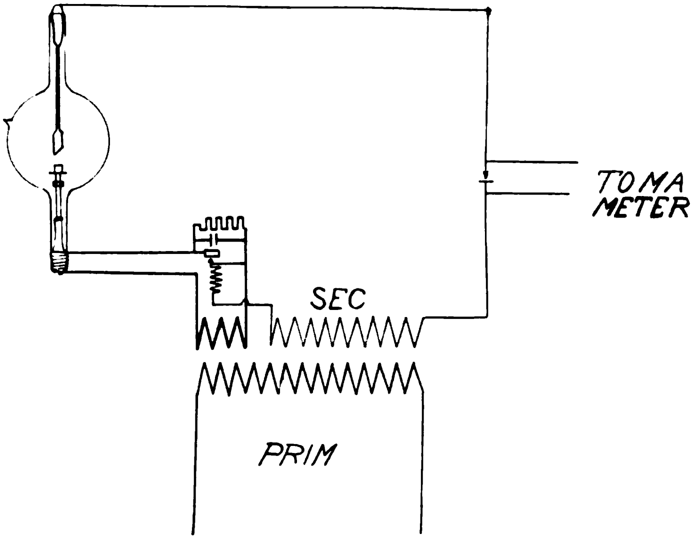

Fig. 3, Wiring diagram

The switchboard contains the customary circuit breakers, operating switch, an auto-transformer for adjusting the primary voltage of the transformer, a voltmeter for reading the primary voltage, a milliammeter and a pressure-stat. This pressure-stat is connected to the outlet end of the water cooled anode of the Coolidge tube. In case the water pressure falls too low, the pressure-stat short-circuits the power line through a small resistance, thus tripping the circuit breakers and protecting the tube from being burned out. The water pressure at the inlet end of the tube should be not less than 25 nor more than 80 lb. per square inch. In setting up the apparatus, enough resistance to flow of water at the outlet is provided so that the pressure-stat opens the protective short-circuit when the inlet pressure is 25 lb. per square inch.

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[2] W. J. Kearsley, Jr., "A New Type of Stabilizer for Use with the Coolidge Tube," *Journal of Radiology*, July, 1921.

A small centrifugal blower, mounted alongside the transformer, provides an air blast against the cathode neck of the Coolidge tube. This prevents local heating of the glass by radiation from the cathode during continuous operation, which might otherwise evolve gas and thus destroy the space-charge characteristics of the tube, with consequent loss of the tube due to overload.

THE SLIT SYSTEM

The slit system is suspended from a disk of ¼-inch plate glass which has a hole 6¼ inches in diameter cut in the center. The system is composed of 15 radial sets of 3 slits each,—the first two of which define the beam, while the third cuts off most of the secondary rays produced by the edges of the second slit. The whole slit system is formed on the surface of a cone whose basal angle is 5°. In this way each of the 15 slits utilizes the full projection of the focal spot of the Coolidge tube. The first slits are separated from each other by small brass partitions hung from the glass disk. The upper edges of these slits are formed by the edge of the glass disk itself. The lower edges are formed by the edges of small sectors of glass suspended between these partitions and the second slit. In this way the first slit is completely insulated from ground so as to avoid corona between it and the bulb of the Coolidge tube with consequent loss of the tube. If the first slit had not been insulated, it would have been necessary to have placed it further from the wall of the Coolidge tube, or to have inserted sheets of insulation (such as insulating paper or fiber) between the slit and the tube. In either case the time required to photograph the diffraction pattern would have been increased.

The second and third slits are composed entirely of brass, which seems to be about as good as lead for the wave length employed. They are supported from the glass disk, but are electrically connected to the grounded cylinder of the table top by small set screws which prevent displacement of the slit system. The slit system is shown in Fig. 4.

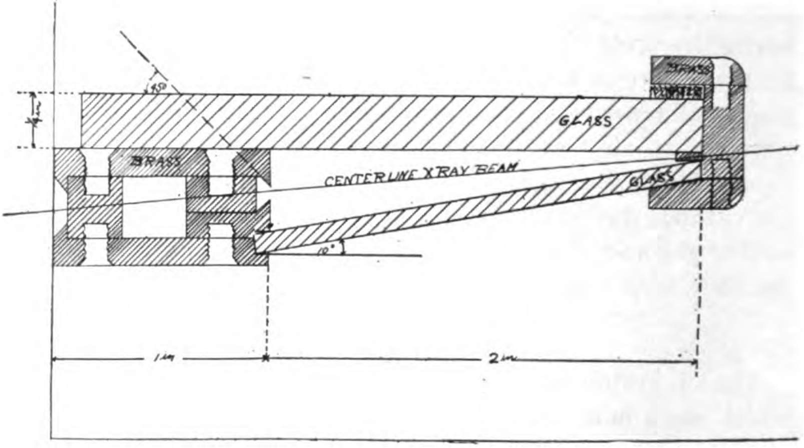

Fig. 4. Diagram of slit system

One of the filters mentioned above is fastened to the inner ring of slits, but is not shown in the diagram. Except for 15 rectangular spaces immediately in front of the slits, the whole surface of the filter is protected by nickel-plated brass from deterioration due to heat from the cathode reflected from the focal spot. At the slits, the white cardboard on which the filter is mounted is used for the same purpose.

The writer is well aware that numerous special types of slit systems have been devised by means of which X-Ray diffraction patterns may be photographed in less time than with the system described above. Although their merits for some highly specialized investigations are undoubted, they are hardly suited for general crystal research, nor for apparatus intended for most forms of routine factory-control work, either because of the large quantity of specimen required, or the cost of preparing the specimen in some special shape, or because certain lines in the pattern are enhanced at the expense of others.

The Cassettes, Films, and Specimens

The cassettes (Fig. 5) serve not only as holders for the films, but also as holders for the specimens. The powdered specimens (preferably 200 mesh) may be mixed with pyroxylin and formed into thin, flat sheets, or they may be packed into thin-walled tubes of about 1/32 inch inside diameter, of paper, celluloid or special glass. The glass tubes are preferred for most substances because of the possibility of sealing the specimens so that they can not be affected by oxygen or moisture. Whether the specimen is prepared in the form of a flat sheet or packed in a tube, the specimen-holder on the cassette automatically holds it in the path of the X-Ray beam. In case the specimen contains elements of high atomic weight, it should be mixed with corn-starch or other amorphous substance, to decrease the capacity of the total mass.

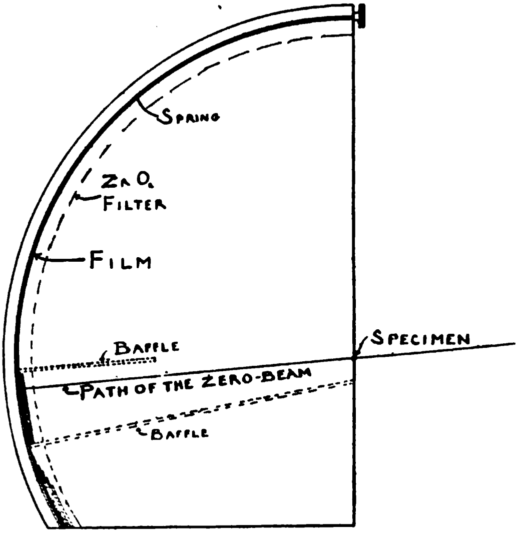

Fig. 5. Cassette

A strip of thin, black celluloid is fastened to the cassette in the form of an arc, 8 inches in radius, with the specimen at the center. The photographic film is held against this by a wide flat brass spring which is drawn up tight by a screw. A lightbaffle over the spring prevents the film from becoming light-struck. The most satisfactory film for X-Ray diffraction work seems to be the Eastman "Dupli-tized X-Ray" film.

A "staircase" of copper is placed as an absorber in the path of the zero-beam. In this way there is always some part of the zero line on the film which has an exposure comparable to that of the first lines on the diffraction pattern. A molybdenum staircase is provided below the zero-beam for calibrating the photographic film.

A septum in the median plane of the cassette divides it into two symmetrical chambers, so that diffraction patterns of two substances may be taken on the same film for purposes of comparison. This is done by filling the glass specimen-tube half full of one substance, inserting a tiny plug of cotton, and then filling the remainder of the tube with the second substance. In work requiring the measurement of inter-planer distances with great accuracy it is recommended that one-half of the specimen-tube be filled with NaCl. The theoretical spacings of NaCl (side of elementary cube = 2.810Å) may then be used to give a correction curve for the film, thus avoiding errors due to any possible changes in the film itself.

A "range finder" is provided for adjusting the height of the Coolidge tube. It is built to fit the guides on the table top so that it may be slipped into place opposite any one of the 15 slits. A tungsten wire acts as the "specimen." A fluorescent screen is mounted at the outside end in a position corresponding to that of the film in the cassette. The screen is faced with glass as a partial X-Ray protection to the operator. The Coolidge tube is at the proper height when the fluorescent screen shows that the shadow of the tungsten wire is in the center of the zero-line.

THE DIFFRACTION-PATTERN

The photograph of the X-Ray diffraction pattern looks much like the familiar picture of a line spectrum. The diffraction pattern of NaCl, taken with this apparatus, is shown in Fig. 6. A full account of the mathematical theory of the pattern is given elsewhere by A. W. Hull.[3]

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[3] A. W. Hull, loc. cit.

Using the classical equation for diffraction,

$$2d = n\lambda \sin \theta$$

a scale has been calculated by which each line on the film may be directly interpreted in terms of the distance between the planes of atoms which produced the line. The apparatus has thus made a record of the diffraction pattern of the specimen and has enabled the experimenter to record in Ångstrom units the spacings of all planes in the crystal which are more than .55 Å apart. Theoretically a diffraction pattern should be given by all planes having a spacing greater than $\lambda/2$ (= .356 Å), but except with very long exposure-time the lines from planes less than .55 Å are so faint as to be useless. Indeed, where a solution of the crystal structure can be found at all, it is usually possible to make a valid solution using only lines corresponding to planer distances of more than .80 Å. When the doublet is resolved ($\lambda = .710$ Å, $\lambda_2 = .714$ Å) the readings of the two lines as given by the scale will be in the ratio of 1 : 1.005. In such a case the pattern is to be considered as showing a single line situated half-way between the two actual lines. The order of accuracy of readings of lines by means of the scale increases from the left hand end of the scale (large values) to the right hand end (small values). At 1.00 Å it is of the order of 0.1 per cent. The limiting feature is the accuracy of the investigator in estimating the center of the line on the film. In case a solution of the crystal structure is found by the graphical method mentioned below, the final result is usually accurate to about ¼ per cent. It can then be checked by numerical calculation to about 0.2 per cent.

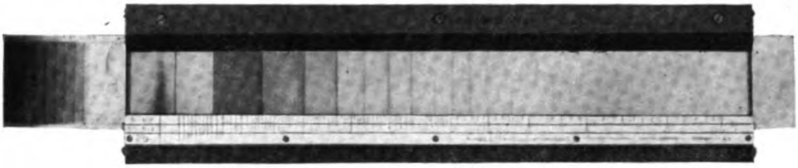

Fig. 6. Diffraction pattern of NaCl on scale, ready for measurement

USES

The apparatus described above has two fields of usefulness,—(1) in the identification of materials, (2) in the determination of crystal structure.

(1) All solid metals and their alloys, and most solid compounds, are crystalline and show X-Ray diffraction patterns. Waxes at ordinary temperature, and glasses are amorphous and show no diffraction patterns. The diffraction pattern of a given substance, as measured quantitatively on the scale described above, is as characteristic of that substance as the density, solubility, melting-point or ability to form chemical compounds. As far as is now known, no two substances have identically the same diffraction pattern. At first it was thought that the patterns of Mo and W were identical, but Hull has since shown[4] that the measurements of their diffraction patterns differ by about ¼ per cent. All other substances so far investigated which happen to have diffraction patterns of similar appearance are found, upon measurement with the scale described above, to differ from each other by several per cent. The apparatus therefore offers a convenient method for the qualitative analysis of crystaline substances. Any substance is completely identified when its pattern, measured on the scale, exactly matches the pattern of some substance whose identity is known. A large mass of such data is already available and will be published shortly in compact form.

This method of analysis has three advantages,—(a) only .001 cc. of the specimen (ground to 200 mesh) is required; (b) the original sample is still available after the analysis is completed; (c) in a mixture of two compounds, the *state of combination* may be determined[5]—i.e.,

$NaF+KCl$ can be distinguished from
$NaCl+KF$

The ease with which substances may be identified should make the method useful as a means of laboratory control of factory products, especially in the case of metals and their alloys. The method of crystal analysis offers the most accurate method known for the determination of coefficient of expansion and density. In both cases a solution of the structure of the crystal is necessary, as described below.

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[4] A. W. Hull. *Physical Review*, 17, p. 5, 1921.

[5] A. W. Hull, "A New Method of Chemical Analysis," *Jour. Am. Chem. Soc.* 41, p. 1168, 1919.

(2) It is often desirable, not only in purely scientific investigations, but also in factory-control work, to determine the arrangement of atoms in the crystal. In case the crystal happens to belong to the cubic, hexagonal, or tetragonal systems (and most metals, alloys, halogen compounds, nitrates of monovalent metals, and carbonates of bivalent metals do), the crystal structure may be solved directly by the following method.[6]

The edge of a strip of paper is laid along the logarithmic scale of abscissae which is found at the bottom of each of the plots of Fig. 7, and the distance between planes in the crystal, as read off from the film in Ångstrom units, is plotted on this edge. The strip of paper is now moved across the various plots keeping the edge always parallel to the axes of abscissae. When an exact match is found between the plot and the pattern marked on the edge of the paper then the crystal belongs to the system and subdivision marked on the plot, and the correct axial ratio is given by the intersection of the edge of the paper with the axis of ordinates. It is necessary that every line in the experimental pattern on the strip of paper be represented in the plot. If there is a single experimental line left over, the solution is not valid, no matter how good a match is obtained with the rest of the lines. The only exception to this is when several lines left over in this way can be shown to all fit some other portion of one of the plots, thus indicating that two crystal forms are present. If the plot predicts lines which are not found in the experimental pattern it is necessary to show either that those lines ought, theoretically, to be too faint to be seen on the film, or that the crystal structure is more complicated than that for which the plot was made, so that certain lines disappear by interference. (Thus any pattern given on the triangular close-packing and rhombohedral plots will fit certain lines of the simple triangular plots at the same axial ratio, and all the patterns of the body-centered and face-centered tetragonal structures will similarly fit portions of the simple tetragonal plots.)

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[6] For a more complete description of the method see *Hull and Davey*, "Graphical Determination of Hexagonal and Tetragonal Crystal Structures from X-Ray Data," *Physical Review*, 17, p. 5, 1921.

Plots of the cubic system will be found at an axial ratio of 1.00 in the plots of the tetragonal system.[7] The diamond division of the cubic system may be found by omitting the 2nd, 5th, 8th, 13th, and 19th lines, counting from the right hand side of the plot, from the plot of the face-centered division of the cubic system.

After an exact match is found for the experimental pattern, it should be verified by computing the density of the specimen in the following manner,—

(a) If the match is in the cubic system, the density of the substance is given by

$$\rho = n\frac{M\times 1.649\times 10^{-24}}{(d\times 10^{-8})^3}$$

where

ρ is the density

M is the molecular weight of the substance; 1.649×10^{-24} is the mass in grams of one unit of molecular (or atomic) weight;

d is the side of the elementary cube,—(it is the distance in Ångstroms between the 1 0 0 planes in the crystal);

n is the number of points associated with a unit cube in the crystal lattice. n is

1 for the simple cubic lattice
2 for the body-centered cubic lattice
4 for the face-centered cubic lattice
8 for the diamond-cubic lattice.

If the line corresponding to the "100 planes" is absent (as in the case of the body-centered and diamond divisions) then twice the distance corresponding to the second-order line, called in the plots "100(2)—100," must be used as the side of the elementary cube.

(b) If the match is found in the tetragonal system, the density is obviously

$$\rho = \frac{M\times 1.649\times 10^{-24}}{c(d\times 10^{-8})^3}$$

where c is the axial ratio, and n has the same values as before.

(c) If the match is found in the hexagonal system, find the distance corresponding to the "10·0 planes." (If the match is

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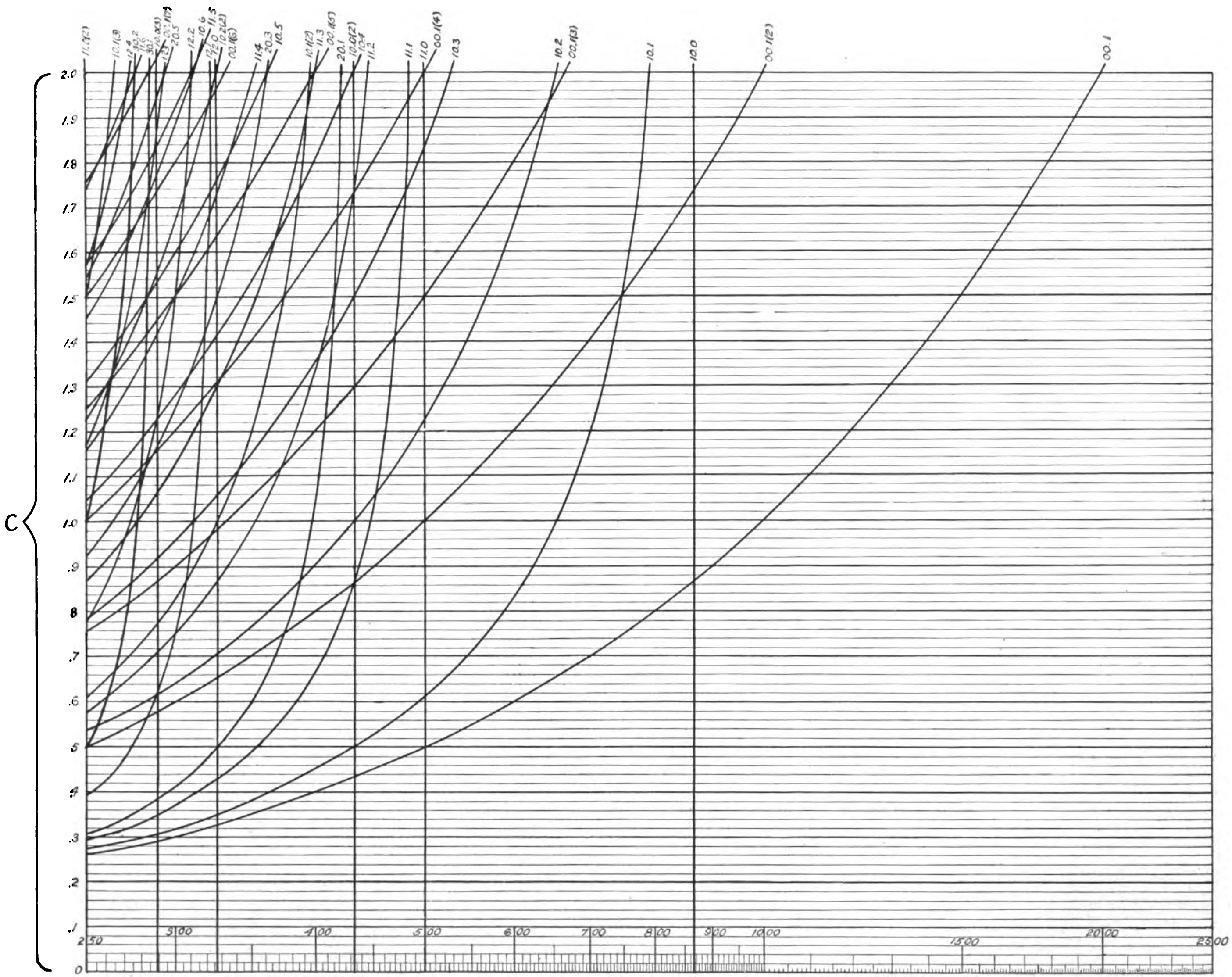
[7] They will also be found in the plots of the rhombohedral division of the hexagonal system.


11.0(2)
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1.9
1.8
1.7
1.6
1.5
1.4
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.4
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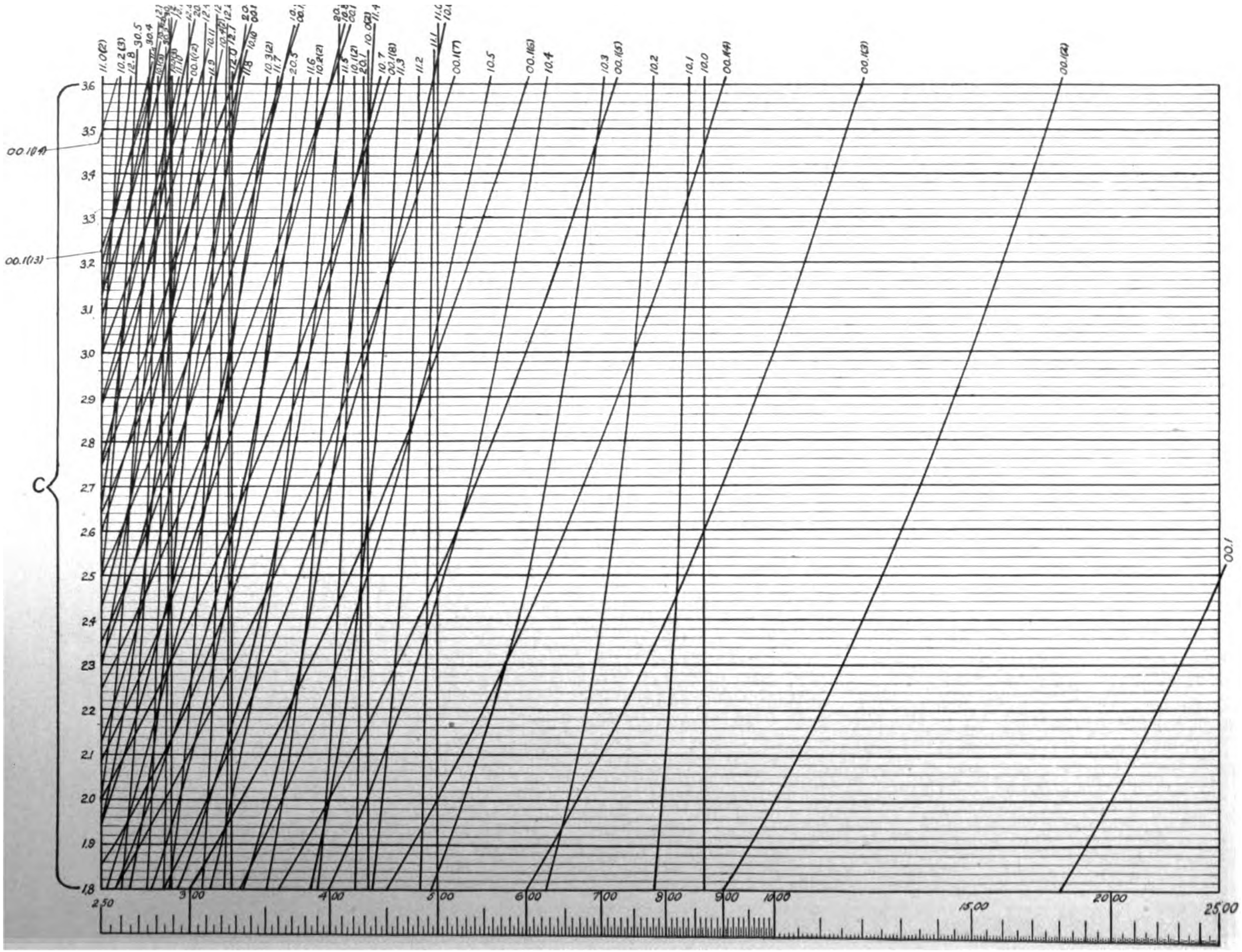
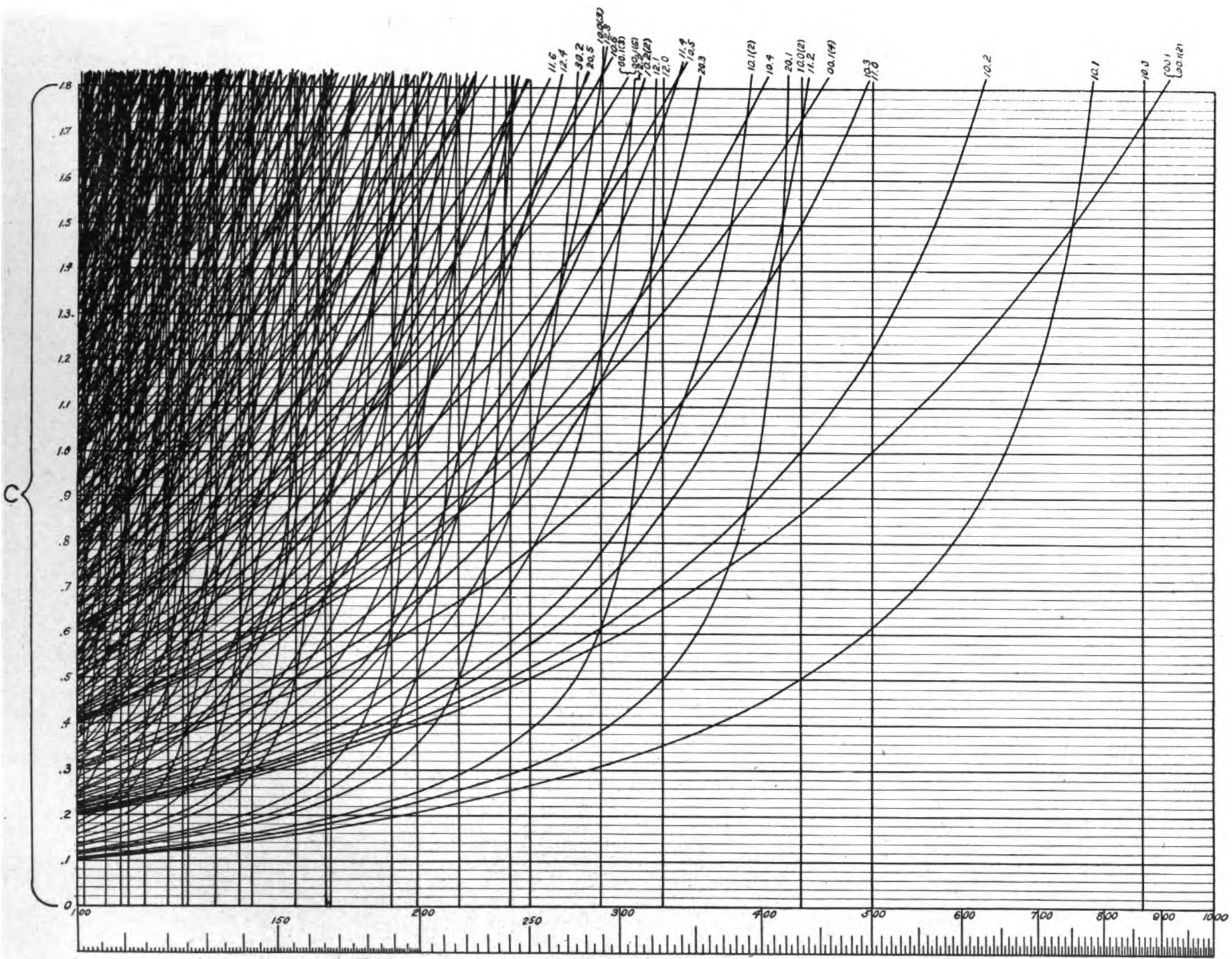


FIG. 7B SIMPLE TRIANGULAR LATTICE (HEXAGONAL SYSTEM)



TRIANGULAR CLOSE PACKING LATTICE (HEXAGONAL SYSTEM)

Fig. 7C

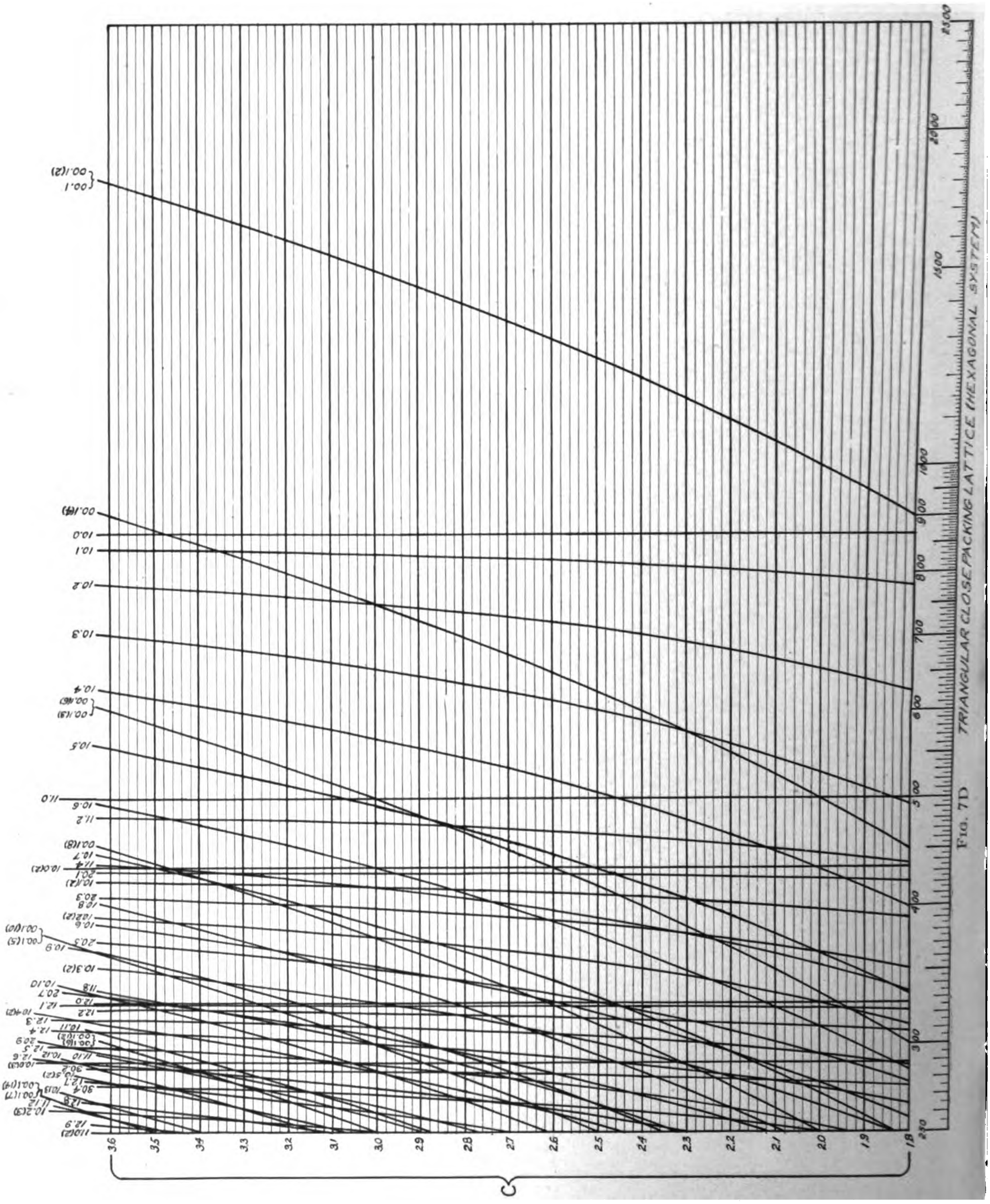
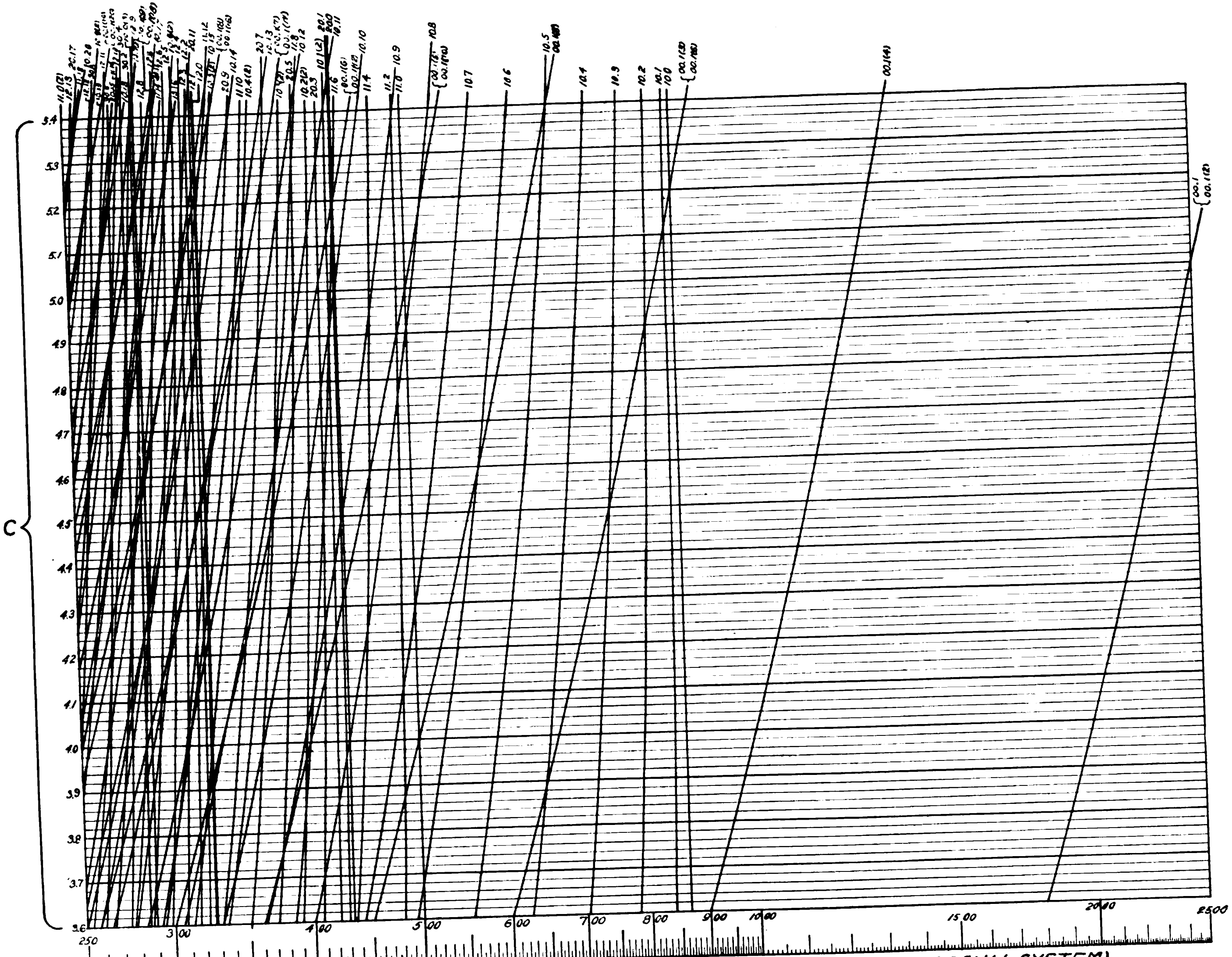


Fig. 7D

TRIANGULAR CLOSE PACKING LATTICE (HEXAGONAL SYSTEM)



TRIANGULAR CLOSE PACKING LATTICE (HEXAGONAL SYSTEM)

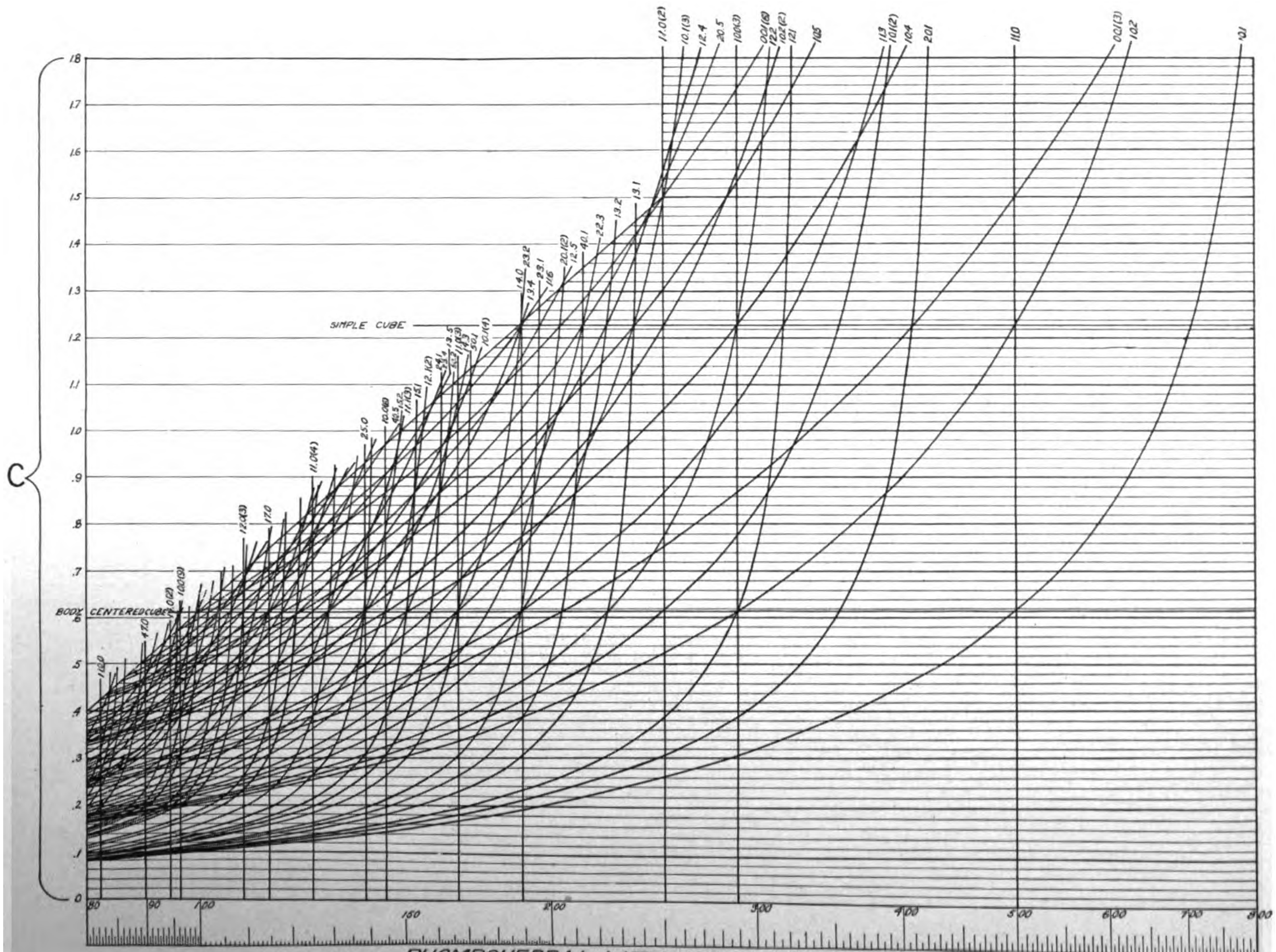


Fig. 7F RHOMBOHEDRAL LATTICE (HEXAGONAL SYSTEM)

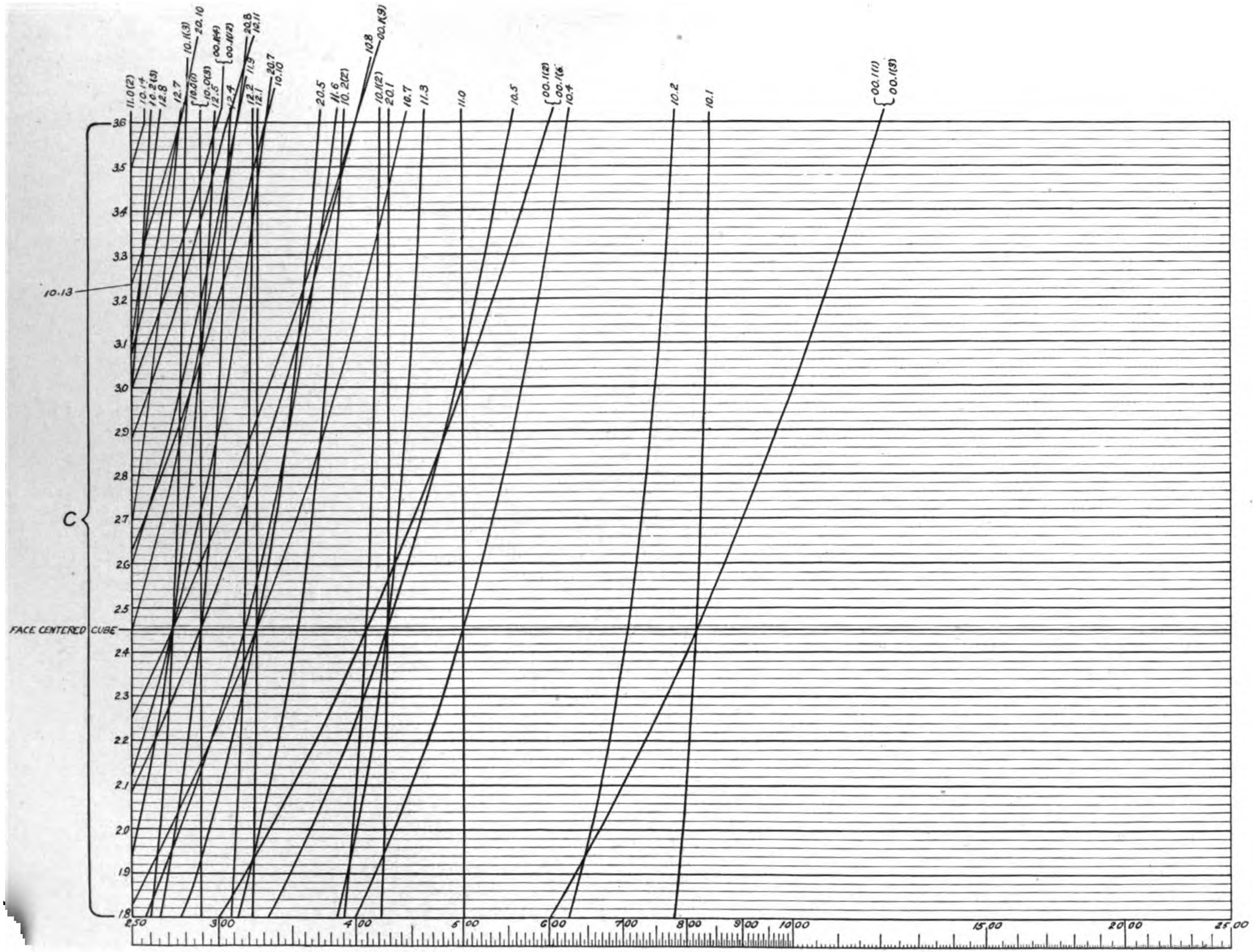


FIG. 7G RHOMBOHEDRAL LATTICE (HEXAGONAL SYSTEM)

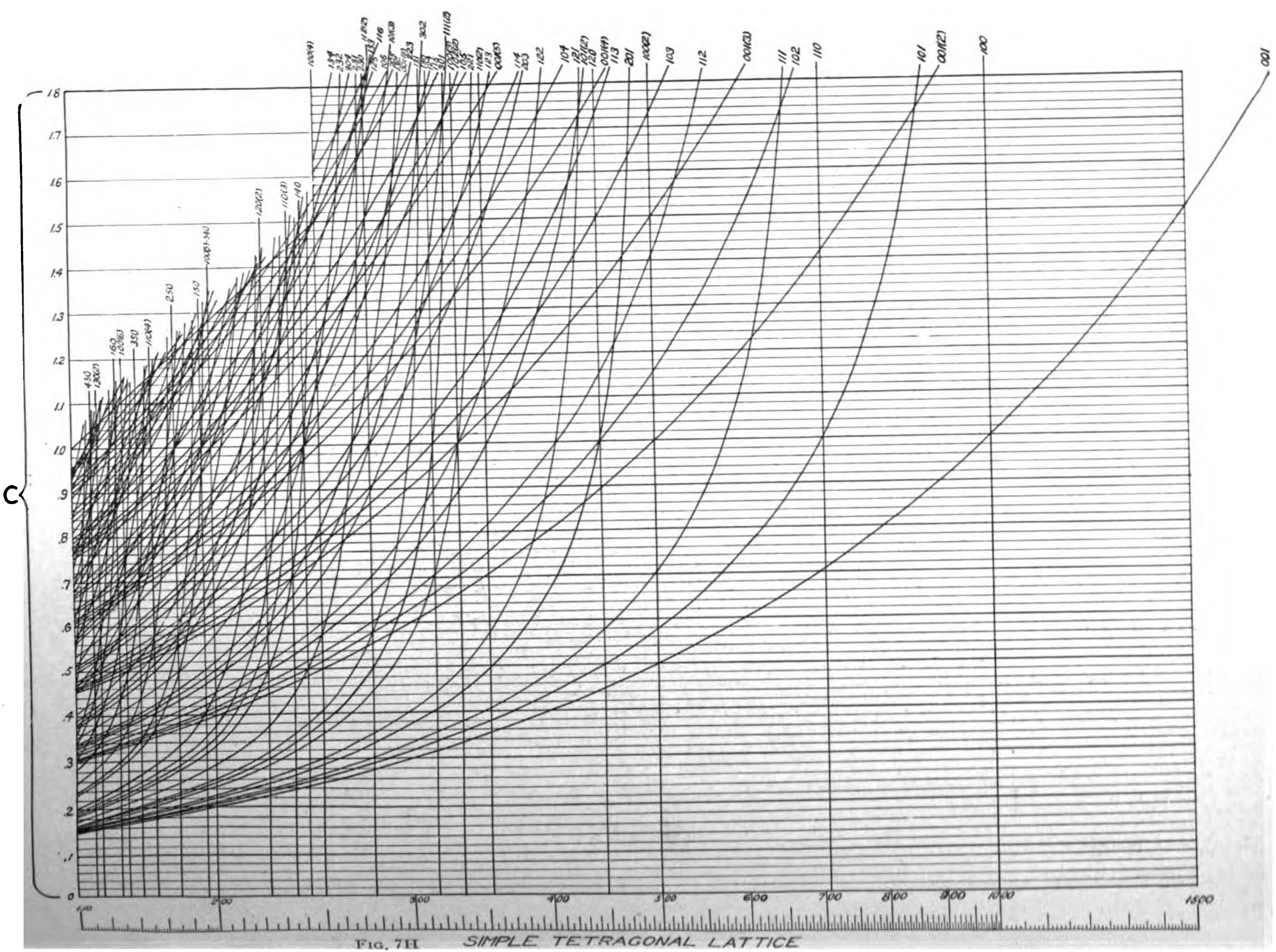
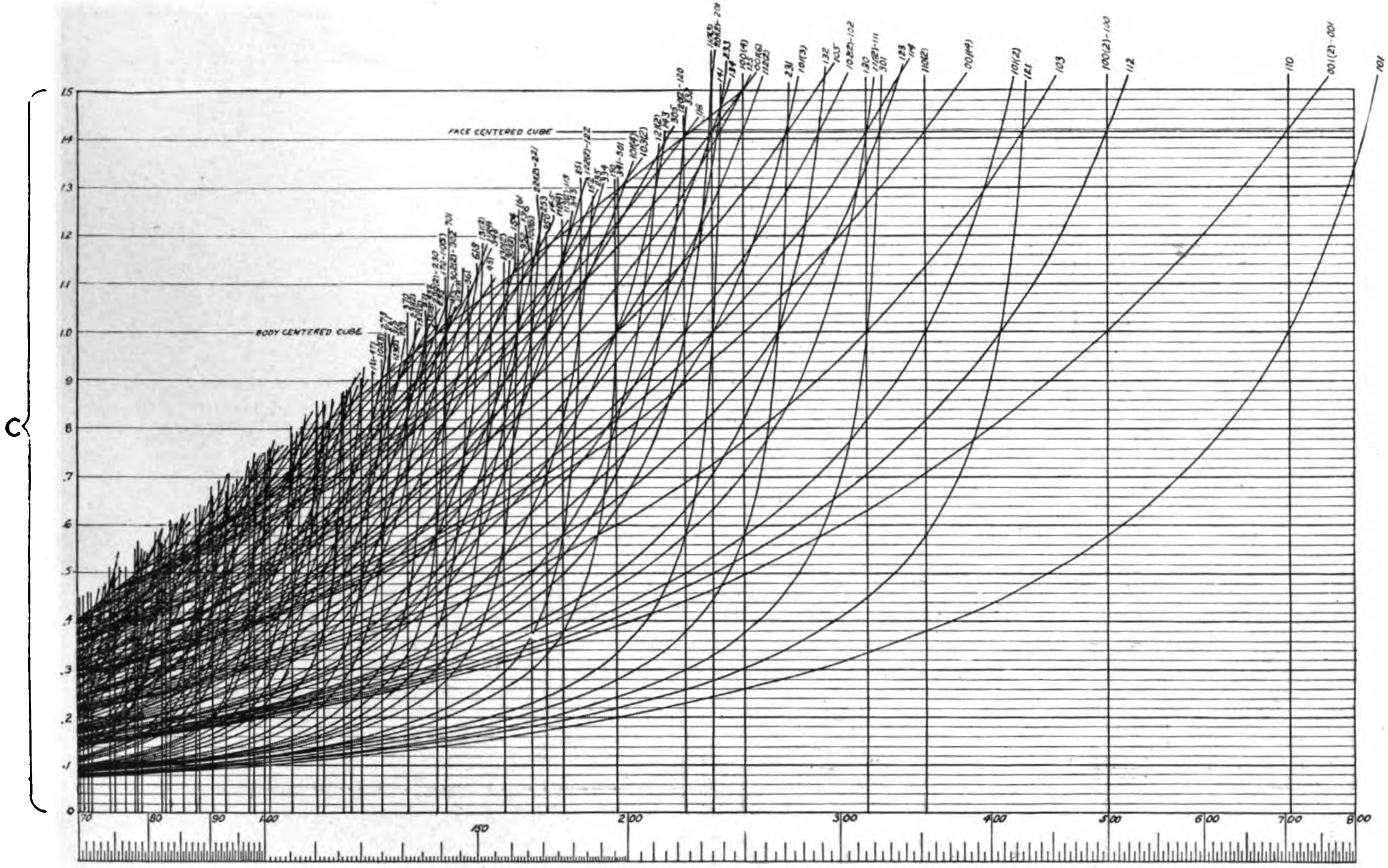


FIG. 711 SIMPLE TETRAGONAL LATTICE



BODY CENTERED TETRAGONAL LATTICE

FIG. 71

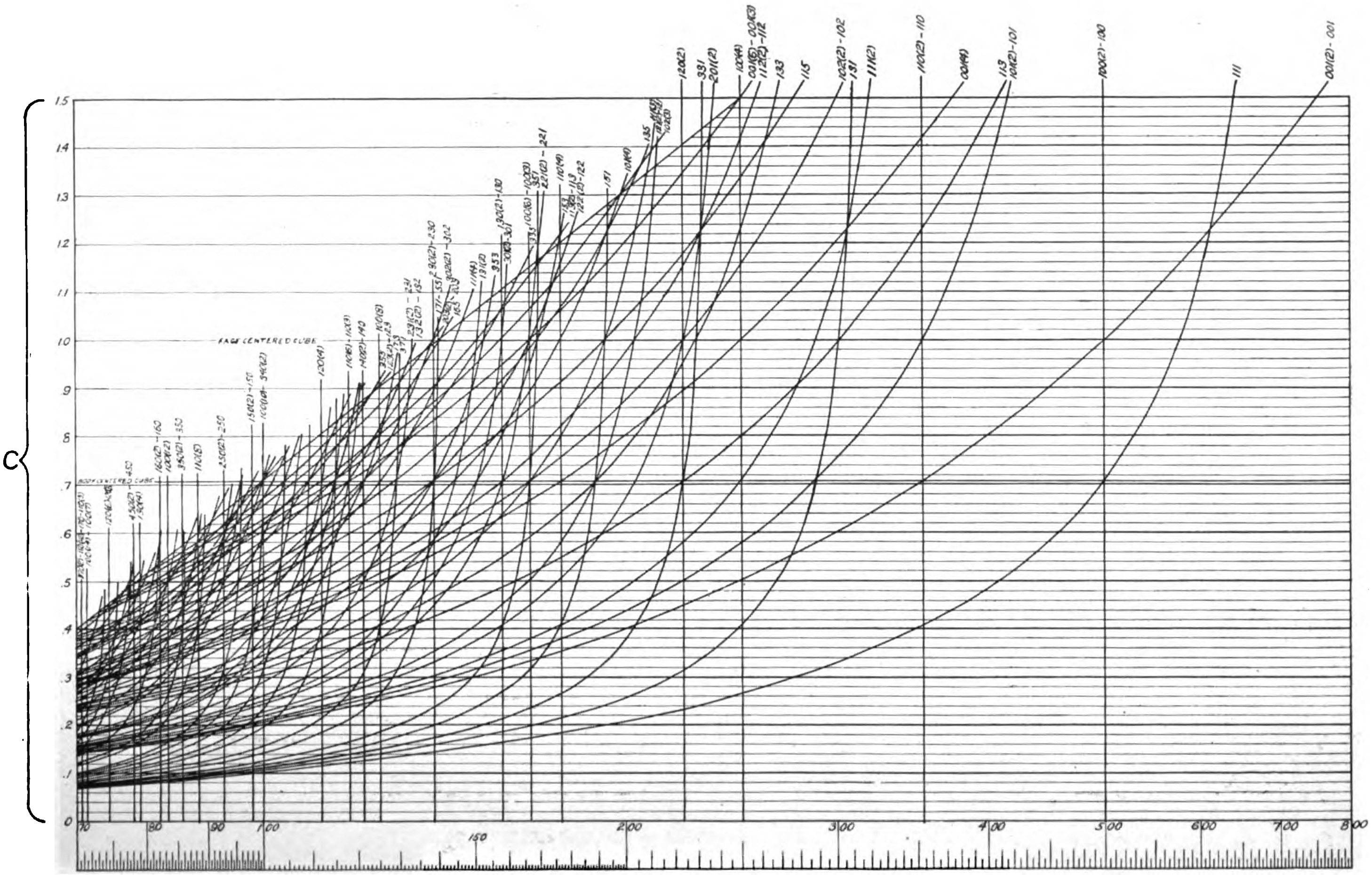


FIG. 7J FACE CENTERED TETRAGONAL LATTICE

found in the rhombohedral division, three times the distance corresponding to the third-order line, called in the plots $\left\{\begin{matrix}10.0(1)\\10.0(3)\end{matrix}\right.$ must be used.) This is the altitude of the equilateral triangle which forms the base of the unit prism. The side of the unit triangle is $2/\sqrt{3}$ times this distance. The density of the specimen is therefore

$$\rho = n\frac{M \times 1.649 \times 10^{-24}}{\sqrt{3/4}\,c\,(a \times 10^{-8})^3}$$

where *a* is the side of the unit triangle.

n is ½ for a simple triangular lattice
1 for a triangular close-packed lattice
1½ for a rhombohedral lattice.

It is a pleasure to acknowledge the suggestions of A. W. Hull during the course of this work; also the assistance of E. O. Hoffman and H. A. Smith in the design of the slit system and the development of the water-cooled tube; R. Hergenrother in the laborious calculation and drawing of the plots; and Wm. F. Winter in the design and construction of the cassettes and the construction of the slit system.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

1920 REPORT OF COMMITTEE ON PYROMETRY

BY

W. E. FORSYTHE

The following Report for 1920 has been formed by the Chairman following the suggestions and criticisms of members of the Committee especially C. E. Mendenhall. This Report will be divided into three parts:

I. The first part embodies some suggestions for nomenclature and standards.
II. The second part embodies a discussion of ways and means of using an optical pyrometer.
III. The third part embodies a review of the literature on pyrometry for the year.

I. NOMENCLATURE AND STANDARDS

The most important single item in pyrometry and radiation is temperature. No attempt will be made to define temperature at this point since this idea is pretty well understood and very seldom do we find in the literature that there is any mistake in its use.

Temperature (T), true temperature or shortened to temperature, is to be used when the ordinary idea of temperature is meant. Care always should be taken to specify the scale used. This is a point that is often neglected both by engineers and physicists. Sometimes it is even hard to tell from the context what scale has been used.

The temperatures are to be measured on the International temperature scale. This scale for all practical purposes is the thermodynamic scale of Kelvin. A table will be found in last year's report[1] giving a number of standard temperatures.

Apparent Temperatures. There are several methods of measuring temperatures based upon the assumption that the body being studied radiates like a black body. Since there is no known substance that radiates like a black body, numbers so obtained are always different from the true temperature. These different temperatures were at first all referred to as "black-body temperatures," but since there are now at least five such "temperatures," some more definite nomenclature is necessary.

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[1] JOUR. OF OPTICAL SOCIETY, *4*, No. 5, p. 310, 1920.

The first of these temperatures is the one that is obtained when measuring non-black bodies with an optical pyrometer calibrated to give the correct temperature of a black body. This temperature is always less than the true temperature and depends upon a measure of the brightness for a particular wave-length interval. It is, in general, different for different wave-length intervals, thus the wave-length to which this temperature corresponds should be specified. Since this temperature is determined from a brightness measure it has been called the *brightness temperature*. The symbol for this is S_λ, where the subscript (λ) is to show the wave-length at which the brightness has been measured.

Another useful temperature is one that is determined from a measurement of the total light that is radiated from the source being investigated, as though the source radiated like a black body. This temperature should be known as total brightness temperature and should be designated by S_t.

The next temperature of interest is the one that is obtained from the color of the radiated light. Not only is the brightness of a substance for any particular wave-length interval different from that of a black body, but in general the emissive powers for the non-black body will vary from wave-length to wave-length throughout the spectrum. It has been found experimentally that practically all metals when hot radiate in such a manner that the color of the light can be matched by that from a black body at some particular temperature, which temperature has been called the color temperature of the hot metal. Since the light from all radiating substances cannot be accurately matched in color with that of a black body at a particular temperature, it is much more accurate to define the color temperature as that temperature of a black body at which the ratio of the radiant flux at some two arbitrarily chosen wave-lengths, λ_1 and λ_2, is the same as that for the radiating substances being investigated. This definition includes the above, but does not demand that the body under investigation should match the black body at all wave-lengths.

Color temperature measurements can be quite accurately made with an ordinary photometer. The emissive powers of the metals that have been studied are larger for the short than for the long wave-lengths; thus the color temperature is higher than the true temperature. Color temperature is to be designated by $T_c(\lambda_1\lambda_2)$. The wave-lengths λ_1 and λ_2 are to be given if the temperature refers to two wave-lengths, but when the integral light is used the wave-length may be omitted.

Another "temperature" that has been used in the past but is very little used now is the one that is determined from one of Wien's displacement laws. That is $\lambda_m T = b$. To determine this temperature the wave-length (λ_m) for maximum emission is determined and the value of b, the same as that for a black body, is used and thus a temperature may be computed. No name has been suggested for this temperature.

The fifth temperature that is met with in practice is the one obtained with a total radiation pyrometer from readings of a non-black body. This is called radiation temperature and is to be designated by T_R. This temperature is always less than the true temperature.

II. OPTICAL PYROMETERS

Optical pyrometers, from their construction, are divided into four classes and for convenience of reference are to be classified on this basis.

1. The pyrometer in which the comparison source is a lamp filament located at the focus of the objective lens shall be designated the disappearing filament pyrometer.

2. There does not seem to be any good descriptive name for the pyrometer, of which there are several forms, in which the comparison source is off at a right angle to the direct line of sight and the comparison is made on a photometer cube or some sort of a mirror.

3. The pyrometer, in which the light from the two sources is by some means polarized at right angles to each other and the final balance is made by rotating a nicol in the eyepiece or elsewhere, shall be designated as the polarization pyrometer.

4. All optical pyrometers in which there is no comparison source shall be listed in a separate class and different forms of this class shall be described, if it is found necessary.

The scale for optical pyrometers shall be extended above, or below that of the standard temperature by means of Wien's equation, using for c_2 14350μ deg. If monochromatic light is used, the wave-length will be obtained from the spectroscope, but if a so-called monochromatic screen is used, the effective wave-length as defined below is to be used.

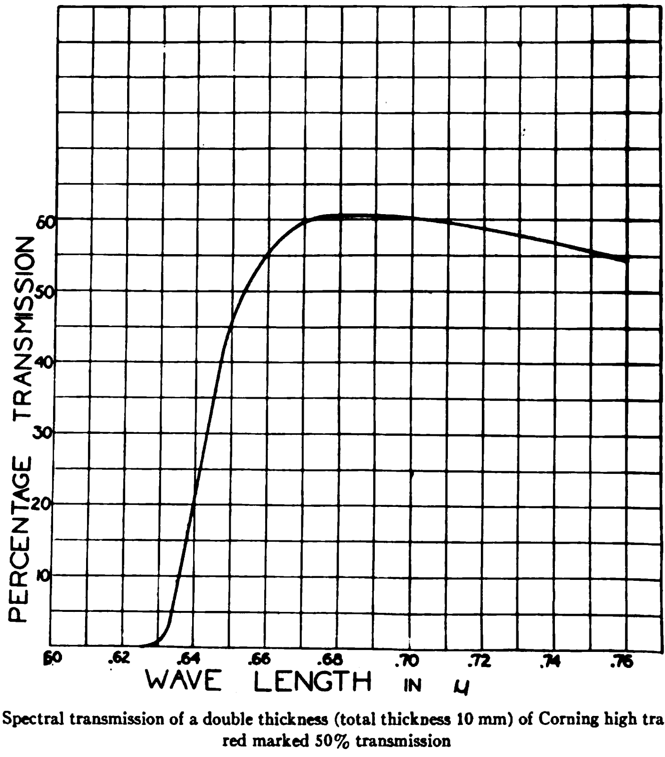

FIG. 1. Spectral transmission of a double thickness (total thickness 10 mm) of Corning high transmission red marked 50% transmission

Monochromatic Screens. For the most part it seems best to use as the monochromatic screen in the eyepiece of the pyrometer, a red glass rather than a screen of some other color. It is recommended that for this a red glass be used that has very nearly the same spectral transmission as has two thicknesses (total thickness, 10 mm) of the Corning high transmission red, marked "50% transmission," a spectral transmission curve of which is given in Fig. 1.

If the pyrometer is constructed for temperatures around 1500°K, it is difficult to get enough light for accurate work at very low temperatures. If one of the red glasses is removed from the eyepiece about twice the amount of light will be transmitted, while if both glasses are removed, about fifteen times as much light will reach the eye as with two glasses. It is very convenient to have the eyepiece so constructed that a large sized opening can be used for low temperatures. For extremely low temperatures there will be a very small difference in the readings whether no red glass, one red glass, or two red glasses are used. In Table 1 are given the readings obtained in each case for several temperatures. For the entire range of temperatures given in the Table, the number of red glasses used, i.e., none, one, or two, would make but a small difference in temperature while for the lowest temperature it would make practically no difference.

For the temperature given in the Table a change of .0003 ampere in the current through the pyrometer corresponds to about one degree change in temperature.

TABLE 1. *Current through Pyrometer Filament for Brightness Match for No Red Glass, One Red Glass, and Two Red Glasses for Different Temperatures*

Temperature degree K	No Monochromatic Screen	One Red Glass	Two Red Glasses
1404	.3167	.3149	.3148
1280	.2786	.2775	.2773
1217	.2613	.2604	.2605
1141	.2423	.2419	.2422

The effective wave-length of the monochromatic screen is defined as the wave-length that is necessary to use to calculate temperatures by means of Wien's equation for brightness measurement with the monochromatic screen in the eye-piece. This is the wave-length for which the same ratio will be obtained using Wien's equation for a certain temperature interval as is obtained by actual measurements from a black body over the same temperature range using the monochromatic screen in question.

Basis of High Temperatures. The basis of all high temperature measurements is the work with the gas thermometer[2] that fixed the value of some of the standard melting points. It has often been suggested that the Stefan-Boltzmann law, which states that the total energy radiated by a black body varies as the fourth power of the temperature, be used as the basis of high temperature measurements. This law would give the absolute thermodynamic temperature and would be just as accurate and would depend upon just as sound reasoning as the gas thermometer. In addition, it would be much easier to use the method based upon the fourth power law than the gas thermometer. The container for the gas is the source of difficulty with the gas thermometer, and it seems that there would be very great experimental difficulty with that alone if an attempt were made to extend the gas scale above the present limit reached by Day and Sosman who succeeded in carrying the gas thermometer up to the temperature of melting palladium. The method based on the fourth power could be carried as high as it would be possible to operate a black-body furnace. To be sure, there would be some difficulties with this method, but they do not seem to be insurmountable.

There have been several attempts to compare the scale of temperature as defined by Wien's Equation with that defined by the Stefan-Boltzmann Law. Lummer and Pringshein[3] made a comparison up to 2200°K, but the accuracy was not very high. Gillette[4] has compared the two scales up to 2700°K and states that they are in agreement, but does not state how good the agreement was nor how accurately the two scales could be reproduced. Mendenhall and Forsythe[5] compared the two scales up to 3100°K and obtained the agreement shown in Table 2. These differences are all within the experimental error. The plan is to repeat this comparison under better conditions and to see if the range cannot be extended and also the accuracy increased.

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[2] Day & Sosman, Am. Jour. Science, *24*, p. 93, 1910.
[3] Verh. Deut. Phys. Ges. *1*, p. 3, 1903.
[4] Jour. Phys. Chem., *15*, p. 213, 1911.
[5] Phys. Rev., N. S., *4*, p. 62, 1914.

TABLE 2. *Comparison of the Stefan-Boltzmann and Wien Scale*

Temperature	$T_W - T_{S.B.}$
2025°K	less than 0.5°C.
2475	less than +2
2775	about +2
3095	about +4

Radiation from Non-Black Bodies. In order to express the relation between the emission of energy from metals and the temperature, a modification of the Wien-Planck equation for a black body is generally used. When the first work was done on the radiation of metals the relation was expressed as

$$E_\lambda = c_1\lambda^{-a}e^{-\frac{c'_2}{\lambda T}} \tag{1}$$

where an attempt was made to determine a and c_2. It was thought for a time that such an equation would represent the facts but later work has shown that a and c'_2 are not constant but may be a function of both the temperature and wave-length.

At the present time the modification adopted is

$$E_\lambda = \epsilon_\lambda c_1\lambda^{-5}e^{-\frac{c_2}{\lambda T}} \tag{2}$$

where the added factor ϵ_λ which is called the spectral emissive power may be a function of both the wave-length and the temperature. According to Kirchhoff's law the emissive power for any wave-length for any body is equal to its absorption factor. If radiation falls upon any body a part is reflected, a part transmitted, and the rest absorbed. The relation between these various quantities is,

$$1 - r_\lambda - t_\lambda = a_\lambda \tag{3}$$

where t_λ = transmission, r_λ = reflection factor and a_λ = the absorption factor. If ϵ_λ is independent of the wave-length the body is said to be gray. As was mentioned above the value of the spectral emissive power for the metals investigated has been found to increase for shorter wave-lengths.

The true temperature and brightness temperature are related as follows:

$$\frac{1}{T} - \frac{1}{S_\lambda} = \frac{\lambda \log \epsilon_\lambda}{C_2 \log e} \tag{4}$$

It has been stated sometimes that for a gray body the brightness temperature is independent of the wave-length. From equation (4) it can be seen that this is not the case, since for a constant brightness temperature $\log e$ must vary inversely as the wave-length. For gray bodies the color temperature is the same as the true temperature.

Use of an Optical Pyrometer. It often happens that when it is necessary to measure the temperatures of a hot radiating body with an optical pyrometer the operator does not know just how to proceed. Although optical pyrometers are readily purchasable, if it is necessary to construct one, it will usually be much easier to build one of the disappearing-filament type. All that is required for this is an objective lens about 3 cm in diameter and about 25 cm focal length, a piece of red glass, a long-focus eyepiece, a pyrometer lamp, an ammeter of the right range, a resistance, and a battery.

An ordinary spectacle lens of the right focal length can be used for the objective lens and two such lenses of the proper focal length will do for the eyepiece. For ordinary work it is not necessary to have an achromatic lens since a red glass is used in the eyepiece. The optical parts may all be mounted on some sort of an optical bench or if they are to be used permanently, it is well to have them mounted inside of a tube after the manner of a telescope. The pyrometer lamp is to be so mounted with respect to the objective lens that an image of the source being investigated can be brought to a focus in the plane of the filament and this image and the pyrometer filament both observed through the eyepiece. When using this instrument, the different parts are adjusted so that the images of both the source being investigated and the pyrometer filament are as well defined as possible. The arrangement of the different parts of the disappearing-filament optical pyrometer is shown in Fig. 2. The particular way in which the pyrometer is built depends upon the purpose for which it is intended. If it is desired to measure the temperature of a large source, it will not be necessary to have very large magnification. The distances between the pyrometer lamp and the eyepiece depend upon the focal length of the lens in the eyepiece F, (in the case illustrated in Fig. 2, a telescope of short focus). For a pyrometer set up to measure the temperature of small filaments the dimensions given below Fig. 2 have been found satisfactory. The main requirement is for the image of the source in the plane of the pyrometer filament to be much larger than the pyrometer filament, since it has been found by experiment that accurate matches cannot be made unless the background image is somewhat larger than the pyrometer filament. Due to diffraction, there will be dark or bright streaks along the edge of the pyrometer filament, and these streaks will be observed if an eyepiece with sufficient resolving power is used. If they can be seen, they interfere with the accuracy of the setting. In order to avoid this difficulty, reduce the resolving power of the eyepiece, either by using an eyepiece in the telescope with a smaller resolving power or using a cap with a small hole over the objective lens of the eyepiece telescope.

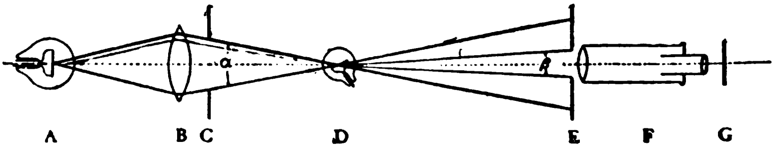

Fig. 2. Diagram showing arrangement of apparatus. A, background; B, objective lens; C, entrance cone diaphragm; D, pyrometer filament; E, eyepiece diaphragm; F, eyepiece; G, monochromatic filter. (AB=25 cm; BD=128 cm; DE=185 cm; diameter of aperture at E=9 mm)

With an optical pyrometer the intensity of the light per unit area, that is, the brightness, for a particular wave-length interval, is compared with the standard. To calibrate the pyrometer to measure temperatures, it is necessary to find a relation between this brightness and the temperature. Such a relation is given by Wien's equation which shows the relation between the energy for a particular wave-length interval, the wave-length, and the temperature. Thus, if the wave-length is known for which the energy, E_λ, is compared, the temperature can be calculated. E_λ is compared with that of the standard source either by a spectrometer, which determines the wave-length, or by the use of a particular monochromatic screen in the eyepiece. In the former case, if proper precautions are taken to have slit-widths of the proper size, the wave-length can be definitely determined. When a so-called monochromatic screen (usually a red glass) is used, its effective wave-length must be known. This effective wave-length defined above is slightly different for different temperature intervals, and is the wave-length that must be used for the different temperature intervals so that calculations may be made.

Calibration of Optical Pyrometers. Despite all that has been written about the calibration of an optical pyrometer, a question is very often asked concerning the starting point for the calibration of the pyrometer. To make a completely independent calibration of any pyrometer would be a very great undertaking. Very fortunately it is not necessary or desirable that each one should make an absolutely independent calibration. If one had to calibrate an ordinary mercury thermometer at about 100°C, it would be calibrated by immersing the thermometer into steam. If a higher point was wanted some material with a melting or boiling point in the proper region might be used, and the calibration made at the melting or boiling point as the case might be. The temperature of boiling water is defined as 100°C but the temperature of the other points in this region are used with just as little question and with just about as much confidence as this one. To probably a lesser extent the same condition exists at high temperatures. The temperature of either melting gold or palladium is very constant and is well known. It only remains to bring the standard radiator, that is the black body, to either one of these definite temperatures and from this to calibrate the pyrometer. In this case, the pyrometer filament is matched in brightness with the image of the black body across which it is seen through the eyepiece of the pyrometer. How this is done, together with a description of the black body, is given in the next few paragraphs.

Black Body. The Kirchhoff ideal black body consists of a cavity having uniformly heated walls with an opening into the cavity, through which the radiation is studied. It has been shown that if this opening is very small with respect to the size of the walls of the cavity, the error that will be introduced by this opening will be negligible. Many different attempts have been made to realize in a practical way the black body. The first attempts were made by surrouding variously shaped cavities by molten materials. This method enables a cavity to be heated quite satisfactorily. A long tube uniformly heated gives very good black-body conditions. By inserting a few diaphragms in this uniformly heated tube with a small opening at the end, it was found to give quite satisfactory black-body conditions. Such tubes can be made from some suitable refractory materials wound with either a wire or ribbon resistor. If the black body thus wound is intended for use at the temperature of the gold point (1336°K) a resistor made of nichrome wire or ribbon is found to work quite well. However, for operating in the open air for any length of time at temperatures much higher than this the only satisfactory material from which the resistor can be made is platinum. For this work, in general, platinum ribbon about 2 cm wide and about 0.01 mm thick is used. A description of a black body that has been found quite satisfactory has been given in the Astrophysical Journal, Vol. 51, p. 247, 1920.

When a furnace is used as a standard of radiation for calibration of an optical pyrometer, it is held at either the temperature of melting gold or melting palladium. In order to be sure that the furnace is at the temperature desired, a sample of the metal is melted inside the furnace. To do this the sample of gold or palladium is mounted between platinum wires supported by two refractory tubes and the circuit completed through a battery and bell or other indicating instrument. It is necessary to mount the specimen in such a manner that the central part of the sample cannot touch either of these two tubes, as it has sometimes been found that if a single tube is used the melted metal will touch and cling to the end of the tube so that the electric circuit is not broken at the instant of melting and thus a high value is obtained. With the central part of the sample free, it is assumed that the circuit is broken at the instant of melting. When a melt is obtained, the furnace is held at this temperature, using as an indicator the emf of the thermocouple, and then a number of readings made with the optical pyrometer sighted upon the black body. Another melt is then obtained and the process repeated. This procedure is necessary because it has been found that the emf of a platinum platinum-rhodium thermocouple, unless the most exceptional precaution to eliminate impurities in its manufacture and to prevent contamination during use, have been taken, changes constantly when used at the high temperatures.

The standard radiator at the standard temperature gives one point on the calibration. To obtain another point,[6] take readings of the current through the pyrometer filament for an apparent brightness match with a rotating sector, or absorbing glass of known transmission, between the pyrometer lamp and the standard black body. This will give a measure, in terms of the pyrometer current, of a brightness that is some known fraction of that of the standard radiator at the standard temperature. If monochromatic radiation is used, it is easy to calculate the temperature T_2 of the black body corresponding to this current through the pyrometer filament, that is to this measured brightness, from T_1 the standard temperature by the following formula derived from Wien's equation:

$$\frac{1}{T_1}-\frac{1}{T_2}=\frac{\lambda \log R}{c_2 \log e} \tag{5}$$

where R is the transmission of the sector, and λ the wave-length used. If the measurements are made with a red glass in the eyepiece, the temperature that would correspond to this fraction of the brightness of the standard black body can be calculated just as before excepting that in this case the effective wave-length of the red glass for the temperature interval is to be used.

Extending Temperature Scale. If a rotating sector or an absorbing glass of known transmission is used between the source being investigated and a calibrated pyrometer lamp, temperatures higher than that of the standard black body can be measured. In this case, the temperature is to be calculated from the temperature corresponding to the current through the pyrometer filament and the transmission of the sector or absorbing glass used by means of equation (3) above, excepting in this for R, use the reciprocal of the transmission of the sector or glass, and then T_2

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[6] Mendenhall, Phys. Rev., *33*, p. 74, 1911.

will come out greater than T_1. A very convenient method is to work out such extrapolated temperatures for the various sectors and absorbing glasses that are to be used, and plot the extrapolated temperatures against the temperatures as determined from the pyrometer reading. Such curves can then be used with any pyrometer using the same red glass, providing the same sectors or absorbing glasses are used.

Wien's equation is generally used to calculate temperatures from brightness measurements. This equation does not fit the experimental evidence excepting for small values of the product λT. Table 3 has been prepared to show just what errors in both the true and color temperature are obtained in the use of this equation rather than Planck's more exact form. The results for true temperature were obtained by calculating the higher temperature from 1828°K and wave-length of $\lambda = 0.665\mu$.

For color temperature the values given were obtained by calculating the color temperature from the ratio of the red ($\lambda = .665\mu$) to the blue ($\lambda = .467\mu$).

TABLE 3. *Corrections to be added to Temperatures obtained from Wien's Equation to Reduce them to what would be obtained from Planck's Equation*

Temperature Degree K	Corrections to	
	True Temperature	Color Temperature
2000	$->0.01$	$>.01$
2600	$-$ 0.05	.1
3000	$-$ 0.3	.7
3600	$-$ 1.5	3.2
5000	$-$ 15.6	30.7

From Table 3 it can be seen that Wien's equation is quite satisfactory for temperatures up to about 3600°K, since the error obtained by its use is less than the experimental error. It is to be noted that the error obtained in color temperature is about twice the magnitude of the error in the brightness temperature and in the opposite direction.

It has been found in photometry that two adjacent surfaces can be brought very accurately to the condition of the same brightness by eye observations if there is no visible line separating them. If the disappearing-filament pyrometer is properly lined up and the proper magnification and resolving power is used, there will be no visible line separating the pyrometer filament from the image of the background, thus the conditions for brightness matches are good. If properly adjusted the polarization type of pyrometer has a very good field since the line separating the two parts disappears entirely. Other types of pyrometers can be so constructed that the line of separation between the two parts is quite satisfactory.

DEPENDENCE OF THE TEMPERATURE ON VARIOUS CONDITIONS

(a) *The Constant* c_2. As was noted in the 1920 report, the value of c_2 that then seemed to fit all the experimental data is 14350μ degrees. Later data indicate that the value of c_2 should be reduced somewhat. Coblentz gives as the best value $14\,320\mu$ deg. In calculating temperature from brightness comparison the results obtained depend upon the value of c_2 used. It is often necessary for purpose of comparison to change the values of a particular temperature to that which would have been obtained if a different value of c_2 had been used. Temperatures are in general calculated from a standard temperature using Wien's equation. From this equation the following relation is obtained between two temperatures, T'_2 and T_2 obtained by use of two different values for c_2, from an initial temperature T_1, and from a particular ratio of brightness and wave-length λ:

$$(6) \qquad \frac{1}{T_2}-\frac{1}{T_1}=\frac{c'_2}{c_2}\left(\frac{1}{T'_2}-\frac{1}{T_1}\right)$$

Using this equation with $T_1=1336°K$ (gold point) the corrections were calculated for various temperatures and various values of c_2. These corrections have been plotted as shown in Fig. 3. From these curves a temperature corresponding to a particular c_2 can be corrected to any desired value of c_2. As an illustration of the use of this set of curves, a correction will be obtained. Suppose that a temperature of 2500°K is found for $c_2=14\,350$ and the correction is desired for $c_2=14200$. From the curve marked 14 200, the correction for 2500°K is seen to be +25°. Thus the temperature should be 2525 for $c_2=14200$.

(b) *Brightness*. A very great advantage of the optical pyrometer in measuring temperature is due to the relation between a change in the brightness and the corresponding change in temperature.

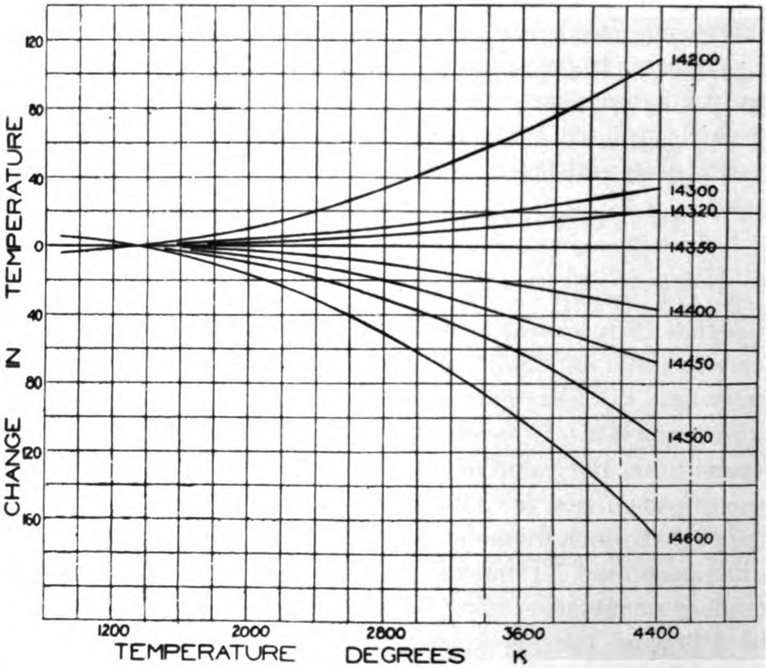

Fig. 3. Curves showing differences obtained in calculating temperature from T_1=1336°K when different values of c_2 are used

At 1000°K for red radiation, ($\lambda=0.665\mu$) the brightness varies about 22 times as fast as the temperature. At 2000°K for red radiation the ratio is about 11. From Wien's equation the following is the relation between the change in energy and the change in temperature:

$$\frac{dE_\lambda}{E_\lambda}=\frac{dT}{T}\frac{c_2}{\lambda T} \tag{7}$$

The relation between a change in temperature and the corresponding change in brightness for the red ($\lambda=0.665\mu$), green ($\lambda=0.57\mu$) and blue ($\lambda=0.475\mu$) has been computed for temperature ranges, and the results platted as shown in Fig. 4. From these curves the ratio of the change in brightness to the change in temperature for any of the three wave-lengths and any temperature can be found. For example at a temperature of 2500° for $\lambda = .57$, the brightness changes about ten times as fast as the temperature.

When measuring the temperature of a source with an optical pyrometer, there are some precautions that should be taken. Very fortunately the maker of the pyrometer has been able to take care of some of these in the construction of the instrument.

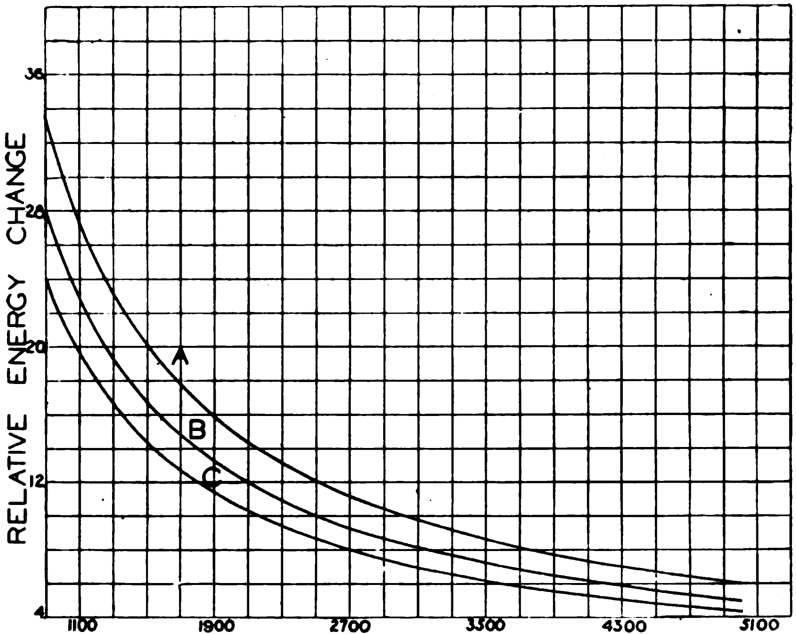

FIG. 4. Curves showing the relative change in energy for percent change in temperature curve A, for Blue (=0.475); curve B for Green (=0.57); curve C for Red (=0.665)

(c) *Clean Optical Parts.* The first thing to keep in mind is that it is a calibrated instrument and must be handled as such. All the optical parts of the instrument should be kept clean. The actual error due to dirt on the objective lens is not very great. A lens has to be very dirty to cause an error of 10° at 1500°K. However, errors much larger than this are possible if the lenses are allowed to become too dirty. An error of 15° at 1500°K would be caused by allowing the lens to become so covered with dirt that its transmission would be reduced to 85% of its value when clean. If the lenses and pyrometer lamp bulb are allowed to become dirty, it interferes with the clearness of the images and thus lessens the accuracy of the setting. They should be kept clean but in cleaning them care is required or scratches will be produced which will again interfere with the definition.

(d) *Comparison Source.* Very great care must be taken not to change the comparison source in any way. In some pyrometers even the position of the comparison lamp must not be changed. In the disappearing-filament type small changes in the position of the part of the filament under observation will cause no appreciable error. In no case must the filament be over-heated, since this may very greatly change its characteristics. The pyrometer lamp, of tungsten, if not used at a temperature higher than 1828°K will have a very long life. The lamps have been aged for a considerable time at a temperature much higher than this, so a current somewhat above that for which it is calibrated will not change its calibration. A good plan is to allow no current higher than that for which it has been calibrated to pass through the filament.

(e) *Reproducing Temperatures.* It is quite often necessary to bring a lamp filament to a particular temperature so that a particular brightness condition can be reproduced. In many cases, all that is wanted is some method by which different filaments can be brought to the same brightness. In general, it is better to use the true temperature if it can be obtained; however, for this particular case either the brightness or the color temperature will answer the purpose quite well.

The brightness temperature has been defined above. The color temperature is defined as the temperature of a black body at which it has in general the same integral color as the source being investigated. These color matches can, generally, be made very easily and very accurately with an ordinary contrast Lummer-Brodhun photometer. To do this the method of substitution is used, the lamp being studied first put in place and set at the desired voltage. The resistance that controls the current through the comparison lamp is then changed by small amounts, the observer at the same time keeping an intensity match by moving either the photometer or one of the lamps. By thus changing the comparison lamp the two lamps can be brought very accurately to the same color. The unknown lamp is then removed, and the standard put in its place, and now the comparison kept at the voltage previously determined, and the voltage of the standard changed until there is again a color match. It is to be noted that there is to be an intensity match at the same time that a color match is obtained, since it is impossible to set for color match unless there is at the same time a very good intensity match.

Such color matches can be made for the various filament lamps and the black body and most of the flame standards of light. Such color matches are impossible between any of the above standards and the Welsbach gas mantle or any source that does not have a continuous distribution of energy through the visible spectrum.

TABLE 4. *Relation between Lumens per Watt and Color Temperature for a Tungsten Lamp*

Lumens per Watt	Color Temperature
0.5	1644
1.0	1777
1.5	1866
2.0	1939
2.5	1998
3.0	2050
3.5	2096
4.0	2138
4.5	2175
5.0	2208
5.5	2241
6.0	2269
6.5	2299
7.0	2327
7.5	2354
8.0	2380
8.5	2406
9.0	2431

To obtain the color temperature, it is necessary to have a standard of color temperature,[7] that is, a lamp that has been compared in color with the black body at various temperatures. The relation between color temperature and lumens per watt has been very carefully worked out for a wide range of temperatures. It has been found that this relation holds very well for different lamps of about the same type. In Table 4 are given these relations for tungsten for a range of efficiencies from 0.5 lumens per watt to 9.0 lumens per watt.

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[7] Hyde, Cady, and Forsythe, Phys. Rev. N. S., *10*, p. 395, 1917.

These values are for the ordinary type of vacuum lamps. Thus they would not hold if applied to a gas-filled lamp. If a standard of color temperature is not available, quite accurate results can be obtained by measuring the lumens per watt and applying the above relations.

III. RECENT LITERATURE ON PYROMETRY

J. A. Anderson[8] at the Mt. Wilson Observatory reached a very high temperature by electrically exploding very small wires. He exploded these wires by discharging through them a condenser of large capacity. According to his estimate the brightness reached corresponded to a temperature of about 20,000°K.

In September 1919, there was held in Chicago, in connection with the American Institute of Mining and Metallurgical Engineers, a Symposium on Pyrometry. The papers and discussions given at this Symposium have been published in book form. In this book there are to be found about sixty papers that cover almost the entire field of pyrometry. While there is some overlapping in the ground covered, this is not of such a nature as to decrease the value of the book.

Priest,[9] describes an interesting instrument called the leucoscope. With this instrument, he is able to construct an apparently quite accurate pyrometer without the use of any comparison source. From the description of the measurements made by Mr. Priest, it seems that he has uncovered quite a valuable instrument for various investigations in the study of color.

Technologic Paper (No. 170) of the Bureau of Standards by Foote, Fairchild, and Harrison, entitled "Pyrometric Practice," a book of 326 pages and 187 figures, gives quite a complete survey of the field. In this are found descriptions of methods of calibrating the various forms of pyrometers, together with the methods for their use.

LABORATORY OF PURE SCIENCE,
NELA RESEARCH LABORATORIES, CLEVELAND, OHIO,
October 1, 1921.

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[8] Astrophysical Journal, *51*, page 37, 1920.

[9] JOURNAL OF OPTICAL SOCIETY, 4, p. 448, 1920.

SUPPLEMENTARY NOTE ON THE FREQUENCIES OF COMPLEMENTARY HUES

BY

IRWIN G. PRIEST

Referring to the "Note on the Relation Between the Frequencies of Complementary Hues,"[1] Dr. Selig Hecht has kindly pointed out in a letter to the author, that, if the relation between frequencies is hyperbolic, it follows of mathematical necessity that the relation between wave-lengths is also hyperbolic. This is true. It might therefore be thought a matter of indifference as to whether frequency or wave-length is considered in presenting such data. There is, however, another aspect of the question: *If frequency scales be used, the experimental data fall symmetrically about the vertex of the hyperbola.* (J. Op. Soc. Am., *5*, p. 404, Fig. 2). *If wave-length scales be used, two-thirds of the same data fall on one side of the vertex.* (J. Op. Soc. Am., *5*, p. 404, Fig. 1.)

It appears therefore that, as regards the treatment of experimental data, the relation can be more conveniently and accurately demonstrated on frequency scales. These are the points worthy of notice. The author regrets that in emphasizing them he was led to imply the contradiction of a mathematical theorem.

It may be noted, that the asymptotes of these hyperbolic equations correspond roughly to the "neutral points" in the spectra of the "red-green blind" and the "yellow-blue blind" respectively,[2] thus:

In vibrations per trillionth of one second,

Asymptotes of Hyperbola	Neutral Points
530	522 yellow-blue blind (not well known)
608	612–601 red-green blind.

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[1] J. OPTICAL SOC. AM., *5*, p. 403; Sept. 1920.

[2] Koellner: Stoerungen des Farbensinnes, pp. 42 and 49.

In words, the spectral region lying between the "neutral points" for these two general classes of "color blinds" is approximately coextensive with the region for which the normal eye finds no spectral complementary.

BUREAU OF STANDARDS
Nov. 5, 1921.

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